for ${}_{\Lambda}H^3$ correspond effectively to complete suppression of a short-range ΛN tensor force. This seems surprising in view of the short range of the force and of the open structure of ${}_{\Lambda}H^3$, and is in contrast to the results for the Λ well depth.

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[15]One has $g_\sigma{}^2 = g_{NN\sigma} \times g_{\Lambda\Lambda\sigma}$ in terms of the $NN\sigma$ and $\Lambda\Lambda\sigma$ constants and with neglect of some small recoil terms. If it is assumed that the σ meson is a unitary singlet, then $g_{NN\sigma} = g_{\Lambda\Lambda\sigma}$. For our purposes this assumption is unnecessary. However, equality of the two coupling constants is in fact consistent with a phenomenological analysis of the singlet ΛN and $\Lambda\Lambda$ interactions (D. A. Rote and A. R. Bodmer, unpublished).

[16]The mass $\bar{M}_K$ (not M_K) occurs because this is an exchange interaction, the Λ particle and the nucleon being interchanged by the exchange of the kaon. This is also the reason for the presence of the operator $-P_x P_\sigma$.

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Properties of the K Matrix in Nuclear-Reaction Theory*

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We derive expressions for the K matrix for the general case where the Hamiltonian is separated as $H = H_0 + V$, and H_0 can have discrete as well as continuum states. It is shown that the correct handling of the bound states in the continuum eliminates one of the correction terms proposed by Tobocman and Nagarajan. In addition, some of the properties of the K matrix evaluated at complex energies are discussed.

I. INTRODUCTION

During the past few years there has been an increased use of the K matrix for both the theoretical and experimental study of nuclear reactions.[1-9] One reason for this increased use is that the K matrix treatment of nuclear reactions involves operators whose matrix representations are real, and this property simplifies numerical calculations. Furthermore, simple approximations and parametrizations for the K matrix do not destroy the

unitarity of the S matrix.

The K matrix, however, has some properties which are considerably different from those of the T matrix used in the earlier works on nuclear-reaction theory. Because of the different prescriptions for handling the singularities in the K matrix and T matrix, there has been some confusion about the proper form of the final expression for the reduced K matrix. To point out the source of this confusion and to resolve these ambiguities, we discuss some properties of the K matrix in the case where the Hamiltonian is separated into two parts, H_0+V, where H_0 can have discrete as well as continuum states. Several reaction theories use this approach, and while our results are of general validity, we limit our discussion to the shell-model treatment of nuclear reactions.

In this derivation we first include the effects of the continuum states in an effective interaction and then look for resonances from bound states of H_0 in the usual way.[2] Tobocman and Nagarajan (TN)[10] have claimed that the expressions derived by MacDonald and Mekjian (MM)[2] and Bloch and Gillet (BG)[11] are incomplete and must be supplemented by a correction term. For the case of a separable interaction this correction term did not seem significant for the MM treatment; however, it could be very important for the BG method. While it is reassuring to know that the effect of the correction term is probably not significant for the MM approach to shell-model calculations, the presence of this term is still disturbing; especially since it appears to have a fairly complicated form which could be difficult to evaluate for a realistic calculation. The correction term that TN associated with the MM approach arises from the application of a principal-value prescription to terms containing discrete states of H_0. In this paper we show that there is no principal value associated with the discrete states of H_0, and, consequently, no correction term is present.

It should be emphasized that there is a principal value associated with the continuum states of H_0, and hence the correction term discussed by TN[10] is necessary for the BG formalism.

Also, we discuss what happens when complex energies are introduced into the expressions for the K matrix. These effects can be important, since the expressions for the energy-averaged cross section are derived by evaluating the S matrix at the complex energy $E+iI$. Care must be taken in relating this S matrix to the K matrix because, strictly speaking, the K matrix is only defined for real energies and it is not an analytic function of the energy. In Sec. III we examine some consequences of this property for the case of a separable interaction and show how discontinuities can occur.

