# Two-Quasiparticle States in $Tb^{158}$ †

H. D. Jones* and R. K. Sheline
*The Florida State University, Tallahassee, Florida 32306*
(Received 13 April 1970)

Thirty-four levels in $Tb^{158}$ have been observed up to an excitation energy of 1166 keV utilizing 12-MeV deuterons and the reaction $Tb^{159}(d,t)Tb^{158}$. The ground-state $Q$ value was determined to be -1870 ± 15 keV. The spectrum has been interpreted in terms of the coupling of the [411↑] Nilsson proton orbital with the neutron orbitals prominent in the $(d,t)$ spectrum of $Gd^{157}$. This interpretation has resulted in the determination of relative energies due to the residual neutron-proton interaction for eight different configurations. The observed singlet-triplet splitting energies for the [521↑], [402↓], and [400↑] neutron orbitals coupled to the [411↑] proton orbital were measured to be +132, −111, and +136 keV, respectively. Theoretical calculations of these energies made for a zero-range spin-dependent central potential gave values of +174, −186, and +136 keV, respectively.

## INTRODUCTION

The energy spectrum of the odd-odd deformed nucleus $Tb^{158}$ can be successfully interpreted as consisting of rotational bands superimposed upon intrinsic two-quasiparticle states in which the individual orbits of the odd proton and odd neutron are described by the Nilsson model.[1] For each pair of proton and neutron orbits there are two states which can be formed in the odd-odd nucleus – the so-called singlet and triplet states. These states are not degenerate but are split both by the rotational Hamiltonian and by the residual interaction between the odd particles. The rotational contribution can be subtracted to yield experimental residual-interaction singlet-triplet splitting energies, which can be used to evaluate calculations made for a particular choice for the two-body potential. In this paper we report the results of calculations of these energies for $Tb^{158}$ utilizing a zero-range spin-dependent central potential and single-particle wave functions calculated in a modified[2] Nilsson model which employs a more realistic Woods-Saxon potential, rather than a harmonic-oscillator potential.

The energy levels of $Tb^{158}$ have been studied by means of the reaction $Tb^{159}(d,t)Tb^{158}$. This reaction selectively populates states in which the proton orbital is the same as that of the odd proton in the $Tb^{159}$ target. Furthermore, neutron orbitals which are below the Fermi surface in the target are more likely to be populated. Even with these restrictions the level density is high, and very few of the levels were completely resolved with the resolution [11 to 18 keV full width at half maximum (FWHM)] we obtained. Our interpretation of the energy levels of $Tb^{158}$ is based on the established systematic behavior of Nilsson states in deformed nuclei. By relating the Hamiltonian of $Tb^{158}$ with that of $Gd^{157}$, the energy of each band head in $Tb^{158}$ was estimated from the known energy of the related state in $Gd^{157}$. The portion of the spectrum of $Tb^{158}$ near this energy was then searched for a well-characterized rotational band with the appropriate intensity pattern or "fingerprint," which is characteristic of the angular momenta involved in the reaction and the Nilsson state of the transferred neutron. By requiring the relative energies to fit the rotational formula, and the relative intensities to fit the fingerprint, almost all assignments of spins and parities can be made unambiguously.

## EXPERIMENTAL METHOD AND RESULTS

Terbium targets with thicknesses of approximately 200 $\mu g/cm^2$ were bombarded with 12-MeV deuterons produced by the Florida State University tandem Van de Graaff accelerator. Tritons from the $Tb^{159}(d,t)Tb^{158}$ reaction were recorded by Eastman Kodak nuclear-emulsion plates which were spring-fitted to the focal plane of a broad-range magnetic spectrograph, which is a scaled-up (6 : 5) copy of one built by Browne and Buechner.[3]

The results of four exposures are displayed in Figs. 1 and 2. These plots are of the usual type made for magnetic-spectrograph data (counts per $\frac{1}{2}$-mm strip versus plate distance) except that the horizontal scales have been adjusted so that triton peaks corresponding to the same $Q$ value are aligned. The horizontal scale is thus used to indicate the excitation energy relative to the ground state, whose $Q$ value was determined to be −1870 keV. From an analysis of inherent errors in this experiment, an error of ±15 keV can be placed on this value.[4]

The observed average excitation energies and relative intensities are listed in Table I. The excitation-energy errors were estimated from the spread in values obtained in the different exposures. The experimental intensities are normalized to the total theoretical cross section in $\mu$b/sr of all observed levels resulting from the coupling of the [411↑] proton orbital with the [521↑] neutron orbital. The accuracy of the experimental relative intensities is believed to vary from about ±15% for the strongest levels to perhaps a factor of 2 for those near the lower limit of observation.

## THEORY

This paper is one of a series on odd-odd deformed nuclei. The first of the series,[5] which deals with the levels of $Lu^{174}$, goes into more detail on the theory of the spectra of odd-odd deformed nuclei. In this article, therefore, we present only those general features of the theory which are essential for an understanding of the results we have obtained.

For the low-energy part of the spectrum where core excitations and collective vibrations can be ignored, the Hamiltonian can be assumed to be

$$H = T_{\mathrm{ROT}} + H_{\mathrm{sp}}(p) + H_{\mathrm{sp}}(n) + H_{\mathrm{INT}}, \tag{1}$$

where $T_{\mathrm{ROT}}$ is the rotational kinetic energy, $H_{\mathrm{sp}}(p)$ and $H_{\mathrm{sp}}(n)$ are the Nilsson single-particle Hamiltonians for the proton and neutron, respectively, and $H_{\mathrm{INT}}$ is the residual interaction between the odd particles. In order to obtain information on the energy resulting from $H_{\mathrm{INT}}$, the contributions from the first three terms of Eq. (1) must be known.

Since the proton orbital is the same for all the states we observe in the $(d,t)$ reaction, the effect of $H_{\mathrm{sp}}(p)$ is simply to shift the origin of the excitation-energy scale and can thus be ignored. The contributions from $T_{\mathrm{ROT}}$ and $H_{\mathrm{sp}}(n)$ can be obtained from a Nilsson calculation, but these energies may be in error by 100 keV or more. A much more accurate procedure is to use the experimental energies of the odd-$A$ isotone of the odd-odd nucleus which has the odd proton removed. Since these energies result from $T_{\mathrm{ROT}}$ and $H_{\mathrm{sp}}(n)$ alone, estimates of the contribution from these terms in Eq. (1) can be made. These estimates should be accurate within 20 keV, or so, provided vibrational mixing does not greatly affect the odd-$A$ excitation energy.

The expectation value of any residual interaction $H_{\mathrm{INT}}$ can be written in the form[6]

