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Magnetic Dipole Transition Probabilities of Deformed Odd-Mass Nuclei*

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Based on the ground-state magnetic moments, the magnetic dipole transition probabilities in the ground-state rotational bands of deformed odd-mass nuclei are analyzed assuming that the rotational gyromagnetic ratios are given by a theoretical expression derived previously.

According to the collective model of Bohr and Mottelson,[1,2] the magnetic properties of the ground-state rotational band (with ground-state angular momentum $I_0 \neq \frac{1}{2}$) of a deformed odd-mass nucleus are characterized by the intrinsic gyromagnetic ratio g_K and the rotational gyromagnetic ratio g_R. These two parameters are derived primarily from the ground-state magnetic moment μ_0 and the magnetic dipole transition probability $B(M1;\, I_i \rightarrow I_f)$ between any two states in the rotational band and with angular momenta I_i and I_f, respectively, through the following model-dependent relations[2,3]:

$$\mu_0 = \frac{I_0}{I_0+1} g_R + \frac{I_0^{\,2}}{I_0+1} g_K \,, \tag{1}$$

$$M_0 = I_0 (g_K - g_R) \,, \tag{2}$$

$$M_0 = \pm \left[\frac{4\pi B(M1;\, I_i \rightarrow I_f)}{3\langle I_i 1 I_0 0 | I_i 1 I_f 0\rangle^2} \right]^{1/2} , \tag{3}$$

where $\langle I_i 1 I_0 0 | I_i 1 I_f 0\rangle$ is a vector-addition coefficient. The sign of M_0 in Eq. (3) is determined by

$$\operatorname{sgn}\epsilon = \operatorname{sgn} \frac{g_K - g_R}{Q_0} \,, \tag{4}$$

where Q_0 is the intrinsic quadrupole moment and ϵ the ratio between the electric quadrupole and magnetic dipole matrix elements for the transition.

The values of g_K and g_R derived from Eqs. (1)–(4) based on the experimental values of μ_0 and M_0 serve two practical purposes. Firstly, they may be used in predicting the magnetic moments of the excited states and the probabilities for other magnetic dipole transitions in the ground-state rotational band. These predictions are, so far, in fairly good agreement with measurements and consequently lend strong support to the collective model.[4,5] Secondly, they may be taken as empirical values for testing any theoretical calculations of g_K and g_R. (Here we reserve the term empirical value for any quantity not directly measured but semiempirically calculated.) These theoretical calculations are more or less independent of Eqs. (1)–(4) and therefore may add insight in understanding the nuclear structure.

Theoretical calculations of g_R for odd-mass nuclei have been considered by several investigators.[6-8] However, in this paper, we are particularly interested in the following macroscopic expression[8]:

$$g_R = \frac{3}{5J} Z M_p R_0^{\,2} \delta (1 - \tfrac{2}{3}\delta + \cdots) \,, \tag{5}$$

where Z is the atomic number, M_p the mass of the proton, R_0 the mean radius of the charge distribution, J the moment of inertia, and δ the deformation parameter. The remarkable feature of this expression is that g_R, J, and Q_0 are interrelated in a self-consistent way.[8] Furthermore, this expression is supposed to be valid for both even-even and odd-mass nuclei.[8]

The reliability of Eq. (5) has been empirically tested previously,[8] following the conventional analysis as discussed in the first two paragraphs of the present paper. However, in order to evaluate various theoretical calculations of g_R more effectively, we choose a different procedure of analysis which is described below.

By taking advantage of the fact that the experimental value of μ_0 is in general very accurate in comparison with the experimental accuracy of M_0, we first calculate g_K from Eq. (1), using the experimental value of μ_0 and the theoretical value of g_R [Eq. (5)]. Then, with this empirical value of g_K, we derive M_0 from Eq. (2) and compare it with the experimental value. Since the experimental uncertainty of M_0 is now completely separated out from the semiempirical calculation, the accuracy of this prediction may be a much more meaningful measure of the reliability of the theoretical calculation of g_R. This simple procedure of analysis needs no further elaboration and will be directly applied in predicting M_0 except for a slight modification due to the inclusion of a spin-orbit correction for odd-proton nuclei (see below).

