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Binding Energies of the p-Shell $\Lambda\Lambda$ Hypernuclei*

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Using the results of a shell-model analysis of the p-shell hypernuclei by the authors, the binding energies of the two Λ particles in the p-shell $\Lambda\Lambda$ hypernuclei are calculated. A consistent account of the binding energies of the two known $\Lambda\Lambda$ hypernuclei, ${}_{\Lambda\Lambda}^{6}$He and ${}_{\Lambda\Lambda}^{10}$Be (or ${}_{\Lambda\Lambda}^{11}$Be) is obtained. Close agreement with the results of other investigators who used different methods is also pointed out. The effects of the noncentral potentials in the effective two-body Λ-N interaction are discussed.

1. INTRODUCTION

An intensive study of the Λ-Λ interaction and the structure of the $\Lambda\Lambda$ hypernuclei has been made,[1] mainly through the variational analyses of the two discovered $\Lambda\Lambda$ hypernuclei, namely, one ${}_{\Lambda\Lambda}^{10}$Be, or ${}_{\Lambda\Lambda}^{11}$Be, event by Danysz *et al.*[2] and one ${}_{\Lambda\Lambda}^{6}$He event by Prowse.[3] Detailed analyses by Tang and Herndon[4] and also Ali and Bodmer[5] showed that if the event by Danysz *et al.* were ${}_{\Lambda\Lambda}^{10}$Be, the binding energies of both discovered events could not be accounted for consistently unless the range of the Λ-N interaction is shorter than the one-kaon Compton wavelength. However, if the event by Danysz *et al.* was alternatively interpreted as ${}_{\Lambda\Lambda}^{11}$Be, as originally suggested by Ali and Bodmer,[5] calculations by Ali and Kok[6] gave a consistent result for the usual two-pion as well as one-kaon range of the Λ-N forces when the effect of core compressibility of ${}_{\Lambda\Lambda}^{11}$Be was considered. The problem of the range of the Λ-N forces was further investigated by Ali, Kok, and Grypeos[7] in the study of ${}_{\Lambda\Lambda}^{14}$C, the results of which, however, are yet to be checked by further observation of ${}_{\Lambda\Lambda}^{14}$C events. Calculations of the binding energy of ${}_{\Lambda\Lambda}^{14}$C, as well as ${}_{\Lambda\Lambda}^{18}$O, have also been made by Anantharayanan[8] using an entirely different method.

In this paper we will make a shell-model analysis of the p-shell $\Lambda\Lambda$ hypernuclei, based on the effective Λ-N interaction deduced from the shell-model analysis of the ordinary hypernuclei recently made by the authors.[9] The success of the latter analysis lies in the fact that (1) the binding energies of the p-shell hypernuclei are accounted for consistently by fitting a few parameters independently of the specific form of the Λ-N potentials, namely, the potential integrals of the Λ-N interaction; and (2) that the exceptional behavior of the binding energy for the "spinless"-core hypernuclei, e.g., ${}_{\Lambda}^{9}$Be and ${}_{\Lambda}^{13}$C, is explained as due to the existance of noncentral, especially tensor, forces in the effective two-body Λ-N interaction, without the need for specific consideration of the size effect or distortion of the cores. For $\Lambda\Lambda$ hypernuclei, an additional parameter is introduced, the potential integral of the Λ-Λ interaction. Thus the further success of the shell-model analysis relies heavily on consistently accounting for the binding energies of the p-shell $\Lambda\Lambda$ hypernuclei.

In Sec. 2 it is shown that a consistent account of the two discovered $\Lambda\Lambda$ hypernuclei is obtained with the same value of the Λ-Λ potential integral. The binding energies of other p-shell $\Lambda\Lambda$ hypernuclei are evaluated, and a comparison with the results of calculations by other authors is made. In Sec. 3 the effect of the noncentral potentials in the Λ-N interaction on the binding energy as well as on the core-nuclear structure are discussed. In Sec. 4 a concluding remark is made concerning the shell-model analysis.