$$\langle H_{\mathrm{INT}} \rangle = A + (-1)^{I} B \delta_{K0}, \tag{2}$$

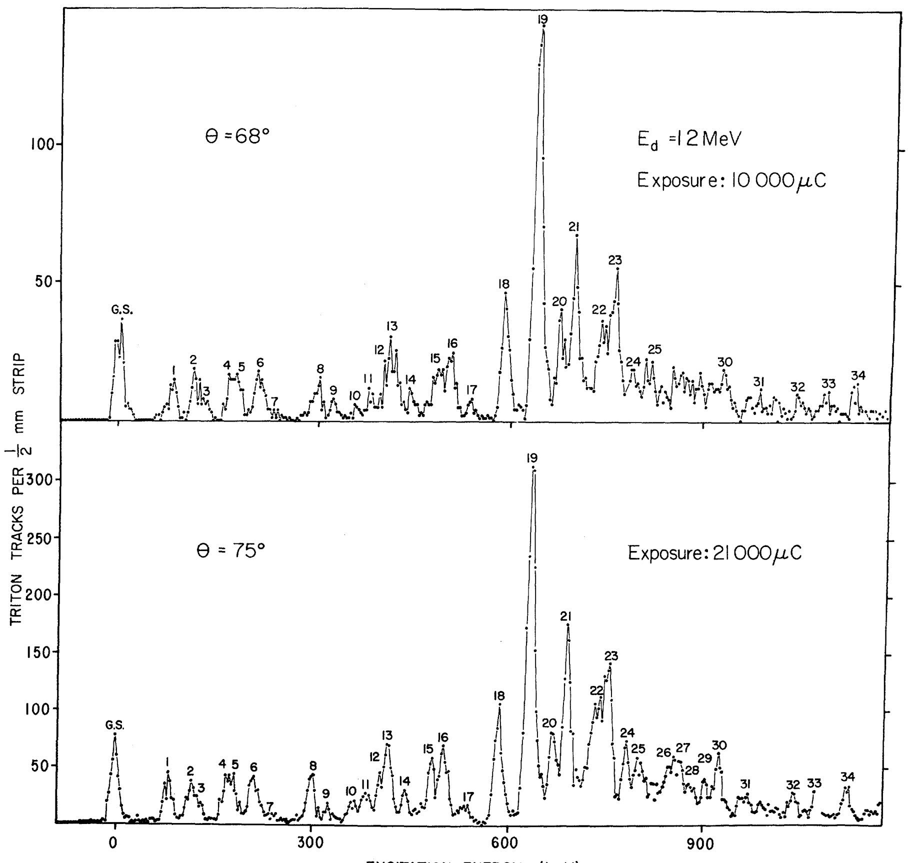


FIG. 1. Spectra obtained from the $Tb^{159}(d,t)Tb^{158}$ reaction at 68 and 75°.

where all of the $I$ dependence is written explicitly. This form results from the symmetry of the odd-odd wave function. $A$ and $B$ can be calculated from the particular type of interaction chosen and the Nilsson single-particle wave functions of the odd proton and odd neutron.

The angular momentum coupling scheme for odd-odd deformed nuclei is shown in Fig. 3. The projection of the total angular momentum on the symmetry axis is

$$K=|\Omega_p+\Omega_n| \, . \tag{3}$$

Since both the projections $\Omega_p$ and $\Omega_n$ can be either positive or negative, two states occur – one with a $K$ value of

$$K_1=\big|\,|\Omega_p|-|\Omega_n|\,\big| \tag{4}$$

and the other with a $K$ value of

$$K_2=|\Omega_p|+|\Omega_n| \, . \tag{5}$$

In the band with $K=K_1$, $\Omega_p$ and $\Omega_n$ have opposite signs. In the band with $K=K_2$, the two projections have the same sign.

If we call the member of a band with $I=K$ the band head, then, except possibly for $K=0$, the band head will have the lowest energy. Using the expression for the total energy, it can be shown[5] that the energy of one band head relative to the other is, for nonzero $K_1$,

$$E(K_1)-E(K_2)=\frac{\hbar^2}{2\mathscr{I}}(K_1-K_2)+A(K_1)-A(K_2)\,, \tag{6}$$

where $\hbar^2/2\mathscr{I}$ is called the moment of inertia in this paper. For $K_1=0$, the odd-even shift also contributes to $E(K_1)-E(K_2)$. Thus the rotational term tends to cause the state with the smaller value of $K$ to lie lower in energy. However, the term arising from the residual interaction is usually larger and thus determines which state will have the lower energy. This can be predicted with remarkable accuracy using the Gallagher-Moszkowski coupling rule.[7] If we define the total spin projection by

$$\Sigma=|\Sigma_p+\Sigma_n|\,, \tag{7}$$

where $\Sigma_p$ and $\Sigma_n$ are the properly signed projections of the spins of the proton and neutron, respectively, then this rule simply states that the lower-energy state is the one which has $\Sigma=1$. We call this the triplet state. The other member of the Gallagher-Moszkowski pair is then the singlet state, since it has $\Sigma=0$.

The value of $A(K_1)-A(K_2)$ in Eq. (6) can be deter-

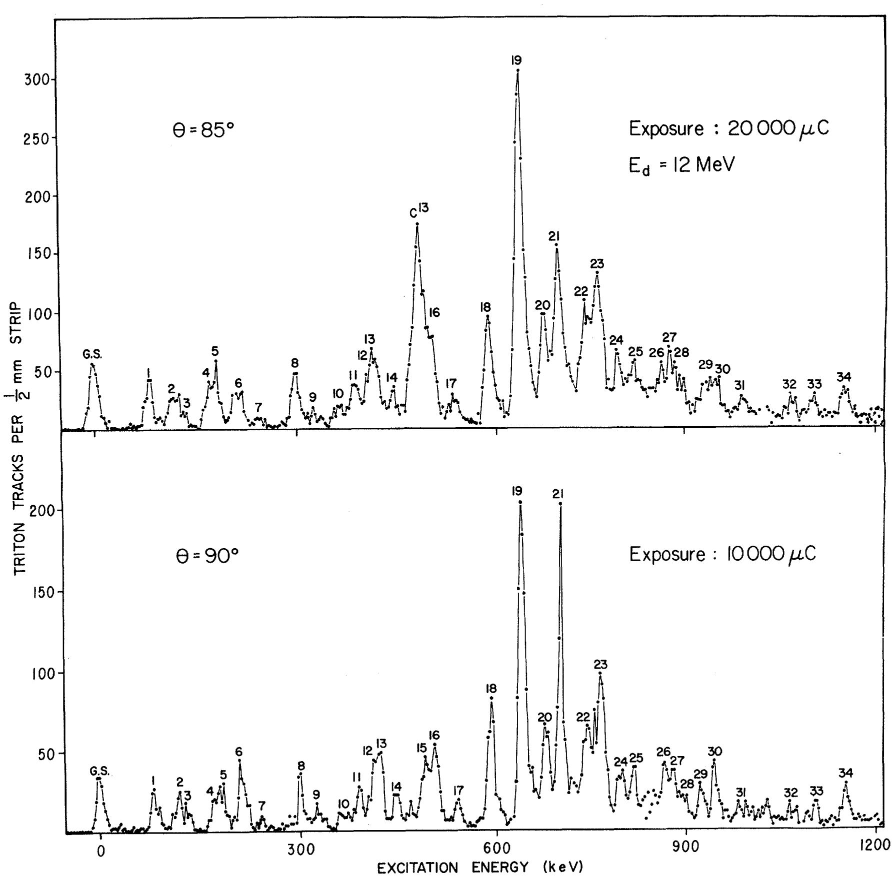


FIG. 2. Spectra obtained from the $Tb^{159}(d,t)Tb^{158}$ reaction at 85 and 90°.

mined accurately from a knowledge of just the excitation energies in the odd-odd nucleus. In this paper we refer to this value as the singlet-triplet splitting energy. The theoretical values of the singlet-triplet splitting energies for two-quasiparticle states in $Tb^{158}$ have been calculated for a zero-range spin-dependent central potential and compared with the observed energies.

