ΛN Tensor Forces for Scattering and for the Λ-Particle Binding in Nuclear Matter*

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The effect of ΛN tensor forces on the Λ-particle binding in nuclear matter is studied with the use of second-order perturbation theory and the Brueckner-Bethe reaction-matrix approach in the g-matrix approximation. The g matrix is calculated self-consistently by use of the Kallio-Day version of the reference-spectrum method. The free kinetic energies are assumed for the unoccupied states. One-boson-exchange (OBE) models indicate that the ΛN tensor force is expected to be of short range and moderate strength. For short-range tensor forces the dominant momentum components are very large, and the effects of such forces are only slightly modified by the nuclear medium. On the other hand, if the ΛN tensor forces were of rather long range, they would be quite strongly suppressed in nuclear matter. These features are very clearly exhibited by consideration of the effective nonlocal central potentials that represent the (s-state) effect of tensor forces for nuclear matter and for scattering. The ratio of the (nuclear-matter) expectation values of these two effective potentials is a good measure of the suppression. The expectation value of the effective potential for nuclear matter is just the second-order perturbation-theory energy. Reaction-matrix calculations show that higher-order effects may become quite important for shorter ranges. Such calculations have, in particular, been made for various mixtures of central and tensor forces chosen to give a constant s-wave scattering length. Yukawa shapes corresponding to the kaon and one- and two-pion masses were used, as well as "realistic" OBE potentials with a hard core and a tensor component due to kaon exchange (and also approximately due to η exchange). For a particular mixture, the suppression is measured by the reduction in the well depth relative to the depth for a purely central potential which has the same hard core and the same scattering length and effective range as the mixture. For the short-range tensor forces there is rather little suppression even for very strong tensor forces which account for all the triplet scattering. Different assumptions about the d-state interaction have an almost negligible effect on the s-state well depth if the same assumption is made for both scattering and nuclear matter. Similar considerations are made for the effect of tensor forces in the p wave, for which we find very little suppression ($\lesssim$1 MeV). We conclude that if central and short-range tensor forces are chosen to compensate each other for low-energy scattering, they will also compensate each other quite closely for nuclear matter. In particular, for the OBE potentials with strengths consistent with the phenomenological values of the $\Lambda N\bar{K}$ coupling constants, the reduction in the well depth is at most about 4 MeV. The conclusions about the ΛN interaction obtained from a comparison of the calculated and phenomenological well depths are, therefore, effectively unchanged by the presence of a ΛN tensor force. Consequently, in order to bring the two numbers into agreement, it is necessary to invoke a substantial short-range repulsion, a rather weak p-state interaction, and suppression of the ΛN-ΣN coupling and/or repulsive ΛNN three-body forces.

1. INTRODUCTION

The effect of tensor forces on nuclear binding energies is a problem of long standing. Of particular interest is the relation between the effect of a tensor force for scattering and for binding energies – in particular, the question of the so-called suppression of a tensor force in nuclei. In this paper we study these questions for a Λ particle in nuclear matter. Thus we consider the role of ΛN tensor forces for the binding energy of a Λ particle in nuclear matter (i.e., for the Λ well depth D), using both perturbation theory and Brueckner-Bethe reaction-matrix methods. Apart from the specific interest of the Λ-well-depth problem we have tried – in the context of this problem – to obtain a better understanding of the role of tensor forces. We believe our approach has some novel features and is also of wider interest.

The Λ well depth (D) is of fundamental interest for hypernuclei and plays a role for these which is analogous to the role of nuclear matter for ordinary nuclei.[1] On the one hand, a comparison of the phenomenological value[2] $D \approx 30$ MeV with calculated values may shed some light on the ΛN interaction. A number of studies with this emphasis have been made, especially for central ΛN forces,[3] which are related to the Λp scattering data and to the binding energies of the s-shell hypernuclei. In particular, recent studies for hard-core potentials have used current developments of the reaction-matrix method[1,2,4,5] and also variational procedures which use Jastrow-type wave functions.[6,7]

On the other hand, the nuclear many-body problem of the Λ particle in nuclear matter can serve as a theoretical laboratory for problems of hypernuclear structure that are also of interest for finite hypernuclei. Thus, because of our lack of detailed knowledge about the ΛN tensor force, we try to obtain useful and rather generally valid results (e.g., about the equivalence of a tensor force for scattering and for a Λ particle in nuclear matter) for ΛN tensor forces which have reasonable ranges of strengths and shapes consistent with theoretical expectations.

Although we have no direct phenomenological information about the ΛN tensor forces, we do have some theoretical indications about the strengths and shapes of these interactions. OBE models of the ΛN interaction[8] show that, if charge symmetry is assumed, the longest-range tensor-force contributions in the ΛN channel will be due to the pseudoscalar K and η mesons. Thus, in contrast to the NN interaction, for which one-pion exchange is possible, the tensor force in the ΛN channel is expected to be of short range. However, it could be fairly strong if one uses some of the recently determined values of the $\Lambda N\bar{K}$ coupling constant.

The effect of ΛN tensor forces on hypernuclear binding energies was first considered by Buxton and Schrils[9] for ${}_\Lambda H^3$ and by Law, Bhaduri, and Gunye,[10] for ${}_\Lambda He^5$. The latter authors used an effective local interaction obtained from the appropriate second-order perturbation expression for a Λ particle in nuclear matter. The first authors who used reaction-matrix methods to study the effect of tensor forces on the well depth were Bodmer and Rote,[1] who also investigated the use of perturbation theory and emphasized the importance of the range of the tensor forces. Considerations based on the use of Jastrow-type wave functions together with a perturbation-theory treatment of the tensor forces were made by Mueller and Clark.[7] Since then, Dabrowski and Hassan[11] have also made calculations using reaction-matrix methods with effectively the same assumptions as those of Ref. 1. Goodfellow and Nogami[12] have studied the effect of separable tensor potentials. Nearly all the results of these studies indicate a small suppression of the ΛN tensor force, with a correspondingly small reduction in the well depth of not more than a few MeV for the short ranges and moderate strengths appropriate to the ΛN interaction.[13] The present work is an extended and more complete version of the relevant portions of Ref. 1; in particular, we attempt to obtain a more general understanding of the role of tensor forces.

Section 2 deals with tensor forces and ΛN scattering, and describes the potentials which we subsequently use. These potentials are mixtures of central and tensor forces. For each set of mixtures the potential shapes are the same but the ratio of tensor-to-central-force strength is varied so as to give a fixed scattering length. We consider both Yukawa and OBE potentials with several different tensor-force ranges for the former. In Sec. 3 we consider perturbation theory (to second order) followed in Sec. 4 by a discussion of the related effective local central potentials and the associated average energies which have frequently been used to represent the effect of a tensor force. Section 5 discusses the reaction-matrix procedure for ΛN potentials that include a tensor component. This is followed in Sec. 6 by a discussion of the effective nonlocal central potentials for scattering and for a Λ particle in nuclear matter. The use of these potentials can be regarded as an approximation to the complete reaction-matrix treatment, and they are very useful, in particular, for a comparative understanding of the effect of tensor forces for scattering and nuclear matter. In Secs. 7 and 8 we present the complete reaction-matrix results for Yukawa and OBE potentials, respectively; in particular, we also discuss the limitations of the perturbation-theory and effective-potential approximations. In Sec. 9 we consider tensor-force effects in the p wave. Finally, in the last section we summarize our results and also give some discussion of their limitations and of their relation to the effect of the NN tensor force for ordinary nuclear matter. A summary is also given of the conclusions obtained about the ΛN interaction from a comparison of the calculated and phenomenological well depths.

An important aspect of our work is the careful attempt to obtain a meaningful measure of the suppression of a tensor force in nuclear matter. To do this we have obtained purely central potentials with the same hard core and the same scattering length and effective range as the central-plus-tensor-force mixtures. The well depths corresponding to these equivalent central potentials are then considered to give the "unsuppressed" values, and the suppression is measured by the reduction in the well depth for the mixture relative to the corresponding "unsuppressed" depth.

2. ΛN SCATTERING AND TENSOR FORCES

Tensor Forces and Scattering

With a tensor force, the triplet ΛN interaction

$\overline{V}_t$ is

$$\overline{V}_t = V_t(r) + S_{\Lambda N} V_T(r), \tag{2.1}$$

where $V_t(r)$ is the triplet central potential, $S_{\Lambda N}$ is the ΛN tensor operator, and $V_T(r)$ is the shape function of the tensor potential. The singlet interaction $V_s(r)$ is of course purely central. The effect of the tensor force is to couple different partial waves. In particular, the (triplet) s-state wave function u is now coupled to the (triplet) d-state wave function w, and for a free ΛN pair, one has the coupled equations:

$$\frac{d^2u}{dr^2} + k^2 u = V_t u + \sqrt{8}\, V_T w, \tag{2.2a}$$

$$\frac{d^2w}{dr^2} + \left(k^2 - \frac{6}{r^2}\right) w = V_d w + \sqrt{8}\, V_T u, \tag{2.2b}$$

where (as in all of the following) the potentials are in units of $\hbar^2/2\mu$, $\mu = M_\Lambda M_N/(M_\Lambda + M_N)$ is the reduced mass, and V_d denotes the net (diagonal) d-state potential, which for the case of a *local* potential is given by $V_d = V_t - 2V_T$. In particular, we are interested in the low-energy s-wave scattering – especially in the s-wave scattering length a_t and the effective range r_{0t} as given by the s-wave dominant solution. It is clear that the tensor force affects the s-state wave function u only through the coupling to w, and hence can affect s-state properties only through V_T^2 or higher powers of V_T. We shall discuss this in more detail in Secs. 6 and 7. We note here that if V_d is independent of V_T (and in particular if $V_d = 0$) all the s-wave results are independent of the sign of V_T. Of course if one uses $V_d = V_t - 2V_T$, then the sign of V_T does matter. This is explicitly brought out by the form of the effective potential $V_{\text{eff}}^{(\text{SCAT})}$ which is given by Eqs. (6.3), (6.4), and (7.1) and which, when used in the s-state equation, reproduces the s-wave scattering exactly.

The corresponding p-state expressions and considerations are given in Sec. 9.

We recall that the analysis of the Λp scattering data gives about −2 F for both singlet and triplet scattering lengths and about 3 F for the effective ranges and that there is tentative evidence of a rather weak p-state interaction about half that of the s-state one.[3,14]

ΛN Potentials

Subsequently we shall use mixtures of central and tensor forces, as in Eq. (2.1). For a given set of mixtures the potential shapes are fixed and the ratio of tensor-to-central-force strength is varied so as to give the same (s-state) scattering length for all members of the set. We have used two general types of potentials. Those of the first type are Yukawa potentials. These we consider also for illustrative purposes. For these potentials, both $V_t(r)$ and $V_T(r)$ have Yukawa shapes $Ve^{-\mu r}/\mu r$. For all the Yukawa mixtures the range of the central force is the same, namely, $\mu_C^{-1} = 0.7$ F. For $V_T(r)$ we consider the three ranges $\mu_T^{-1} = 0.4$, 0.7, and 1.4 F. These correspond approximately to a kaon mass, a mass of two pions, and a pion mass,

TABLE I. Results for mixtures of triplet central and tensor Yukawa potentials which give a scattering length of −0.75 F. The central part $V_t e^{-\mu r}/\mu r$ has a range $\mu^{-1} = 0.7$ F. Two ranges $\mu_T^{-1} = 1.4$ and 0.4 F are considered for the tensor potential $S_{12} V_T e^{-\mu_T r}/\mu_T r$. The triplet effective range is denoted by r_{0t}. $\overline{E}_T^{(\text{SCAT})}$ is the energy in the effective central local potential, given by Eq. (3.13), which reproduces the effect of the tensor potential. The s-state well depths D_s are for the corresponding triplet potentials ***plus*** a singlet potential which is equal to the triplet potential in the limit $V_T = 0$. The values of D_s are for $k_F = 1.4$ F^{-1}, $M_N^*/M_N = 0.64$, $\Delta_N = 85.4$ MeV and $\Delta_\Lambda = 30$ MeV. The results depend only slightly on Δ and thus are very close to the self-consistent results.

		$\mu_T^{-1} = 1.4$ F					$\mu_T^{-1} = 0.4$ F		
V_t (MeV)	V_T (MeV)	r_{0t} (F)	$\overline{E}_T^{(\text{SCAT})}$ (MeV)	D_s (MeV)	V_t (MeV)	V_T (MeV)	r_{0t} (F)	$\overline{E}_T^{(\text{SCAT})}$ (MeV)	D_s (MeV)
				$V_d = 0$					
0.00	22.80	2.68	580	20.0	0.00	412.4	0.72	10 300	32.0
20.40	18.11	3.17	513	25.8	20.40	328.4	2.05	7 870	33.4
40.80	11.12	3.66	450	31.5	40.80	206.2	3.22	5 530	34.6
48.96	6.63	3.84	424	33.9	48.96	121.8	3.70	4 500	34.9
53.28	0.0	3.94	415	35.1	53.28	0.0	3.94	3 900	35.1
				$V_d = V_t - 2V_T$					
0.00	24.00	3.04	642	20.4	0.00	444.0	0.81	11 950	32.3
20.40	18.70	3.32	550	26.5	20.40	346.1	2.02	8 900	33.5
40.80	11.32	3.69	475	31.5	40.80	212.7	3.21	5 825	34.6
48.96	6.67	3.84	450	33.9	48.96	122.6	3.70	4 600	34.9
53.28	0.0	3.94	440	35.1	53.28	0.0	3.94	4 100	35.1

TABLE II. The same as for Table I except that the potentials give a scattering length of −2 F. The well depths are again for $\Delta_\Lambda = 30$ MeV and in this case are about 1 MeV greater than the self-consistent values.

$\mu_T^{-1} = 1.4$ F				$\mu_T^{-1} = 0.4$ F			
V_t (MeV)	V_T (MeV)	r_{0t} (F)	D_s (MeV)	V_t (MeV)	V_T (MeV)	r_{0t} (F)	D_s (MeV)
			$V_d = 0$				
0.00	30.95	2.68	36.0	0.00	480.3	0.67	50.2
20.40	27.28	2.64	41.5	20.40	424.0	1.08	52.7
40.80	22.90	2.58	47.1	40.80	356.0	1.50	55.1
61.20	17.14	2.51	52.6	61.20	266.8	1.91	57.2
81.60	7.51	2.40	58.3	81.60	115.4	2.29	59.2
86.29	0.0	2.37	59.7	86.29	0.0	2.37	59.7
			$V_d = V_t - 2V_T$				
0.00	33.20	2.90	37.0	0.00	524.2	0.69	50.8
20.40	28.87	2.78	42.2	20.40	455.3	1.12	53.1
40.80	23.92	2.66	47.5	40.80	375.6	1.52	55.3
				61.20	276.3	1.90	57.3
				86.29	0.0	2.37	59.7

respectively. The last range is unreasonably long for the ΛN interaction and is considered for illustrative purposes only.

For each range μ_T^{-1}, pairs of values of V_t and V_T are determined such that the total triplet interaction $\overline{V}_t$ always gives the same scattering length a_t. This procedure thus determines V_T as a function of the central strength V_t, such that a_t has a constant value. In particular, at one extreme $V_T = 0$ and all the scattering is due to the central part V_t, while at the other extreme $V_t = 0$ and all the (triplet) scattering is due to the tensor force. The singlet potential is the same for a given set of mixtures, and is chosen equal to $V_t(r)$ in the limit $V_T = 0$. Two values of a_t were considered, namely, $a_t = -0.75$ and −2 F. With $V_T = 0$ and $V_s = V_t$ and with $\mu_C^{-1} = 0.7$ F, the value −0.75 F is appropriate to the spin-average strength determined from $B_\Lambda({}_\Lambda He^5)$. The value −2 F is roughly consistent with the scattering data and leads to potentials which overbind ${}_\Lambda He^5$. Our potentials and the associated low-energy scattering parameters are shown in Tables I and II for $a_t = -0.75$ and −2 F, respectively, and for two different assumptions about V_d, namely, $V_d = 0$ and $V_d = V_t - 2V_T$. For $V_d = 0$, the sign of V_T is irrelevant; for $V_d = V_t - 2V_T$, the results correspond to the choice $V_T < 0$. Of course, for a given a_t, the effective range r_{0t} depends on the range μ_T^{-1} and on the ratio of V_T to V_t.

In particular, for the longer range $\mu_T^{-1} = 1.4$ F – and especially for $a_t = -2$F – the values of r_{0t} do not change much as the ratio of tensor-to-central force is varied. Thus for this case we have in effect constructed mixtures of central and tensor forces that give almost the same values of a_t *and* r_{0t}, namely, −2 and ≈2.5 F, respectively. Also for $a_t = -0.75$ F, the variation of r_{0t} over the whole range of mixtures is relatively small. However, for the short range $\mu_T^{-1} = 0.4$ F the values of r_{0t} become quite (unreasonably) small for large V_T.

This behavior of r_{0t} may be understood as fol-

TABLE III. Results for central Yukawa potentials: $V(r) = \infty$ for $r \le c$, and $-V_0 e^{-\mu r}/\mu r$ for $r > c$. The strengths V_0 are determined as a function of μ so as to give a constant scattering length a; r_0 is the effective range. The s-state well depths $\overline{D}_s^{(C)}$ are for ***equal*** singlet and triplet strengths; the values are for $k_F = 1.4$ F^{-1}, $M_N^*/M_N = 0.64$, $\Delta_N = 85.4$ MeV (Ref. 23), and $\Delta_\Lambda = 30$ MeV for $c = 0.0$ and $\Delta_\Lambda = \overline{D}_s^{(C)}$ (i.e., self consistent) for $c = 0.43$ F.