2. CALCULATIONS

For the binding-energy calculation we choose the basis states for the construction of the wave function of a $\Lambda\Lambda$ hypernuclei (isospin T, spin J) to be those formed from the product of the ${}^1S_0$ state of the two Λ particles and the various core states (βTJ), where β designates various energy levels of the core. The binding energy then corresponds to the lowest eigenvalue of the following energy matrix elements:

$$H_{TJ}(\beta TJ, \bar{\beta}TJ) = \epsilon_{\Lambda\Lambda}\delta_{\beta\bar{\beta}} + E(\beta TJ)\delta_{\beta\bar{\beta}} + \frac{1}{2J+1}\sum_{J_\Lambda = J\pm\frac{1}{2}}(2J_\Lambda+1)[\mathcal{V}_{TJ_\Lambda}(\beta TJ, \bar{\beta}TJ) - B_s\delta_{\beta\bar{\beta}}]\,. \quad (1)$$

Here, $\epsilon_{\Lambda\Lambda}$ denotes the potential integral of the Λ-Λ interaction with the two Λ particles in the ${}^1S_0$ state; $E(\beta TJ)$ is the excitation energy of the core for the state (βTJ); the $\mathcal{V}_{TJ_\Lambda}(\beta TJ, \bar{\beta}TJ)$ represent the matrix elements of the Λ-N potentials between the p-shell nucleons and a Λ particle and are expressed in terms of the potential integrals of the Λ-N interaction in Ref. 9; and B_s denotes the binding energy of $^{5}_{\Lambda}$He, i.e., 3.08 MeV.

In the shell-model approach we except $\epsilon_{\Lambda\Lambda}$ to be nearly constant throughout the p shell, as is the case for the potential integrals of the Λ-N interaction. In fact, from the binding-energy data of the two discovered events we obtain: $\epsilon_{\Lambda\Lambda} = -4.6 \pm 0.6$ MeV from $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\mathrm{He}) = 10.8 \pm 0.6$ MeV,[2] and $\epsilon_{\Lambda\Lambda} = -4.8 \pm 0.5$ MeV or -5.0 ± 0.6 MeV from $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{10}\mathrm{Be}) = 17.6 \pm 0.5$ MeV[3] [based on the recent value $B_{\Lambda}({}_{\Lambda}^{9}\mathrm{Be}) = 6.63 \pm 0.04$ MeV][10] or $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{11}\mathrm{Be}) = 20.2 \pm 0.6$ MeV[3] [based on $B_{\Lambda}({}_{\Lambda}^{10}\mathrm{Be}) = 9.10 \pm 0.64$ MeV].[11] Hence within the present experimental accuracy we may take $\epsilon_{\Lambda\Lambda}$ to be constant throughout the p shell, although a slight increase in the absolute value of $\epsilon_{\Lambda\Lambda}$ of a few tenths of 1 MeV for heavier $\Lambda\Lambda$ hypernuclei may be expected, due to the effect of the increasing number of nucleons. Accordingly, we will take ${}_{\Lambda\Lambda}^{6}$He as the reference point for calculating the binding energies $B_{\Lambda\Lambda}$ of the p-shell $\Lambda\Lambda$ hypernuclei. The results are shown in Table I. Calculations for $A = 7$ and 8 $\Lambda\Lambda$ hypernuclei are not given, because of the poorer determination, experimentally as well as theoretically, of the binding energies of the $A = 6$ and 7 hypernuclei.

First of all, it is interesting to notice that the shell-model calculation consistently accounts for the binding energy of the event observed by Danysz *et al.* by interpreting it either as ${}_{\Lambda\Lambda}^{10}$Be or as ${}_{\Lambda\Lambda}^{11}$Be with a single Λ-N interaction. This is in contrast to the results of calculations by variational methods[4-6] mentioned in the preceding section, in which different ranges for Λ-N interaction have to be considered in the two interpretations. In fact, the ambiguity in the interpretation arises because the determination of the binding energy is mainly from the decay process

$$ {}_{\Lambda\Lambda}^{A}Z \rightarrow \pi^- + p + {}^{A-1}_{\Lambda}Z \tag{2} $$