The form chosen for the two-body potential is

$$H_{\mathrm{INT}} = V(|\vec{r}_p - \vec{r}_n|)(1 - \alpha + \alpha\vec{\sigma}_p \cdot \vec{\sigma}_n), \tag{8}$$

and for simplicity we take

$$V(|\vec{r}_p - \vec{r}_n|) = -4\pi g \delta(\vec{r}_p - \vec{r}_n), \tag{9}$$

where $g$ is the interaction parameter, $\alpha$ determines the amount of spin-spin force in $H_{\mathrm{INT}}$, and $\vec{\sigma}_p$ and $\vec{\sigma}_n$ are the proton and neutron spin operators. The calculations were performed using the equations developed by Pyatov.[8]

The parameter $g$ does not have the dimensions of energy, and it is customary to consider instead the parameter

$$W = g(2\nu^3/\pi)^{1/2}, \tag{10}$$

where $\nu$ is a quantity which appears in the expressions for the radial wave functions. The splitting energy depends only on the product of the parameters $\alpha$ and $W$. We have made a least-squares fit of all values of splitting energy for deformed odd-odd nuclei in the rare-earth nuclei including the assignments for $Tb^{158}$. The 20 experimental splitting energies obtained can be fitted with a standard deviation of 40 keV. The value obtained for $\alpha W$ is 0.85 MeV. This compares with the value of Pyatov of 0.24 MeV made using the same formalism but with much less complete data.

Theoretical cross sections for states in $Tb^{158}$ were calculated from the equations developed by Satchler[9] using Rost[2] wave functions for improved accuracy. The intrinsic single-particle cross sections were calculated in the distorted-wave Born-approximation (DWBA)[10,11] using the code DWUCK on the Florida State University CDC 6400 computer. The normalization used was that suggested by Bassel.[12] The optical-potential parameters used were those which best fit the measured angular distribution of tritons from the $Gd^{160}(d,t)Gd^{159}$ reaction and were taken from the work of Jaskola *et al.*[13]

TABLE I. Relative energies and intensities observed in the $Tb^{159}(d,t)Tb^{158}$ reaction.

| Peak number | Energy (keV) | Error (keV) | Angle in degrees 68 | 75 | 85 | 90 |
|---|---|---|---|---|---|---|
| 0 | 0 | ··· | 107 | 80 | 68 | 64 |
| 1 | 80 | 1 | 42 | 47 | 44 | 40 |
| 2 | 114 | 1 | 44 | 42 | 38 | 35 |
| 3 | 130 | 1 | 21 | 16 | 16 | 19 |
| 4 | 168 | 2 | 34 | 38 | 38 | 35 |
| 5 | 180 | 2 | 34 | 39 | 40 | 38 |
| 6 | 209 | 3 | 43 | 52 | 38 | 50 |
| 7 | 219 | 3 | 25 | 12 | 18 | 18 |
| 8 | 299 | 2 | 43 | 49 | 58 | 44 |
| 9 | 323 | 4 | 22 | 14 | 16 | 19 |
| 10 | 362 | 3 | 14 | 23 | 26 | 21 |
| 11 | 386 | 3 | 29 | 34 | 54 | 45 |
| 12 | 408 | 3 | 64 | 35 | 40 | 55 |
| 13 | 420 | 3 | 52 | 77 | 60 | 75 |
| 14 | 447 | 3 | 29 | 34 | 43 | 41 |
| 15 | 487 | 3 | 61 | 60 | ···[a] | 84 |
| 16 | 507 | 3 | 76 | 78 | 113[a] | 83 |
| 17 | 540 | 2 | 22 | 24 | 37 | 34 |
| 18 | 593 | 3 | 135 | 110 | 112 | 128 |
| 19 | 641 | 3 | 424 | 331 | 357 | 335 |
| 20 | 678 | 3 | 103 | 87 | 111 | 120 |
| 21 | 702 | 3 | 181 | 182 | 185 | 210 |
| 22 | 744 | 3 | 116 | 129 | 122 | 133 |
| 23 | 767 | 4 | 175 | 179 | 184 | 196[a] |
| 24 | 795 | 4 | 66 | 80 | 80 | 73 |
| 25 | 819 | 4 | 38 | 70 | 76 | 66 |
| 26 | 864 | 3 | ··· | 58 | 50 | 67 |
| 27 | 878 | 4 | 70 | 67 | 75 | 57 |
| 28 | 898 | 4 | 51 | 42 | 44 | 37 |
| 29 | 925 | 4 | 47 | 41 | 30 | 45 |
| 30 | 949 | 4 | 63 | 66 | 73 | 59 |
| 31 | 988 | 4 | 29 | 18 | 24 | 25 |
| 32 | 1072 | 4 | 32 | 36 | 33 | 25 |
| 33 | 1112 | 4 | 38 | 33 | 41 | 29 |
| 34 | 1166 | 4 | 33 | 30 | 22 | 50 |

[a]Obscured or partly obscured by the $C^{13}(d,t)C^{12}$ impurity peak or plate edge.

TABLE II. Interpretation (see Ref. 18) of intrinsic states in $Gd^{157}$.

| $K\pi$ | Band head energy (keV) | Character | Configuration |
|---|---|---|---|
| $\frac{3}{2}-$ | 0 | ground state | 521↑ |
| $\frac{5}{2}+$ | 63 | hole | 642↑ |
| $\frac{11}{2}-$ | 426 | hole | 505↑ |
| $\frac{5}{2}-$ | 435 | particle | 523↓ |
| $\frac{3}{2}+$ | 475 | hole | 402↓ |
| $\frac{1}{2}+$ | 686 | hole | 400↑ |
| $\frac{3}{2}-$ | 700 | hole | 532↓ |
| $\frac{1}{2}-$ | 704 | particle | 521↓ |
| $\frac{1}{2}-$ | 809 | hole | 530↑ |
| $\frac{5}{2}-$ | 1391 | particle | 512↑ |
| $\frac{7}{2}+$ | 1825 | hole | 404↓ |

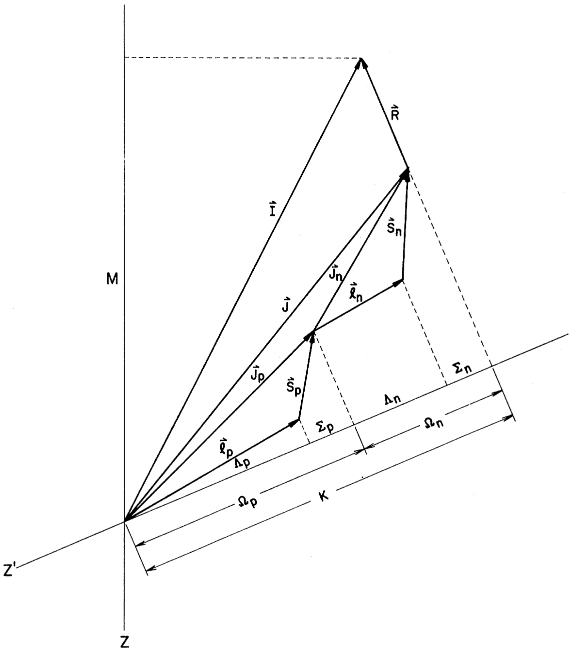


FIG. 3. Angular momentum coupling scheme for odd-odd deformed nuclei. The symmetry axis of the nucleus is labeled $Z'$. The other axis shown is the space-fixed $Z$ axis.

## ANALYSIS

### Predicted Spectrum

The Nilsson diagram[1] shows that for a deformation of $\beta = 0.3$, the 65th proton is expected to occupy the [411↑] Nilsson state in the ground-state configuration. Since the neighboring odd-$A$ isotopes of $Tb^{158}$ have ground-state spins of $\frac{3}{2}+$,[14] the proton assignment for all the two-quasiparticle states that we observe in $Tb^{158}$ with the $(d,t)$ reaction is [411↑]. The neighboring odd-$A$ isotones of $Tb^{158}$ are $Gd^{157}$ and $Dy^{159}$, both of which have ground-state spins of $\frac{3}{2}-$.[14] The Nilsson diagram shows that this spin results from the [521↑] neutron orbital. The ground state of $Tb^{158}$ may therefore reasonably be expected to be formed from the coupling of the [411↑] proton orbital to the [521↑] neutron orbital.