Before taking into account the spin-orbit correction, let us consider the theoretical calculation of g_K. According to Nilsson's formalism, g_K in the ground-state rotational band (with $I_0 \neq \frac{1}{2}$) is given by[9,10]

$$g_K = g_l + I_0^{-1}(g_s - g_l)\langle s_{z'}\rangle \,. \tag{6}$$

Here g_l and g_s are the gyromagnetic ratios associated with the orbital motion and the spin $\vec{s}$ of the unpaired nucleon, and $\langle s_{z'}\rangle$ is the expectation value of the component of $\vec{s}$ along the nuclear symmetry axis z'.

In principle, we can certainly carry out the above analysis starting with the theoretical value of g_K or, much better, using the theoretical values of g_K and g_R to check the experimental values of μ_0 and M_0. This is, however, not yet possible in practice, because the empirical values of g_K deviate considerably from the theoretical predictions[11] if g_s in Eq. (6) is set equal to the free-particle value g_s^{free} ($g_s^{\text{free}} = 5.59$ for a proton and -3.83 for a neutron), and there seems to be no complete theory which can predict accurately what value g_s should have (except perhaps for the conjecture $g_s \leq g_s^{\text{free}}$, which may be valid in general).

Nevertheless, the aforementioned discrepancy between the predicted and the empirical values of g_K is usually attributed to the polarization of the unsaturated spins in the core,[11,12] which is supposed to reduce the magnitude of g_s below that of the free-particle value. Previous analyses[11,12] indicated that $g_s \simeq 0.6 g_s^{\text{free}}$ might be a good approximation, often taken as an indication of the success of the explanation. It will be seen, however, that even from a purely empirical point of view the accuracy of this approximation appears to be doubtful.

Furthermore, the theory of nuclear magnetic moments is not entirely satisfactory at this stage. In particular, the contributions from the two-body interaction currents such as meson exchange,[13] polarization, two-body spin-orbit interaction,[13,14] etc. may not be calculated without uncertainty, and the entangling of these two-body contributions is probably an extremely difficult task. Therefore, treating all possible two-body contributions in an empirical way by regarding g_s as a parameter in Eq. (6) (this is essentially the point of view taken by Boer and Rogers[11] and Bochnacki and Ogaza[12]), we shall limit ourselves to the contribution from the one-body spin-orbit interaction in the average potential (referred to as the spin-orbit correction), which may be less uncertain and probably the only appreciable one-body contribution we may think of at present.

Following a previous treatment of the magnetic moment correction due to spin-orbit interaction in deformed nuclei,[15] we may easily show that including spin-orbit correction Eq. (1) and Eq. (2) should be replaced by

$$\mu_0 = \frac{I_0}{I_0+1} g_R + \frac{I_0^2}{I_0+1}(g_K + g_{so}) \,, \tag{7}$$

$$M_0 = I_0(g_K + g_{so} - g_R) \,. \tag{8}$$

Here g_{so} is the effective gyromagnetic ratio associated with the spin-orbit interaction.[15]

It is thus clear that the effect of the spin-orbit interaction is simply to modify the value of g_K (see Ref. e, Table I), and therefore should be taken into account in evaluating other contributions such as the polarization effect. It is to be noted, however, that so far as the predicted value of M_0 is concerned, Eqs. (1), (2) and Eqs. (7), (8) are actually indistinguishable.

The numerical results are presented in Table I, in which two sets of theoretical values[10] of g_K are also shown for comparison. In view of the experimental uncertainties involved (not only of M_0 but also of Q_0 and J), the agreement between the predicted and the experimental values of M_0 may be regarded as fairly reasonable. Furthermore, the general trend of the experimental data of M_0 seems to be well accounted for.

The calculated value of M_0 in the above analysis turns out to be sensitive to the "input" value of g_R, and therefore a more effective test of the theoretical calculation of g_R may be obtained. It is easy to

TABLE I. Magnetic dipole transition probabilities of deformed odd-mass nuclei. The first column lists the nuclei for which magnetic dipole transition probabilities ($|M_0/I_0|$) in the ground-state rotational bands have been analyzed. Column two and column three give the ground-state angular momentum and the ground-state magnetic moment, respectively. Column four gives the magnetic moment correction due to spin-orbit interaction with the corresponding gyromagnetic ratio in parentheses (these values are taken from Ref. 15 with improved values for Am^{241} and Am^{243}). Column five lists the theoretical values of the rotational gyromagnetic ratio g_R according to Eq. (5), taken from Ref. 8. Column six gives the empirical values of the intrinsic gyromagnetic ratio g_K calculated from Eq. (1) or Eq. (7). Columns seven and eight list two sets of theoretical values of g_K taken from Ref. 10, except for Re^{187} (Ref. 11). The predicted values of $|M_0/I_0|$ according to Eq. (2) or Eq. (8) are listed in column nine. The experimental values of $|M_0/I_0|$ given in column ten and column eleven are those derived from the measured magnetic dipole transition probability between the first excited state and the ground state, and between the second excited state and the first excited state, respectively. The experimental errors are not shown. The experimental values of $|M_0/I_0|$ are taken from Ref. 5 except for Hf^{177}, Hf^{179}, Re^{185}, and Re^{187}.