II. THEORY

In the shell-model theory of nuclear reactions,[9] the Hamiltonian of the A-particle system is divided into a shell-model Hamiltonian H_0 and a residual interaction V; that is, one writes

$$H=H_0+V, \tag{1a}$$

where

$$H_0=\sum_{i=1}^{A}(T_i+U_i)=T+U. \tag{1b}$$

The shell-model Hamiltonian has both continuum and discrete (i.e., bound) states; by continuum states we mean at least one particle is in a continuum state of U. The A-particle bound states of H_0 can have energies corresponding to continuum-state energies, and it is these bound states in the continuum which are usually associated with the experimentally observed resonances. It is the presence of these bound states at positive energies which has caused difficulties and confusion in the K-matrix approach to nuclear reactions. Therefore, we give a careful derivation of the K-matrix equations for the general case where $H=H_0+V$ and H_0 can have discrete states. Lane and Robson[12] have shown how to define H_0 for the various nuclear-reaction theories.[13] Using our general results, we show that proper handling of the bound states of H_0 insures that no correction terms are present in the final expression for the K matrix.

We define the projection operators P_s and P_d onto the scattering states χ_c and the discrete states χ_d of H_0. Since these states form a complete set,

$$P_s+P_d=1. \tag{2}$$

An outgoing-wave scattering state for the total Hamiltonian is found from the Lippmann-Schwinger equation

$$\psi_c^{(+)}=\chi_c^{(+)}+\frac{1}{E-H_0+i\epsilon}V\psi_c^{(+)}, \tag{3}$$

where the scattering from the potential U is contained in the $\chi_c^{(+)}$, and the scattering from the residual interaction is generated in Eq. (3). Using the two-potential formula,[14] the T matrix for scattering from channels c to c' is

$$T_{c'c}=\langle\chi_{c'}^{(-)}|U|\varphi_c\rangle+\langle\chi_{c'}^{(-)}|V|\psi_c^{(+)}\rangle; \tag{4}$$

here φ_c is the incident plane-wave state in channel c, and

$$\chi_c^{(\pm)}=\varphi_c+\frac{1}{E-H_0\pm i\epsilon}U\varphi_c. \tag{5}$$

If we define the background-scattering phase shift by

$$\pi\langle\chi_{c'}^{(-)}|U|\varphi_c\rangle=\delta_{c'c}(e^{i\delta_c}\sin\delta_c), \tag{6}$$

the S matrix can be written in the form

$$S_{c'c} = e^{i\delta_c}(\delta_{c'c} - 2\pi i T_{c'c}) e^{i\delta_c}, \tag{7}$$

where the reduced T matrix $T_{c'c}$ is defined as

$$\begin{aligned} T_{c'c} &= e^{-i(\delta_c + \delta_{c'})} \langle \chi_{c'}^{(-)} | V | \psi_c^{(+)} \rangle , \\ &= e^{-i(\delta_c + \delta_{c'})} \langle \chi_{c'}^{(-)} | T | \chi_c^{(+)} \rangle . \end{aligned} \tag{8}$$

Using Eqs. (3) and (8) we find

$$T = V + V \frac{1}{E - H_0 + i\epsilon} T . \tag{9}$$

The $i\epsilon$ in the denominator is a result of the boundary condition imposed on $\psi_c^{(+)}$ in Eq. (3). To study the effects of the bound states we rewrite Eq. (3) as

$$\psi_c^{(+)} = \chi_c^{(+)} + \sum_{c'} \int \rho_{E'} dE' \frac{\chi_{c'}^{(+)}(\chi_{c'}^{(+)}, V\psi_c^{(+)})}{E - E' + i\epsilon} + \sum_d \frac{\chi_d(\chi_d, V\psi_c^{(+)})}{E - E_d + i\epsilon} . \tag{10}$$

The $i\epsilon$ specifies how to handle the pole in the integral over the continuum states of H_0, and the limit $\epsilon \to 0$ is to be taken after the integration is performed. The presence of the $i\epsilon$ insures that the wave function has the proper asymptotic behavior, that is, only outgoing waves occur. The bound-state wave function χ_d is zero in the asymptotic region and it is independent of the boundary conditions on the continuum wave functions. Therefore, we do not need a label on χ_d to indicate the boundary conditions used.

