PHYSICAL REVIEW C VOLUME 2, NUMBER 4 OCTOBER 1970

Levels in P^{33} of $E_{ex} < 4.3$ MeV*

W. R. Harris, K. Nagatani, and J. W. Olness
Brookhaven National Laboratory, Upton, New York 11973
(Received 29 April 1970)

Bound states of P^{33} below 4.3-MeV excitation energy have been investigated via the $P^{31}(t,p\gamma)P^{33}$ reaction at triton bombarding energies of 2.45 and 3.10 MeV. Proton and γ-ray singles measurements, together with p-γ coincidence measurements, have determined the principal γ-ray decay modes for the eight states previously reported below 4.3-MeV excitation energy in P^{33}. Evidence is also obtained for the γ decay of a previously unreported state at 4.19 MeV. Angular-correlation measurements, in conjunction with lifetime restrictions obtained from Ge(Li) studies of the γ-ray Doppler shifts, lead to the following determinations of spin parity for these levels [MeV(J^π)] in P^{33}: 0.0($\frac{1}{2}^+$), 1.43($\frac{3}{2}^+$), 1.85($\frac{5}{2}^+$), 2.54($\frac{3}{2}^+$), 3.28-($\frac{3}{2}$, $\frac{5}{2}$), 3.49($\frac{3}{2}$ or $\frac{1}{2}$), 3.63($\frac{1}{2}^+$), 4.05($\frac{3}{2}$, $\frac{5}{2}$), 4.19($\frac{5}{2}$, $\frac{7}{2}$), and 4.22($\frac{7}{2}^-$). These results serve in part to confirm the previously published assignments for the ground state and first three excited states, and establish new information for the higher-lying states. Restrictions on multipole mixing in the deexcitation transitions are summarized and compared with previous results. Finally, the experimental spectrum of P^{33} levels thus obtained is compared with applicable shell-model calculations for these low-lying levels.

I. INTRODUCTION

The shell-model calculations of Glaudemans, Wiechers, and Brussaard[1] have predicted the excitation energies, spins, and parities of 17 levels of P^{33} lying below an excitation energy of 11.3 MeV. More recently, Bouten, Elliot, and Pullen[2] have extended an intermediate-coupling calculation to high enough excitation energy in the $A = 33$ system to include also the first three $T = \frac{3}{2}$ states. Thus a reasonably complete set of low-lying states in P^{33} have been predicted from a theoretical basis.

At the time the present investigation was initiated, the experimental information on the P^{33} level structure was rather sparse. In the region of excitation energy in P^{33} below 4.3 MeV, which is the range of investigation we presently report, levels had been observed[3-5] at excitation energies of 1.43, 1.85, 2.54, 3.50, and 3.63 MeV via the $Si^{30}(\alpha,p)P^{33}$ and $S^{34}(d,He^3)P^{33}$ reactions. The γ-ray deexcitations of the first three of these states was studied by Moss, Poore, Tilley, and Roberson,[3] who thus determined spin-parity assignments of $\frac{3}{2}^+$, $\frac{5}{2}^+$, and $\frac{3}{2}^+$ for the levels at 1.43, 1.85, and 2.54 MeV, respectively. The lifetimes of the 1.43- and 1.85-MeV levels have been measured by Currie *et al.*[6] using the Doppler-shift attenuation method.

The existence of these low-lying levels has since been confirmed by Davies, Harvey, and Darcey[7] who have studied proton angular distributions in the $P^{31}(t,p)P^{33}$ reaction. In addition to providing a rigorous confirmation for a $\frac{1}{2}^+$ assignment for the P^{33} ground state, these latter authors report states in P^{33} at excitation energies of 3.27, 4.04, and 4.22 MeV, plus 10 states of $E_{ex} > 4.3$ MeV. For the 4.22-MeV level the angular distributions determine that the angular momentum transfer is $L = 3$, and thus $J^\pi = \frac{5}{2}^-$, $\frac{7}{2}^-$, for this level. These states have all been observed even more recently in the $Si^{30}(\alpha,p)P^{33}$ reaction by Berkowitz *et al.*,[8] who report 25 states in P^{33} below an excitation energy of 6.6 MeV.

In summary, for states of P^{33} previously reported for $E_{ex} < 4.3$ MeV, the excitation energies and spin-parity assignments [MeV(J^π)] are as follows: 0.0($\frac{1}{2}^+$), 1.43($\frac{3}{2}^+$), 1.85($\frac{5}{2}^+$), 2.54($\frac{3}{2}^+$), 3.28, 3.50, 3.63, 4.05, and 4.23($\frac{5}{2}^-$, $\frac{7}{2}^-$).