The coupling of these orbitals produces a triplet state with $K = 3$ and a singlet state with $K = 0$. The Gallagher-Moszkowski coupling rule predicts that the $K = 3$ band is the ground state with the $K = 0$ band a low excited state. An $M3$ isomeric transition of 110 keV has been observed in $Tb^{158}$ which can be interpreted as the transition from the 0−, 0 state to the 3−, 3 state.[15,16] This interpretation implies that there are no low-spin excited states below 110 keV to which the 0−, 0 level can decay. In

TABLE III. Theoretical cross sections in $\mu$b/sr of levels in $Tb^{158}$ at 75 deg. Cross sections do not include the occupation probability. States with cross sections less than 1 $\mu$b/sr are omitted. The quantity $E_0$ is the observed or estimated band-head excitation energy.

| Nilsson neutron orbital | $K\pi$ | $E_0$ (keV) | Total angular momentum in units of $\hbar$: 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 521↓ | 2− | 700 | | | 202 | 30 | 37 | 17 | 2 | ··· | ··· |
| 521↓ | 1− | 700 | | 148 | 63 | 33 | 29 | 11 | 1 | ··· | ··· |
| 521↑ | 3− | 0 | | | | 190 | 89 | 91 | 5 | 2 | ··· |
| 521↑ | 0− | 110 | 40 | 71 | 71 | 83 | 58 | 17 | 2 | ··· | ··· |
| 523↓ | 4− | 435 | | | | | 38 | 21 | 13 | 1 | ··· |
| 523↓ | 1− | 435 | | 15 | 17 | 17 | 14 | 6 | 1 | ··· | ··· |
| 642↑ | 4+ | 64 | | | | | 4 | 14 | 18 | 5 | 4 |
| 642↑ | 1+ | 64 | | 1 | 1 | 10 | 14 | 10 | 6 | 3 | 1 |
| 505↑ | 7− | 426 | | | | | | | | 34 | ··· |
| 505↑ | 4− | 426 | | | | | 26 | 7 | 1 | ··· | ··· |
| 402↓ | 3+ | 593 | | | | 281 | 17 | 2 | ··· | ··· | ··· |
| 402↓ | 0+ | 420 | 78 | 144 | 87 | 24 | 3 | ··· | ··· | ··· | ··· |
| 651↑ | 3+ | 500 | | | | 2 | 7 | 14 | 12 | 3 | 2 |
| 651↑ | 0+ | 500 | 0.004 | 1 | 2 | 7 | 12 | 10 | 5 | 2 | 1 |
| 400↑ | 2+ | 641 | | | 468 | 52 | 14 | 1 | ··· | ··· | ··· |
| 400↑ | 1+ | 767 | | 305 | 130 | 26 | 6 | ··· | ··· | ··· | ··· |
| 660↑ | 2+ | 700 | | | 1 | 4 | 9 | 13 | 8 | 2 | 1 |
| 660↑ | 1+ | 700 | | 0.4 | 2 | 5 | 11 | 12 | 6 | 2 | 1 |
| 532↓ | 3− | 700 | | | | 55 | 46 | 13 | 5 | ··· | ··· |
| 532↓ | 0− | 700 | 7 | 26 | 36 | 27 | 14 | 6 | 2 | ··· | ··· |
| 530↑ | 2− | 678 | | | 99 | 104 | 29 | 16 | 3 | 1 | ··· |
| 530↑ | 1− | 720 | | 57 | 94 | 56 | 24 | 11 | 2 | ··· | ··· |

particular, this implies that the 1− member of the $K=0$ band is above the 0− member.

Neutron orbitals which give rise to prominent states in $Tb^{158}$ should be the same orbitals which are prominent in the odd-$A$ isotone $Gd^{157}$. This nucleus has been studied extensively and interpretation has been made in terms of the Nilsson model.[17,18] The band-head energies and interpretation of intrinsic states in $Gd^{157}$ are listed in Table II. Table III lists the unmixed theoretical cross sections of levels resulting from all orbitals which may be important for $Tb^{158}$. The particle-hole or quasiparticle nature of each of the configurations is taken into account with the appropriate values and use of $V^2$, the "occupancy" parameter, in calculating the $(d,t)$ cross sections.

### Discussion of the Level Scheme

#### *Levels Resulting from the* [521↑] *Orbital*

As mentioned previously, the ground state of $Tb^{158}$ is predicted to be the triplet ($K=3$) band head resulting from the coupling of the [411↑] proton and the [521↑] neutron. In $Gd^{157}$, the observed moment-of-inertia parameter for this orbital is 11.0 keV.[18] Using this value, the second member of the $K=3$ band should be near 88 keV with an intensity at 75° which is about half that of the ground state. The level at 80 keV has an intensity relative to that of the ground state which varies from 0.4 at 68° to 0.6 at 90°. The 80-keV level is therefore assigned as the 4− member of the ground-state rotational band.

As shown by Table III, the third member of the ground-state rotational band is also expected to be about half as strong as the ground state. The moment-of-inertia parameter derived from the energies of the first two members of the ground-state rotational band is $10.00\pm0.14$ keV. Using this value in the rotational formula, we expect the energy of the 5−, 3 level to be $180\pm3$ keV. The triton group in the experimental spectrum centered at about 175 keV is an obvious doublet. A least-squares fit to the data yields energy levels with roughly equal intensities at $168\pm2$ and $180\pm3$ keV. The intensities of each of these levels relative to that of the ground state varies from 0.3 at 68° to 0.6 at 90°. Since the 180-keV level is much closer to the expected energy, it is assigned as the 5−, 3 state.

The expected energy of the 6−, 3 state is $300\pm4$ keV, but its intensity should be only about $\frac{1}{40}$ that of the ground state. Since there is a strong level at 299 keV, it is impossible to directly observe the weak 6−, 3 level. Higher members of this band have even smaller cross sections and are not expected to be observed.

As mentioned previously, the $M3$ isomeric transition observed in $Tb^{158}$ places the 0−, 0 level at an energy of 110 keV. As shown in Table III, the intensity of this level relative to that of the ground state should be about 0.2. There is a level observed in the $(d,t)$ reaction spectrum at $114\pm1$ keV. Since the energy of this level is a little higher than that of the 0−, 0 level and since its intensity is 2 to 3 times as great as the expected intensity of the 0−, 0 level alone, it is proposed that there is another level with an energy slightly greater than that of the 0−, 0 level which is responsible for most of the observed intensity. The amount of intensity to be accounted for at 75° is 0.37, relative to the intensity of the ground state. As mentioned previously, the only levels expected in this energy region are those resulting from the [521↑] and [642↑] Nilsson orbitals. All strong levels in the $K=3$ [521↑] band have already been accounted for, and all of the [642↑] levels are expected to have intensities which are less than 0.1, relative to that of the ground state. They are thus too weak to account for the intensity of the 114-keV level. The only remaining alternative which is consistent with previous assignments is to assume that the odd-even shift lowers the odd $I$ levels in the $K=0$ [521↑] band so that the 1−, 0 level is at 114 keV. The expected intensity of the 1−, 0 level relative to that of the ground state at 75° is 0.37, which can be seen from Table III. Since this is exactly the intensity we need to account for, the assignment of the 114-keV level as containing both the 0−, 0 and 1−, 0 levels is probably correct.

Using the moment-of-inertia parameter found for the $K=3$ band, the 2−, 0 level is expected at an energy of 170 keV. The intensity of this level is expected to be about 0.4 relative to that of the ground state. The unassigned level at 168 keV is very close to the expected energy and has an intensity relative to that of the ground state which varies from 0.3 at 68° to 0.5 at 90°. The 168-keV level is therefore assigned as the 2−, 0 level. The energy of this level implies a moment-of-inertia parameter of $9.67\pm0.37$ keV for the $K=0$ band. The odd-even shift strength $B$ is then determined to be $8\pm2$ keV.