Furthermore, it is clear that there is no need for the $i\epsilon$ in the sum over discrete states, since there is no integration to be performed and the last term in Eq. (10) is not singular at E equal to E_d. That there is no singularity can be seen by using the relation

$$\begin{aligned} (\chi_d, V\psi_c^{(+)}) &= (\chi_d, (H - H_0)\psi_c^{(+)}) , \\ &= (E - E_d)(\chi_d, \psi_c^{(+)}) , \end{aligned} \tag{11}$$

where the crucial point in the above relation is that χ_d is a discrete state and thus asymptotically goes to zero. To clearly indicate that the $i\epsilon$ is needed only for the integrals over the continuum, we rewrite the reduced T matrix as

$$T = V + V \frac{P_s}{E - H_0 + i\epsilon} T + V \frac{P_d}{E - H_0} T . \tag{12}$$

Because the $i\epsilon$ has no effect on the sum over discrete states, Eqs. (3) and (10), as well as Eqs. (9) and (12), are equivalent. However, a similar result does not hold for the K matrix.

To derive the expression for the K matrix, we modify Eq. (10) to impose standing-wave boundary conditions on the eigenfunctions of H, that is, we write

$$\psi_c^{(0)} = \chi_c^{(0)} + \sum_{c'} \mathrm{P} \int \rho_{E'} dE' \frac{\chi_{c'}^{(0)}(\chi_{c'}^{(0)}, V\psi_c^{(0)})}{E - E'} + \sum_d \frac{\chi_d(\chi_d, V\psi_c^{(0)})}{E - E_d} , \tag{13}$$

where P means that the principal value of the integral is taken and $\chi_c^{(0)}$ is the standing-wave solution for the shell-model Hamiltonian. Just as in Eq. (10) we do not need to specify the asymptotic boundary condition on χ_d, since it goes to zero in the asymptotic region. The principal-value prescription for handling the poles of the integrand is specified by the standing-wave boundary conditions on the scattering wave function $\psi_c^{(0)}$. Equation (13) can be written in the operator form

$$\psi_c^{(0)} = \chi_c^{(0)} + \mathrm{P} \frac{P_s}{E - H_0} V\psi_c^{(0)} + \frac{P_d}{E - H_0} V\psi_c^{(0)} . \tag{13'}$$

This is not the same as

$$\begin{aligned} \psi_c &= \chi_c^{(0)} + \mathrm{P} \frac{1}{E - H_0} V\psi_c , \\ &= \chi_c^{(0)} + \mathrm{P} \frac{P_s}{E - H_0} V\psi_c + \mathrm{P} \frac{P_d}{E - H_0} V\psi_c , \end{aligned} \tag{14}$$

because in Eq. (13) the principal value only appears in the integral over the continuum states of H_0. Of course, if H_0 has no bound states, Eqs. (13) and (14) are the same.

We define the reduced K matrix in the usual manner as

$$K\chi_c^{(0)} = V\psi_c^{(0)} , \tag{15}$$

then using Eqs. (13) and (15), we find

$$K = V + V\mathrm{P} \frac{P_s}{E - H_0} K + V \frac{P_d}{E - H_0} K . \tag{16}$$

Since Eqs. (13) and (14) are not the same when H_0 has bound states, the reduced K matrix in Eq. (16) is not the same as

$$\begin{aligned} \bar{K} &= V + V\mathrm{P} \frac{1}{E - H_0} \bar{K} , \\ &= V + V\mathrm{P} \frac{P_s}{E - H_0} \bar{K} + V\mathrm{P} \frac{P_d}{E - H_0} \bar{K} , \end{aligned} \tag{17}$$

when H_0 has bound states. However, the form in Eq. (17) has appeared in the literature, and the principal-value prescription for discrete states has led to a final expression for the reduced K matrix which differs by a correction term from the result of MM. We show that using Eq. (16), one obtains the MM expression for the reduced K matrix in terms of the effective interaction

$$\hat{K} = V + V\mathrm{P} \frac{P_s}{E - H_0} \hat{K} . \tag{18}$$

However, before doing this, we first present an alternate derivation of Eq. (16) which also gives

the relationship between the reduced T matrix and the reduced K matrix. To do this, we use in Eq. (12) the relation

$$\frac{P_s}{E-H_0+i\epsilon}=P\frac{P_s}{E-H_0}-i\pi\delta(E-H_0)P_s, \tag{19}$$

which is valid only when the operators appear in an integral over the continuum eigenvalues of H_0. The result of the substitution of Eq. (19) into Eq. (12) is

$$T=V+V\mathrm{P}\frac{P_s}{E-H_0}T-i\pi V\delta(E-H_0)P_sT+V\frac{P_d}{E-H_0}T. \tag{20}$$