It is on the basis of the above information that the present investigation of P^{33} via the $P^{31}(t,p\gamma)P^{33}$ reaction is presented. In Sec. II, we report the results of proton and γ-ray singles measurements designed to further investigate the P^{33} level scheme. Proton-γ angular-correlation studies are reported in Sec. III. A synthesis of these and of previous results are summarized in Sec. IV, which includes also the recently reported results of Hardie *et al.*[9] on the γ-ray deexcitation of several of the states which we have studied. Finally, the experimental description of P^{33} is compared with theoretical predictions for $A = 33$.

II. PROTON AND γ-RAY SINGLES MEASUREMENTS

For all of the present work the $P^{31}(t,p\gamma)P^{33}$ reaction was initiated using the triton beam from the BNL 3.5-MV Van de Graaff accelerator. Targets were prepared by evaporating zinc phosphide (Zn_3P_2) onto thin backing foils. For the particle singles measurements, a silicon surface-barrier detector, 2000 μ thick, was used to detect the reaction products resulting from triton bombardment of a 200-$\mu g/cm^2$ Zn_3P_2 target prepared on a 100-

μg/cm² carbon backing. The detector was placed at a distance of 3 in. from the target with a 0.1-in. vertical slit limiting the angular dispersion.

Figure 1 summarizes the results of some measurements made at a triton bombarding energy of 3.0 MeV and a detection angle of 165°. Figure 1(b) shows the spectrum measured with no absorber foil in front of the detector, showing various particle groups from the $P^{31}(t,p)P^{33}$ ($Q = 9.558$ MeV) and $P^{31}(t,\alpha)Si^{30}$ ($Q = 12.527$ MeV) reactions. Proton groups 1–7 from the (t,p) reaction are labeled according to the level schemes given previously[7,8] for P^{33}, where the numbering of the α groups from the (t,α) reaction follows that given by Endt and Van der Leun.[10] Figure 1(a) shows the spectrum measured under identical conditions but with a 12-mg/cm² aluminum foil interposed between target and detector, such that the α groups are either stopped or sufficiently degraded in energy that they no longer appear in this portion of the spectrum. Thus Fig. 1(a) shows only the proton groups from the (t,p) reaction, although with a markedly poorer resolution due to the energy straggling introduced by the absorber foil.

The data shown in Fig. 1 were next used to deduce the excitation energies of the P^{33} states populated at this bombarding energy. The results are summarized in Table I. Our values were obtained from a computer fit to determine the "peak channels" of the various proton and α groups evident in Figs. 1(a) and 1(b). Energy calibration for these data was based on the energies of the tritons elastically scattered from zinc and phosphorous, [not shown in Fig. 1(b)] and on the energies of α groups 1–11. The α energies were calculated from the Si^{30} excitation energies of Ref. 10. The energy separations for p_0, p_1, p_2, and p_3 are accurately given from the Ge(Li) results, and these separations were also used as inputs to the calibration data. Thus, Table I gives mainly the excitation energies for states of $3 \leq E_{ex} \leq 4.3$ MeV.

Also shown in Table I are weighted averages of the excitation energies quoted in Refs. 3–9. We note that the values given in Refs. 3, 8, and 9 in-