Nucleus (Ref. a)	I_0 (Ref. b)	μ_0 (Ref. c)	$\mu_{so}(g_{so})$	g_R Eq. (5) (Ref. d)	g_K Eqs. (1), (7) (Ref. e)	g_K^{theor} [Eq. (6)] $g_s = g_s^{free}$	$g_s = 0.6 g_s^{free}$	$\|M_0/I_0\|$ Eqs. (2),(8) (Ref. d)	$\|M_0/I_0\|$ Exp. (Ref. f)	
${}_{64}Gd^{155}$	$\frac{3}{2}$	−0.242	···	0.26	−0.44 [g]	−0.376	−0.226	0.70	0.807	0.95
Gd^{157}	$\frac{3}{2}$	−0.3225	···	0.24	−0.52 [g]	−0.438	−0.263	0.76	0.794	0.77
${}_{66}Dy^{161}$	$\frac{5}{2}$	−0.472	···	0.14	−0.32	−0.424	−0.254	0.46	···	0.55
Dy^{163}	$\frac{5}{2}$	0.635	···	0.23 [h]	0.26 [h]	0.313	0.188	0.03	0.021	···
${}_{68}Er^{167}$	$\frac{7}{2}$	−0.5647	···	0.19	−0.26	−0.378	−0.227	0.45	0.441	0.447
${}_{70}Yb^{173}$	$\frac{5}{2}$	−0.6775	···	0.28	−0.49	−0.528	−0.317	0.77	0.768	0.773
${}_{71}Lu^{175}$	$\frac{7}{2}$	2.2300	0.43 (0.16)	0.29	0.58	0.404	0.695	0.45	0.417	0.378
${}_{72}Hf^{177}$	$\frac{7}{2}$	0.61	···	0.28	0.14	0.397	0.238	0.14	~0.09 (average) [i]	
Hf^{179}	$\frac{9}{2}$	−0.47	···	0.25	−0.18	−0.348	−0.209	0.43	0.470 [j]	0.375 [j]
${}_{73}Ta^{181}$	$\frac{7}{2}$	2.3600	0.43 (0.16)	0.31	0.62	0.403	0.694	0.47	0.487	0.46
${}_{75}Re^{185}$	$\frac{5}{2}$	3.1720	−0.38 (−0.21)	0.32	1.86	1.893	1.457	1.33	···	1.19 [j]
Re^{187}	$\frac{5}{2}$	3.2040	−0.38 (−0.21)	0.35	1.87	1.88	···	1.31	1.35 [j]	1.21 [j]
${}_{95}Am^{241}$	$\frac{5}{2}$	1.4000	0.31 (0.17)	0.23	0.52	0.436	0.711	0.46	···	···
Am^{243}	$\frac{5}{2}$	1.4000	0.31 (0.17)	0.23	0.52	0.439	0.713	0.46	···	···

[a] Excluded here are a few cases of odd-proton nuclei (in particular, Eu^{153}, Tb^{159}, and Ho^{165}) for which the microscopic calculations are reasonably successful but present calculations have failed. Why the unpaired protons in these few cases make such a great difference is not yet clearly understood.

[b] Taken from Nuclear Data Sheets, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Sciences-National Research Council, Washington, D. C.).

[c] Taken from Ref. 4 except for Ta^{181}, Re^{185}, Re^{187}, Am^{241}, and Am^{243} (Nuclear Data Sheets).

[d] If the theoretical values of g_R in column five are substituted by the microscopic calculations (alternative I) of Ref. 7, we would have $|M_0/I_0| = 0.70$, 0.89, 0.32, 0.08 (negative sign of M_0), 0.49, 0.81, 0.39, 0.14, 0.27, 0.40, 1.15, 1.12, etc. in column nine. These predictions are obviously in much poorer agreement with the experimental values. This remark essentially holds also for the microscopic calculations of Ref. 6 and those according to alternative II in Ref. 7. It thus appears that the accuracy of the predictions of Eq. (5) is in general much better than the accuracy of the microscopic calculations (however, see Ref. a).