Using the moment-of-inertia parameter of 9.67 keV and taking account of the odd-even shift, the 3−, 0 level is expected at an energy of $211\pm4$ keV. As shown in Table III, its intensity relative to that of the ground state should be about 0.44 at 75°. The triton group centered at about 213 keV appears to be a doublet. A least-squares fit to the data yields energy levels at 209 and 219 keV. The 209-keV level has the larger intensity. Since it is also closer to the expected energy, it is the most likely candidate for the 3−, 0 state. The total intensity of the two levels relative to that of the ground state varies from 0.6 at 68° to 1.0 at 90°. Since the intensity is much greater at 90 than 68°, a sizable part of the intensity could be due to one of the levels resulting from the [642↑] orbital. The strength of these levels comes primarily from the single-particle cross section for $l=6$. The relative magnitude of this single-particle cross section to the ones which contribute most of the strength of the [521↑] levels ($l=1$ and $l=3$) increases by roughly a factor of 2 from 68 to 90°. It is thus proposed that the 209-keV level is the 3−, 0 state and the 219-keV level is one of the stronger levels resulting from the [642↑] orbital.

The expected energy of the 4−, 0 level is $303\pm7$ keV, and its intensity relative to that of the ground state should be about 0.3. The intensity of the experimental level very near this energy varies from 0.4 at 68° to 0.7 at 90°. Since the relative intensity of this level increases sharply from 68 to 90°, it is possible that some of the intensity is due to one of the levels resulting from the [642↑] orbital. It is thus proposed that the 4−, 0 level is at 299 keV and accounts for most of the observed intensity of that level. It should be remembered that the 6− member of the ground-state band also occurs near 300 keV, and thus accounts for part of the observed intensity.

The 5−, 0 level is expected at an energy of 385 $\pm 11$ keV, but its expected intensity is less than $\frac{1}{10}$ that of the ground state. The experimental level at 386 keV is several times stronger than the 5−, 0 level should be. Consequently, the 386-keV level is not assigned as the 5−, 0 level, but a small part of the observed intensity of the 386-keV level should be due to the 5−, 0 level.

### *Levels Resulting from the* [642↑] *Orbital*

The only remaining unassigned levels below 300 keV are at 130 and 219 keV. As mentioned earlier, these levels probably result from the [642↑] orbital. This orbital has a large Coriolis mixing coefficient connecting it with the [633↑] orbital. This mixing is expected to cause the apparent moment-of-inertia parameter for bands resulting from the [642↑] orbital to be smaller than those for other rotational bands. In $Gd^{157}$, the apparent moment-of-inertia parameter of the [642↑] band is 7.5 keV.[18] If we assume that the 130- and 219-keV levels are adjacent members of the same rotational band and that the moment-of-inertia parameter is 7.5 keV, then the 130-keV level should have an angular momentum of 5 and the 219-keV level should have an angular momentum of 6. The intensities of these levels are consistent with those of the 5+ and 6+ members of the $K=4$ band resulting from the [642↑] orbital. Since this band is the triplet state, it is expected at a lower energy than the $K=1$ band, which is the singlet state. These assignments then are quite reasonable. The moment-of-inertia parameter derived from the energies is then 7.42 $\pm 0.26$ keV. Using this value, the $K=4+$ band head is expected at an energy of $56\pm3$ keV, and it is expected to be extremely weak. Although there are a few triton tracks near this energy, experimental evidence for the 4+, 4 state is not conclusive.

The 7+ member of this band is expected at an energy of $323\pm9$ keV. Its intensity should be roughly $\frac{1}{3}$ that of the 5+, 4 level at 130 keV. The level observed at 323 keV is at the expected energy but is considerably stronger than the 7+, 4 level alone should be. Consequently, the 7+, 4 state has not been assigned.

The $K=1$ band resulting from the [642↑] orbital is expected at a higher energy than the $K=4$ band, since it is the singlet state. As Table III shows, there are four possibly observable members of this band with roughly equal intensities. There are a number of weak unassigned states between 300 and 500 keV. However, they are all much stronger than the members of the $K=1$ band should be. Consequently, no assignments have been proposed for the $K=1$ band resulting from the [642↑] orbital.

### *Levels Resulting from the* [402↓] *and* [400↑] *Orbitals*

These orbitals have been observed in several odd-$A$ nuclei but usually with cross sections which are much less than would be expected for the pure [402↓] and [400↑] orbitals.[19] Part of the high $(d,t)$ reaction cross section of the [402↓] and [400↑] states is transferred to the [651↑] and [660↑] states, respectively, so that four strong hole states are usually seen in the spectra of odd-$A$ nuclei rather than just two. This has been observed in several odd-$A$ nuclei with neutron numbers between 91 and 97.[19]

In $Gd^{157}$, the intensity of the [400↑] orbital is very high and essentially equal to the theoretical intensity.[18] This is in contrast to the situation in most nuclei, where this orbital is not as strong as theory predicts. The [402↓] orbital, on the other hand, is only about 60% as strong as the pure [402↓] orbital is expected to be.[18] In view of these observations, the levels resulting from the [400↑] orbital in $Tb^{158}$ should be essentially as strong as the theoretical prediction, but the intensities of the levels resulting from the [402↓] orbital should be roughly 60% of the theoretical prediction. Of course, these predictions will be true only if the relative order of the levels which are mixed by the $\Delta N=2$ matrix elements is the same in $Tb^{158}$ as in $Gd^{157}$. When the relative order of the energy levels being mixed by any perturbation is reversed, the signs of the mixing amplitudes (which determine the mixed reduced widths) are reversed. This reversal of signs causes the "flow of intensity" from one level to the other to reverse direction. In view of the fact that the residual-interaction energy can cause such a reversal, it is even possible that one of the two bands resulting from a given orbital could be made weaker by the mixing while the other band is not. These possibilities must be remembered when attempting to assign the levels resulting from the $N = 4$ orbitals.

The two rotational bands resulting from the [400↑] orbital have $K = 2$ and $K = 1$. Since the $K = 2$ band is the triplet state, it is expected to have the lower energy. Since the occupation probability of the [400↑] orbital is approximately 0.92, the $K = 2$ band head should have a relative intensity of about 430 $\mu$b/sr at 75°. The intensity of the level at the 641-keV level is much greater than any others in the experimental spectrum. The intensity is 331 $\mu$b/sr at 75° and 424 $\mu$b/sr at 68°. Since no other level is expected to be this strong, the assignment of the 641-keV level as the 2+, 2 state is considered certain. The observed intensity indicates that $\Delta N = 2$ mixing does not reduce the intensity of the 2+, 2 level, just as the intensity of the $\frac{1}{2}$+, $\frac{1}{2}$ level in $Gd^{157}$ is not reduced by the mixing. The second member of the $K = 2$ band should be roughly 60 keV above the 641-keV level. The level observed at 702 keV is over three times as strong as the 3+, 2 level should be. Clearly, the 3+, 2 state cannot account for all of the intensity of the 702-keV level. The same is also true of the other nearby levels at 678 and 744 keV, which are other possible candidates for the 3+, 2 state. Since the 702-keV level corresponds to a more reasonable moment-of-inertia parameter, we tentatively assign the 3+, 2 state an energy of 702 keV and note the probability that there is another stronger level which is also near that energy. Higher members of the $K = 2$ band have much lower theoretical cross sections and are not expected to be observed.