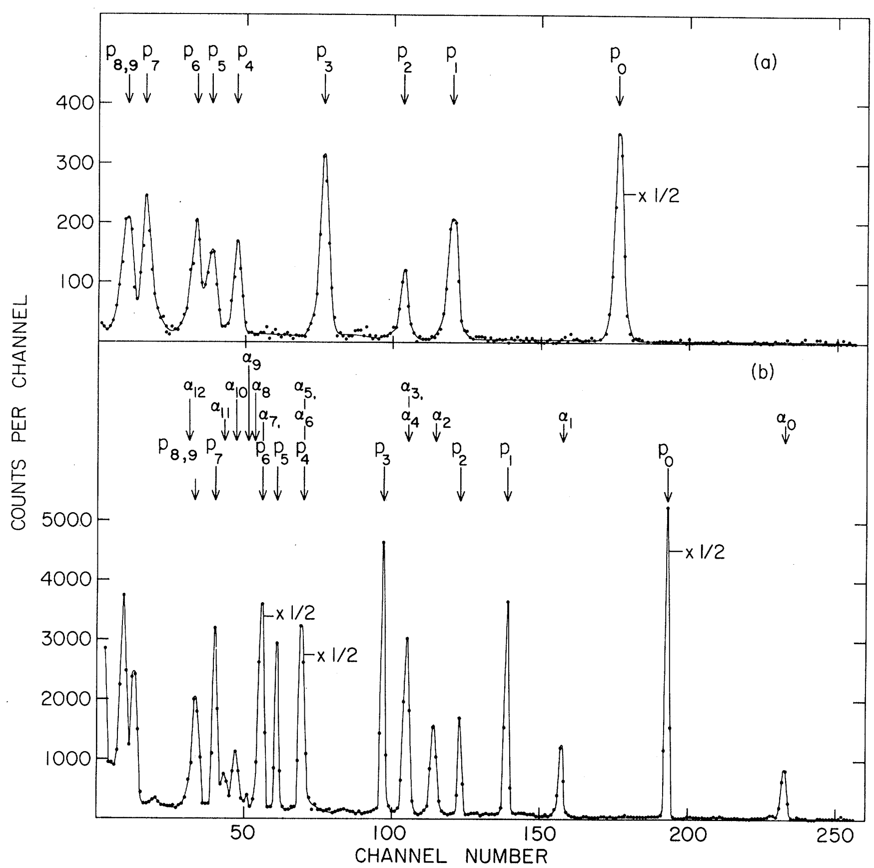

FIG. 1. Particle spectra resulting from 3-MeV triton bombardment of a Zn_3P_2 target, measured with a surface-barrier detector at $\theta = 165°$. In the lower plot (obtained with no absorber foil) proton and α groups from the $P^{31}(t,p\gamma)P^{33}$ and $P^{31}(t,\alpha)Si^{30}$ reactions are labeled according to the previously established level schemes for the final nuclei, as given in the text. In the upper plot a 12-mg/cm² aluminum absorber has been used to stop the α particles, thus showing more clearly the proton spectra from the (t,p) reaction.

TABLE I. Excitation energies (given in keV) for levels of P^{33}. Columns two and three list the excitation energies deduced in the present experiment from particle spectra and from Ge(Li) measurements of γ-ray spectra in the $P^{31}(t,p)P^{33}$ reaction. Column four gives weighted averages of the values quoted previously in Refs. 3–9, and summarized in Ref. 7. The last column lists an average of all results.

Level No.	Present results Particle spectra	Ge(Li)	Previous values	Adopted average value
1	(1434)[a]	1432 ± 3	1435 ± 3	1434 ± 3
2	(1849)[a]	1848 ± 3	1849 ± 3	1849 ± 3
3	(2542)[a]	2538 ± 4	2542 ± 3	2542 ± 3
4	3280 ± 12	3274 ± 4	3280 ± 4	3279 ± 4
5	3492 ± 10	3492 ± 5	3496 ± 4	3494 ± 4
6	3636 ± 13	3627 ± 5	3631 ± 4	3631 ± 4
7	4050 ± 10	4050 ± 5	4056 ± 4	4053 ± 4
8	4181 ± 14	···	···	4189 ± 12[b]
9	4217 ± 14	···	4228 ± 4	4225 ± 4

[a]Used as calibration points: values taken from column five.
[b]Value for level 8 obtained relative to level 9.

volve the smallest uncertainties, and hence these results are more heavily weighted. The agreement between the values given in columns two and four lead us to conclude that we are populating the same states in P^{33} observed previously.

The lone exception is for groups p_8 and p_9. Because of the possible presence in the spectrum [Fig. 1(b)] of α_{12}, the increased breadth of the group marked $p_{8,9}$ does not in itself allow the conclusion that this is a doublet level in P^{33}. The markedly poorer resolution obtained in Fig. 1(a) precludes its observation here. The strongest evidence for the existence of this 4.19-MeV state is obtained from the p-γ coincidence measurements, and is discussed in detail in Sec. III. These latter results, as will be shown, lead to an energy difference of 36 ± 10 keV for the members of this 4.2-MeV doublet.

In order to check this point, additional particle-singles data were measured at $E_t = 3.1$ MeV and $\theta_p = 135°$ using a 900-μ detector of improved energy resolution [~28-keV full width at half maximum (FWHM)]. Measurements were made both with and without a 250-μg/cm^2 aluminum absorber in front of the detector, in order to distinguish α particles from protons. The foil was thin enough that it did not introduce measurable straggling – yet the energy losses for protons and α particles were markedly different. In the spectra thus obtained, α-particle groups were readily distinguishable from proton groups. This is demonstrated (for example) in Fig. 1(b), where the characteristic breadth of the α groups is approximately twice that of the proton groups. These results show that, in the region of 4.2-MeV excitation in P^{33} [see Fig. 1(b)], there are two proton groups and a single α group. The corresponding excitation energies in P^{33} are 4.19 and 4.22 MeV, while that in Si^{30} corresponding to the measured α energy is 5.951 MeV. The latter value is in excellent agreement with the value 5.948 ± 10 MeV quoted[10] for the Si^{30} 12th excited state. The results for these two levels quoted in Table I are finally taken as an average of these values and those obtained in Sec. III.