$a = -2$ F, $c = 0.43$ F				$a = -2$ F, $c = 0.0$ F				$a = -0.75$, $c = 0.0$ F			
V_0 (MeV)	μ (F^{-1})	r_0 (F)	$\overline{D}_s^{(C)}$ (MeV)	V_0 (MeV)	μ (F^{-1})	r_0 (F)	$\overline{D}_s^{(C)}$ (MeV)	V_0 (MeV)	μ (F^{-1})	r_0 (F)	$\overline{D}_s^{(C)}$ (MeV)
89.0	1.00	7.53	27.8	6.8	0.5	8.18	48.3	9.6	0.75	10.16	31.8
327.5	1.50	4.97	40.9	36.7	1.0	3.82	58.5	20.5	1.00	7.51	35.4
880.0	2.00	3.80	46.9	97.0	1.5	2.21	59.6	37.6	1.25	4.85	36.0
3245.0	2.84	2.88	50.5	188.7	2.0	1.51	58.0	60.6	1.50	3.64	35.9
8340.0	3.58	2.46	51.4	311.7	2.5	1.14	55.9	126.8	2.00	2.33	35.5
37 035.0	4.97	2.04	51.5	467.3	3.0	0.90	54.2	221.7	2.50	1.66	34.3
				874.0	4.0	0.64	51.5	346.4	3.00	1.27	28.7
				1410.0	5.0	0.49	49.7				

lows. The range of the effective central potential $V_{\rm eff}^{(\rm SCAT)}$ (that reproduces the effect of a tensor force for the *s* state) is expected to be about half of μ_T^{-1}. Hence for $\mu_T^{-1}=1.4$ F, the range of $V_{\rm eff}^{(\rm SCAT)}$ will be quite close to the range $\mu^{-1}=0.7$ F of the central part V_t, and one obtains mixtures of V_t and V_T which all correspond to an effective central potential $V_t+V_{\rm eff}^{(\rm SCAT)}$ of about the same range, $\mu^{-1}\approx 0.7$ F. The same reasoning also immediately explains the very small values of r_{0t} which are obtained for the short range $\mu_T^{-1}=0.4$ F for large strengths V_T.

Table III shows *purely central* Yukawa potentials of varying range which give either $a=-0.75$ or -2 F. These results enable one to find a central Yukawa potential *that has both the same (s-wave) scattering length and effective range* as any of the mixtures of central and tensor forces that we consider. Also shown for these central potentials are the corresponding *s*-state well depths D_s (appropriate to the interaction acting only in *s* states) for equal singlet and triplet potentials.

The second type of ΛN potentials is more "realistic." For these potentials we consider a simplified OBE model which has the essential meson-theoretical features – in particular, tensor forces with the meson-theoretical shape and of the expected range. Our model has two mesons, as well as a hard core of radius $c=0.43$ F. This hard core is similar to that of the *NN* interaction and may be considered as representing the repulsive effect of the vector mesons.

One of the mesons is a scalar, isoscalar ($T=0$, $J=0^+$) σ meson. This gives a purely central Yukawa potential

$$V^{(\sigma)}=-g_\sigma^2 M_\sigma e^{-\mu_\sigma r}/\mu_\sigma r\,, \tag{2.3}$$

where $\mu_\sigma=M_\sigma/\hbar c$. For the mass of the σ meson, we use $M_\sigma=420\text{ MeV}\approx 3M_\pi$. This is roughly consistent with the mass needed for the *NN* interaction. In fact, for our purposes the precise range of $V^{(\sigma)}$ is unimportant. The strength of $V^{(\sigma)}$ is determined by the effective coupling constant g_σ^2.[15]

The other meson is the kaon with $M_K=494$ MeV. This gives the well-known kaon-exchange potential $V^{(K)}$ (appropriate to pseudoscalar-meson exchange), which has a (spin-dependent) central part and a tensor part

$$V^{(K)}=g_{N\Lambda\overline{K}}^2(-P_xP_\sigma)\frac{\overline{M}_K^3}{12M_NM_\Lambda}[\vec{\sigma}_N\cdot\vec{\sigma}_\Lambda f(\mu_K r)+S_{\Lambda N}h(\mu_K r)], \tag{2.4}$$

where $f(x)=e^{-x}/x$, $h(x)=(1+3x^{-1}+3x^{-2})e^{-x}/x$, $\mu_K=\overline{M}_K/\hbar c$ with $\overline{M}_K=[M_K^2-(M_\Lambda-M_N)^2]^{1/2}=462$ MeV, and P_x and P_σ are the space-exchange and spin-exchange operators, respectively.[16] For the triplet state one has $-P_xP_\sigma=-1$ and $\vec{\sigma}_N\cdot\vec{\sigma}_\Lambda=1$. The strength of $V^{(K)}$ is determined by $g_{\Lambda N\overline{K}}^2$, which is the square of the (pseudoscalar) $N\Lambda\overline{K}$ coupling constant.

Since we cannot be sure of the exact strengths (or shapes) of the potentials, we proceed in analogy to what we did for a mixture of central and tensor Yukawa potentials. Thus we do not fix g_σ or $g_{N\Lambda\overline{K}}$, but determine these such that $V^{(\sigma)}+V^{(K)}$ always gives the same triplet scattering length $a_t=-2$ F. In this way $g_{N\Lambda\overline{K}}$ is determined as a function of g_σ, the value of which then characterizes the interaction; in particular, for $g_{N\Lambda\overline{K}}=0$, the whole triplet potential is due to the central potential $V^{(\sigma)}$ plus the hard core. The central part of the triplet potential (for $r>c$) is $V_t=V^{(\sigma)}+V_t^{(K)}$, where $V_t^{(K)}$ is the triplet central part of $V^{(K)}$. The tensor part is entirely due to $V^{(K)}$. The OBE potentials are shown in Table IV for the two assumptions $V_d=0$ and V_t-2V_T for $r>c$. The self-consistent well depths D_s are also shown. These values of D_s are for the singlet potential equal to V_t in the central-force limit $g_{N\Lambda\overline{K}}=0$.

Some *p*-wave results for an OBE potential with $g_{N\Lambda\overline{K}}^2=23.9$ are given in Sec. 9, in particular, in Table VIII.

Dispersion-theory analyses of $\overline{K}N$ scattering give widely differing values of $g_{N\Lambda\overline{K}}$ (to a large extent because of the differing detailed assumptions made to extrapolate into the unphysical region). The values obtained from the existing analyses, recently compiled by Levi-Setti,[17] group into two regions. The lower group of values are $g_{N\Lambda\overline{K}}^2\approx 4-7$, and the higher ones about 11–15. The larger values together with the associated values obtained for $g_{N\Sigma\overline{K}}$ – and also with use of $g_{NN\pi}$ – are consistent

TABLE IV. Results for OBE potentials which have a hard core of radius $c=0.43$ F and which are mixtures of σ-meson and kaon exchange potentials: $V^{(\sigma)}+V^{(K)}$ [see Eqs. (2.3) and (2.4)]. The mixtures are determined so as to all give a scattering length $a=-2$ F; this determines $g_{N\Lambda\overline{K}}^2$ and r_0 as functions of g_σ^2. The values of V_d are for $r>c$, with $V_d=\infty$ for $r\leq c$. The D_s are the self-consistent *s*-state well depths for the same nuclear-matter parameters as in Table III, and assuming the singlet potential equal to the triplet one for $g_{N\Lambda\overline{K}}=0$.

g_σ^2	$g_{N\Lambda\overline{K}}^2$	V_d	r_0 (F)	D_s (MeV)
2.62	0.0	V_t-2V_T	3.60	47.8
2.16	16.0	V_t-2V_T	3.40	46.1
1.79	23.9	V_t-2V_T	3.20	44.3
1.24	33.4	V_t-2V_T	2.87	41.8
2.18	16.0	0	3.41	46.4
1.78	23.9	0	3.18	44.5
1.14	33.4	0	2.76	40.8

with $SU(3)$ for an F-D mixing ratio $\alpha \approx 0.4$, where $\alpha = F/(F+D)$. These larger values of $g_{N\Lambda\overline{K}}$ correspond to a moderately strong ΛN tensor force. Thus with $g_{\Lambda N\overline{K}}{}^2 = 16$, one obtains $a_t = -1.36$ F if one uses only V_t, compared with $a_t = -2$ F for the full triplet potential $V_t + V_T^{(K)}$.

Table III shows the purely central potentials which have the same hard core as the OBE potentials, but which have attractive Yukawa parts of varying range and which all give $a = -2$ F. Similarly as for the Yukawa potentials, these results allow us to find a central hard-core potential $V_s^{(C)}$ which has the same scattering length and effective range as any of the OBE mixtures we consider. Also shown are the corresponding self-consistent s-state well depths $D_s^{(C)}$ – again for equal singlet and triplet potentials.

The η meson ($M_\eta = 580$ MeV) is only slightly heavier than the kaon. Thus for the long-range part of the ΛN tensor force one should also consider the contribution due to η exchange. The η meson has $J^P = 0^-$, $I = 0$, and therefore it gives rise to a potential which is similar to that of Eq. (2.4) except that $\overline{M}_K$ is replaced by M_η and $g_{N\Lambda\overline{K}}{}^2(-P_x P_\sigma)$ is replaced by $g_{NN\eta} g_{\Lambda\Lambda\eta}$. Because M_η and $\overline{M}_K$ are not very different, one may obtain a rough representation of the effect of η exchange on the ΛN tensor force by approximating $M_\eta{}^3 h(\mu_\eta r)$ by $\overline{M}_K{}^3 h(\mu_K r)$. One may then still use Eq. (2.4) for the tensor force in our OBE model, but with $g_{N\Lambda\overline{K}}$ replaced by $g_{N\Lambda\overline{K}}{}^2 - g_{NN\eta} g_{\Lambda\Lambda\eta}$ (in the triplet state). If one further assumes the $SU(3)$ relations between the coupling constants, then $g_{N\Lambda\overline{K}}{}^2$ in Eq. (2.4) is replaced by $C(\alpha) g_{N\Lambda\overline{K}}{}^2$, where $C(\alpha) = 1 + 2(1-\alpha)(4\alpha - 1)(1+2\alpha)^{-2}$ and where α is the F-D mixing ratio. One has $C = -1$, 0.25, 0.84, 1.22, 1.23, 1.13, and 1 for $\alpha = 0$, 0.1, 0.2, 0.4, 0.6, 0.8, and 1, respectively. Thus, for most values of α, the coefficient C is quite close to 1 and, furthermore, its largest value is only slightly greater than 1; the contribution from the η meson is thus considerably less than that from the kaon for most values of α. For very small α, the value of C becomes negative (and the η contribution relatively very important), but even then never larger than 1 in magnitude.

3. PERTURBATION THEORY

Perturbation theory gives the Λ-particle binding energy as an expansion in powers of the ΛN potential strength. The convergence, and therefore usefulness, of such an expansion is dependent on the use of weak ΛN potentials – in particular potentials without strongly repulsive cores. In fact, when purely attractive central potentials of reasonable range are fitted to the binding energies of the s-shell hypernuclei, or even to the scattering data, their well-depth parameters are less than unity. For such central potentials one expects fairly rapid convergence.[18] For tensor forces there is no first-order contribution, and if the forces are relatively strong the corresponding convergence is less certain. We shall have some comments on this later (in particular in Sec. 7) but in this section we simply give the second-order results which in any case are instructive. In fact, no higher-order terms have so far been calculated.

We thus consider the first two terms of the perturbation expansion $D = D^{(1)} + D^{(2)} + \cdots$. The expressions for $D^{(1)}$ and $D^{(2)}$ are

$$D^{(1)} = -\sum_{n \le k_F} \langle \vec{n}\vec{\lambda} | V | \vec{n}\vec{\lambda} \rangle \,, \tag{3.1}$$

$$D^{(2)} = \sum_{n \le k_F} \sum_{\substack{\lambda' \\ n' > k_F}} \frac{|\langle \vec{n}'\vec{\lambda}' | V | \vec{n}\vec{\lambda} \rangle|^2}{E_{n'} + E_{\lambda'} - E_n - E_\lambda} \,. \tag{3.2}$$

The momentum $\vec{\lambda}$ is that of the Λ particle, and $\vec{n}$ is that of an occupied state in the nuclear Fermi sea. The latter is represented by a Fermi gas with Fermi momentum k_F, the density being $\rho = 2k_F{}^3/3\pi^2$. The state $|\vec{n}\vec{\lambda}\rangle$ is a product of plane-wave states for the Λ and the nucleon. For the ground state, $\lambda = 0$. For both $D^{(1)}$ and $D^{(2)}$, there is a sum over all occupied nuclear states $n \le k_F$. (Spin and isospin summations are not explicitly indicated, and V denotes the total spin-dependent ΛN potential.)

For an ordinary central potential (i.e., one which is local and the same in all angular momentum states), the first-order term is[19]

$$D^{(1)} = D_C^{(1)} = \rho \overline{U} \,, \quad \overline{U} = \tfrac{1}{4}(U_s + 3U_t) \,, \tag{3.3}$$

where $\overline{U}$ is the spin-average volume integral of the central part of the interaction and U_s and U_t are the singlet and triplet volume integrals corresponding to V_s and V_t, respectively. There is no first-order contribution due to the tensor force.

The total second-order contribution is $D^{(2)} = D_C^{(2)} + D_T^{(2)}$, where $D_C^{(2)}$ and $D_T^{(2)}$ are the second-order central- and tensor-force contributions, respectively. The momenta $\vec{\lambda}'$, $\vec{n}'$ are those for the intermediate state $|\vec{n}'\vec{\lambda}'\rangle$. The interaction conserves momentum; thus $\vec{n} + \vec{\lambda} = \vec{n}' + \vec{\lambda}'$, and the momentum transfer to the Λ particle in the intermediate state is $\vec{q} = \vec{\lambda}' - \vec{\lambda} = \vec{n} - \vec{n}'$. Because of the exclusion principle, one has $n' > k_F$ for the intermediate-state sum; of course, no such restriction is placed on λ'.

The energy denominator in Eq. (3.2) is the difference between the intermediate- and initial-state energies (or unoccupied and occupied states). The relevant single-particle energies are $E_k = (k^2/2M) + U(k)$, where $U(k)$ is the appropriate single-particle potential. In the effective-mass approximation, this has a quadratic momentum dependence. We then consider the following effective-mass expressions in which the initial and intermediate energies are separated by a gap, namely,

$$E_{n'} = (n'^2/2\overline{M}_N) + \Delta_N, \quad E_n = n^2/2M_N^*,$$
$$E_{\lambda'} = (\lambda'^2/2\overline{M}_\Lambda) + \Delta_\Lambda, \quad E_\lambda = \lambda^2/2M_\Lambda^* . \tag{3.4}$$

In general, different effective masses may be assumed for the initial and intermediate states; $\Delta = \Delta_N + \Delta_\Lambda$ is the total gap. These expressions are usually more general than is required by perturbation theory, and in their general form are appropriate for the reaction-matrix calculations for potentials with a strongly repulsive core, which we consider in Secs. 5–8. In perturbation theory the single-particle potentials are given by the appropriate first-order expressions. This implies that for not too highly excited intermediate states (in the vicinity of k_F) one should use $\overline{M}_N \approx M_N^*$, $\overline{M}_\Lambda \approx M_\Lambda^*$, and $\Delta_N = \Delta_\Lambda = 0$, and thus single-particle energies which are continuous between the initial and intermediate states.

If one replaces the sums in Eq. (3.2) by integrals in the usual way and makes use of Eq. (3.4), one has (for $\lambda = 0$)

$$D^{(2)} = \frac{\langle U^2\rangle}{(2\pi)^6}\frac{2\overline{\mu}}{\hbar^2}\int_{n\le k_F} d^3n \int_{|\vec{n}-\vec{q}|>k_F} d^3q\, |V(q)|^2 \Big/ \left[\vec{q}\cdot\left(\vec{q}-\frac{2\overline{\mu}}{\overline{M}_N}\vec{n}\right)+\frac{2\overline{\mu}}{\hbar^2}\Delta+\left(\frac{\overline{\mu}}{\overline{M}_N}-\frac{\overline{\mu}}{M_N^*}\right)n^2\right], \tag{3.5}$$

where $\overline{\mu} = \overline{M}_N\overline{M}_\Lambda/(\overline{M}_N+\overline{M}_\Lambda)$ is the reduced mass appropriate for the *intermediate states*, and where for the central-force contribution $D_C^{(2)}$ one has

$$\langle U^2\rangle = U_s^{\,2} + 3U_t^{\,2}, \tag{3.6}$$

$$V(q) = V_C(q) = 4\pi\int_0^\infty j_0(qr)\mathcal{V}_C(r)r^2dr. \tag{3.7}$$

For the tensor-force contribution $D_T^{(2)}$, one has

$$\langle U^2\rangle = 3\times 8U_T^{\,2}, \tag{3.8}$$

$$V(q) = V_T(q) = 4\pi\int_0^\infty j_2(qr)\mathcal{V}_T(r)r^2dr. \tag{3.9}$$

Here $\mathcal{V}_C$ is the normalized central potential (the same shape is assumed for the singlet and triplet central potentials) and U_T and $\mathcal{V}_T$ are the volume integral and normalized potential, respectively, of $V_T(r)$.