of which the Q values, or rather the binding energies of the second Λ particle,

$$ B_{\Lambda}^{*}({}_{\Lambda\Lambda}^{A}Z) = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{A}Z) - B_{\Lambda}({}^{A-1}_{\Lambda}Z) = 37.6 \text{ MeV} - Q\,, \tag{3} $$

for both ${}_{\Lambda\Lambda}^{10}$Be and ${}_{\Lambda\Lambda}^{11}$Be are accidentally coincident (experimental values[3] being 11.0 and 11.1 MeV as compared with the calculated ones, 10.8 and 10.7 MeV). Notice that a similar situation occurs in the three $\Lambda\Lambda$ hypernuclei, ${}_{\Lambda\Lambda}^{12}$B, ${}_{\Lambda\Lambda}^{13}$B, and ${}_{\Lambda\Lambda}^{14}$B, which have nearly the same B_{Λ}^{*} values at about 12.0 MeV. Hence further identification of these $\Lambda\Lambda$ hypernuclei should be made from other kinematical determinations.

The calculated binding-energy values $B_{\Lambda\Lambda}$ in the table can not be compared with experiment until more events are discovered. However, a comparison with a few theoretical estimates of the "spinless"-core $\Lambda\Lambda$ hypernuclei by other authors using different approaches is also of great interest. For ${}_{\Lambda\Lambda}^{14}$C, our predicted value, $B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{14}\mathrm{C}) = 24.8 \pm 0.6$ MeV, is found to be quite compatible with that of an earlier calculation by Ali, Kok, and Grypeos[7] (25.2 ± 0.5 MeV for the two-pion range of the Λ-N interaction, and 24.5 ± 0.5 MeV for the one-kaon range) in a three-body model which did not consider core distortion. Allowing an increase of a few tenths of 1 MeV for the absolute value of $\epsilon_{\Lambda\Lambda}$, as suggested at the beginning of this section, our calculation thus gives strong support for the longer range of the Λ-N interaction. As will be shown in the next section, our calculation also confirms the assumption of a fair rigidity of the core, ^{12}C. It is also to be noted that our value for $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{14}\mathrm{C}) = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{14}\mathrm{C}) - 2B_{\Lambda}({}_{\Lambda}^{13}\mathrm{C}) = 3.8 \pm 0.6$ MeV is in good agreement with that estimated by Ananthanarayanan[8] (i.e., 3.75 MeV). For ${}_{\Lambda\Lambda}^{18}$O, however, the latter author gave an estimate $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{18}\mathrm{O}) = 3.5$ MeV, which is about 1 MeV smaller than our calculated value.

TABLE I. Binding energy and its contributing terms for the p-shell $\Lambda\Lambda$ hypernuclei. Column two denotes the isospin spin. Column three gives the binding energies of the two Λ's. Column four gives twice the binding energy for the ordinary hypernuclei, on which the calculations of column three are based. Column five denotes the contribution from the Λ-N interaction, whereas column six denotes the effect of the excitation of the core from the ground-state structure.

${}_{\Lambda\Lambda}^{A}Z$	T J	$B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{A}Z)$ (MeV)	$2B_{\Lambda}({}^{A-1}_{\Lambda}Z)$ (MeV)	$B_{\Lambda N}({}_{\Lambda\Lambda}^{A}Z)$ (MeV)	$B_{NN}({}_{\Lambda\Lambda}^{A}Z)$ (MeV)
${}_{\Lambda\Lambda}^{9}$Li	$\frac{1}{2}$ $\frac{3}{2}$	15.7	13.6	11.3	−0.2
${}_{\Lambda\Lambda}^{10}$Li	1 2	18.2	16.5	13.9	−0.3
${}_{\Lambda\Lambda}^{10}$Be	0 0	17.4	13.3	13.1	−0.3
${}_{\Lambda\Lambda}^{11}$Be	$\frac{1}{2}$ $\frac{3}{2}$	19.8	18.2	15.5	−0.3
${}_{\Lambda\Lambda}^{12}$B	0 3	22.1	20.3	17.8	−0.3
${}_{\Lambda\Lambda}^{13}$B	$\frac{1}{2}$ $\frac{3}{2}$	23.4	22.2	19.2	−0.4
${}_{\Lambda\Lambda}^{14}$B	1 1	24.4	25.0	20.0	−0.2
${}_{\Lambda\Lambda}^{14}$C	0 0	24.8	21.0	20.8	−0.6
${}_{\Lambda\Lambda}^{15}$C	$\frac{1}{2}$ $\frac{1}{2}$	26.3	26.8	22.0	−0.3
${}_{\Lambda\Lambda}^{16}$N	0 1	27.2	32.0	22.6	−0.0
${}_{\Lambda\Lambda}^{17}$O	$\frac{1}{2}$ $\frac{1}{2}$	27.0	27.6	22.4	−0.0
${}_{\Lambda\Lambda}^{18}$O	0 0	26.8	22.2	22.2	−0.0