As a final note, angular-distribution data measured at $E_t = 3.1$ MeV provide additional confirmation for the $\frac{1}{2}^+$ assignment[5,7] for the P^{33} ground state. The angular distribution of proton group p_0 leading to the P^{33} ground state exhibits a strong maximum at $\theta_p = 0°$, with a secondary maximum $\theta_p \sim 45°$. A simple distorted-wave Born approximation (DWBA) calculation assuming the two nucleons transferred in the $d_{3/2}$ orbit revealed qualitative agreement with $L = 0$ transfer, which, due to uncertainties in the distortion potentials at such a low incident trition energy, is as good as can be expected. However, there is no doubt about the $L = 0$ assignment; and thus we obtain independent confirmation of the $J^\pi = \frac{1}{2}^+$ assignment for the P^{33} ground state.

γ rays from the $P^{31}(t,p\gamma)P^{33}$ reaction were observed in a separate experiment using a 30-cc Ge(Li) detector having a measured resolution of 3 keV FWHM for the 1.33-MeV γ rays of Co^{60}. The main purpose of this work was to determine the γ-ray transition energies with sufficient accuracy to check the P^{33} excitation energies given in Table I, and also to obtain some information on the lifetimes of these states through the observation of possible Doppler effects in order to aid in the interpretation of the proton-γ angular-correlation results described below. The Ge(Li) detector was placed at 7 cm from the target, which was for this work a 600-μg/cm^2 layer of Zn_3P_2 evaporated onto a 0.002-in. molybdenum backing. Measurements of the γ-ray spectra at $\theta_\gamma = 0$ and 90° were made at $E_t = 2.55$ and 3.53 MeV, using beam currents of ~0.04 μA. Pulses from the Ge(Li) detector, after amplification utilizing standard pole-zero and derestoration techniques, were analyzed in a 4096-channel analyzer.

From an internal energy calibration of the $\theta_\gamma = 90°$ data, based on known[10] transitions in S^{33} and also γ rays from Co^{60} and RaTh C'', we have identified transitions from the first seven of the P^{33} states listed in Table I. This identification was based on the decay modes for the initial states, as will be discussed in Sec. III, and subsequently on the agreement between the measured excitation energies deduced from this work and those quoted in Table I. A most important conclusion follows from these measurements: The γ rays from all of the P^{33} levels of $E_{ex} < 4.1$ MeV exhibit Doppler effects, indicating the levels have mean lifetimes in the range $\tau \lesssim 2$ psec.

The 1.43- and 1.85-MeV levels have only small Doppler shifts, consistent with the previously reported[6] mean lives $\tau = 0.79$ psec and $\tau = 1.36$ psec, respectively. We therefore adopt these values for the discussion of the correlation analysis to be presented in Sec. III. For the remaining levels of $E_{ex} \lesssim 4.1$ MeV (with the exception of the 4.05-MeV level) we can set a lower limit on the measured attenuation factors $F(\tau) > 0.4$, corresponding to a restriction on the mean lives of $\tau < 0.3$ psec. (We have here used for the characteristic stopping time a value of $\alpha = 0.76$ psec appropriate to P^{33} ions stopping in Zn_3P_2.) The exception is for the 4.05-MeV level, which appears to have a significantly longer lifetime. For this level, we take $F(\tau) > 0.2$ and thus $\tau < 0.8$ psec. Although these limits are quite conservative, they will nevertheless be useful in limiting possible multipole mixings in the deexcitation γ rays from the P^{33} levels studied.

III. PARTICLE-γ COINCIDENCE MEASUREMENTS

A. Experimental Methods

Proton-γ angular correlations in the $P^{31}(t, p\gamma)P^{33}$ reaction were measured using a 500-μg/cm^2-thick target of Zn_3P_2 evaporated onto a 0.0001-in. nickel backing. Protons were detected in an annular surface-barrier detector centered at a scattering angle of 180° with respect to the incident beam. The detector center was 2.5 cm from the target and the sensitive area of the detector allowed for detection of protons emitted at scattering angles between 165.2 and 172.8°. A 12-mg/cm^2-thick Al absorber was placed over the sensitive area of the detector to prevent elastically scattered tritons from striking the detector and to degrade the energies of α particles sufficiently to ensure that they did not mask the proton spectra. The triton beam emerging from the target was stopped by a 1-mil Mo foil. A permanent magnet of approximately 400 G was inserted between target and detector to prevent secondary electrons from striking the detector.

γ rays were detected by a 5×6-in. NaI(Tl) detector whose front face was 18 cm from the target. Pulse-height spectra from the proton and γ-ray detectors, gated by an external proton-γ coincidence requirement, were analyzed by a 16 384-channel analyzer operated in the 128×128-channel configuration. Two-parameter spectra were thus measured for γ-ray detection angles in random order of 0, 30, 45, 60, and 90°. Two runs of 5–7-h duration were made at each angle, using a beam current of ~120 nA and a coincidence resolving time of ~70 nsec. Under these conditions, the ratio of reals-to-randoms was better than 20:1.