In Eq. (3.8), the factors 3 and 8 come from the triplet-state statistical weight and the square of the tensor-force matrix element, respectively. The functions $j_0(x)$ and $j_2(x)$ are the zeroth-order and second-order spherical Bessel functions, respectively, and $V_C(q)$ and $V_T(q)$ are the momentum transforms appropriate to $\mathcal{V}_C$ and $\mathcal{V}_T$, respectively.

Using the methods of Euler,[20] one finally obtains

$$D^{(2)}(\lambda=0) = \langle U^2\rangle I^{(2)},$$
$$I^{(2)} = \frac{\overline{\mu}}{\hbar^2}\frac{k_F^{\,4}}{4\pi^4}\int_0^\infty dt\, V^2(q)I_2(t), \tag{3.10}$$

where $t = q/k_F$. The function $I_2(t)$ also depends on Δ and on certain mass factors that arise from the kinematics associated with unequal masses in the intermediate state. $I_2(t)$ may also be obtained analytically in closed form and is given explicitly in the Appendix. The correction due to the difference between the Λ particle and nucleon masses in the intermediate states turns out to be quite small. Thus, even when $\overline{M}_N = \frac{1}{2}M_N$, the results for $I^{(2)}/\overline{\mu}$ are almost the same as for $\overline{M}_N = M_N$. Thus, to a very good approximation, $D^{(2)}$ simply scales with the appropriate reduced mass.

It is instructive to consider the average excitation energy $\overline{E}$ which represents the energy denominators in Eq. (3.2). If one replaces these by $\overline{E}$, use of closure gives

$$\overline{E} = \frac{\rho}{4I^{(2)}}\int_0^\infty \mathcal{V}^2(r)r^2dr. \tag{3.11}$$

Thus $D^{(2)}$ may be obtained by the use, to *first order*, of an effective local potential

$$V_{\rm eff}(r) = -(\langle U^2\rangle/\overline{E})\mathcal{V}^2(r). \tag{3.12}$$

In particular, the effective central (and *local*) potential, which gives $D_T^{(2)}$ if *used* (as a triplet central potential) *to first order* is

$$V_{\rm eff}^{(T)}(r) = -8V_T^{\,2}(r)/\overline{E}_T, \tag{3.13}$$

where $\overline{E}_T$ is again given by Eq. (3.11) together with the expression for $I^{(2)}$ appropriate to tensor forces. This is also the effective potential used by Law, Gunye, and Bhaduri[10] in their study of the effect of a ΛN tensor force on ${}_\Lambda He^5$. It should be noted that the range of $V_{\rm eff}^{(T)}(r)$ is only about half that of $V_T(r)$.

We discuss results for the Yukawa potentials described in Sec. 2. Figure 1 shows $\mathcal{V}^2(q)I_2(t)$ for two Yukawa ranges and for both central and tensor potentials. As one expects, the shorter-range potentials involve higher-momentum components – the average momentum involved for central potentials is of the order of magnitude of the inverse range μ. On the other hand, the function $I_2(t)$ starts from zero and rapidly levels out for $q > 2k_F$; this behavior corresponds to the decreasing effect of the exclusion principle for large q. Thus one expects the

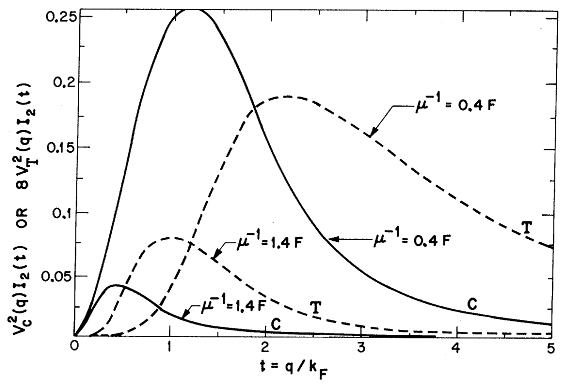

FIG. 1. The products $\mathcal{V}_c{}^2(q)I_2(t)$ and $8\mathcal{V}_T{}^2(q)I_2(t)$, relevant for the second-order energy $D^{(2)}$, as a function of q/k_F. The results are for Yukawa potentials of the indicated ranges μ^{-1}. The full and dashed lines are for central and tensor potentials, respectively. The values are for $k_F=1.4$ F^{-1}, $M_N^*=\overline{M}_N=M_N$, $M_\Lambda^*=\overline{M}_\Lambda=M_\Lambda$, and $\Delta=0$.

exclusion principle to be less important for shorter ranges and, conversely, to be more effective in suppressing the relatively low-lying intermediate states which predominate for longer-range potentials. Also $\mathcal{V}^2(q)I_2(t)$, and hence $I^{(2)}$, are larger for shorter ranges. For a given value of $\overline{U}$, and hence of $D^{(1)}$, the second-order contribution $D^{(2)}$ will therefore be larger for shorter ranges.

These features are illustrated by Figs. 2 and 3. These show the average excitation energy $\overline{E}$ as a function of ρ and of Δ, respectively. If one wants to compare the values of $\overline{E}$ with the Fermi energy E_F, then one should divide the former by about 2, corresponding to the ratio of the actual to the reduced masses in the intermediate states. Equations (3.11) and (3.12) show that, apart from the trivial proportionality to ρ, the effect of the nuclear medium on $D^{(2)}$ enters only through $\overline{E}$. Thus if $\overline{E}$ were independent of the nuclear medium, then $D^{(2)}$ would be given by use of V_{eff} to first order with V_{eff} independent of the nuclear medium, and $D^{(2)}$ would be proportional to ρ just as $D^{(1)}$ is. Indeed, as expected, $\overline{E}$ is larger for the shorter ranges and for these, furthermore, depends less sensitively on ρ or Δ than for the longer ranges.

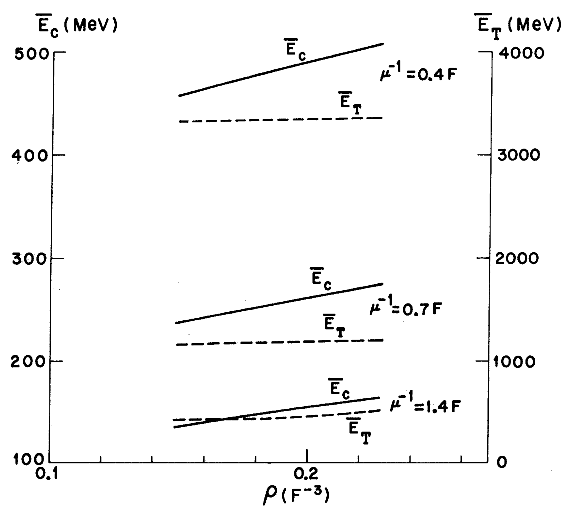

FIG. 2. The average closure energies $\overline{E}_C$ and $\overline{E}_T$, appropriate to central and tensor Yukawa potentials, respectively, as a function of the density ρ for $M_N^*=\overline{M}_N=M_N$, $M_\Lambda^*=\overline{M}_\Lambda=M_\Lambda$, and $\Delta=0$. The full and dashed lines are for central and tensor potentials, respectively.

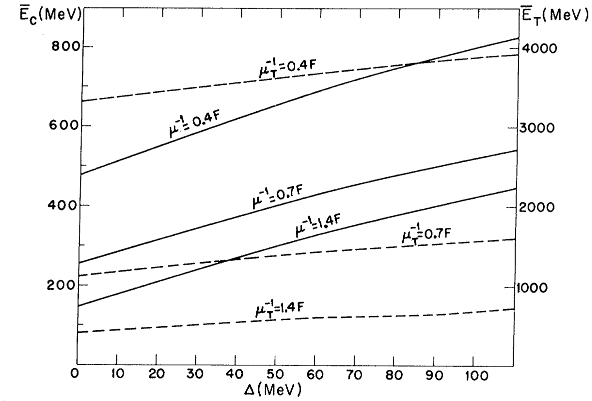

FIG. 3. Same as Fig. 2 except that the energies $\overline{E}_C$ and $\overline{E}_T$ are shown as a function of the gap Δ for $k_F=1.4$ F^{-1}.

It is most important to note that for $D_T^{(2)}$ the relevant transform $V_T(q)$ involves the second-order (spherical) Bessel function $j_2(qr)$ instead of $j_0(qr)$ as for central forces. The former peaks at a finite value of r – in contrast to $j_0(qr)$, which has its maximum value at $r=0$. As Fig. 1 shows, this implies that, for the same shape, $V_T(q)$ involves much higher-momentum components than does $V_C(q)$. Hence, one expects $D_T^{(2)}$ to be much less sensitive than $D_C^{(2)}$ to the properties of the nuclear medium. Thus the values of $\overline{E}_T$ are much larger (by a factor of about 5) than the corresponding values of $\overline{E}_C$, and the dependence of $\overline{E}_T$ on Δ or ρ is much less than that of $\overline{E}_C$ (and the same is true of the dependence on the exclusion principle). Again, as for central forces, $\overline{E}_T$ is larger for shorter ranges and correspondingly less dependent on Δ or ρ (i.e., on the nuclear medium) for these ranges.

The increase of $\overline{E}_T$ with Δ or with ρ corresponds to a suppression of the effect of a tensor force in nuclear matter, since increasing Δ or ρ inhibits transitions to intermediate states. Suppression effects are more important for longer ranges, as expected, because of the correspondingly lower-lying intermediate states. On the other hand, for short-range tensor forces the dominant momentum components are so high that, to a good approximation, $\overline{E}_T$ is independent of the nuclear medium and one can use an effective potential $V_{\text{eff}}(r)$ that is independent of this medium.

Since also for the low-energy s-wave scattering V_T enters at least in second order, one may conjecture that – for short ranges – the effect of a tensor force is similar both for scattering and for nuclear matter, and that one may use an effective central potential which may then also be used (to a good approximation) for a Λ particle in nuclear matter. Below and in Secs. 4, 6, and 7, we shall clarify the relation between the effect of V_T on a free ΛN pair and on one in nuclear matter.

In this connection it is interesting to consider perturbation-theory results for some of the mixtures of central and tensor Yukawa potentials discussed in Sec. 2. Thus Fig. 4 shows D as a function of V_t for several ranges μ_T^{-1} and for $a_t = -0.75$ and -2 F. The first-order contribution $D^{(1)}$ varies linearly with V_t, since $\overline{U} = \frac{1}{4}(U_s + 3U_t)$. Thus, when the triplet interaction is all tensor, $D^{(1)}$ is only one quarter of its value for $V_T = 0$. The second-order central contribution $D_C^{(2)}$ is small, consistent with other results.[1,18] The striking constancy of D for the shorter ranges shows how well $D_T^{(2)}$ compensates very large changes in $D^{(1)}$. Consistent with our earlier discussion, this compensation is considerably less for the long one-pion range ($\mu_T^{-1} = 1.4$ F) for which there is an appreciable decrease of D for large values of V_T (which is evidence for important suppression effects). Further and related results for the Yukawa potentials will be discussed in Secs. 6 and 7.

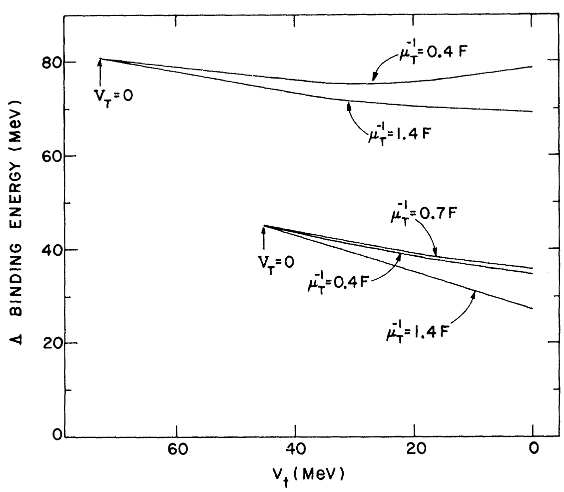

FIG. 4. The total (singlet-plus-triplet) well depths calculated by first- plus second-order perturbation theory for mixtures of central and tensor Yukawa potentials, as a function of the strength V_t of the triplet central potential. The indicated ranges μ_T^{-1} are for the tensor part. For all cases the range of the central potential is $\mu^{-1} = 0.7$ F; the strength of the singlet potential is kept fixed and equal to the value of V_t for $V_T = 0$, i.e., for no tensor force. The upper and lower sets of curves are for mixtures which give $a_t = -2$ and -0.75 F, respectively. The results are for $k_F = 1.4$ F^{-1}, $M_N^* = \overline{M}_N = M_N$, $M_\Lambda^* = \overline{M}_\Lambda = M_\Lambda$, and $\Delta = 0$.

The results just discussed assumed an effective nucleon mass $\overline{M}_N = M_N$ for the intermediate states. Since $D^{(2)}$ effectively scales with the reduced mass $\overline{\mu} = M_\Lambda \overline{M}_N/(M_\Lambda + \overline{M}_N)$, the compensation between $D_T^{(2)}$ and $D^{(1)}$ depends on the assumption that $\overline{M}_N = M_N$. For potentials that involve excitations close to k_F, one would expect $\overline{M}_N \approx M_N^* \approx 0.65 M_N$, which would reduce $D^{(2)}$ by a factor of about 0.775. However, for tensor potentials the relevant excitations are much higher than k_F and, just as for hard-core potentials, the relevant value of $\overline{M}_N$ is probably close to M_N for such highly excited states. Some further discussion of this is given in Sec. 10.

4. EFFECTIVE LOCAL POTENTIALS AND AVERAGE CLOSURE ENERGIES FOR BINDING AND s-WAVE SCATTERING

It is of interest to comment on the values of $\overline{E}_T^{(\mathrm{SCAT})}$ that reproduce the (s-wave) scattering and to compare these with the values of $\overline{E}_T$ discussed in Sec. 3. These considerations are also closely related to those of Secs. 6 and 7.

We determine the values of $\overline{E}_T^{(\mathrm{SCAT})}$ by requiring that for a given mixture of central and tensor forces, the same scattering length is obtained with the corresponding effective purely central local potential $V_t + V_{\mathrm{eff}}^{(T)}(r)$, where $V_{\mathrm{eff}}^{(T)}(r) = -8V_T^2(r)/\overline{E}_T^{(\mathrm{SCAT})}$. Results for $\overline{E}_T^{(\mathrm{SCAT})}$ are given in Table I for the Yukawa mixtures.

The energy $\overline{E}_T^{(\mathrm{SCAT})}$ increases with V_T so that $V_{\mathrm{eff}}^{(T)}(r)$ increases less rapidly than V_T^2; furthermore, this increase of $\overline{E}_T^{(\mathrm{SCAT})}$ is relatively greater for the shorter range. This may be understood as follows. The values of $\overline{E}_T^{(\mathrm{SCAT})}$ have been chosen so that use of the corresponding local effective potential $V_{\mathrm{eff}}^{(T)}(r)$ reproduces the exact s-wave results obtained with the coupled-channel equations. However, the solution of these, for $V_d = 0$, is exactly equivalent to using the s-wave nonlocal *central* potential $V_{\mathrm{eff}}^{(\mathrm{SCAT})}(r,r';k)$ given by Eqs. (6.3) and (6.4). Because of the presence of the Green's function $G_2(r,r';k)$ in Eq. (6.3), the range of this nonlocal potential does not decrease as rapidly with the range of $V_T(r)$ as does the range of the local effective potential. Since higher-order effects are more important for shorter ranges, the higher-order effects of $V_{\mathrm{eff}}^{(T)}(r)$ are larger for shorter ranges of $V_T(r)$ and, more especially, are relatively larger than the higher-order effects due to the longer-range nonlocal potential which gives the exact result. Thus, to reproduce this exact result, the value of $\overline{E}_T^{(\mathrm{SCAT})}$ must increase with V_T and, furthermore, must increase relatively more for the shorter ranges.

The perturbation-theory results for the energies $\overline{E}_T$ are obtained from the second-order energy $D_T^{(2)}$

and thus correspond to the limit of small V_T and, furthermore, also to $V_d = 0$. Thus, for a significant comparison of $\overline{E}_T^{(\mathrm{SCAT})}$ with $\overline{E}_T$, we should use the values of $\overline{E}_T^{(\mathrm{SCAT})}$ for $V_T \to 0$ and also for $V_d = 0$.

The necessity for $V_d = 0$ may be seen from Eq. (7.1), which gives the correction to the nonlocal effective potential arising from V_d. In particular, with $V_d = V_t - 2V_T$, this additional term gives a correction which is proportional to $V_T^2 V_t$ for $V_T \to 0$. Thus, in general, with $V_d \neq 0$ there will be additional terms proportional to V_T^2 which will affect the value of $\overline{E}_T^{(\mathrm{SCAT})}$ that is obtained in the limit $V_T = 0$.

Quite generally it is seen that the various higher-order effects can appreciably change the values of $\overline{E}_T^{(\mathrm{SCAT})}$. This indicates that also for nuclear matter, one should be quite careful in using the perturbation-theory results for $\overline{E}_T$ to represent the effect of a tensor force through use of an effective local potential.