3. DISCUSSION

The $\Lambda\Lambda$-hypernuclei wave function is, by construction, the sum of the products of the wave functions of the two Λ particles at the $^{1}S_0$ state and those of the core nucleus at various energy states. It is found from the above binding-energy calculations that the core nuclei for all the p-shell $\Lambda\Lambda$ hypernuclei remain almost at their ground states when coupled to the two Λ particles (the probability of the core being at the ground state in each case is above 96%). This is expected in the shell model, since the two Λ's form a closed 1s shell, and therefore the interaction of the p-shell nucleons with the two Λ particles is effectively central.[12] The contribution to $B_{\Lambda\Lambda}$ of the slight "distortion" of the core nucleus from its ground state can be estimated by taking the expectation value B_{NN} of the second term in Eq. (1). The values of B_{NN} for the p-shell $\Lambda\Lambda$ hypernuclei are given in the table, and are indeed found to be extremely small.

The contribution to $B_{\Lambda\Lambda}$ of the Λ-N interaction is given by the expectation value, $B_{\Lambda N}$, of the third term in Eq. (1). As was shown in Ref. 9, the Λ-N interaction is highly noncentral and the noncentral part contributes a few MeV to the hypernuclei binding energy B_Λ. When a second Λ particle is added to the hypernuclei, the effect of the noncentral Λ-N potentials, as stated above, diminishes. Therefore, $B_{\Lambda N}$ will, in general, be smaller than $2B_\Lambda$ by a few MeV. This is, in fact, shown in the results of the calculation of $B_{\Lambda N}$ in the table. However, for the "spinless"-core $\Lambda\Lambda$ hypernuclei, such as ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$, ${}^{14}_{\Lambda\Lambda}\mathrm{C}$, and ${}^{18}_{\Lambda\Lambda}\mathrm{O}$, the $B_{\Lambda N}$ have values nearly equal to $2B_\Lambda$. This is due to the fact that the contribution of the noncentral Λ-N potentials to B_Λ in the corresponding "spinless"-core hypernuclei, i.e., ${}^{9}_{\Lambda}\mathrm{Be}$, ${}^{13}_{\Lambda}\mathrm{C}$, and ${}^{17}_{\Lambda}\mathrm{O}$, is exceptionally small, as was discussed in Ref. 9.

The apparent binding energy between the two Λ particles, i.e., $\Delta B_{\Lambda\Lambda}=B_{\Lambda\Lambda}-2B_\Lambda$, is thus given by

$$\Delta B_{\Lambda\Lambda} \simeq -\epsilon_{\Lambda\Lambda}-(2B_\Lambda-B_{\Lambda N}) . \tag{4}$$

For the "spinless"-core $\Lambda\Lambda$ hypernuclei, we have $\Delta B_{\Lambda\Lambda}\simeq -\epsilon_{\Lambda\Lambda}$, whereas for the remaining p-shell $\Lambda\Lambda$ hypernuclei, the values of $\Delta B_{\Lambda\Lambda}$ generally are less than $-\epsilon_{\Lambda\Lambda}$ by a few MeV and are found to decrease with the increase of the mass number of the $\Lambda\Lambda$ hypernuclei.[5,13]