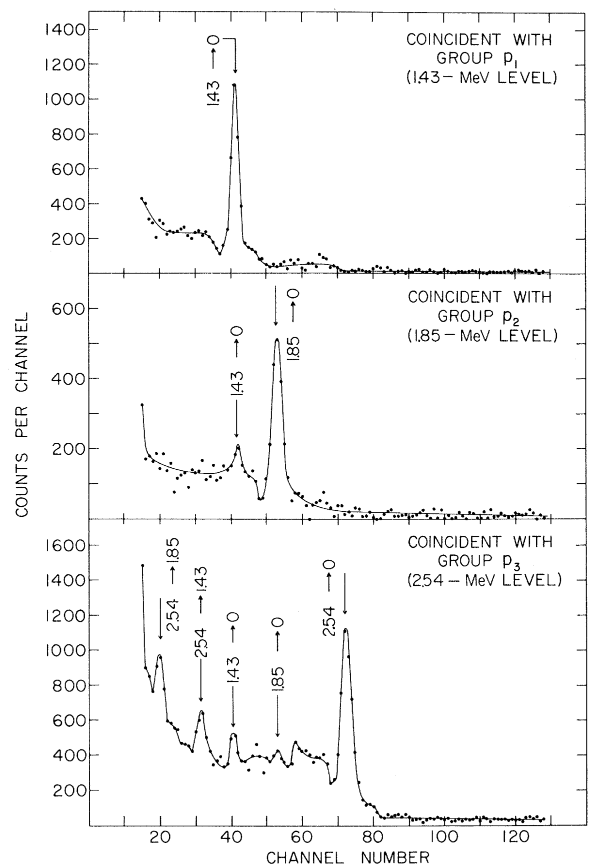

FIG. 2. Spectra of γ rays from the $P^{31}(t, p\gamma)P^{33}$ reaction measured in coincidence with proton groups 1, 2, and 3 – illustrating the γ-ray deexcitation of the first three excited states of P^{33}. γ-ray peaks are identified according to the levels between which the transitions occur. These data were obtained as part of a correlation measurement at $E_t = 2.45$ MeV, using an annular particle detector centered at $\theta_p = 180°$, and a 5×6-in. NaI(Tl) detector for γ rays.

The 10 sets of data thus obtained were normalized according to the charge collected on the beam stop. A preliminary γ-decay scheme was determined from an analysis of the sum of such data matrices. Corrections were later applied to the γ-ray branching ratios thus obtained as determined from the correlation analysis. These corrections were small (<10%).

Two such angular-correlation measurements were carried out at triton bombarding energies of 2.45 and 3.1 MeV. At $E_t = 2.45$ MeV the range of proton analysis was set to cover states in P^{33} of $1.4 \leq E_{ex} \leq 4.3$ MeV, while for $E_t = 3.1$ MeV the range was set for $E_{ex} > 4$ MeV. Thus two sets of data were obtained for proton groups 7, 8, and 9.

B. Results: Branching Ratios and Correlations

The first step in the analysis was to examine the spectrum of protons measured in coincidence with all γ rays. The resulting spectrum, similar to that shown in Fig. 1(a), showed that groups 1–7 were sufficiently well resolved that there could be no ambiguities involved in extracting the γ spectra corresponding to the decay of each P^{33} state.

Figure 2 shows the γ-ray spectra measured in coincidence with proton groups leading to the first three excited states of P^{33}. These data, obtained from the "summed" spectrum for $E_t = 2.45$ MeV, indicate that both the 1.85- and 2.54-MeV levels decay primarily to the ground state, but with weaker branches to available lower-lying levels. The angular dependence of the stronger branches was