For a still more meaningful comparison between $\overline{E}_T$ and $\overline{E}_T^{(\mathrm{SCAT})}$ one should probably consider the energies $\overline{E}_{T(s)}$ that correspond to the second-order energies $D_{T(s)}^{(2)}$ appropriate to exclusively s-state interactions, since $\overline{E}_T^{(\mathrm{SCAT})}$ is obtained entirely from s-wave scattering. The energy $D_{T(s)}^{(2)} = -\langle V_{\mathrm{eff}}^{(\mathrm{NM})}\rangle$ is just the expectation value of the (s-state) effective nonlocal potential $V_{\mathrm{eff}}^{(\mathrm{NM})}(r,r';k)$ appropriate for nuclear matter and is discussed in Sec. 6. One again obtains $\overline{E}_{T(s)}$ from $D_{T(s)}^{(2)}$ by the use of $V_{\mathrm{eff}}^{(T)}(r)$ to first order, except that now the expectation value $\langle V_T^2\rangle_s$ appropriate to a pure s-state interaction is used instead of the value $\langle V_T^2\rangle$ appropriate to equal interactions for all L – which was used to calculate $\overline{E}_T$.

The relevant results are given in Table V. The ratio $\langle V_T^2\rangle_s / \langle V_T^2\rangle$ is close to unity even for the longer range $\mu_T^{-1} = 1.4$ F, because even for this range $V_T^2(r)$ is a quite short-range potential for which the interactions for $L > 0$ are relatively unimportant. The ratio, in this case, is only slightly greater than the corresponding ratio $D_{T(s)}^{(2)}/D_T^{(2)}$, and thus $\overline{E}_{T(s)}$ is only slightly larger than $\overline{E}_T$. For $\mu_T^{-1} = 0.4$ F the effect of the interactions for $L > 0$ is seen to be quite small.

Within the context of effective local potentials, a measure of the suppression for a given $V_T(r)$ is expected to be the ratio $\overline{E}_T^{(\mathrm{SCAT})}/\overline{E}_{T(s)}$, since this measures the ratio of the strengths of $V_{\mathrm{eff}}^{(T)}(r)$ for nuclear matter and for scattering. Clearly, this ratio becomes smaller for larger Δ and ρ. In particular, it is smaller for longer ranges, which corresponds to stronger suppression for these. A better measure of the suppression is probably the ratio $-D_{T(s)}^{(2)}/\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$, where $\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$ is the (s-state) expectation value of the nonlocal potential that reproduces the s-wave scattering. In fact this ratio, which is also given in Table V and which is further discussed in Sec. 7, shows the same qualitative features as $\overline{E}_T^{(\mathrm{SCAT})}/\overline{E}_{T(s)}$ but is numerically somewhat different.

5. REACTION-MATRIX FORMALISM FOR ΛN POTENTIALS WHICH INCLUDE A TENSOR COMPONENT

A perturbation treatment is, of course, no longer possible for forces with strongly repulsive cores (except insofar as one uses the long-range Moskowski-Scott separated potential). Furthermore, even without strongly repulsive cores, one would like to know something about the importance of higher-order effects, i.e., about the goodness of the second-order result $D_T^{(2)}$, especially for large values of V_T. One would also like to clarify the relation between the effects of a ΛN tensor force for scattering and for a Λ particle in nuclear matter.

To deal with these questions, we turn to the reaction-matrix (or g-matrix) approximation which gives the leading term in an expansion in powers of the density. In Sec. 6 we shall also consider another, more novel, approach which involves effective nonlocal central potentials and which is, in fact, an approximation to the complete g-matrix solution. This approach also forms a convenient bridge between the perturbation and the g-matrix results and is very helpful in relating the scattering to the nuclear-matter case.

As in Ref. 1 we obtain the g matrix by use of the Kallio-Day version[21] of the reference-spectrum method. We use the free kinetic energies for the unoccupied states[5] which corresponds to the use of

TABLE V. Results for closure energies and related quantities for pure Yukawa tensor potentials of range μ_T^{-1}. The values of $\overline{E}_T^{(\mathrm{SCAT})}$ are for $V_d = 0$ and $V_T \to 0$. The nuclear-matter results are for $k_F = 1.4$ F^{-1} and $M_N^*/M_N = 1$.

μ_T^{-1} (F)	Δ (MeV)	$\overline{E}_T^{(\mathrm{SCAT})}$ (MeV)	$\langle V_T^2\rangle_s / \langle V_T^2\rangle$	$\overline{E}_T$ (MeV)	$\overline{E}_{T(s)}$ (MeV)	$D_{T(s)}^{2}/D_T^{(2)}$	$\overline{E}_T^{(\mathrm{SCAT})}/\overline{E}_{T(s)}$	$-D_{T(s)}^{(2)}/\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$
1.4	60	415	0.90	627	705	0.80	0.59	0.475
	120			782	857	0.82	0.48	0.39
0.4	60	3900	0.945	3688	3636	0.96	1.07	0.80
	120			3973	3920	0.96	0.98	0.74
0.2								0.9

Eqs. (3.4) with the free masses for the unoccupied states, i.e., we use $\overline{M}_N = M_N$, $\overline{M}_\Lambda = M_\Lambda$. This method involves the use of integrodifferential equations rather than of integral equations as in the Brueckner-Gammel procedure.[22] The latter is used by Dabrowski and Hassan.[4,11] With the same assumptions about the single-particle spectra, the two methods are effectively equivalent and indeed give the same numerical results.[2,4]

The procedure we use for tensor forces is then a straightforward generalization of that for central ΛN forces which was described in Ref. 2. Instead of the single integrodifferential equation required for central forces, one now has for the triplet case ($S=1$) a pair of coupled equations for the partial-wave correlated wave functions. These equations, which correspond to the use of the angle-average Pauli operator $\overline{Q}$ and to an initial relative ΛN momentum k_0 are

$$\left[\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} - \gamma^2\right] u_{LJ}^{(L'')}(r,k_0) = -(\gamma^2 + k_0^2) r j_{L''}(k_0 r)\delta_{LL''} + \overline{Q}\left[V_t u_{LJ}^{(L'')} + V_T \sum_{L'} s_{LJ,L'} u_{L'J}^{(L'')}\right] . \tag{5.1}$$

For $\lambda = 0$, $\vec{k}_0$ is completely determined by $\vec{n}$ through $\vec{n} = (M_N/\mu)\vec{k}_0$.

For a given total angular momentum J, the orbital angular momentum quantum number L can have the two values $L = J \pm 1$, so there is a pair of coupled equations. The quantum number L'' labels the solutions and refers to the (dominant) partial wave that corresponds to the unperturbed solution (i.e., to the appropriate partial-wave component of a plane wave). Thus the fact that the solution must asymptotically approach the unperturbed one is built into the equations through the presence of the inhomogeneity in Eq. (5.1). For a given J, there are then two solutions corresponding to $L'' = J \pm 1$. The coefficients $s_{LJ,L'}$ are the usual matrix elements of the tensor operator S_{12}. The quantity γ^2 is discussed below.

For the s-wave dominant solution for the coupled sd case, which we emphasize here, Eqs. (5.1) are explicitly

$$\left(\frac{d^2}{dr^2} - \gamma^2\right) u = -(\gamma^2 + k_0^2) r j_0(k_0 r) + \overline{Q}(V_t u + \sqrt{8}\, V_T w) , \tag{5.2a}$$

$$\left(\frac{d^2}{dr^2} - \frac{6}{r^2} - \gamma^2\right) w = \overline{Q}(V_d w + \sqrt{8}\, V_T u) . \tag{5.2b}$$

These are to be compared with Eqs. (2.1) for the scattering case. Of course for the singlet case ($S=0$), one has $J = L$ and thus only one equation – just as for the case of a purely central force. The appropriate considerations for the p wave are given in Sec. 9.

In all the above equations $\overline{Q}$ is the angle-average approximation to the complete Pauli operator Q. For this, one explicitly has

$$\overline{Q}[\Phi_L] \equiv \int_0^\infty dr' \left[\frac{2}{\pi}\int_0^\infty dk\, k^2 r j_L(kr)\overline{Q}(k,P) r' j_L(kr')\right]\Phi_L(r') . \tag{5.3}$$

[For Eqs. (5.2a) and (5.2b), one has $L=0$ and 2, respectively.] $\vec{P} = \vec{n} + \vec{\lambda}$ is the total c.m. momentum (conserved in the interaction), and $\vec{k}$ is the intermediate relative ΛN momentum. The definitions of Q and $\overline{Q}$ are given in Ref. 2. The angle-average Pauli operator $\overline{Q}(k,P)$ depends only on the magnitudes k and P, whereas the exact operator $Q(\vec{k},\vec{P})$ depends on the angle between $\vec{k}$ and $\vec{P}$. The effect of approximating Q by $\overline{Q}$ is to decouple the different partial waves (except of course for the coupling due to the tensor force), and one is then left with only a single pair of coupled equations to solve. In Ref. 2 the angle-average approximation was shown to be excellent for central potentials, and we shall assume this to be true also for tensor potentials; in particular, the use of $\overline{Q}$ is exact for interactions only in the s state.

The correlated wave function must asymptotically approach the unperturbed solutions, i.e., $u_{LJ}^{(L'')}(r) \sim r j_{L''}(k_0 r)\delta_{LL''}$. Thus the correlated wave functions must "heal," and the rate of healing is largely determined by the decay distance γ^{-1}. With a hard core of radius c, the correlated wave functions must also satisfy the boundary condition $u_{LJ}^{(L'')}(r) = 0$ for $r \le c$. The procedure used for solving the coupled integrodifferential equations is a straightforward extension of that used for central forces and is described in Ref. 21. The solution is obtained by iterating on the integral term (corresponding to the contribution $\overline{Q} - 1$), and the iteration process converges rapidly.

The partial-wave g matrix element is given by

$$g_{LJ}^{(L'')}(k,k_0) = \int r\,dr\, j_L(kr)\left[V_t u_{LJ}^{(L'')}(r,k_0) + V_T \sum_{L'} s_{LJ,L'} u_{L'J}^{(L'')}(r,k_0)\right] . \tag{5.4}$$

The total diagonal g matrix element, which also includes the singlet contribution, is then

$$\langle \vec{k}_0 | g | \vec{k}_0 \rangle = 4\pi \sum_{J,L} (2J+1) g_{LJ}^{(L)}(k_0,k_0) . \tag{5.5}$$

The well depth is finally given by

$$D = -\frac{\hbar^2}{2\mu}\frac{1}{(2\pi)^3}\int_0^{k_F} d^3n\, \langle \vec{k}_0 | g | \vec{k}_0 \rangle . \tag{5.6}$$

For $\lambda = 0$, the quantity γ^2 in Eqs. (5.1) and (5.2) is given by

$$\gamma^2 = \frac{2\mu}{\hbar^2}\Delta - \frac{M_N}{M_N^*}\left(1 + \frac{M_N - M_N^*}{M_\Lambda}\right)k_0^{\,2} \ . \tag{5.7}$$

(Recall that $\Delta = \Delta_N + \Delta_\Lambda$.)

Since D depends on Δ_Λ (through the effect of γ^2) and since Δ_Λ is identified with D, one has a self-consistency condition for the determination of D, namely,

$$D = D(\Delta_\Lambda) = \Delta_\Lambda \ . \tag{5.8}$$

For a given k_F, the effect of the nuclear medium enters via γ^2 through the appropriate values of M_N^* (the nucleon effective mass for the occupied states) and through Δ_N. These values are taken from the results of nuclear-matter calculations; in particular, we have used the results of Bhargava and Sprung.[23]

The fact that the free kinetic energies have been assumed for the energies of the unoccupied states enters Eqs. (5.1) and (5.2) through the fact that the potentials are expressed in units of $\hbar^2/2\mu$ just as for the free-scattering case of Eq. (2.1). If the masses $\overline{M}_N$ and $\overline{M}_\Lambda$ for the unoccupied states were different from the free masses, then all the potentials in Eqs. (5.1) and (5.2) should be multiplied by $\overline{\mu}/\mu$, where $\overline{\mu}$ is the reduced mass for the unoccupied states and where the potentials are again expressed in units of $\hbar^2/2\mu$. Equation (5.7) would also have to be appropriately modified. The *changes* in the wave functions due to the presence of the potentials are proportional to the strengths of these potentials. In particular, all of w – which determines the effect of the tensor potential – is proportional to the potential strength, in particular, to V_T. Thus the contribution to g, and hence to D, from V_T is expected to be approximately proportional to $\overline{\mu}$, just as for the second-order energy which was discussed in Sec. 3. (The first-order energy is independent of $\overline{\mu}$, since it involves just the unperturbed wave functions – which are independent of the potentials.) Thus if $\overline{\mu} \neq \mu$, the results for the tensor-force contribution to D can be expected to be obtainable, to a good approximation, from the results appropriate to μ by multiplying the latter by $\overline{\mu}/\mu$.

6. EFFECTIVE POTENTIALS, PERTURBATION THEORY, AND THE EQUIVALENCE OF ΛN TENSOR FORCES FOR s-WAVE SCATTERING AND FOR NUCLEAR MATTER

For simplicity of presentation and also because this is the only case we consider in detail in this connection, we discuss only effective s-state potentials for the coupled sd-wave case. The generalization to other partial waves is trivial. For both a free ΛN pair and one in nuclear matter, one may obtain the effect of the tensor force on the s-wave u in terms of an effective s-state potential. This is obtained by expressing w in terms of u by use of the equation for w and then substituting this expression into the equation for u. This may be done most simply for $V_d = 0$, i.e., for the case of no net d-state potential. (This is, in general, a quite good approximation because the d-state function is being driven by the dominant s-state function and also because of the shielding effect of the strong d-state centrifugal barrier, as discussed below.)

With $V_d = 0$, one may then use the Green's functions which correspond to the left-hand side of the d-state equations (2.2b) and (5.2b) and to the appropriate boundary conditions (i.e., one uses the appropriate noninteracting d-state propagators) to obtain w in terms of u. Substituting in the s-wave equations then gives

$$\left(\frac{d^2}{dr^2} + k^2\right)u(r) = V_t(r)u(r) + \int V_{\rm eff}^{(\rm SCAT)}(r,r';k)u(r')dr' \tag{6.1}$$

for scattering, and

$$\left(\frac{d^2}{dr^2} - \gamma^2\right)u(r) = -(\gamma^2 + k^2)rj_0(kr) + \overline{Q}[V_t(r)u(r) + \int V_{\rm eff}^{(\rm NM)}(r,r';k)u(r')dr'] \tag{6.2}$$

for nuclear matter. The integral operator $\overline{Q}$ is defined by Eq. (5.3). The effective potential $V_{\rm eff}(r,r';k)$ is in both cases proportional to $V_T^{\,2}$ and is nonlocal and energy dependent but central, being given by

$$V_{\rm eff}(r,r';k) = -8V_T(r)G_2(r,r';k)V_T(r') \ . \tag{6.3}$$

The Green's function for scattering is

$$G_2^{(\rm SCAT)}(r,r';k) = -krr'j_2(kr_<)n_2(kr_>) \ , \tag{6.4}$$

where j_2 and n_2 are the d-state spherical Bessel and Neumann functions, respectively, and where $r_>$ and $r_<$ are the greater and lesser of the distances r and r', respectively. For nuclear matter with neglect of the exclusion principle (i.e., with $Q \equiv 1$), the Green's function is

$$G_2^{(\rm NM)}(r,r';k) = -\gamma rr'j_2(i\gamma r_<)h_2^{(1)}(i\gamma r_>) \ , \tag{6.5}$$

where $h_2^{(1)}$ is the spherical Hankel function of the first kind and γ is given by Eq. (5.7). For nuclear matter with use of the angle-average exclusion-principle operator $\overline{Q}$, one has

$$G_2^{(\rm NM)}(r,r';k) = \frac{2}{\pi}\int_0^\infty dk'k'^2 rj_2(k'r)\frac{\overline{Q}(k',P)}{k'^2 + \gamma^2}r'j_2(k'r') \ . \tag{6.6}$$

This reduces to Eq. (6.5) for $\overline{Q} \equiv 1$.

As indicated, the Green's functions are not only nonlocal but also depend on the *initial* relative ΛN momentum k. For $G_2^{(\mathrm{NM})}$ this dependence arises through γ^2 which is a function of k.

If there is a hard core in the central part of the interaction, this will force the wave functions to zero at the hard-core radius, and the Green's functions must then be appropriately modified. This may be done by following the procedure of Brueckner and Gammel.[22]

The s-wave contribution $D_{T(s)}^{(2)}$ to the second-order tensor-force perturbation result is given by use of the nonlocal potential $V_{\mathrm{eff}}^{(\mathrm{NM})}(r,r';k)$ to *first* order. Thus

$$\begin{aligned} D_{T(s)}^{(2)} &\equiv -\langle V_{\mathrm{eff}}^{(\mathrm{NM})}(r,r';k)\rangle \\ &= -\sum_{n\le k_F} \langle \lambda n | V_{\mathrm{eff}}^{(\mathrm{NM})}(r,r';k) | \lambda n\rangle \\ &= -\frac{2\mu}{\hbar^2}\frac{6}{\pi}\int_0^{k_F} dn\, n^2 \int_0^\infty dr\, r j_0(kr) \\ &\quad \times \int_0^\infty dr' V_{\mathrm{eff}}^{(\mathrm{NM})}(r,r';k) r' j_0(kr') . \end{aligned} \tag{6.7}$$

That $D_{T(s)}^{(2)}$ is the s-wave contribution to $D_T^{(2)}$ may also be shown directly by making a partial-wave analysis of $D_T^{(2)}$. The ratio $D_{T(s)}^{(2)}/D_T^{(2)}$ becomes closer to 1 for shorter ranges, since then the partial waves with $L \geq 1$ become less important. This is illustrated by the appropriate results of Table V. Thus, especially for the shorter ranges, the effects of including the tensor force also in the higher partial waves are quite small *if there is no hard core.* In Sec. 9 we discuss g matrix results for the p-wave effects of a tensor force when a hard core is present.

