PHYSICAL REVIEW C VOLUME 2, NUMBER 4 OCTOBER 1970

New Theoretical Approach to Nuclear Heavy-Ion Scattering*

L. Wilets

Department of Physics, University of Washington, Seattle, Washington 98105

and

A. Goldberg and F. H. Lewis, Jr.

Lawrence Radiation Laboratory, University of California, Livermore, California 94550

(Received 21 April 1970)

Some recent calculations based on the statistical model have employed the adiabatic assumption inconsistently. In particular, the high central repulsive core in ion-ion collisions is due to a trial density function which leads to unphysically large densities in the close-in region.

The purpose of this note is to point out certain internal inconsistencies in a recent paper by Brueckner, Buchler, and Kelly (BBK).[1] We concern ourselves here with the theoretical calculation of the ion-ion potential, and not with the phenomenological analysis of the scattering data.

The starting point of the BBK calculation is the representation of the energy of the nuclear system as a functional of the density, further expanded in terms of gradients of the density.[2] In simplified notation,

$$E=\int\left\{\epsilon(\rho)+f(\rho)\,|\nabla\rho|^2+\tfrac{1}{2}e\rho_P\phi_c\right\}d\tau\,. \qquad (1)$$

The ground-state energy of the system is given by minimizing $E\{\rho(\vec{r})\}$ with respect to $\rho(\vec{r})$. The formal justification of this procedure has been given by Hohenberg and Kohn[3] who proved that the *ground-state* energy and density are obtained by minimizing a functional of the density. (This is an existence proof, since the functional is not given.) The object of BBK is to study E during a collision of two nuclei (e.g., ^{16}O-^{16}O) at relatively low velocity. Although not stated in the following terms, they treat the problem analogously to an atom-atom collision in which $E(R)$, obtained in the Born-Oppenheimer approximation, is the energy of the electrons as a function of the internuclear distance. The distinctions in the present case are:

(1) The energy E is obtained from Eq. (1) rather than by solving a Schrödinger equation.

(2) It is necessary to define some appropriate collision coordinate. At large distances the relative separation $\vec{R}$ of the projectile and target centers-of-mass is appropriate, and this is what BBK use. In the interaction region, this prescription becomes ambiguous and we must be prepared to use some general deformation parameters. For example, the matter quadrupole moment $Q=\langle 3z^2-r^2\rangle$ can describe not only the composite system but also the separated system where $Q\rightarrow cR^2$, where $c=2A_1A_2/(A_1+A_2)$. We could *define* $R=(Q/c)^{1/2}$ and use this R in all regions.[4] Let us do so below for the conceptual convenience of having a definitely defined coordinate. Thus it appears to us that BBK seek to obtain $E(R)$ as the adiabatic (ground-state) energy of the system as a function of $\vec{R}$ – i.e., under the constraint that the two composite particles have some prescribed separation or deformation. We concur that this is a sensible program.

In executing the above program, BBK do not in fact minimize $E\{\rho(\vec{r})\}$ with respect to $\rho(\vec{r})$ but rather evaluate $E\{\rho(\vec{r})\}$ under the assumption that

$$\rho(\vec{r})\approx\rho_1(\vec{r}-\tfrac{1}{2}\vec{R})+\rho_2(\vec{r}+\tfrac{1}{2}\vec{R})\,,$$

where ρ_1 and ρ_2 are the separated spherically symmetric nuclear densities (they further considered identical colliding nuclei). Their justification of this program is in the paragraph following their Eq. (5), which we quote verbatim:

"We now assume that the energy functional is given by our statistical approximation (1) and that the density function $\rho(r)$ is the optimal ground-state function (4) as obtained by the minimization of (1). We have thus assumed that the densities of the interacting nuclei superimpose adiabatically without distortion during the collision so that the total energy of the system is still expressed by the same energy functional (3). A justification of the assumption of adiabaticity lies in the fact that the kinetic energy of the nucleons inside the nucleus (≃42 MeV) is much larger than the kinetic energy associated with their relative motion during the collision (≃1 MeV). The omission of polarization effects in the elastic range may be justified by the large excitation energy of the 3^- state in O^{16} (6.13 MeV), which prevents easy distortion during the collision."