We define the on-shell T matrix t as

$$t=\delta(E-H_0)P_sT. \tag{21}$$

Now Eq. (20) can be rewritten in the form

$$T=V(1-i\pi t)+V\mathrm{P}\frac{P_s}{E-H_0}T+V\frac{P_d}{E-H_0}T. \tag{20'}$$

Using Eq. (20′) we obtain the result

$$K=T(1-i\pi t)^{-1},$$

$$=V+V\mathrm{P}\frac{P_s}{E-H_0}K+V\frac{P_d}{E-H_0}K, \tag{22}$$

which is the same as Eq. (16). Combining Eqs. (21) and (22), the relationship between the reduced K matrix and the reduced T matrix is found to be

$$T=K-i\pi Kt,$$

$$=K-i\pi K\delta(E-H_0)P_sT. \tag{23}$$

This is the usual relationship except that now the projection operator onto continuum states of H_0 appears explicitly in the nonlinear term. However, this is equivalent to

$$T=K-i\pi K\delta(E-H_0)T, \tag{23'}$$

because from Eq. (11)

$$\delta(E-H_0)P_dT|\chi_c^{(+)}\rangle=0.$$

Returning to Eq. (16), we now show that the K matrix can be expressed in the standard form with no correction term. Rewriting Eq. (16) in the form

$$\left(1-V\mathrm{P}\frac{P_s}{E-H_0}\right)K=V+V\frac{P_d}{E-H_0}K, \tag{16'}$$

and solving for K gives

$$K=\hat{K}+\hat{K}\frac{P_d}{E-H_0}K, \tag{24a}$$

where

$$\hat{K}=\left(1-V\mathrm{P}\frac{P_s}{E-H_0}\right)^{-1}V,$$

$$=V+V\mathrm{P}\frac{P_s}{E-H_0}\hat{K}. \tag{18'}$$

Equation (24a) may be solved formally for K to give

$$K=\left(1-\hat{K}\frac{P_d}{E-H_0}\right)^{-1}\hat{K}. \tag{24b}$$

Now, using the operator identity

$$\frac{1}{A-B}=\frac{1}{A}+\frac{1}{A}B\frac{1}{A-B}, \tag{25}$$

Equation (24b) can be rewritten

$$K=\hat{K}+\hat{K}\frac{P_d}{E-H_0}\left(1-\hat{K}\frac{P_d}{E-H_0}\right)^{-1}\hat{K},$$

$$=\hat{K}+\hat{K}P_d\frac{1}{E-H_0-\hat{K}P_d}\hat{K}. \tag{26}$$

This is usually written in the more symmetric but equivalent form

$$K=\hat{K}+\hat{K}P_d\frac{1}{E-H_0-P_d\hat{K}P_d}P_d\hat{K}. \tag{27}$$

We now have K expressed in a simple form in terms of $\hat{K}$, the solution of Eq. (18). We have shown that in contrast to some previous derivations,[9,10] there is no principal-value prescription or correction term associated with the discrete states in Eq. (27).

III. K MATRIX AT COMPLEX ENERGIES

Equation (18) for $\hat{K}$ involves taking a principal value of the integral over the continuum states of H_0; therefore, this equation cannot be analytically continued to complex energies. Because $K(E+iI)$ and consequently $\hat{K}(E+iI)$ have been used in formal derivations of the relations between experimentally observed quantities, it is of interest to investigate the interpretation of $K(E+iI)$.