It must be emphasized that, for $V_d = 0$, the use of V_{eff} in the s-wave equations (6.1) and (6.2) gives the *exact* s-wave solutions which would be obtained by solving the corresponding coupled s- and d-wave equations. With $V_d = 0$, the (s-wave) g matrix approximation thus corresponds to summing all the terms in an expansion in powers of V_{eff} and therefore in powers of V_T^2 (since V_{eff} is proportional to V_T^2). The leading contribution, proportional to V_T^2 in this expansion, is just the second-order contribution $D_{T(s)}^{(2)}$. It should also be noted that the range of $V_{\mathrm{eff}}(r,r';k)$ becomes less as the range of $V_T(r)$ becomes less. [However, this decrease in range is not as strong as for the local effective potential of Eq. (3.13) because of the presence of $G_2(r,r';k)$ for $V_{\mathrm{eff}}(r,r';k)$. This is also discussed in Sec. 7 and in the treatment of $\overline{E}_T$ in Sec. 4.]

The difference between the effects of a given $V_T(r)$ on the s-wave interaction of a free ΛN pair and of one in nuclear matter (for $V_d = 0$) is reflected entirely in the difference between $V_{\mathrm{eff}}^{(\mathrm{SCAT})}$ and $V_{\mathrm{eff}}^{(\mathrm{NM})}$ – and this difference between effective potentials arises entirely from the difference between the corresponding Green's functions. Thus a comparison of $G_2^{(\mathrm{SCAT})}(r,r';k)$ and $G_2^{(\mathrm{NM})}(r,r';k)$ will indicate the difference between the effects of a tensor force for a free ΛN pair and for one in nuclear matter.

A more quantitative indication of this difference is given by comparing $-D_{T(s)}^{(2)} = \langle V_{\mathrm{eff}}^{(\mathrm{NM})}\rangle$ with $\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$. The latter energy is just the second-order s-wave contribution obtained by using Eq. (6.7), except that $V_{\mathrm{eff}}^{(\mathrm{NM})}$ is replaced by the *effective potential* $V_{\mathrm{eff}}^{(\mathrm{SCAT})}(r,r';k)$ *appropriate for scattering.*

Figure 5 shows $G_2(r,r';k)$ as a function of r' for two values of r and for $k = 0.61$ F^{-1}. The discontinuity at $r = r'$ is of course characteristic of the Green's function. Also shown are two Yukawa potential shapes, one for the short (kaon) range $\mu^{-1} = 0.4$ F and the other for the long (pion) range $\mu^{-1} = 1.4$ F. The scattering and nuclear-matter Green's functions are very nearly the same for short distances. However, for larger distances $G_2^{(\mathrm{SCAT})}$ is *appreciably larger* than $G_2^{(\mathrm{NM})}$. This difference corresponds to suppression of the effect of a tensor force in nuclear matter, especially for longer ranges. The effect of the exclusion principle and of an increase in the gap is, as expected, to reduce $G_2^{(\mathrm{NM})}$ – appropriate to more suppression.

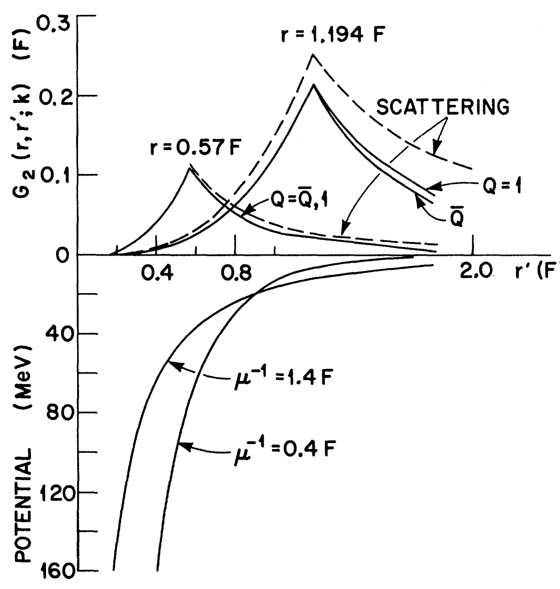

FIG. 5. The d-state Green's functions appropriate to the effective potentials of Eqs. (5.3)–(5.6) and for $k = 0.61$ F^{-1}. The full lines are for nuclear matter ($k_F = 1.4$ F^{-1}, $M_N^* = \overline{M}_N = M_N$, $\overline{M}_\Lambda = M_\Lambda$, and $\Delta = 49$ MeV), and the dashed lines for scattering. The lower part of the figure shows two Yukawa potentials with the indicated ranges.

The dominant feature of the functions $G_2^{(\mathrm{SCAT})}(r,r';k)$ and $G_2^{(\mathrm{NM})}(r,r';k)$ is the sharp peak at $r=r'$, which has a magnitude of $\frac{1}{5}r$, and the rapid falloff from this peak towards the origin. This rapid falloff is a reflection of the d-state centrifugal barrier. Outside the peak, $G_2^{(\mathrm{NM})}(r,r';k)$ goes into a damped long-wavelength oscillation or decays exponentially, depending on whether or not the Pauli principle is included. The wavelengths are about 4 F – i.e., about λ_F – consistent with a modulation which corresponds to the exclusion of intermediate nucleon momenta less than k_F. The wavelength of the oscillations is thus much longer than the range of our potentials, and hence has a negligible effect on the energy. Outside the peak, the function $G_2^{(\mathrm{SCAT})}(r,r';k)$ has a node followed by undamped oscillations. For the values of k of interest, these are again effectively outside the range of the potentials.

These features of $G_2^{(\mathrm{NM})}$ and $G_2^{(\mathrm{SCAT})}$ make it very apparent [in the light of Eq. (5.3)] that $V_{\mathrm{eff}}^{(\mathrm{SCAT})}$ and $V_{\mathrm{eff}}^{(\mathrm{NM})}$ will be quite similar for short-range tensor forces for which only the short-distance parts of the Green's functions are important. For longer range potentials, on the other hand, $V_{\mathrm{eff}}^{(\mathrm{NM})}$ will be significantly smaller (in magnitude) than $V_{\mathrm{eff}}^{(\mathrm{SCAT})}$ and, correspondingly, $\langle V_{\mathrm{eff}}^{(\mathrm{NM})}\rangle$ will be appreciably less than $\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$ in magnitude.

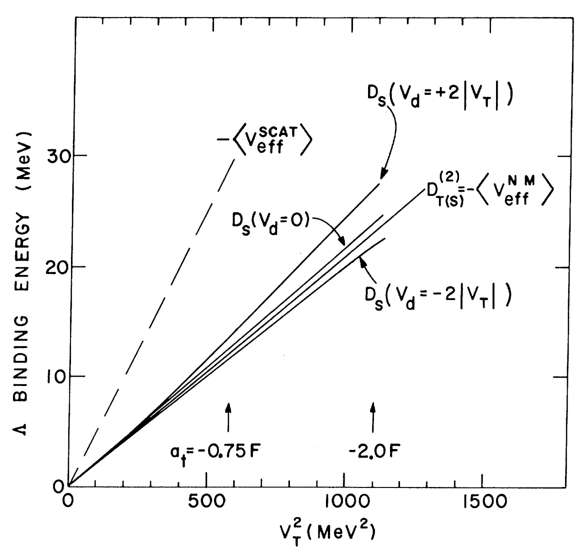

FIG. 6. The (triplet) s-state well depths as a function of V_T^2 for a pure tensor Yukawa potential of strength V_T and of range $\mu_T^{-1}=1.4$ F. ($k_F=1.4$ F^{-1}, $\Delta=103$ MeV, $M_N^*=M_N$.) The strengths that give the indicated scattering lengths (for $V_d=0$) are shown by arrows. The dashed line corresponds to using the effective potential $V_{\mathrm{eff}}^{(\mathrm{SCAT})}$, appropriate for scattering, and should be compared with the second-order result $D_T^{(2)}$ which is obtained by use of the effective potential $V_{\mathrm{eff}}^{(\mathrm{NM})}$ appropriate for nuclear matter. The lines labeled D_s are coupled-channel reaction-matrix results for the indicated values of the d-state potential V_d; $V_d=+2|V_T|$ and $-2|V_t|$ correspond to $V_t<0$ and $V_t>0$, respectively (making the assumption $V_d=-2V_T$ appropriate to a local force with $V_t=0$).

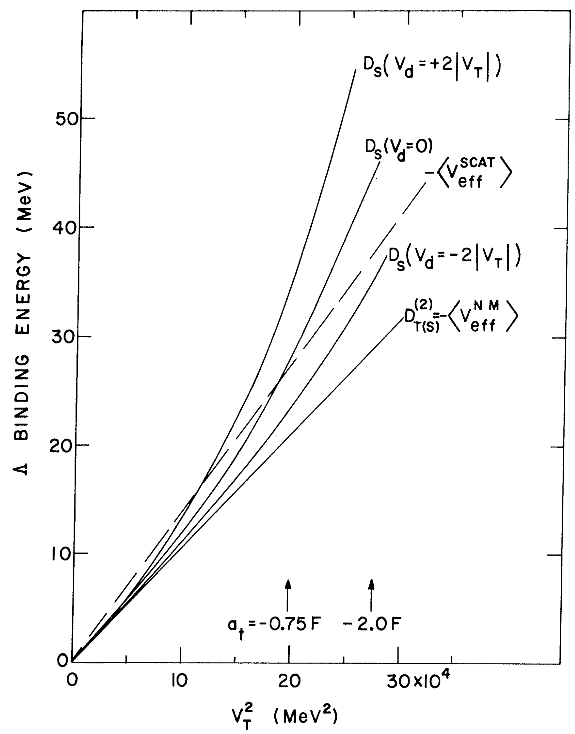

FIG. 7. Same as Fig. 6 except for a Yukawa range $\mu_T^{-1}=0.4$ F.

These considerations are illustrated by Figs. 6 and 7, as well as by some of the results of Table V. The figures show the s-wave well depths D_s as a function of V_T^2 for purely tensor potentials ($V_t=0$) of Yukawa shape. For $\mu^{-1}=0.4$ F, the ratio of $-D_{T(s)}^{(2)}=\langle V_{\mathrm{eff}}^{(\mathrm{NM})}\rangle$ to $\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$ is 0.8 and fairly close to the value 1 (which corresponds to no suppression). As Table V shows, the ratio $\langle V_{\mathrm{eff}}^{(\mathrm{NM})}\rangle/\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$ for the even shorter range $\mu^{-1}=0.2$ F is 0.9 and thus very close to 1. However, for the long range $\mu^{-1}=1.4$ F, the ratio is less than 0.5, appropriate to strong suppression.

These results demonstrate very graphically the dependence of suppression effects on the range of the tensor potential; in particular, for short ranges very little suppression is expected, whereas for long ranges there may be large suppression. A reasonable measure of the expected (s-wave) suppression for a tensor potential would seem to be the ratio $\langle V_{\mathrm{eff}}^{(\mathrm{NM})}\rangle/\langle V_{\mathrm{eff}}^{(\mathrm{SCAT})}\rangle$.

7. *g*-MATRIX APPROXIMATION AND HIGHER-ORDER EFFECTS WITH RESULTS FOR YUKAWA TENSOR FORCES

Figures 6 and 7 also show the s-wave results – for pure tensor forces – of the exact g matrix calculations which are obtained by use of the coupled reference-spectrum equations. A comparison of these results with the perturbation results $\langle V_{\rm eff}^{(\rm NM)}\rangle$, which are proportional to V_T^2, leads to an understanding of the higher-order effects of the tensor force in the g-matrix approximation.

For small values of V_T, the g-matrix results are proportional to V_T^2 and agree with the perturbation results – as they must. (This is in fact a good check on the coupled g-matrix calculations.) For larger values of V_T, we first look at the results for $V_d = 0$. The *g-matrix results are then larger than the perturbation-theory results, and this difference is greater for the shorter range.* We may understand this as follows. We recall that if $V_d = 0$, the s-wave equation (6.2) with $V_{\rm eff}^{(\rm NM)}$ gives the exact s-wave solution in the g-matrix approximation, and this exact solution then corresponds to an expansion in powers of $V_{\rm eff}^{(\rm NM)}$, i.e., in powers of V_T^2, the leading term in V_T^2 being $D_{T(s)}^{(2)}$. The terms of higher order in V_T^2 are associated with the change that $V_{\rm eff}$ causes in the s-state wave function. Thus the fourth-order term in V_T will be given by the second-order term in $V_{\rm eff}$ and is associated with the change of u to first order in $V_{\rm eff}$.

As is generally the case for the higher-order perturbation terms these (especially the second-order term in $V_{\rm eff}$) increase the binding – which is consistent with our results. Furthermore, for central potentials the second-order term not only enhances the binding but also becomes larger for shorter ranges. We thus expect that also for the central, but nonlocal, potential $V_{\rm eff}^{(\rm NM)}$ the second-order terms (proportional to V_T^4) will be larger for shorter ranges of $V_{\rm eff}^{(\rm NM)}$, i.e., for shorter ranges of $V_T(r)$. This is indeed the case and is another example of the general result that higher-order effects are larger for shorter ranges.

It is interesting to attempt to estimate the contribution in V_T^4 by using the *local* effective potential $V_{\rm eff}(r)$ of Eq. (3.13) to *second order*. One then obtains contributions in V_T^4 which are in general much too large, especially for the shorter ranges. The explanation for this is in essence the same as that given in Sec. 4 to account for the increase in the values of $\overline{E}_T^{(\rm SCAT)}$ with V_T. Thus $V_{\rm eff}(r)$ is proportional to $V_T^2(r)$, and its range [which is only about half that of $V_T(r)$] decreases much faster with the range of $V_T(r)$ than does the range of the nonlocal potential $V_{\rm eff}^{(\rm NM)}(r,r';k)$, which has the Green's function built in and which gives the exact results. This difference in range accounts for the unreasonably large second-order terms due to the local $V_{\rm eff}(r)$; the increase of these second-order terms as the range of V_T decreases is much more rapid for the local potential than for the corresponding nonlocal one.

If one turns on the d-state interaction V_d, then the effective nonlocal s-state potential will no longer be given by Eqs. (6.3)–(6.6), but now has additional higher-order terms which involve V_d. Of course, the effect of these higher-order terms is also automatically included in an exact coupled g-matrix calculation. Thus the exact expression for $V_{\rm eff}(r,r';k)$ when $V_d \neq 0$ is obtained by modifying the expression for $V_{\rm eff}$ previously given by including the additional term

$$8V_T(r)\left[\int dr'' G_2(r,r'';k)V_d(r'')G_2^{(d)}(r'',r';k)\right]V_T(r'), \tag{7.1}$$

where G_2 is the appropriate Green's function given by Eq. (6.4) or Eqs. (6.5) and (6.6) for scattering or nuclear matter, respectively; $G_2^{(d)}$ is the corresponding Green's function appropriate to the total d-state Hamiltonian *including* V_d. Since $G_2^{(d)} = G_2 + G_2 V_d G_2^{(d)}$, one may expand $G_2^{(d)}$ in terms of the noninteracting Green's function G_2 and so obtain an expansion for $V_{\rm eff}$ in powers of V_d. Thus the first-order term in V_d is just given by Eq. (7.1) with $G_2^{(d)}$ replaced by G_2. This lowest-order additional term to $V_{\rm eff}$ is proportional to $V_T^2 V_d$. We note that $V_{\rm eff}$ *is independent of the sign of* V_T, even if $V_d \neq 0$, so long as V_d is considered independent of V_T – i.e., not given by $V_d = V_t - 2V_T$ as for a local potential. (The sign of the d-state wave function is of course changed if V_T changes sign.)

With the assumption $V_d = V_t - 2V_T$, one has $V_d = -2V_T$ for the case of pure tensor forces which is appropriate to Figs. 6 and 7. The lowest-order additional contribution is then proportional to $-V_T^3$ and is repulsive or attractive depending on whether V_T is negative or positive. This is nicely confirmed by the appropriate curves (Figs. 6 and 7) depicting the exact g-matrix results for $V_d = \pm 2|V_T|$. Thus $D_{T(s)}$ is indeed larger or smaller than the value for $V_d = 0$ if V_d is positive or negative, respectively. The effects of V_d are rather small for long ranges, but these particular higher-order effects are again seen to be more important for the shorter range. In what follows, it is also important to note that the effect of V_d is similar for scattering and for nuclear matter. The relative insensitivity of the results to V_d is due to the presence of the d-state centrifugal barrier which shields the d-state wave function from the d-state potential V_d.

Figures 8 and 9 show the (s-state) g-matrix well depths D_s for the mixtures of central and tensor Yukawa potentials discussed in Sec. 2. These exact g-matrix results are consistent with the above discussion and show the same general features as were already apparent from the perturbation-theory results.