Since the $K = 1$ band resulting from the [400↑] orbital is the singlet state, it is expected at a higher energy than the $K = 2$ band. As shown in Table III, the intensity of the $K = 1$ band head should be near 300 $\mu$b/sr. Since there are no unassigned levels with intensities over 200 $\mu$b/sr, the $\Delta N = 2$ mixing probably reduces the intensity of the $K = 1$ band head. If this mixing affects the first two members of the $K = 1$ band the same way, then the $K = 1$ band should appear as a strong level (the 1+, 1 state) with one which is a little more than $\frac{1}{3}$ as strong approximately 40 keV higher in energy. Since the strongest remaining unassigned level has just these characteristics and occurs at a higher energy than the $K = 2$ band head, it is assigned as the $K = 1$ band head. Thus the 767-keV level is assigned as the 1+, 1 state. Accordingly, the level at 795 keV is assigned as the 2+, 1 state. These assignments are obviously not as certain as that

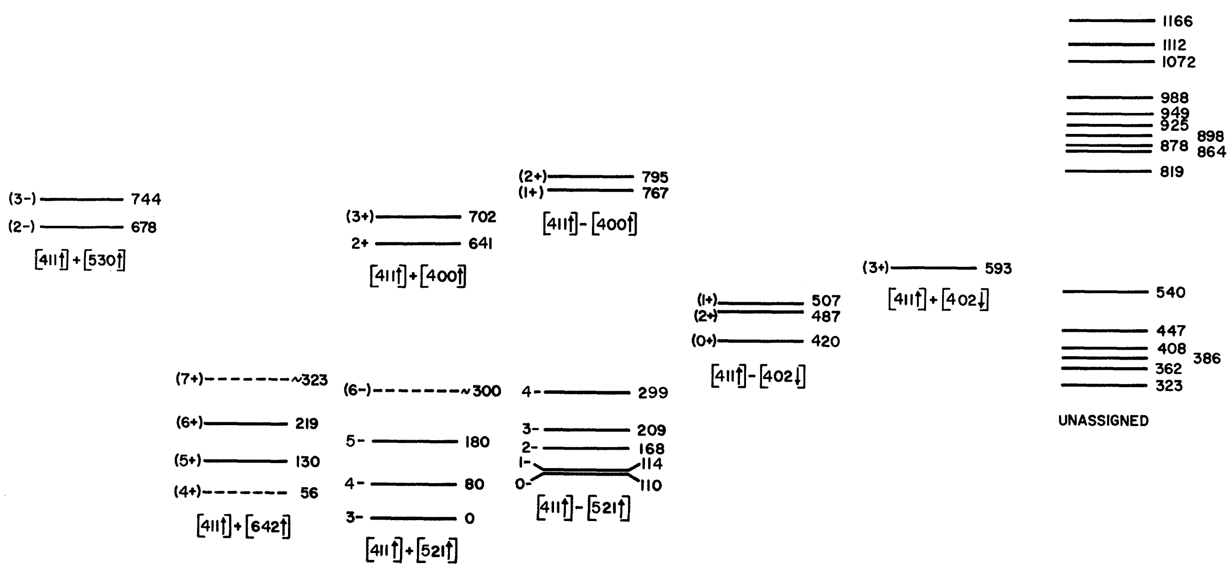


FIG. 4. Level scheme for $Tb^{158}$. Tentative spin assignments are enclosed in parentheses. In the notation for the configurations, the first set of quantum numbers refers to the proton and the second set refers to the neutron. The algebraic signs are the signs of the projections of the particle angular momenta.

of the $K=2$ band head.

The two rotational bands resulting from the [402↓] orbital have $K=3$ and $K=0$. Since the $K=0$ band is the triplet state, it is expected at the lower energy. Since the [402↓] orbital is about 200 keV lower than the [400↑] orbital in $Gd^{157}$, it is expected that the $K=0$ band, at least, should occur below the levels resulting from the [400↑] orbital. Thus the strongest levels below 641 keV should result from the [402↓] orbital. However, since the intensity of the [402↓] orbital is reduced by the $\Delta N=2$ mixing in $Gd^{157}$, the [402↓] levels may not be especially strong. If this mixing reduces the intensity by 40%, as it does in $Gd^{157}$, then the intensity of the $K=0$ band head ($I=0$) should be about 42 $\mu$b/sr. The strong unassigned levels below the 2+, 2 state are at ~414 (a doublet), 487, 507, and 593 keV. Since we expect four strong levels in the two bands resulting from the [402↓] orbital, we propose tentative assignments for these levels in terms of this orbital. The logical assumption is that the three lowest of these levels are the 0−, 1−, and 2− members of the $K=0$ band. The intensities of these levels are reasonably close to the expected values. The 2− level should be approximately 60 keV higher than the 0− level. Because of the odd-even shift, the relative energy of the 1− level cannot be predicted. The triton group centered near 414 keV is an obvious doublet. A least-squares fit to the data yields levels at 408 and 420 keV with the 420-keV level the stronger of the two. Since one of the other strong levels occurs 67 keV higher in energy than the 420-keV level, we tentatively assign the 420-keV level as the 0−, 0 state and the 487-keV level as the 2−, 0 state. The level at 507 keV is then the obvious choice for the 1−, 0 level.

The remaining unassigned level below the 2+, 2 state is at 593 keV and is much stronger than any of the levels assigned to the $K=0$ band. The ob-

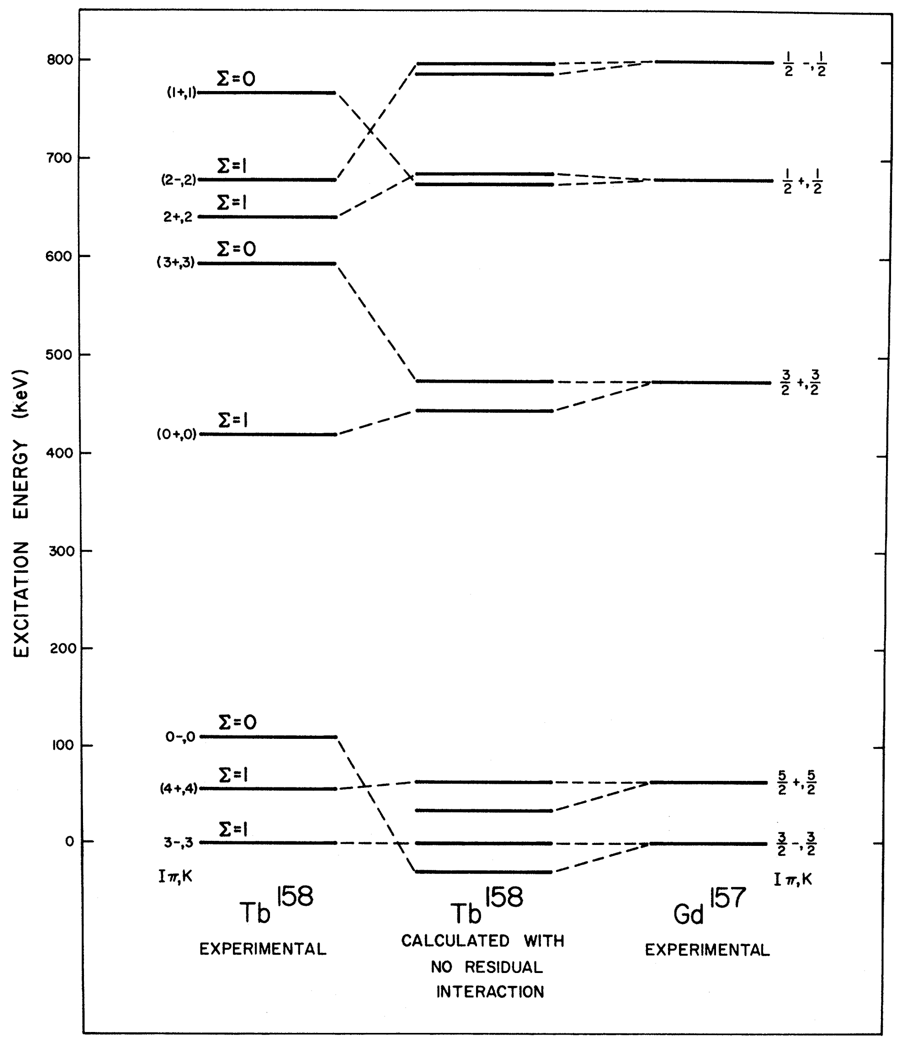


FIG. 5. Correspondence between one-quasiparticle states in $Gd^{157}$ and two-quasiparticle states in $Tb^{158}$.