[e] These empirical values of g_K do not differ appreciably from previous tabulations (Refs. 3 and 11) except for odd-proton nuclei in which part of the role of g_K is taken up by g_{so}. It may be of interest to note that spin-orbit interaction appears to have the tendency to restore g_s for odd-proton nuclei to the free-particle value g_s^{free}. This is particularly noticeable in Re^{185} and Re^{187}. No contradiction with the sign of M_0 appears (however, see Ref. h).

[f] The constancy of $|M_0/I_0|$ for all transitions in a rotational band is actually a test of the validity of the collective model. It may be of interest to note that best agreement between the predicted and the experimental values of $|M_0/I_0|$ just occurs in such cases (column ten and column eleven).

[g] A comparison of these two empirical values of g_K with their corresponding theoretical values listed in column seven and column eight leads to the virtually absurd conclusion, namely, $g_s/g_s^{free} > 1$. This difficulty cannot be resolved with previous empirical values of g_K which do not differ appreciably from present results. However, the large discrepancies between some of the theoretical values of g_K listed in column seven and those reported in Ref. 11 should be noted, which may partly arise from the inclusion of Y_4^0 deformation in Ref. 10. For Gd^{155} and Gd^{157}, $g_K^{theor} = -0.79$ with $g_s = g_s^{free}$ was reported in Ref. 11.

[h] The predicted positive sign of M_0 is in agreement with Ref. 4. This is probably a case of particular interest (in which g_K is positive and the difference between g_K and g_R is so small) and should be experimentally checked.

[i] Taken from A. J. Haverfield *et al.*, Nucl. Phys. A94, 337 (1967).

[j] Taken from Ref. 11.

check that much poorer agreement between the predicted and the experimental values of M_0 would result if the above analysis was carried out with the theoretical values of g_R according to the microscopic calculations by Grin' and Pavlichenkov[6] and Prior, Boehm, and Nilsson[7] (see Refs. a and d, Table I).

Now, by substituting the theoretical value of g_K from column seven of the table (or from column eight of the table) and $g_s = g_s^{\text{free}}$ (or $g_s = 0.6 g_s^{\text{free}}$) into Eq. (6) and assuming the free-particle value for g_l ($g_l = 1$ for a proton and zero for a neutron), the value of $\langle s_{z'} \rangle$ can easily be calculated. Then, by using this calculated value of $\langle s_{z'} \rangle$ and the empirical value of g_K (column six of the table) and again assuming the free-particle value for g_l, the empirical value of g_s can easily be calculated from Eq. (6). It is a simple matter to check that the ratio g_s / g_s^{free} so determined varies appreciably in going from nucleus to nucleus. It is of particular interest to note that for Dy^{163}, Er^{167}, Yb^{173}, Ta^{181}, and Re^{187}, we have $g_s / g_s^{\text{free}} = 0.83$, 0.69, 0.93, 0.70, and 0.99, respectively, and in each of these cases the predicted and the experimental values of M_0 agree very well. It thus appears that the usual approximation $g_s \simeq 0.6 g_s^{\text{free}}$ may be rather inadequate and more effort should be directed to the understanding of the intrinsic gyromagnetic ratio g_K.

In conclusion, we may say that the present analysis does lend support to Eq. (5), the simplicity of which may facilitate further investigation of the magnetic properties of deformed nuclei.

However, a few additional remarks are in order. First of all, the reasonable accuracy of Eq. (5) as exhibited in the preceding paper[8] and the table should not be mistaken as validity in the absolute sense; or, in other words, the possible improvement of Eq. (5) should not be excluded. For instance, rotation-vibration interaction and high-order deformations may be taken into account, and a more accurate calculation of the mean radius of the charge distribution may be made. Secondly, it would be equally unreasonable to conclude that the microscopic calculations are invalid, although they may not be very accurate at present, as demonstrated in the present analysis. On the other hand, the basic assumptions made in the derivation of Eq. (5) have no apparent harmony with the microscopic theory.[8] Therefore, nontrivial questions concerning the reconciliation of these two different approaches certainly arise, which may have the advantage of providing insight in understanding our problem.

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