The above paragraph properly justifies the adiabatic assumption, but that is inconsistent with the assumption of superposition of densities of undeformed interacting nuclei. To illustrate the problem dramatically, consider $R=0$. Their approximate density (for $\rho_1=\rho_2$) is just $\rho(\vec{r})=2\rho_1(\vec{r})$. A variational calculation would yield a central density $\rho(0)\simeq\rho_1(0)$, not twice this value. In general, a variational calculation will not lead to densities much above normal. Their unrealistic *ansatz* is what is responsible for the large core in $E(R)$. Not only should there not be a large core, but $E(R=0)$ should be lower than $E(R=\infty)$ because fusion is exothermic in this region. The BBK interaction energy is $W(R)=E(R)-E(\infty)$; the nuclear part alone (neglecting Coulomb energy) should be negative at $R=0$.

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*Work performed under the auspices of the U. S. Atomic Energy Commission.

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New Low-Lying States in ^{57}Ni Observed in the ^{58}Ni(^{3}He, α)^{57}Ni Reaction*

R. W. Zurmühle, D. P. Balamuth, and James E. Holden
University of Pennsylvania, Philadelphia, Pennsylvania 19104
(Received 29 June 1970)

States in ^{57}Ni have been observed at excitations of 2.45 ±0.01 and 3.00 ±0.01 MeV using the ^{58}Ni(^{3}He, α)^{57}Ni reaction, confirming previous indications of their existence from ^{54}Fe($\alpha, n\gamma$)-^{57}Ni studies.

The study[1] of neutron-pickup reactions from ^{58}Ni and a recent angular-correlation study of ^{57}Ni performed at this laboratory using the ^{58}Ni(^{3}He, $\alpha\gamma$)-^{57}Ni reaction[2] showed that the low-lying states of ^{57}Ni excited in these reactions can be classified into two types. The first three states consist primarily of single neutrons occupying the $p_{3/2}$, $f_{5/2}$, and $p_{1/2}$ orbitals, respectively, outside a closed ^{56}Ni core. The other type of state observed consists of a neutron hole in the $f_{7/2}$ shell, and presumably a pair in the $p_{3/2}$, $f_{5/2}$, or $p_{1/2}$ shells. Naturally these hole states are strongly populated by neutron-pickup reactions.

Recently groups at Duke[3] and Oxford[4] Universities have measured lifetimes of states in ^{57}Ni using the ^{54}Fe($\alpha, n\gamma$)^{57}Ni reaction and Doppler-shift-attenuation techniques. In these experiments there were strong indications of γ rays resulting from levels in ^{57}Ni at 2.444 and 3.012 MeV; these γ rays cannot be accommodated within the level scheme obtained from the (^{3}He, $\alpha\gamma$) study.

Additional evidence for a level at about 3.0 MeV is also present in the decay of a $\frac{3}{2}^+$ state at 6.00 MeV, as observed in the (^{3}He, $\alpha\gamma$) experiment.[2] A 3.0-MeV γ ray appears in the spectrum; this can presumably result from a cascade $6.0 \to 3.0 \to$ g.s.

The present work was undertaken to see whether any evidence for the existence of these additional states could be obtained from the (^{3}He, α) reaction. Even though these states may not be populated strongly in direct pickup, it was believed that a compound-nuclear or two-step-reaction amplitude, which is small compared with the distorted-wave Born-approximation amplitude *for good single-hole states*, might still be observable.

Accordingly, we have measured spectra of α particles emitted at lab angles of 30, 70, and 90° in the reaction ^{58}Ni(^{3}He, α)^{57}Ni at a beam energy of 15 MeV. Backward angles were chosen to reduce pileup and to attenuate the contributions from the strongly forward-peaked hole states. A 55-μg/cm^2 self-supporting Ni foil enriched to 99.98% ^{58}Ni was used as the target. α particles were detected in three 1-mm silicon surface-barrier detectors. Signals from each detector were digitized in a separate 4096-channel analog-to-digital converter; a 1024-channel segment of this spectrum, repre-