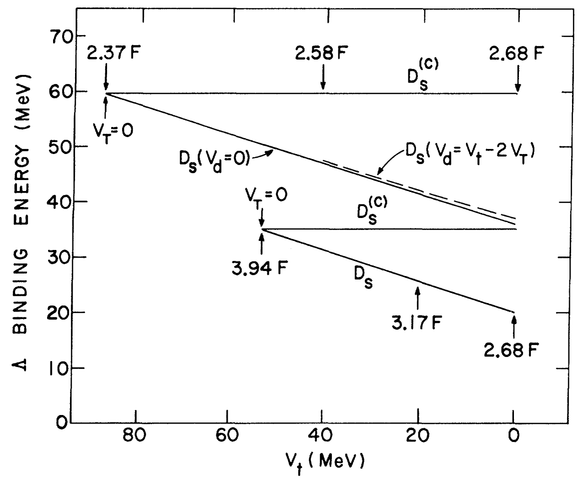

FIG. 8. The s-state (singlet-plus-triplet) well depth D_s obtained by use of the reaction-matrix procedures for mixtures of central and tensor Yukawa potentials as a function of the strength V_t of the triplet central potential. The range of the tensor potential is $\mu_T^{-1}=1.4$ F. The range of the central potential is $\mu^{-1}=0.7$ F and the strength of the singlet potential is fixed and equal to the value of V_t for $V_T=0$ (i.e., for no tensor potential). The curves labeled $D_s^{(C)}$ are the (s-state) well depths for the equivalent purely central potentials whose triplet parts give the same scattering length a_t and effective range r_{0t} as the corresponding central-plus-tensor-force mixtures; the singlet potential is the same as for the corresponding mixtures. Values of r_{0t} (for $V_d=0$) are indicated by the arrows and the associated numbers. The reduction of the well depth due to suppression of the tensor force corresponding to a given value of V_t is given by the corresponding value of $D_s^{(C)}-D_s$. The upper and lower sets of curves correspond to mixtures which give $a_t=-2$ and -0.75 F, respectively. The assumptions made for the d-state potential V_d (for both scattering and nuclear matter) are indicated. For $a_t=-0.75$ F the results for $V_d=0$ and V_t-2V are indistinguishable. All the results are for $k_F=1.4$ F^{-1}, $\Delta=115.4$ MeV, and $M_N^*/M_N=0.638$.

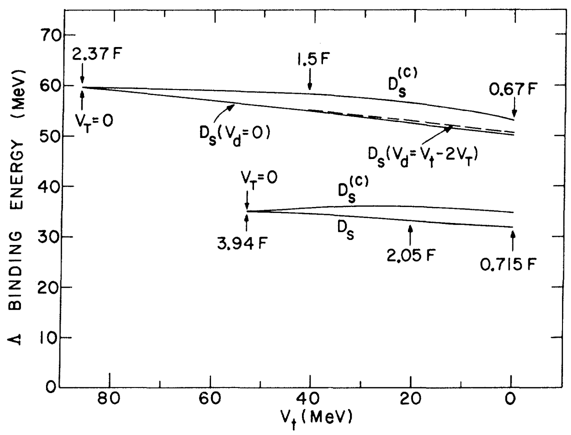

FIG. 9. Same as Fig. 8 except for a Yukawa tensor potential of range $\mu_T^{-1}=0.4$ F.

Figures 8 and 9 also show the corresponding well depths $D_s^{(C)}$ obtained with the *purely central* Yukawa potentials $V_s^{(C)}$ (Table III) whose triplet parts give both the same scattering length and effective range as those of any given mixture which is characterized by the values of the triplet central strength V_t pertaining to the mixture. The singlet potential is fixed and the same for both the mixture and the corresponding purely central potential.[24] Thus for a given mixture, determined by V_t, the figures depict both the corresponding well depth D_s and the well depth $D_s^{(C)}$ appropriate to the purely central potential that has the same low-energy scattering parameters. The appropriate triplet effective ranges are indicated in the figures.

In connection with the results for $D_s^{(C)}$ it is interesting to note that even where – for a given a_t – the effective range r_{0t} varies quite appreciably, the value of $D_s^{(C)}$ does not change too much.[25] (On the other hand, changes of the hard-core radius produce large effects.)

The suppression for a given V_t is measured by the difference $\delta D_s=D_s^{(C)}-D_s$. In particular, just as for the perturbation-theory results, there is rather little suppression for the shorter ranges of $V_T(r)$ even if these tensor potentials are of very large relative strengths. For these short ranges there is thus very good compensation between the central and tensor forces if these have been chosen to compensate each other exactly for the low-energy scattering. Thus, as is most graphically demonstrated by consideration of the Green's functions G_2 and the associated effective potentials, the effect of a short-range tensor force is quite similar for nuclear matter and for scattering.

However, as we should expect, there is large suppression for the long one-pion range $\mu_T^{-1}=1.4$ F, as shown in Fig. 8. For this long range, the well depth is very considerably reduced for strong tensor forces. Thus in the extreme limit in which $V_t=0$ (and all the triplet scattering is due to V_T), the tensor-force contribution to D_s is only about half of the corresponding (triplet) central-force contribution for $V_T=0$.

We recall that for $\mu_T^{-1}=1.4$ F, the effective range r_{0t} varies only slightly over the whole range of mixtures, in particular, if $a_t=-2$ F. Correspondingly, the value of $D_s^{(C)}$ is very nearly constant over the whole range of V_t. On the other hand, for the short range $\mu_T^{-1}=0.4$ F, the range r_{0t} varies greatly as the ratio of tensor force to central force is varied and, in particular, r_{0t} becomes very short for large V_T. The corresponding variation of $D_s^{(C)}$ is therefore more appreciable than for $\mu_T^{-1}=1.4$ F – although still not large. The very small values of r_{0t} are of course quite unreal-

TABLE VI. Effect of the gap and the exclusion principle on the s-state well depth D_s for $k_F = 1.4\ \mathrm{F}^{-1}$ and for potentials which give $a_t = -2$ F. $D_s(Q)$ and $D_s(1)$ correspond to the use of the exclusion-principle operator Q and to neglect of the exclusion principle (i.e., $Q \equiv 1$), respectively. $D_{T(s)}^{(2)}$ denotes the second-order perturbation-theory result. The upper part of the table shows the results for purely tensor potentials of Yukawa shape with range μ_T^{-1} and strength V_T and for $M_N^*/M_N = 1.0$. The lower part is for mixtures of σ-meson and kaon exchange potentials and for $M_N^*/M_N = 0.638$.

μ_T^{-1} (F)	V_T (MeV)	Δ (MeV)	$D_s(Q)/D_s(1)$	$D_{T(s)}^{(2)}(Q)/D_{T(s)}^{(2)}(1)$
1.4	22.8	50	0.78	0.81
1.4	22.8	100	0.83	0.84
0.4	412.41	50	0.89	0.99
0.4	412.41	100	0.94	1.00

g_σ^2	$g_{N\Lambda\bar{K}}^2$	Δ (MeV)	$D_s(Q)/D_s(1)$
2.18	16.0	65.0	0.92
2.18	16.0	115.4	0.96
1.137	33.4	65.0	0.90
1.137	33.4	115.4	0.95

istic and very different from the experimental Λp effective ranges.

We have examined the effect of different assumptions about the d-state interaction V_d. Thus Tables I and II show the results for the two assumptions $V_d = 0$ and $V_d = V_t - 2V_T$. (For a given value of a_t but with different assumptions about V_d, one of course obtains different values of V_T for a given value of V_t.) We recall that the effects of V_d are similar for scattering and nuclear matter [as can be seen from Eq. (7.1) and the discussion following it]. To be consistent, one must therefore make the same assumptions for V_d in the two cases. Thus if V_T is obtained (as a function of V_t) so as to give some fixed scattering length with some assumption about V_d, then one should calculate D_s with the same assumption.

One then finds that for a given scattering length and a given value of V_t, i.e., *for a given net effect of the tensor force, the value of D_s is almost independent of the assumption about V_d so long as this is the same for both scattering and nuclear matter.* In fact the values of D_s for $V_d = 0$ and $V_d = V_t - 2V_T$ are almost indistinguishable for both $\mu_T^{-1} = 0.4$ and 1.4 F on the scale of Figs. 8 and 9. This insensitivity to V_d is just due to the d-state centrifugal barrier as already discussed. (The value of r_{0t} is slightly different for different assumptions about V_d, and therefore the equivalent central potential will be slightly different. Thus strictly the value of $D_s^{(C)}$ will depend on V_d; however, this dependence is negligible.)

The effects of the gap Δ and of the exclusion principle (Q) on the (s-state) well depth, when this is due to purely tensor forces of Yukawa shape, are shown in Table VI. As expected from our previous considerations, the effect of Q becomes less as Δ is increased or as the range of the potential is decreased. The effect of Q on the reaction-matrix and on the perturbation-theory results is quite similar for the long-range potential. This is because the contributions to D_s of higher order than V_T^2 are then relatively small. On the other hand, for the short-range potential the effect of the exclusion principle on the perturbation-theory results is considerably less than on the reaction-matrix results. This is presumably because, in this case, the exclusion principle significantly modifies the wave function at quite short distances, and this has the effect of reducing the contributions from higher order than V_T^2, which are quite important for short-range potentials.

8. s-WAVE REACTION-MATRIX RESULTS FOR ONE-BOSON-EXCHANGE POTENTIALS

Although the results just discussed were obtained for Yukawa shapes, they strongly suggest that there

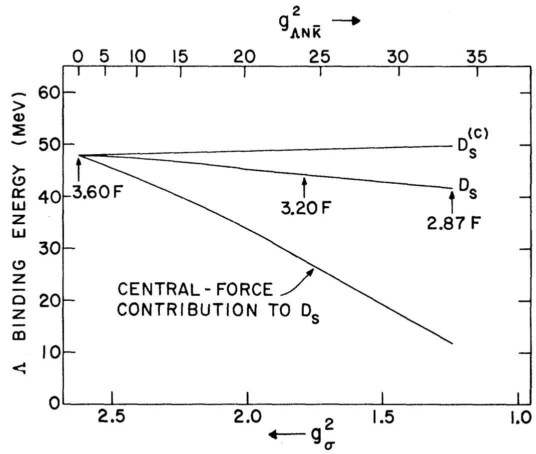

FIG. 10. The self-consistent s-state (singlet-plus-triplet) well depth D_s as a function of g_σ^2 for mixtures of σ-meson and kaon exchange potentials which have a hard core of radius $c = 0.43$ F and which all give $a_t = -2$ F. The appropriate values of $g_{N\Lambda\bar{K}}^2$ are indicated; $g_{N\Lambda\bar{K}}^2 = 0$ corresponds to a purely central potential. The singlet potential is fixed and equal to the triplet potential for $g_{N\Lambda\bar{K}} = 0$. The results for $V_d = 0$ and $V_d = V_t - 2V_T$ are indistinguishable (with the same assumptions for scattering and nuclear matter). The numbers and corresponding arrows indicate the appropriate values of r_{0t} (for $V_d = 0$). The contribution to D_s due to just the central part of the potential is shown. The (s-state) well depths $D_s^{(C)}$ are those for purely central potentials with the same values of c, and of a_t and r_{0t} (and with the same singlet potential) as the corresponding mixtures characterized by g_σ^2. The suppression of the tensor force is again characterized by $D_s^{(C)} - D_s$. The results are for $k_F = 1.4\ \mathrm{F}^{-1}$, $\Delta_N = 85.4$ MeV, and $M_N^*/M_N = 0.638$.

will also be rather little suppression for the more "realistic" hard-core OBE potential mixtures which have short-range tensor forces due to kaon exchange and which were described in Sec. 2. We again choose the singlet potential to be equal to the triplet potential in the limit $g_{N\Lambda\bar{K}}=0$, i.e., when the triplet potential is purely central and entirely due to σ-meson exchange.

Figure 10 shows the (s-state) well depth D_s as a function of g_σ^2 whose value characterizes a given mixture. The contribution $D_s(V_t)$ due to only the central part of the potential is also shown. Because of the hard core, the triplet contribution becomes negative when the triplet attractive tail becomes sufficiently weak, i.e., for sufficiently small g_σ^2. In fact the extreme right of the curves corresponds to a quite small triplet central contribution to the well depth. Again, we also show the well depth $D_s^{(C)}$ for the purely central hard-core potentials of Table III whose triplet parts give the same scattering length ($a_t = -2$ F) and the same effective range as a given mixture.

For discussion of the well depths it is convenient to use $D_s^{(C)}$ as a measure of the "unsuppressed" well depth appropriate to the effect of the tensor force for scattering. The suppression is then characterized by the difference $\delta D_s = D_s^{(C)} - D_s$. It is notable that D_s, $D_s^{(C)}$, and $D_s(V_t)$ all vary almost linearly with $g_{N\Lambda\bar{K}}^2$, as well as with the strength of V_t (i.e., with g_σ^2). A convenient measure of this strength is then $D_s(V_t)$, which is the well-depth contribution due to only V_t. The difference $D_s^{(C)} - D_s(V_t)$ then measures the net "unsuppressed" contribution, appropriate to scattering, due to just the tensor force alone. Thus this difference is the analog of $-\langle V_{\text{eff}}^{(\text{SCAT})}\rangle$ which was used in Sec. 6 as a perturbation-theory measure of the effectiveness of the tensor force for scattering. The actual (suppressed) contribution for nuclear matter is then $D_s - D_s(V_t)$, which is the analog of $\langle V_{\text{eff}}^{(\text{NM})}\rangle = -D_{T(s)}^{(2)}$.

A very convenient measure of the effectiveness of the suppression is then the ratio $\xi = [D_s - D_s(V_t)]/[D_s^{(C)} - D_s(V_t)]$, which, in view of the linearity pointed out above, is essentially independent of the magnitude of the coupling and which corresponds to the ratio $\langle V_{\text{eff}}^{(\text{SCAT})}\rangle / \langle V_{\text{eff}}^{(\text{NM})}\rangle$ previously discussed. Thus for complete suppression one has $\xi = 0$, whereas for no suppression $\xi = 1$. The effectiveness with which the OBE tensor force is suppressed is then characterized by the corresponding value $\xi = 0.79$. This is not too different from unity, indicating that the suppression is relatively slight. It is interesting that this is close to the perturbation-theory ratio for a Yukawa tensor potential with range $\mu_K = 0.4$ F. In this connection it is important to note that *the meson-theory tensor shape is much more singular than the corresponding Yukawa shape and hence of effectively shorter range.* However, the hard core has the opposite effect, and effectively increases the range, since it forces the attractive part of the potential out beyond the hard-core radius.

For the magnitude of the suppression it is then notable that also for the more "realistic" meson-theory potentials with a short-range tensor force, the suppression δD_s is only quite moderate even for the strongest tensor forces considered. This is in spite of the fact that the central-force contribution $D_s(V_t)$ is then only about 10 MeV. However, the tensor force actually expected due to kaon exchange are only moderately strong and correspond to $g_{N\Lambda\bar{K}}^2 \lesssim 15$. For such strengths the corresponding suppression is only $\delta D_s \lesssim 2$ MeV. We also recall that all our OBE potentials have low-energy scattering parameters similar to those obtained from Λp scattering. It should be noted that if the tensor force were completely suppressed in nuclear matter (i.e., $\xi = 0$) then the only contribution would be $D_s(V_t)$, and in that case the reduction in the well depth corresponding to $g_{N\Lambda\bar{K}}^2 \lesssim 15$ would be $\delta D_s \lesssim 9$ MeV.

If one includes the tensor-force contribution due to η-meson exchange in the way discussed in Sec. 2, then the effect is to modify $g_{N\Lambda\bar{K}}^2$ by a factor of at most 1.25 in magnitude. A reasonable upper limit for the suppression would now be $\delta D_s \approx 3$ MeV. (It is clear from our considerations that a change in sign of the tensor force will not alter any of the conclusions about δD_s.) Suppression associated with tensor-force contributions due to heavier mesons, such as the ω meson, is expected to be quite small because of the short ranges involved.

It will be observed – especially clearly in the limit where most of the triplet well depth is due to the tensor force – that there is rather more suppression for the OBE potentials than for the Yukawa potentials with $\mu_T^{-1} = 0.4$ F. (In fact the range of the OBE tensor potential is effectively shorter because this potential is more singular than a pure Yukawa potential corresponding to the same exchanged mass.) This difference arises from the hard core (in the OBE potentials) which increases the effective range for a given shape of the attractive part and for a given scattering length. This is illustrated by the results for r_0 (shown in Table III) corresponding to Yukawa potentials with and without a hard core. These results also show that $D_s^{(C)}$ as a function of r_0 has a maximum which occurs for moderate values of r_0. This dependence of $D_s^{(C)}$ on r_0 is also illustrated by the curves for $D_s^{(C)}$ shown in Figs. 9 and 10. In Fig. 9, $D_s^{(C)}$ decreases as r_0 decreases, because the relevant values of r_0 are rather small in the absence of a hard core. On the other hand, in Fig. 10 the slight increase of $D_s^{(C)}$

as r_0 decreases arises from the hard core, because this gives rise to much larger values of r_0 for which the slope of $D_s^{(C)}$ versus r_0 is reversed.