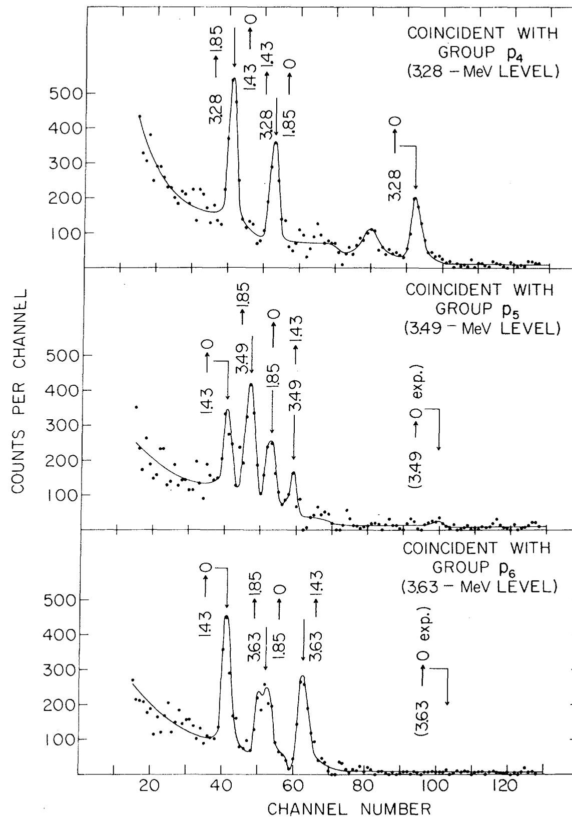

FIG. 3. Partial results of a two-parameter analysis (similar to Fig. 2) of proton-γ coincidences in the $P^{31}(t,p\gamma)P^{33}$ reaction, illustrating the γ-ray deexcitation of the P^{33} levels at 3.28, 3.49, and 3.63 MeV. These data were obtained at $E_t = 2.45$ MeV.

TABLE II. Summary of p-γ correlation measurements in the $P^{31}(t,p\gamma)P^{33}$ reaction.

E_t (MeV)	E_i (MeV)	Transition $E_i \to E_f$	$\nu_{max}=2$: a_2 (%)	χ^2	$\nu_{max}=4$: a_2 (%)	a_4 (%)	χ^2
2.45	1.43	1.43→ 0	+43± 4	0.6	+42± 4	+ 5± 5	0.4
2.45	1.85	1.85→ 0	+48± 5	10.4	+51± 5	−31± 6	1.3
2.45	2.54	2.54→ 0	−10± 3	1.0	−12± 3	+ 5± 4	4.3
2.45	3.28	3.28→ 0	+30± 7	0.2	+31± 7	0± 8	0.2
2.45	3.28	$E_\gamma = 1.85$	+15± 7	0.5	+14± 8	+ 3± 9	0.7
2.45	3.28	$E_\gamma = 1.43$	− 8± 6	0.3	− 8± 7	0± 8	0.4
2.45	3.49	3.49→ 1.43	−62±11	1.5	−60±13	− 2±12	2.2
2.45	3.63	3.63→ 1.43	+14± 7	6.6	+31± 9	−34± 9	0.2
2.45	3.63	1.43→ 0	−65± 8	2.3			
2.45	3.63	3.63→ 1.85→ 0	−10± 8	5.6	+10±10	−33±11	2.0
2.45	4.05	4.05→ 1.43	+ 3± 4	1.7	+ 2± 5	+ 3± 5	2.4
2.45	4.05	1.43→ 0	−65± 7	1.0			
2.45	4.22	4.22→ 1.85	−30±11	1.1	−20±12	−20±13	0.4
2.45	4.22	1.85→ 0	+54± 8	14.5	+64± 8	−58± 9	1.3
3.10	4.05	4.05→ 1.43	0± 4	2.6	+ 1± 5	− 3± 5	3.8
3.10	4.05	1.43→ 0	−38± 5	0.9			
3.10	4.22	4.22→ 1.85	−12± 9	1.2	− 6±10	−15±13	1.0
3.10	4.22	1.85→ 0	+45± 7	0.8	+47± 7	−12± 8	0.1
3.10	4.19	4.19→ 0	+20± 9	4.5	+31± 9	−32±10	2.0

next obtained from the data matrices measured at the various angles θ_γ. The measured intensities were fitted with the Legendre polynomial expansion $W(\theta)=\sum_\nu a_\nu P_\nu(\cos\theta)$ where $a_0\equiv 1$, ν is even, and $\nu_{max}=2$ or 4. The resulting least-squares solutions for the a_ν, and the corresponding values of the normalized χ^2, are summarized in Table II.

Figure 3 shows similar data obtained for the next three excited states. The energy resolution of the NaI(Tl) detector is clearly inadequate to distinguish the cascade deexcitation of the 3.28-MeV level as taking place to either or both of the 1.43- or 1.85-MeV states. Accordingly, we have in Table II indicated the angular-correlation coefficients as determined for the net intensity of the 1.43- and 1.85-MeV γ rays. After corrections for correlation effects, the measured branching ratio for the $3.28\to 0$ transition is $(49\pm 8)\%$, compared with $(51\pm 8)\%$ for the net cascade intensity. Possible summing effects of 1.85- and 1.43-MeV γ rays represent less than 5% of the $3.28\to 0$ intensity and have been included in this calculation. The 3.49- and 3.63-MeV levels are observed to cascade to both the first and second excited states at 1.43 and 1.85 MeV. The two members of the $3.63\to 1.85\to 0$ transition were not resolved by the NaI(Tl) detector, and the "net" angular correlation is given in Table II.

As indicated in Fig. 4, the 4.05-MeV level decays predominantly to the 1.43-MeV level; no evidence is seen for a ground-state transition. These data for $E_t=2.45$ MeV are in excellent agreement with the results for $E_t=3.1$ MeV. The correlation coefficients extracted for both bombarding energies are given in Table II.