The expression $K(E+iI)$ can imply either of two alternatives. The first alternative is to solve Eq. (16) or Eq. (27) with E replaced by $E+iI$. Since the spectrum of H_0 is real, there is no pole in the operator $(E-H_0)^{-1}$ when E is complex, and the principal-value prescription is not needed; consequently, one simply obtains the reduced T matrix for the energy $E+iI$. Another way to see this is to consider the relationship between K and T given in Eq. (23); at complex energies the δ function disappears, and K is equal to T. The T matrix is an analytic function of the energy; thus, the K matrix cannot be, since we know that for real energies the K matrix is not equal to the T matrix. This alternative would lead to the conclusion that the average value of the K matrix parameters are equal to those of the T matrix. Clearly, this is not the desired result.

The second alternative is to solve Eq. (18) for real values of the energy and then to evaluate the

resulting function of the energy at the complex energy $E+iI$. This is the method used in the formal derivations for the distribution of the fine-structure parameters in the presence of a doorway state.[4,5] The expressions obtained using this alternative will not be equal to the T matrix evaluated at the energy $E+iI$; however, the expressions are still not analytic functions of the energy. The function $K(E)$ has singularities at the (unphysical) thresholds of H_0, and care must be taken when using $K(E+iI)$. The presence and effects of these singularities are best illustrated in the following example for a separable interaction.

The shell-model potential and residual interaction we choose is one employed in previous discussions of the shell-model theory of nuclear reactions.[15,16] This model consists of a spinless particle of mass m and zero angular momentum being scattered by an infinite-mass target with states φ_α whose energy is Q_α. The interaction of the particle with the target is of the form

$$U=-\frac{\hbar^2}{2m}\sum_\alpha |\varphi_\alpha\rangle\left(\frac{1}{r}+b_\alpha\right)\delta(r)\langle\varphi_\alpha|\,, \tag{28a}$$

$$V=\frac{\pi\hbar^2}{2m}\lambda\sum_{\alpha,\beta}\left|\frac{\delta(r)}{r}\varphi_\alpha\right\rangle\left\langle\frac{\delta(r')}{r'}\varphi_\beta\right|. \tag{28b}$$

Using the ordinary separable-potential formalism, one obtains

$$\hat{K}_{\alpha\beta}(k,k')=\frac{\lambda f_\alpha(k)f_\beta(k')}{1-\lambda\sum_\gamma M_\gamma(E)}, \tag{29}$$

and

$$K_{\alpha\beta}(k,k')=\frac{\lambda f_\alpha(k)f_\beta(k')}{D(E)}, \tag{30}$$

where

$$f_\alpha(k)=[k/(k^2+b_\alpha{}^2)]^{1/2}, \tag{31a}$$

$$M_\gamma(E)=\mathrm{P}\int\frac{kf_\gamma{}^2(k)dk}{(2m/\hbar^2)(E-Q_\gamma)-k^2}, \tag{31b}$$

$$D(E)=1-\lambda\sum_\gamma M_\gamma(E)-\lambda\sum_\gamma\frac{B_\gamma}{E-E_\gamma}, \tag{31c}$$

and

$$B_\gamma=\pi\lambda\hbar^2 b_\gamma/m\,, \tag{31d}$$

and E_γ is the energy of the bound state in channel γ. The singularities of the reduced K matrix occur because of the behavior of $M_\gamma(E)$ as the energy passes through a threshold Q_γ of H_0. One finds

$$M_\gamma(E)=-\pi(b_\gamma-ik_\gamma)^{-1}, \quad E<Q_\gamma, \tag{32a}$$

$$M_\gamma(E)=-\pi b_\gamma(b_\gamma{}^2+k_\gamma{}^2)^{-1}, \quad E>Q_\gamma, \tag{32b}$$

where

$$k_\gamma=[2m(E-Q_\gamma)]^{1/2}/\hbar\,. \tag{33}$$

For real values of the energy, $M_\gamma(E)$ is continuous across the threshold energy Q_γ; therefore, $K(E)$ is also continuous. For complex values of the energy the discontinuity of $M_\gamma(E+iI)$ across the threshold is given by

$$M_\gamma(Q_\gamma^{>}+iI)-M_\gamma(Q_\gamma^{<}+iI)=\frac{i\pi\kappa_\gamma}{b_\gamma{}^2+\kappa_\gamma{}^2}, \tag{34}$$