It should be noted that most of the difference in the values of δD_s for Figs. 9 and 10 actually comes from the different variation of $D_s^{(C)}$ with r_0 in the two cases, rather than from differences in the variation of D_s with V_t.

Again, just as for the Yukawa mixtures, the effect on the well depth of the two different assumptions about V_d; namely, $V_d=0$ and V_t-2V_T, is negligible – if the same assumption is made for nuclear matter as for scattering.

To relate our OBE results to those for other potentials, we show in Table VII the self-consistent values of D_s for our purely central OBE potential ($g_{N\Lambda\overline{K}}=0$) and for Tang and Herndon's potentials E and H. The latter are consistent with both the Λp scattering data and the binding energies of the $A=3$ and 4 hypernuclei. The value $k_F=1.366$ F^{-1} was used in Ref. 2 and corresponds more closely than 1.4 F^{-1} to the central density of heavy nuclei. The results for the different potentials are mutually consistent and in agreement with the fact that D_s is a strongly decreasing function of the hard-core radius c and an increasing one of the scattering length.

If for our OBE potentials we were to use $c=0.6$ F, which is the value for potential H, instead of $c=0.43$ F, then (for $a\approx -2$ F) all our OBE results would be reduced by about 10 MeV. For a given value of $g_{N\Lambda\overline{K}}{}^2$, which determines the strength of the tensor force, we then expect the magnitude of the suppression to be appreciably less for the larger hard core. This is because the contribution of the tensor force is then reduced (for a given $g_{N\Lambda\overline{K}}$) because a larger hard core cuts out more of the inner part of the tensor potential which is thus effectively weakened and must be compensated for by an increase in the central potential.

Our OBE potential results, together with our previous discussion, thus lead us to expect only very moderate s-state suppression, of at most about 3 MeV, for the meson-theory tensor forces of the expected range and strengths. The s-state effect of a short-range "realistic" tensor force is thus expected to be quite similar for scattering and for nuclear matter. In particular, if for scattering the effect of a ΛN interaction with such a tensor force is reproduced by a phenomenological central potential with the same low-energy scattering parameters (and the same hard-core radius), then the latter potential is expected to a rather good approximation to also reproduce the effects of the tensor force in nuclear matter.

TABLE VII. Self-consistent s-state well depths for some central potentials. The hard-core radius is denoted by c; a is the scattering length appropriate for the spin-average (and charge symmetric) potential. The potentials E and H are those of Tang and Herndon (Ref. 3). ($\Delta_N=85.4$ MeV, $M_N^*/M_N=0.638$.)

Potential	c (F)	a (F)	r_0 (F)	D_s (MeV) $k_F=1.366$ F^{-1}	1.4 F^{-1}
OBE ($g_{N\Lambda\overline{K}}=0$)	0.43	−2.0	3.60	44.5	47.8
E	0.45	−1.7	3.50	41.0	44.1
H	0.60	−2.1	3.42	34.5	37.1

The effects of the exclusion principle Q and of the gap Δ on the OBE potential well depths are shown in Table VI, and are consistent with expectation. The effect of both Q and Δ is to give more suppression as the relative strength of the tensor force is increased (i.e., as $g_{N\Lambda\overline{K}}{}^2$ is increased). This is simply because the contributions to D_s which depend on Q and Δ then become more important. In fact the effect of Q and Δ does not increase very much with $g_{N\Lambda\overline{K}}{}^2$. This is because of the short range of the potentials and also because the associated decrease in $g_\sigma{}^2$ implies that the corresponding Q- and Δ-dependent contributions for the central part of the potential become less.

9. p-WAVE CONTRIBUTION OF THE TENSOR FORCE

If perturbation theory is applicable (i.e., in particular, for potentials without a hard core), then our results for $D_{T(s)}^{(2)}/D_T^{(2)}$ (shown in Table V) indicate that for short-range tensor potentials the tensor-force contribution to D from partial waves with $L\geq 1$ will be quite small – less than about 5%. Any suppression of this contribution will thus have only a very small effect on D.

However, with a substantial hard core, the p-wave contribution D_p may be comparable to the s-wave one (e.g., about 25 MeV for D_p as compared with about 40 MeV for D_s) if the p- and s-wave potentials are comparable (Refs. 2 and 4). The contributions from $L\geq 2$ will be quite small ($\lesssim$2% of D) for any reasonable potentials and need not be considered here. It is therefore of interest to also consider the question of suppression of the tensor force in the p state.

We consider only the $S=1$ p-wave contribution and assume a local (central-plus-tensor) potential $V_t(r)+S_{12}V_T(r)$. (For $S=0$ only the central force contributes.) For $L=1$, in contrast to the coupled sd case, there are diagonal tensor-force contributions. Thus the diagonal p-wave potentials appropriate to a total angular momentum J (=0, 1, 2) are

$$^3V_{J,L=1}\equiv {^3V_{Jp}}=V_t+s_{L=1,J,L'=1}V_T\ , \tag{9.1}$$

where the $s_{L=1,J,L'=1}$ are the relevant tensor-force matrix elements [appearing also in Eq. (5.1)] whose values are shown in Table VIII. We have ignored possible spin-orbit contributions in this paper.

There is also tensor-force coupling of the p to the f wave. However, the contribution of this can be neglected in comparison with the diagonal tensor-force contributions (Ref. 11). This neglect is made plausible later in this section.

For each J we then use the effectively central potential ${}^3V_{Jp}$ to calculate the corresponding well depth ${}^3D_{Jp}$ in the g-matrix approximation, using the reaction-matrix procedures appropriate to a central p-state potential. These procedures are completely analogous to those for the s wave and were described in Sec. 5 and in Ref. 2, where p-wave results were obtained. It should be noted that for not too large hard cores the p-wave contribution D_p is much less sensitive to the gap Δ than is D_s, so that the requirement of self-consistency is unimportant for D_p unless c is quite large.

The total triplet p-wave well depth ${}^3D_p$ is then given by

$$ {}^3D_p = \tfrac{1}{9}\sum_{J=0}^{2}(2J+1)\,{}^3D_{Jp}. \tag{9.2}$$

To first order in the potentials ${}^3V_{Jp}$, there is no net p-wave contribution from V_T [because of the values of $s_{L=1,J,L'=1}$ and the weights $2J+1$ in Eq. (9.2)]. Thus, as one would expect, the tensor force contributes only in second and higher order also for $L=1$. In this sense the situation is quite similar to the coupled sd case, and the same general considerations about the effect of the range on the suppression also apply here. The neglect of the pf coupling is then plausible because this is expected to be much less effective than the pp coupling (i.e., the second- and higher-order diagonal contributions of V_T), since the former involves a large additional angular momentum barrier.

Our p-wave results, shown in Table VIII, are based on our s-wave OBE potential ($V^{(\sigma)}+V^{(K)}$ + hard core of radius $c=0.43$ F) with $g_{N\Lambda\bar{K}}{}^2=23.91$ ($g_\sigma{}^2=1.79$). This potential, described in Sec. 2

TABLE VIII. The p-state results for the OBE potential with $g_{N\Lambda\bar{K}}{}^2=23.9$. The quantities are defined in the text. The well depths are calculated for $k_F=1.4\ \mathrm{F}^{-1}$, $\Delta_N=85.4$ MeV, $\Delta_\Lambda=40$ MeV, and $M_N^*/M_N=0.638$.

J	$s_{L=1,J,L'=1}$	${}^3a_{Jp}$ (F^3)	${}^3r_{Jp}$ (F^{-1})	${}^3D_{Jp}$ (MeV)
0	−4	0.051	38.23	−5.36
1	+2	−0.739	0.42	29.73
2	−0.4	−0.290	5.78	12.69
		${}^3a_p=-0.401$	${}^3r_p=2.46$	${}^3D_p=16.36$
				${}^3D_p^{(C)}=17.93$
				${}^3D_p(V_t)=15.1$

(in particular in Table III), gives $a_t=-2$ F and has a very strong tensor force. More specifically, our p-wave potential is obtained by treating this s-wave potential as if it were local. Thus both the central and tensor p-wave potentials $V_t(r)$ and $V_T(r)$, respectively, are the same as the corresponding s-wave potentials; in particular, $V_T(r)$ has the same shape, strength, and sign as the K-exchange s-wave tensor potential.

According to Eq. (2.4) there is an over-all change of sign of $V^{(K)}$ between the odd states and the even states. Since, however, the effect of a tensor potential is almost independent of its sign (and is attractive), our p-wave tensor force has effectively a meson-theoretical basis.

In fact the main effect of the over-all sign change is to make the central K-exchange contribution $V_t^{(K)}$ repulsive with respect to $V^{(\sigma)}$, which does not change sign. However, since we are specifically interested in the suppression of the tensor force, we have kept the central potential the same as the s-state one, so as to preserve the connection with an s-state potential normalized to give $a_t=-2$ F. (In fact $V_t^{(K)}$ is not too large compared with $V^{(\sigma)}$, and changing its sign does not have too large an effect on the net central potential V_t.)

The well depths ${}^3D_{Jp}$ and ${}^3D_p$ obtained with our OBE potential are shown in Table VIII. The large splitting between the different J values is a reflection of the strong tensor force. However, since the contribution to ${}^3D_p$ (= 16.35 MeV) from the central part V_t (which includes the hard core) is ${}^3D_p(V_t)=15.1$ MeV, the *net* (suppressed) contribution from the tensor force to ${}^3D_p$ is only 1.25 MeV. This is relatively much less than for the corresponding s-state well depth D_s (= 44.3 MeV), for which the net tensor-force contribution is about 16.5 MeV (as seen in Fig. 10). The tensor force is thus much less effective for D_p than for D_s. This is confirmed by the results for the "unsuppressed" tensor-force contribution discussed below.

Just as for the s-wave case, one must be quite careful if one is to obtain a meaningful estimate of the suppression of the tensor force for the p wave. Thus one should compare our OBE potential with a purely central potential which has the same low-energy *p-wave* scattering characteristics and the same hard core. Because for $L=1$ there are three phase shifts ${}^3\delta_{Jp}$, one must use an average phase shift.

Thus for the basic p-wave scattering data, which we suppose in principle to be available, we use the average (low-energy) p-wave phase shift:

$$ {}^3\delta_p = \tfrac{1}{9}\sum_{J=0}^{2}(2J+1)\,{}^3\delta_{Jp}, \tag{9.3}$$

where ${}^3\delta_{Jp}$ are the phase shifts for the individual

J values obtained from the potentials ${}^3V_{Jp}$ appropriate to our OBE potential. The results for ${}^3\delta_p$ (and also for ${}^3\delta_{Jp}$) are conveniently parametrized by use of the p-wave form of the effective-range expansion, namely,

$$k^3 \cot {}^3\delta_p = -\frac{1}{{}^3a_p} + \tfrac{1}{2}\,{}^3r_p k^2 . \qquad (9.4)$$

The values of ${}^3a_p$ and ${}^3r_p$, as well as the values ${}^3a_{Jp}$ and ${}^3r_{Jp}$ appropriate to the individual ${}^3\delta_{Jp}$, are shown in Table VIII.

We may then proceed in exact analogy with the coupled sd-wave case, and obtain an "equivalent" purely central potential $V_p^{(C)}$ which reproduces ${}^3a_p$ and ${}^3r_p$ and also has a hard core of radius $c = 0.43$ F. For the attractive part of $V_p^{(C)}$, we have used a Yukawa potential which then has a strength ${}^3V_p = 1606.2$ MeV and a range parameter ${}^3\mu_p = 2.36$ F^{-1}. Finally, from $V_p^{(C)}$ one obtains the corresponding well depth ${}^3D_p^{(C)}$.

This well depth is to be regarded as the "unsuppressed" p-wave well depth, appropriate to the effect of the tensor force for scattering. The suppression is then given by $\delta D_p = {}^3D_p^{(C)} - {}^3D_p = 1.57$ MeV, which is indeed positive appropriate to suppression. Although fairly small, δD_p is, nevertheless, quite large compared with the net "unsuppressed" contribution of the tensor force, namely, ${}^3D_p^{(C)} - {}^3D_p(V_t) = 2.83$ MeV. In fact, the relative suppression is quite strong if this is, most naturally, defined by $\xi_p = [{}^3D_p - {}^3D_p(V_t)]/[{}^3D_p^{(C)} - {}^3D_p(V_t)]$, which is the ratio of the suppressed to the unsuppressed contribution of the tensor force. Thus one has $\xi_p = 0.45$. ($\xi_p = 1$ means no suppression.)

That the absolute suppression $\delta D_p = 1.57$ MeV is nevertheless quite small, in spite of this large relative suppression and of the large value of $g_{N\Lambda\bar{K}}{}^2$, is due to the relative ineffectiveness of the tensor force in the p state. Thus the "unsuppressed" tensor-force contribution is only 2.83 MeV, whereas the comparable s-wave value is 21.2 MeV.

The above discussion of the suppression is based on the single average phase shift ${}^3\delta_p$ defined by Eq. (9.3) and is consistent with our s-wave approach, as well as with the very limited available scattering data. If the individual phase shifts ${}^3\delta_{Jp}$ were considered as the basic scattering data, then this would imply noncentral forces (in view of the splitting between the ${}^3\delta_{Jp}$); in particular, one could attempt to obtain a tensor force which fits these data and then use this tensor force directly to obtain the well depth. If, nevertheless, one attempts to reproduce the ${}^3\delta_{Jp}$ by use of equivalent central potentials ${}^3V_{Jp}^{(C)}$, then these potentials would give well depths ${}^3D_{Jp}^{(C)}$ which would be very nearly the same as the OBE values ${}^3D_{Jp}$. (Thus for $J = 1$ and 2, one has ${}^3D_{Jp}^{(C)} = 29.63$ and 12.79 MeV, respectively.) This is because both ${}^3V_{Jp}$ and ${}^3V_{Jp}^{(C)}$ are central potentials with the same hard core and the same low-energy scattering parameters, and such equivalent potentials give effectively the same well depths. Such a procedure of fitting the individual ${}^3\delta_{Jp}$ therefore has little point to it.

It is instructive, however, to consider another procedure for obtaining the suppression, especially as the relevant results are also of significance for further discussion of the above results. Thus we may use the purely central s-wave potential $V_s^{(C)}$, which is equivalent to our (local) OBE potential for low-energy *s-wave* scattering, to calculate a p-wave well depth. We obtain ${}^3D_p(V_s^{(C)}) = 17.11$ MeV. Using $\delta D_p = {}^3D_p(V_s^{(C)}) - {}^3D_p$ to measure the suppression, we find $\delta D_p = 0.75$ MeV, which is quite small. The corresponding value of ξ_p, the relative suppression, is now only 0.63.

Since this latter procedure does not use p-wave phase shifts as the basic data, we do not consider it to be as significant as the earlier procedure which does. However, the value of ${}^3D_p(V_s^{(C)})$ is significant in that it indicates, in view of its closeness to ${}^3D_p^{(C)}$, that the s- and p-wave equivalent potentials $V_s^{(C)}$ and $V_p^{(C)}$ are quite similar. This is confirmed by comparison of the values ${}^3a_p = -0.374$ F^3, ${}^3r_p = 2.334$ F^{-1} obtained using $V_s^{(C)}$ with the corresponding values, shown in Table VIII, obtained using $V_p^{(C)}$. The closeness of the two sets of values shows that the two potentials are indeed quite similar, not only for nuclear matter but also for scattering. This implies that the OBE potential has effectively very nearly the same strength for the s and p wave, since a measure of these strengths is just the appropriate phase-shift-equivalent central potentials.

Our results thus correspond to a ratio $p/s \approx 1$ between the effective p- and s-wave strengths. It should also be noted that our value $D_p = {}^3D_p + {}^1D_p \approx 22$ MeV agrees very well with the results obtained for reasonable central potentials with equal p- and s-state strengths.

We may obtain the suppression δD_p corresponding to other values of $g_{N\Lambda\bar{K}}{}^2$ by noting that δD_p, and also the unsuppressed tensor-force contribution ${}^3D_p^{(C)} - {}^3D_p(V_t)$, are to a good approximation proportional to $g_{N\Lambda\bar{K}}{}^2$. {One also has the relation $\delta D_p = (1-\xi_p)[{}^3D_p^{(C)} - {}^3D_p(V_t)]$, where ξ_p is independent of $g_{N\Lambda\bar{K}}$.} Thus, using our results for $g_{N\Lambda\bar{K}}{}^2 = 23.9$, we obtain for $g_{N\Lambda\bar{K}}{}^2 \approx 11–15$ (which are the larger of the two possible sets of phenomenological values) the values ${}^3D_p^{(C)} - {}^3D_p(V_t) \approx 1.3–1.8$ MeV, and for the suppression, the values $\delta D_p \approx 0.7–1$ MeV.

These values of δD_p are expected to be about the same, even for much weaker *net* p-state interactions, so long as we use the same values of $g_{N\Lambda\bar{K}}$.

Thus most of the net p-wave interaction is due to $V^{(\sigma)}$ and relatively little due to $V^{(K)}$, even for very large values of $g_{N\Lambda\bar{K}}$. Since all of the tensor force is due to $V^{(K)}$, the above results for δD_p (and for the unsuppressed tensor-force contribution) are therefore quite consistent also with a much smaller net p-state interaction, which is obtained by reducing just $V^{(\sigma)}$ (but not $V^{(K)}$).