4. REMARKS

In the usual variational analysis of the $\Lambda\Lambda$ hypernuclei[4-6] as well as the ordinary hypernuclei,[14] the importance of taking into account the effects of core distortion in the binding-energy calculations has been emphasized. For example, a specific treatment of the core of ${}^{9}_{\Lambda}\mathrm{Be}$ as well as ${}^{10}_{\Lambda\Lambda}\mathrm{Be}$ has been considered necessary, in view of the fact that the core itself is unbound and the binding energy of ${}^{9}_{\Lambda}\mathrm{Be}$ is exceptionally low compared with those of the neighboring hypernuclei. The Λ-N potentials used in the calculation, on the other hand, are assumed to be of simple form (usually only central). On the contrary, the shell-model approach does not really take the physical core distortion into account, since the potential integrals are assumed to be constant throughout the p shell. Instead, the binding energies for the hypernuclei and the $\Lambda\Lambda$ hypernuclei are consistently accounted for by the introduction of noncentral potentials in the effective two-body Λ-N interaction. Serious consideration of core distortion, such as core compression, can be brought out by allowing the potential integrals to vary with both mass number and the specific structure of each hypernuclei. This, however, requires a larger accumulation of experimental data, such as information about the excited levels of the hypernuclei and the $\Lambda\Lambda$ hypernuclei. Finally, the consideration of the deformation effects[15] of the cores would take the calculation beyond the scope of our shell model, since such effects are not considered in the Cohen-Kurath analysis[16] of the p-shell nuclei, on which our model is based.

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Macroscopic Self-Consistency of the Collective Model of Deformed Nuclei*

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It has been suggested, in effect, that in a deformed nucleus the number of protons (or the amount of charge) which are carried along by the rotational motion may, on the average, be approximated as the product of the atomic number Z and the deformation parameter δ. The theoretical justification of this suggestion is discussed, and a more accurate expression is obtained. Assuming that the interaction of the rotational motion with an external magnetic field is entirely due to the current associated with the amount of charge following the rotational motion, on the average, and also in accordance with the cranking approximation, a macroscopic expression for the rotational gyromagnetic ratio g_R is derived. This expression, supplemented by the usual macroscopic formula for the intrinsic quadrupole moment Q_0, may constitute a macroscopic self-consistency relation among the collective parameters in a rotational band, namely, Q_0, g_R, and the moment of inertia J. The values of g_R calculated from the experimental values of J and Q_0 are tabulated for the ground-state rotational bands of both even-even and odd-mass nuclei. The aforementioned macroscopic self-consistency is then tested by comparing these calculated values of g_R with both empirical values and previous microscopic calculations. According to the present approach, the well-known lowering of g_R from the usual fluid-model value is mainly due to the limitation on the number of protons which can follow the rotational motion, on the average. It is not clear, however, whether there is any direct connection between this limitation and the pairing interaction which seems to play a rather essential role in the current microscopic calculations. This puzzling situation is further illustrated by considering the moments of inertia of even-even nuclei.

I. INTRODUCTION

Rotational band structure has been experimentally established in the excitation spectra of nuclei in several regions of the Periodic Table. According to the collective model of Bohr and Mottelson,[1] the static and dynamic properties of each rotational band (with band-head angular momentum not equal to $\frac{1}{2}$) can be characterized by four parameters, namely, the moment of inertia J, the intrinsic quadrupole moment Q_0, the rotational gyromagnetic ratio g_R, and the intrinsic gyromagnetic ratio g_K, which can be related to certain measurable quantities such as $E2$ and $M1$ transition probabilities and the level spacings in the rotational band.

These characteristic parameters may also be calculated theoretically in terms of the intrinsic properties of the nucleus. Such a microscopic approach has had considerable success in recent years, particularly in the calculation of the moments of inertia and rotational gyromagnetic ratios of even-even nuclei.[2,3]

Although a microscopic approach has the advantage of testing the nuclear model in detail, we attempt to establish, in this paper, certain macroscopic relations among these parameters which may be directly verified with the experimental values. Such macroscopic relations, if they are valid, are certainly useful in the analysis of the experimental results but may also be of interest in evaluating the microscopic theories.

In Nilsson's formalism of the intrinsic motion,[4,5] the single-particle wave function depends critically on the deformation parameter δ which charac-