vious conclusion is that the 593-keV level is the 3+, 3 state, since it is expected to contain essentially all of the strength of the $K=3$ band and should be higher in energy than the $K=0$ band. Although the intensity is somewhat less than expected, this could be due to the $\Delta N=2$ mixing and is not a serious objection to this assignment.

With the exception of the assignment of the 641-keV level, all of the assignments in terms of the [400↑] and [402↓] orbitals are considered tentative.

*Levels Resulting from the [530↑] Orbital*

Since some of the levels resulting from this orbital are expected to have high intensities, it is proposed that the strongest remaining unassigned levels result from this orbital. As shown by Table III, the $K=2$ band resulting from the [530↑] orbit-

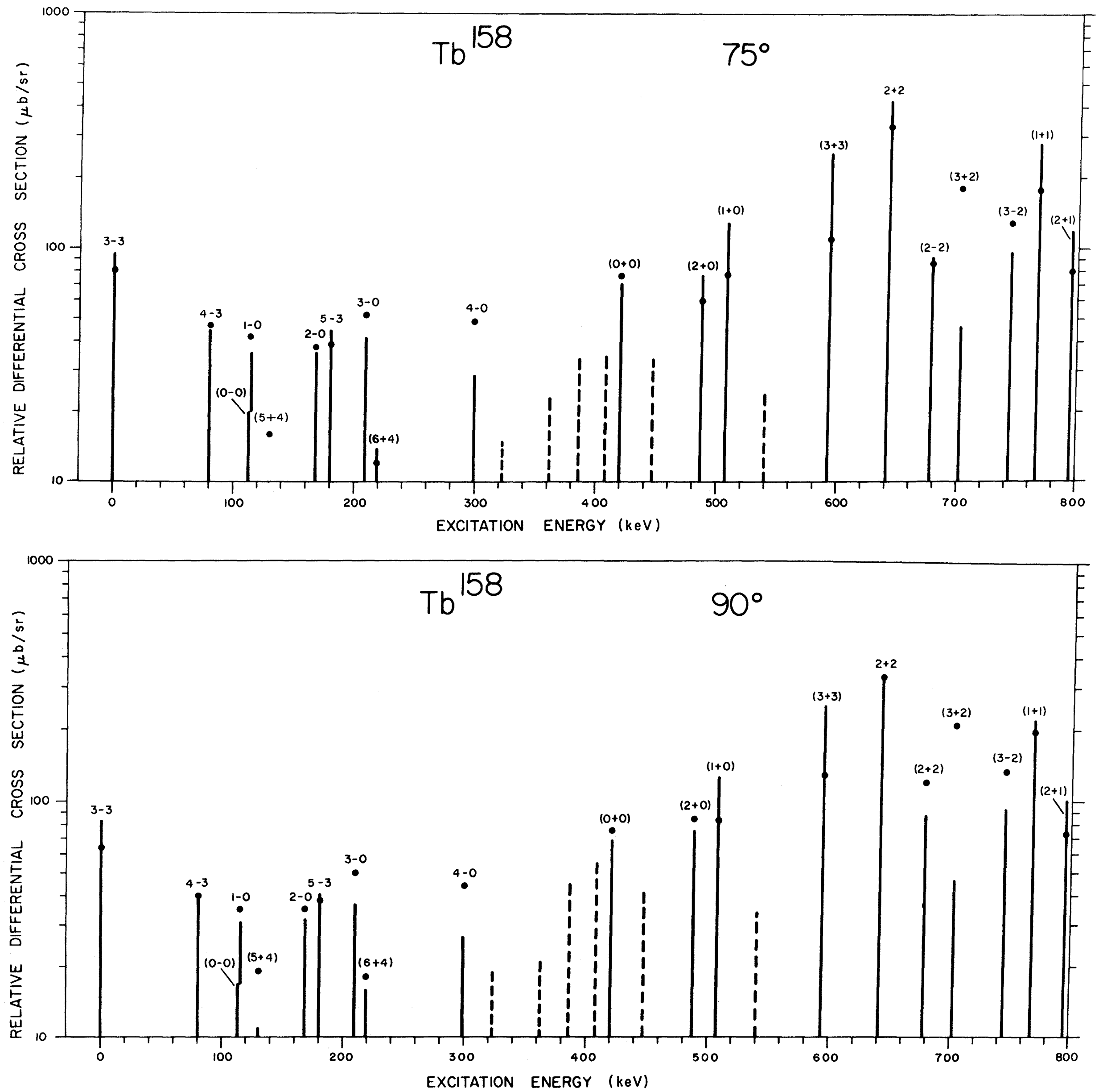


FIG. 6. Comparison of theoretical and experimental cross sections for the $Tb^{159}(d,t)Tb^{158}$ reaction at 75 and 90°. The solid lines represent theoretical cross sections and the large dots represent experimental cross sections. Dashed lines represent the intensities of unassigned levels. The numbers above each level are the assigned values of $I\pi K$. Values of $I\pi K$ enclosed in parentheses are tentative assignments.

al should contain the two strongest levels, which both have intensities of about 100 $\mu$b/sr and should be separated by approximately 60 keV. The two strongest remaining unassigned levels are separated by 66 keV, and both have intensities slightly greater than 100 $\mu$b/sr. Since these levels are at excitation energies comparable to that observed for the [530↑] orbital in $Gd^{157}$, this interpretation is quite reasonable. The levels at 678 and 744 keV are therefore assigned as the 2−, 2 and 3−, 2 levels resulting from the coupling of the [411↑] proton orbital with the [530↑] neutron orbital. In view of the fact that this band can mix via the Coriolis interaction with at least three bands which may be near in energy, these assignments must be considered tentative.

## SUMMARY

Assignments which have been proposed for levels in $Tb^{158}$ are shown in the level scheme in Fig. 4. Tentative assignments are enclosed in parentheses. Figure 5 shows the relationship between the excitation energies of band heads in $Tb^{158}$ and in $Gd^{157}$. The lines in the center $Tb^{158}$ scheme represent the odd-odd excitation energies calculated assuming no residual neutron-proton interaction. The difference then between these levels and the corresponding observed levels is due to the residual interaction and second-order mixing effects. The residual-interaction energies are summarized in Table IV. Figure 6 shows the experimental and theoretical unmixed cross sections for all levels which have been assigned. Theoretical cross sections include the occupation probability which was estimated from the observed excitation energies in $Gd^{157}$. The values used were 0.5, 0.75, 0.90, 0.92, and 0.94 for the [521↑], [642↑], [402↓], [400↑], and [530↑] orbitals, respectively.

From Table IV, it can be seen that the singlet-triplet splitting energies for the [521↑], [402↓], and [400↑] neutron orbitals have experimental values of +132, −111, and +136 keV, respectively. We calculated the corresponding theoretical energies for the two-body potential of Eq. (8) and obtained values of +174, −186, and +136, respectively.