Thus a reasonable upper limit for the p-wave tensor-force suppression is about 1 MeV, even if the net p-wave interaction should be much weaker than the s-state interaction, which is quite possible.

10. CONCLUSION AND DISCUSSION

For local tensor forces of short range (compared with k_F^{-1}) the dominant momentum components are very high, and consequently the effects of such tensor forces are only slightly modified by the nuclear medium. The effect of such short-range tensor forces (for the s and p state) is then very similar for scattering and for nuclear matter. Thus, if central and tensor forces are chosen to compensate each other for low-energy scattering, they also compensate each other very closely for nuclear matter if the range is short. This is brought out particularly clearly by comparing the effective nonlocal and energy-dependent central potentials that represent the effect of tensor forces for scattering on the one hand and for nuclear matter on the other (Sec. 6).

Thus short-range local tensor forces are only slightly suppressed in nuclear matter. However, the effect of long-range tensor forces (as, for example, tensor potentials with a Yukawa shape and with a range corresponding to one-pion mass, which were considered in Secs. 6 and 7) is strongly modified, and such potentials are strongly suppressed in nuclear matter.

In general, a good measure of the expected (s-wave) suppression for interactions without a hard core is the perturbation-theory ratio $\langle V_{\text{eff}}^{(\text{NM})}\rangle / \langle V_{\text{eff}}^{(\text{SCAT})}\rangle$. This is the ratio of the (nuclear-matter) expectation values of the effective nonlocal potentials appropriate for nuclear matter and for scattering, respectively. These expectation values include the effects of the tensor force to second order; thus $\langle V_{\text{eff}}^{(\text{NM})}\rangle$ is just the second-order perturbation result for the energy. Effects of higher-order than V_T^2 (in the reaction-matrix approximation) become more important as the range decreases; in particular, for ranges corresponding to the kaon mass such higher-order effects may become quite large. In that case the use to first order of a local effective potential (and the corresponding average closure energy), which is equivalent to use of the second-order perturbation energy, becomes a poor approximation for relatively small tensor-force strengths. Furthermore, for such short-range potentials the local effective potentials give much too large contributions of higher order. A corresponding measure of the suppression ξ which is suitable for hard-core interactions is defined in Sec. 8.

For the ΛN interaction the tensor forces are in fact expected to be of quite short range, since they are due to the exchange of K, η, and heavier mesons. Our detailed reaction-matrix calculations show that for short-range tensor forces there is indeed rather little suppression, even for very strong tensor forces which account for all the triplet scattering. The suppression is measured by the reduction in the well depth relative to the depth for a purely central potential which has both the same scattering length and effective range as the particular mixture of central and tensor forces being considered. Different assumptions about the d-state interaction have an almost negligible effect on the (s-state) depth if the same assumption is made for both scattering and nuclear matter. We conclude that if central and *short-range* tensor forces are chosen to compensate each other for low-energy scattering, then they will also compensate each other quite closely for nuclear matter. In particular, the results for "realistic" OBE potentials that have a hard core and include tensor forces due to kaon exchange (and also to η exchange in an approximate way) show that there is only rather slight s-state suppression, corresponding to $\xi \approx 0.8$ and to a reduction of the well depth of at most about 3 MeV for a reasonable hard-core radius and current phenomenological values of the $N\Lambda\bar{K}$ coupling constant. Heavier mesons, which could contribute to the ΛN tensor force, are not expected to change this conclusion.

Our p-wave results show that the p-wave suppression is expected to be quite small ($\lesssim 1$ MeV) for reasonable OBE potentials, even though the total p-wave contribution to D can be quite large if there is a substantial hard core. Also one must be very careful here to use suitably defined equivalent central potentials if one is to obtain a meaningful measure of the suppression. (The perturbation-theory results indicate that for short-range tensor potentials *and without a hard core*, the total tensor-force contribution to D from higher partial waves with $L \geq 1$ will be quite small and less than about 5%.)

Thus the total suppression, including that of the p wave, is rather small and probably at most about 4 MeV.

The reasons for the slight suppression of short-range tensor forces in nuclear matter are expected to hold also for finite hypernuclei. In fact, it may be expected that suppression effects will be relatively smaller for the latter because the average density is less – and since suppression becomes less as the density decreases. Of course, to test the validity of these conjectures for finite hypernuclei, one must perform appropriately realistic calculations.

Limitations of the Results

The Brueckner-Bethe reaction-matrix approach gives an expansion of the energy in powers of the density ρ (more precisely the ratio of the correlation volume to the volume per particle), with the leading term given by the g-matrix approximation. Higher-order terms, in particular those proportional to ρ^2, have not been adequately calculated for the Λ case. One may perhaps conjecture that the contribution to these terms from the short-range ΛN tensor force are small if also the associated correlation volumes are small. The latter is expected to be the case if the tensor forces are also not too strong.

A related question is the choice of the single-particle energies (i.e., of the effective masses) for the unoccupied states. For a given choice, the ρ^2 terms should be calculated; in particular, if the choice is fortunate then these terms will be small. The choice of the free kinetic energies is believed to be appropriate for central potentials with a sizable hard core.[5] The presence of strong tensor forces will, of course, affect the choice of the "best" single-particle energies but, as discussed below, is expected to favor the free kinetic energies. In any case, the choice of these is the most neutral one in the absence of estimates of the higher-order terms.

It is important to note that our (numerical) results about the suppression of tensor forces depend on the use of the free kinetic energies for the unoccupied states. If one uses effective masses ($\overline{M}_N,\overline{M}_\Lambda$) that are not equal to the free masses, then, as discussed in connection with both the perturbation theory and reaction-matrix calculations (Secs. 3 and 5), the *tensor-force contribution* is expected to be modified approximately in the ratio $\overline{\mu}/\mu$ of the appropriate reduced masses. This is not true, however, for the central-force contribution, much of which (especially for purely attractive potentials) comes from the first-order perturbation result which is independent of $\overline{\mu}$.

We can use the value $\overline{M}_\Lambda = 0.9\ M_\Lambda$ to obtain an estimate of this effect. This value of $\overline{M}_\Lambda$ is that appropriate to just a particular third-order ρ^2 (g-matrix) term considered by Dabrowski and Köhler.[26] One then has $\overline{\mu} = 0.95\mu$. Thus for the OBE results of Fig. 10, the tensor-force contribution which corresponds to the strongest tensor force considered is now reduced from about 30 to 28.5 MeV; i.e., the suppression is enhanced and corresponds to a further decrease of about 1.5 MeV in the total well depth D_s. For more moderate and realistic strengths, the corresponding reduction would be less than about 0.7 MeV. Thus, if the ratio of the effective masses for the unoccupied states to the free masses is close to unity, then the qualitative conclusion that the suppression is small for realistic ΛN tensor forces is unchanged.

Since the approximate equivalence of short-range tensor forces for scattering and nuclear matter seems physically reasonable, one may perhaps use this as a justification for the choice $\overline{\mu} = \mu$, since the equivalence depends on this choice. However, this argument – if correct – depends on the tensor-force component in the interaction being strong enough to dominate second- and higher-order effects. The argument would have to be qualified if the interaction contains a sizable hard core – which brings us back to the question of the "best" choice of single-particle energies for a general interaction which contains both central and tensor components, and to the associated question of the corresponding value of the ρ^2 terms.

A different limitation of our results is due to the assumption of locality for our tensor potentials. For nonlocal tensor forces,[27] the off-energy-shell matrix elements are in general no longer determined by the range – as is effectively the case for local tensor forces. These matrix elements may then be damped relative to those expected for a local force of the same range. This implies that – in principle – even for short-range forces, one could get quite strong suppression of a nonlocal tensor force in nuclear matter. However, since such tensor forces must be fitted together with central forces to the low-energy scattering, the net result on the binding energy is not obvious. Although OBE potentials in general have momentum-dependent nonlocal components, one expects that the local components – which are given by the static approximation – dominate at longer distances, especially for the pseudoscalar kaon and η-meson exchanges. However, the limitation of our results due to the assumption of locality should be kept in mind.

Comparison of the Hypernuclear and the Nuclear Cases

For the NN interaction the tensor force is strong and of long range and is expected to be strongly suppressed. Such strong suppression is obtained for the Λ case with the long-range, one-pion Yukawa potential. This range is, of course, quite unrealistically long for the ΛN interaction and was considered for illustrative purposes only. (However, it should be remembered that the meson-theory tensor shape is much more singular than the corresponding Yukawa shape and hence is effectively of shorter range than the latter.)

Furthermore, for the purely nuclear case the exclusion principle operates for both particles, and this would increase the suppression relative to that for the hypernuclear case – especially if the range is long. Probably a more significant effect of the exclusion principle is the resulting large average kinetic energy per nucleon. This implies that the total energy per particle – which is the difference between this kinetic energy and a comparable potential energy – is relatively much more sensitive to suppression effects than is the potential energy alone, which is effectively the only energy relevant for the Λ well depth. (This is clear from Fig. 10 – if the binding energy were not the well depth but were this minus a comparable average kinetic energy.) In fact, suppression of the NN tensor force is known to be quite important for the saturation properties of nuclear matter.

ΛN Interaction and the Phenomenological Well Depth

If we accept a value of about 3 MeV as a reasonable upper limit for the reduction of the well depth as a result of suppression of a possible ΛN tensor force, then the presence of this will not significantly change any of the conclusions obtained for the ΛN interaction by a comparison of the calculated with phenomenological well depth. It seems appropriate to summarize these conclusions for a phenomenological well depth of about 30 MeV.

With purely central potentials and for given low-energy ΛN s-wave scattering parameters, the well depth (calculated in the g-matrix approximation) is sensitive predominantly to two properties of the potential, namely, the short-range repulsion and the p-state interaction. Thus, firstly, the s-state contribution D_s to the well depth decreases strongly as the hard-core radius increases. Here, the significant quantity is most probably the correlation volume (or "wound integral") associated with the hard core, and one expects D_s to decrease with this volume in a manner which is roughly independent of the details of the (short-range) repulsion.[28] (Thus this could be due to a hard or a soft core or a nonlocal potential, so long as it gives the required correlations.[29]) Thus for $a \approx -2$ F, $r_0 \approx 3-4$ F, an increase in the hard-core radius c from 0.0 to 0.6 F gives a decrease in D_s from about 55 MeV to about 35 MeV (for $k_F = 1.366$ F^{-1}) – a drop of about 20 MeV. Secondly, the p-state contribution D_p can be quite large and equal to about 20 MeV for a p-state interaction equal to the s-state one. (Thus a state-independent potential without any hard core would give a very large well depth $D \approx 75$ MeV for $a \approx -2$ F, $r_0 \approx 3-4$ F.) One may thus reduce D by weakening the p-state interaction. For example, Tang and Herndon's potential H, for which $c = 0.6$ F and the ratio of p- to s-state strengths is $p/s = 0.6$, gives $D \approx 46$ MeV; but for $p/s = 0$, the result is only $D \approx 36$ MeV.

Higher-order effects, in particular the ρ^2 terms, are rather uncertain and could be significant, especially for large hard cores. If the rearrangement energy dominates the ρ^2 terms, then these could reduce D by about another 10%.[4,26]

Thus central forces (or mixtures of central and tensor forces) consistent with the Λp scattering data could give agreement with the phenomenological well depth if there is a strong short-range repulsion corresponding to a large hard core of radius $c \approx 0.6$ F and if also the p-state interaction is very weak and close to zero. However, such forces will still overbind $_\Lambda He^5$, since only the s-state interaction is effective for this. (As is well known, triplet s-state interactions that fit the scattering give too much binding for $_\Lambda He^5$.) Two mechanisms that have been proposed to deal with this difficulty are repulsive three-body ΛNN forces[30] and suppression of the coupling of the ΛN to the ΣN channel.[31] The two are closely related and to some extent equivalent.[30] Here we consider only the latter.

The coupling of the ΛN to the ΣN channel is quite possibly very important, both through the strong and long-range OPE coupling potential, which has predominantly a tensor character, and perhaps also through the shorter-range but strong central component of the ρ-meson exchange potential. The qualitative considerations concerning the effect of the nuclear medium on these couplings are quite similar to those discussed for tensor forces, except that now one has an additional gap of 77 MeV due to the $\Sigma\Lambda$ mass difference. Reaction-matrix calculations indicate that one could readily obtain an appreciable suppression of the ΛN-ΣN coupling in nuclear matter, corresponding to a reduction in the well depth by as much as 10–15 MeV.[32]

If one accepts suppression of this order of magnitude, then one can tolerate a smaller hard-core radius and a weakened but still appreciable p-state interaction. Thus Tang and Herndon's potential E ($c = 0.45$ F, $p/s = 0.6$) gives $D \approx 52$ MeV. The well depth could then be reduced to $D \approx 37-42$ MeV by suppression of the ΛN-ΣN coupling. Suppression of the ΛN tensor force and the contribution from the ρ^2 terms could perhaps give a further reduction of about 5 MeV, which would bring the calculated well depth into agreement with the phenomenological one.

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APPENDIX. THE SECOND-ORDER PERTURBATION-THEORY EXPRESSION FOR THE WELL DEPTH

Using the methods of Euler[20] allows one to reduce Eq. (3.5) to Eq. (3.10). The latter involves only a one-dimensional integral with an algebraic expression for the factor I_2 in the integrand. This reduction is in fact strictly possible only if $\overline{M}_N = M_N^*$, i.e., if the term involving n^2 in the denominator of Eq. (3.5) vanishes. However, this term if nonzero has usually only a rather small effect, and it is usually a quite good approximation to replace n^2 by its average $\overline{n^2}$ over the Fermi sea, and then combine this term with the gap Δ. This approximation then corresponds to the replacement

$$\Delta \to \overline{\Delta} = \Delta - \frac{\overline{M}_N - M_N^*}{M_N^*}\,\overline{T}_F\,,$$

where $\overline{T}_F = 3k_F^{\,2}/5\overline{M}_N$. (In fact, the numerical results of Sec. 3 are for $\overline{M}_N = M_N^*$ and the correction then vanishes.) For reasonable values of Δ the correction is small.

Since $\overline{M}_\Lambda \neq \overline{M}_N$, it is convenient to introduce the new variable $s = t/\beta$, where $\beta = 2\overline{\mu}/\overline{M}_N$, and also the quantity $\epsilon = 1-\beta$. (Thus for equal masses, one has $s=t$, $\beta=1$, and $\epsilon=0$.) It is also convenient to use the dimensionless gap $\delta = \beta^{-2}(2\overline{\mu}/\hbar^2 k_F^{\,2})\overline{\Delta}$. More explicitly, the function $I_2(t)$ in Eq. (3.10) then also depends on β and δ. Consequently, we define

$$I_2(t) \equiv I_2(t,\beta,\delta) = t\beta^{-1}\,\bar{I}_2(s,\epsilon,\delta)\,.$$

The function $\bar{I}_2$ is given by the following expressions:

For $0 \leq t \leq 2$,

$$\bar{I}_2 = \tfrac{1}{2}s(1-\epsilon)[1+\tfrac{1}{2}s(1+\epsilon)] + \tfrac{1}{2}\delta(1-\epsilon) - \tfrac{1}{2}[(s^2+2\delta)J_1 - (\epsilon^2 s^2 + 2\epsilon\delta)J_2 + (1-\delta^2 s^{-2})J_3]\,,$$

where:

$$J_1 = \ln\left[\frac{1+s+\delta s^{-1}}{\frac{1}{2}(1+\epsilon)s+\delta s^{-1}}\right],$$

$$J_2 = \ln\left[\frac{1+\epsilon s+\delta s^{-1}}{\frac{1}{2}(1+\epsilon)+\delta s^{-1}}\right],$$

$$J_3 = \ln\left[\frac{1+\epsilon s+\delta s^{-1}}{1+s+\delta s^{-1}}\right].$$

For $2 < t < \infty$,

$$\bar{I}_2 = (s+\delta) + \tfrac{1}{2}[1-(s+\delta s^{-1})^2]\ln\left(\frac{s+1+\delta s^{-1}}{s-1+\delta s^{-1}}\right).$$

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[15]One has $g_\sigma{}^2 = g_{NN\sigma} \times g_{\Lambda\Lambda\sigma}$ in terms of the $NN\sigma$ and $\Lambda\Lambda\sigma$ constants and with neglect of some small recoil terms. If it is assumed that the σ meson is a unitary singlet, then $g_{NN\sigma} = g_{\Lambda\Lambda\sigma}$. For our purposes this assumption is unnecessary. However, equality of the two coupling constants is in fact consistent with a phenomenological analysis of the singlet ΛN and $\Lambda\Lambda$ interactions (D. A. Rote and A. R. Bodmer, unpublished).

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Properties of the *K* Matrix in Nuclear-Reaction Theory*

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We derive expressions for the K matrix for the general case where the Hamiltonian is separated as $H = H_0 + V$, and H_0 can have discrete as well as continuum states. It is shown that the correct handling of the bound states in the continuum eliminates one of the correction terms proposed by Tobocman and Nagarajan. In addition, some of the properties of the K matrix evaluated at complex energies are discussed.

I. INTRODUCTION

During the past few years there has been an increased use of the K matrix for both the theoretical and experimental study of nuclear reactions.[1-9] One reason for this increased use is that the K matrix treatment of nuclear reactions involves operators whose matrix representations are real, and this property simplifies numerical calculations. Furthermore, simple approximations and parametrizations for the K matrix do not destroy the