where

$$\kappa_\gamma{}^2=2imI/\hbar^2, \tag{35}$$

and the resulting discontinuity of $K(E+iI)$ is

$$K_{\alpha\beta}(Q_\gamma^{>}+iI)-K_{\alpha\beta}(Q_\gamma^{<}+iI)=\left(\frac{i\pi\kappa_\gamma}{b_\gamma{}^2+\kappa_\gamma{}^2}\right)\frac{\lambda f_\alpha(k)f_\beta(k')}{D(Q_\gamma^{>}+iI)D(Q_\gamma^{<}+iI)}. \tag{36}$$

It is clear from Eq. (36) that $K(E+iI)$ is discontinuous at a threshold of the arbitrary shell-model Hamiltonian H_0, and that this discontinuity disappears as I goes to zero. Since $K(E+iI)$ is used to obtain relationships between experimentally observed parameters which are continuous functions of the energy, care must be exercised in using the reduced K matrix near a threshold of H_0.

The threshold effects discussed above are not physical and consequently should not be present in the expression for the T matrix. To show that this is the case, we use Eq. (12) to find

$$T_{\alpha\beta}(k,k')=\frac{\lambda f_\alpha(k)f_\beta(k')}{D(E)+i\pi\lambda\sum_{\gamma'}f_{\gamma'}{}^2(k_{\gamma'})}, \tag{37}$$

where the sum over γ' includes only the open channels. From Eq. (31a) we see that at threshold

$$f_\gamma{}^2=\kappa_\gamma/(b_\gamma{}^2+\kappa_\gamma{}^2), \quad E=Q_\gamma; \tag{38}$$

therefore, the discontinuity of $M_\gamma(E)$ across Q_γ is just cancelled by the discontinuity in the sum over open channels which occurs when a new channel opens. This cancellation occurs for both real and complex energies; hence, the T matrix given in Eq. (37) is an analytic function of the energy. Thus, the singularities of K at the eigenvalues of H_0 are not physically observable.

IV. CONCLUSIONS

If the Hamiltonian is separated into the two parts H_0+V, as in the shell-model approach to nuclear reactions, the expression for the reduced K matrix contains a sum over the discrete states of the Hamiltonian H_0. We have shown that there should be no principal value associated with this summation and consequently no correction terms are nec-

essary for the MM expression for the K matrix. In addition, we have shown that one must be careful when evaluating the resulting expressions for the K matrix at a complex value of the energy.

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Photodisintegration of ^{9}Be†

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The photoneutron cross section of ^{9}Be is calculated using the Dirac model: It is found to have a form that differs somewhat from the standard Breit-Wigner form near threshold. A good fit to the data is obtained.

I. INTRODUCTION

In an experiment measuring the photoneutron cross section of ^{9}Be, Berman, Hermert, and Bowan,[1] detected a peak thought to correspond to a $\frac{1}{2}^+$ excited state of ^{9}Be a few above the threshold for breakup into $^9\mathrm{Be}(0^+)+n$. Below this threshold, the cross section was found to be very small (<1 μb). They were unable to fit the data with a one-level Breit-Wigner form. Similar earlier calculations by Mahaux[2] and by Barker and Treacy[3] fitted the higher-energy data but exhibited low-energy peaks too far from threshold. Barker and Fitzpatrick[4] managed to force an R matrix fit of the data but only at the expense of making an unwarranted 15% renormalization of the Berman, Hemert, and Bowan data. The experimental results can, however, be understood by utilizing a Dirac model. This method has the advantage that it treats the final state as a three-particle state and takes careful account of the threshold energy dependence. One interesting consequence of using the Dirac model is that the standard Breit-Wigner form must be modified near threshold. This modification arises from an energy-dependent energy shift in the denominator of the resonant part of the amplitude and from an additional term in the numerator which corresponds to direct nonresonant breakup into three particles. These effects are usually important only for low energies.

II. THEORY

To see this, we need to calculate the cross section for the reaction

$$^9\mathrm{Be}(\tfrac{3}{2}^-)+\gamma \rightarrow n+\alpha+\alpha \qquad (1)$$

which has the form