The lower plot in Fig. 4 shows the spectrum measured in coincidence with the proton group or groups corresponding to 4.2-MeV excitation in P^{33}. Only a cascade transition through the 1.85-MeV level is observed, an upper limit of 17% being placed on a possible ground-state decay. In the corresponding spectrum measured at $E_t=3.1$ MeV, shown in Fig. 5, the presence of a ground-state transition is clear, with the intensity ratio ground state/cascade $=(77\pm 16)\%$. The inconsistency evident in the branching ratios measured at these two different energies points to the existence of a doublet level at 4.2 MeV in P^{33}. The inset in Fig. 5 shows the particle spectra for $E_t=3.1$ MeV as measured in coincidence with (a) cascade γ rays, and (b) ground-state γ rays. The observed shift of the proton peak confirms the doublet hypothesis, and leads to a level separation energy of 36 ± 10 keV.

From an analysis of these data, we conclude the following: (1) The 4.22-MeV level decays primarily to the 1.85-MeV level, with a possible ground-

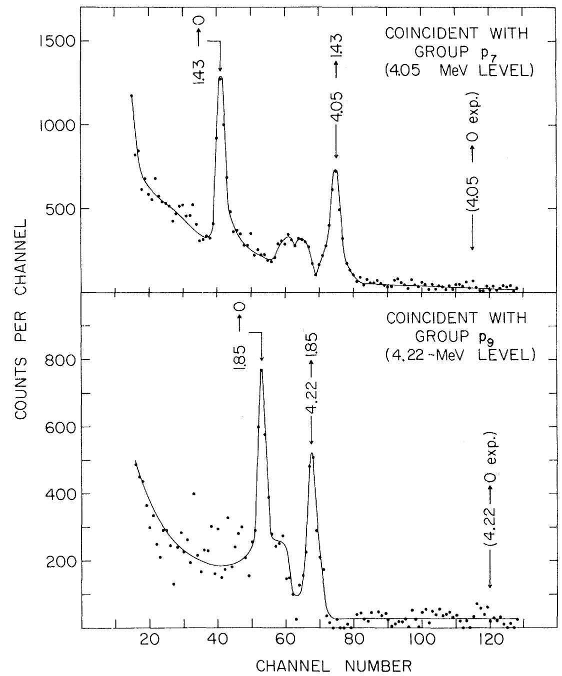

FIG. 4. Partial results of a two-parameter analysis (similar to Fig. 2) of proton-γ coincidences measured in the $P^{31}(t,p\gamma)P^{33}$ reaction, illustrating the γ-ray deexcitation of the P^{33} levels at 4.05 and 4.22 MeV. These data were obtained at $E_t=2.45$ MeV.

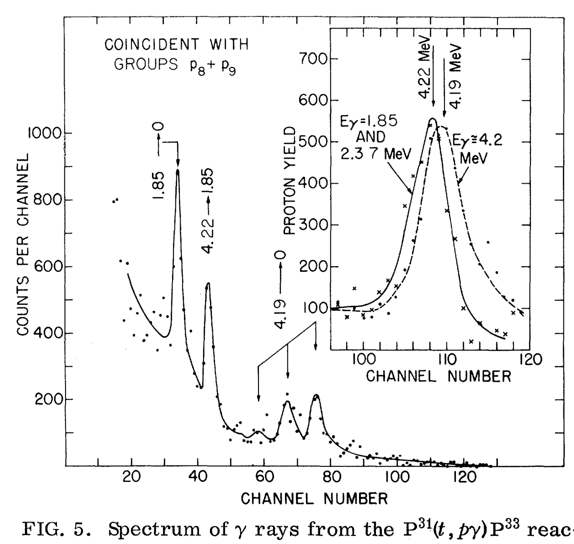

FIG. 5. Spectrum of γ rays from the $P^{31}(t,p\gamma)P^{33}$ reaction measured in coincidence with proton groups p_8 and p_9, illustrating the existence of a "doublet" of levels at ~4.2-MeV excitation in P^{33}. These data were obtained at $E_t=3.1$ MeV – for comparison, see the spectrum measured at $E_t=2.45$ MeV in Fig. 4. The insert shows the proton spectra measured in coincidence with those γ rays characterizing the deexcitation of levels at 4.22 and 4.19 MeV in P^{33}.

state branch of <9%. (2) The 4.19-MeV level decays primarily to the P^{33} ground state, with an upper limit of <50% being placed on a possible cascade to the 1.85-MeV state. From these data, the excitation energies are given as 4.217 ± 0.014 and 4.181 ± 0.017 MeV.