## CONCLUSIONS

The energy levels of $Tb^{158}$ have been observed by means of the $(d,t)$ reaction. Using data on the energy levels of $Gd^{157}$ and the expected systematic behavior of energy levels in deformed nuclei, it has been possible to interpret the energy levels of $Tb^{158}$ below about 1 MeV as resulting from the coupling of the [411↑] proton Nilsson orbital with the neutron orbitals prominent in the low-energy $(d,t)$ spectrum of $Gd^{157}$. This interpretation has resulted in the determination of relative energies due to the residual neutron-proton interaction for all of the assigned two-quasiparticle states.

The three singlet-triplet splitting energies observed were compared with theoretical calculations for a simple zero-range spin-dependent central potential. These calculations predict that the triplet state has the lower residual-interaction energy in each case, in accordance with both the experimental results and the Gallagher-Moszkowski coupling rule. It thus appears that the spin-spin interaction can account for most of the singlet-triplet splitting energy.

TABLE IV. Experimental relative residual-interaction energies for $Tb^{158}$.

| Configuration Proton | Neutron | $K\pi$ | $\Sigma$ | Relative residual-interaction energy (keV) |
|---|---|---|---|---|
| 411↑ | 521↑ | 3− | 1 | 0 |
| 411↑ | 521↑ | 0− | 0 | 132[a] |
| 411↑ | 642↑ | 4+ | 1 | −8 |
| 411↑ | 402↓ | 0+ | 1 | 7[b,c] |
| 411↑ | 402↓ | 3+ | 0 | 118[c] |
| 411↑ | 400↑ | 2+ | 1 | −44 |
| 411↑ | 400↑ | 1+ | 0 | 92[c] |
| 411↑ | 530↑ | 2− | 1 | −119[c] |

[a]The odd-even shift is $B=8$ keV.
[b]The odd-even shift is $B=-32$ keV.
[c]Assignments for these states are uncertain.

## ACKNOWLEDGMENTS

We would like to thank Dr. C. T. Hess and Dr. Y. Shida for helpful discussions of various aspects of this study and Dr. M. M. Minor, who programmed part of the residual-interaction calculations. We thank, also, Kenneth Chellis for preparing the targets used in this research and the plate-counting staff of Mrs. Mary Jones, Mrs. Hazel Benton, Mrs. Veronica Lanier, and Mrs. Ella Jean Wehunt. The help of Robert McDole in drafting for this manuscript is gratefully acknowledged.

†Research supported in part by the U. S. Atomic Energy Commission, by the Nuclear Program of the State of Florida, and by the U. S. Air Force.

*Present address: U. S. Army Nuclear Effects Laboratory, Edgewood Arsenal, Maryland.

[1]S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-

Fys. Medd. 29, No. 16 (1955).
[2]E. Rost, Phys. Rev. 154, 994 (1967).
[3]C. P. Browne and W. W. Buechner, Rev. Sci. Instr. 27, 899 (1956).
[4]R. A. Kenefick and R. K. Sheline, Phys. Rev. 133, B25 (1964).
[5]H. D. Jones and R. K. Sheline, Phys. Rev. C 1, 2030 (1970).
[6]N. D. Newby, Jr., Phys. Rev. 125, 2063 (1962).
[7]C. J. Gallagher and S. A. Moszkowski, Phys. Rev. 111, 1282 (1958).
[8]N. I. Pyatov, Izv. Acad. Nauk SSSR Ser. Fiz. 27, 1436 (1963) [transl.: Bull. Acad. Sci. USSR, Phys. Ser. 27, 1409 (1968)].
[9]G. R. Satchler, Ann. Phys. (N.Y.) 3, 275 (1958).
[10]G. R. Satchler, Nucl. Phys. 55, 1 (1964).
[11]R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL 3240, 1962 (unpublished).
[12]R. H. Bassel, Phys. Rev. 149, 791 (1966).
[13]M. Jaskola, K. Ngboe, P. O. Tjoem, and B. Elbek, Nucl. Phys. A96, 52 (1967).
[14]C. M. Lederer and J. M. Hollander, *Table of Isotopes* (John Wiley & Sons, Inc., New York, 1967), 6th ed.
[15]W. D. Schmit-Ott, F. Smend, and A. Flammersfeld, Z. Physik 184, 310 (1965).
[16]N. B. Gove, R. W. Henry, L. T. Dillman, and R. A. Becker, Phys. Rev. 112, 489 (1958).
[17]Y. Shida, Ph.D. thesis, Florida State University, 1969 (unpublished).
[18]P. O. Tjoem and B. Elbek, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 36, No. 8 (1967).
[19]R. K. Sheline, M. J. Bennett, J. W. Dawson, and Y. Shida, Phys. Letters 26B, 14 (1967).

PHYSICAL REVIEW C VOLUME 2, NUMBER 5 NOVEMBER 1970

# Fission of Odd-$A$ Uranium and Plutonium Isotopes Excited by $(d,p)$, $(t,d)$, and $(t,p)$ Reactions*

H. C. Britt and J. D. Cramer
*Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87544*
(Received 19 January 1970)

Fission-fragment angular correlations and fission probabilities have been measured for a series of odd-$A$ uranium and plutonium isotopes excited by $(d,p)$, $(t,d)$, and $(t,p)$ reactions. The following fissioning nuclei have been studied: (1) $^{235}$U from $(d,pf)$, $(t,pf)$; (2) $^{237}$U from $(d,pf)$, $(t,df)$, $(t,pf)$; (3) $^{239}$U from $(d,pf)$; (4) $^{241}$Pu from $(d,pf)$, $(t,df)$, $(t,pf)$; and (5) $^{243}$Pu from $(d,pf)$, $(t,df)$. Fission probabilities and angular-correlation coefficients for each case are compared with previously reported $(n,f)$ results. The direct-reaction experiments show fission thresholds at excitation energies equal to or less than threshold energies observed in $(n,f)$ experiments. Some qualitative characteristics of the transition-state spectrum for the nuclei studied are determined from comparisons of the direct-reaction and $(n,f)$ results.

## I. INTRODUCTION

Since the concept of transition states was first introduced by Bohr,[1] there have been several attempts to try to deduce the spectra of low-lying transition states for various Th, U, and Pu isotopes from an analysis of cross sections and angular distributions for $(n,f)$ reactions on even-even targets. Some aspects of the transition state spectra for several nuclei were determined by Lamphere[2,3] from a qualitative analysis of the structure apparent in $(n,f)$ cross sections and angular anisotropies. More recently, attempts have been made to obtain information on the low-lying transition states for several nuclei from a simultaneous quantitative fit to $(n,f)$ cross sections and fragment angular distributions.[4-7]

In addition to the $(n,f)$ reaction, it is also possible to excite these same nuclei to energies above the fission barrier using various direct reactions. Results have previously been reported[8] for the angular distributions of fission fragments from the $^{234}$U$(d,pf)$ reaction. A quantitative interpretation of direct-reaction results is difficult because of the uncertainties in the characteristics of the direct-reaction process. However, for excitation energies near the neutron binding energy, the direct-reaction and neutron-capture processes excite a different distribution of angular momentum values. This can create qualitative differences in the fission probabilities and the angular distributions of the fragments for nuclei excited by neutron capture or direct reactions. For example, a low-lying transition-state band of high spin may be observed in the direct-reaction fission experiment, but may not be apparent in neutron-fission results because the neutron-capture reaction is unable to excite states of the appropriate angular momenta.

In this paper results are reported on the fission probabilities and angular correlations for $(d,pf)$,