Finally, we conclude that the spectrum obtained at 2.45 MeV (Fig. 4) is due almost wholly to a $4.22 \rightarrow 1.85 \rightarrow 0$ cascade, an upper limit of 5% being placed on the maximum contribution from a possible $4.19 \rightarrow 1.85 \rightarrow 0$ cascade. Similarly, the contribution of a possible $4.22 \rightarrow 0$ transition to the observed $4.19 \rightarrow 0$ intensity (Fig. 5) is less than 6%. Thus the data obtained at $E_t = 2.45$ MeV (Fig. 4) can be safely used in an analysis of the $4.22 \rightarrow 1.85 \rightarrow 0$ correlations, while that obtained for $E_t = 3.1$ MeV provide information on the $4.19 \rightarrow 0$ correlation.

Figure 6 summarizes our conclusions on the branching ratios for states of P^{33}. The excitation energies given are the weighted average values of Table I. Also indicated are the assignments of spin and parity for these levels, as outlined in the following section.

As a final consideration, we note that the excitation energies given in Table I (column three) are based on the observation in the Ge(Li) spectrum of those deexcitation γ rays having branching ratios >30%. An exception is made for the 3.28-MeV level, for which we observe the $3.28 \rightarrow 0$ transition. To all appearances, the $3.28 \rightarrow 1.85 \rightarrow 0$ and/or $3.28 \rightarrow 1.43 \rightarrow 0$ transitions are degenerate in energy to within the resolution (3 keV FWHM) of the Ge(Li) detector, and hence no additional information was gained on the nature of this cascade or cascades.

C. Angular-Correlation Analysis

The results of even-order Legendre polynomial fits to the measured correlation data are summarized in Table II. The angular-correlation analysis follows method II of Litherland and Ferguson[11] (as given explicitly by Poletti and Warburton[12]). The method depends on the fact that for protons detected at $\theta_p \cong 180°$, the projection of their orbital angular momentum along the beam direction is zero. The spin of both P^{31} and the triton are $\frac{1}{2}$, and hence only the $\pm\frac{1}{2}$ and $\pm\frac{3}{2}$ magnetic substates of the P^{33} levels can be formed in this case. In the present work the particle detector subtended a range $165° \leq \theta_p \leq 173°$ which allows the possibility of population of higher magnetic substates. This we refer to as the finite-size effect. Hence, in analyzing the γ-ray angular-correlation data to determine spins and multipole mixing ratios we have allowed for as much as 10% population of the $\pm\frac{5}{2}$ magnetic substates for initial levels with $J > \frac{3}{2}$. This assumed limit of 10% is more than a factor of 2 greater than we expect for the population of the higher magnetic substate.[11,12]

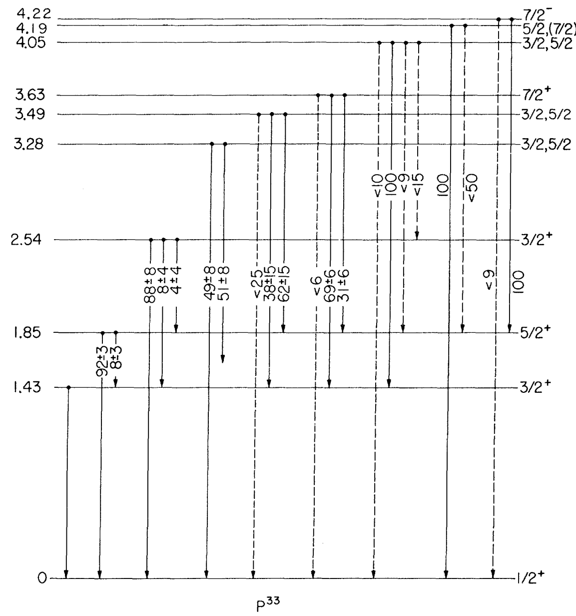

FIG. 6. P^{33} level scheme summarizing information obtained in the present experiment on spins, parities, and branching ratios for levels of $E_{ex} < 4.3$ MeV. This work has confirmed the previously reported spin-parity assignments for the ground state and first three excited states (Refs. 3–7) and also for the states at 3.49 and 3.63 MeV.

The general procedure used in the analysis of a given correlation was as follows: Having *assumed* a set of spins J_i and J_f for the initial and final levels linked by the transition, a least-squares fit to the experimental correlation data was obtained for discrete values of the mixing ratio x, which specifies the $(L+1)/L$ mixing in the $J_i \rightarrow J_f$ transition. The only free parameter in these individual fits is thus the ratio of the population parameters $P(\frac{1}{2})/P(\frac{3}{2})$. x was varied from $-\infty$ to $+\infty$ in selected steps; for each value of x the normalized goodness-of-fit parameter χ^2 was calculated. χ^2 is normalized so that its expectation value for an allowed set J_i, J_f, and x is unity. Probability tables were subsequently used to rule quantitatively against those cases for which χ^2 considerably exceeds unity. In the following we give several examples of this approach, as pertaining to the more complicated cases of multiple cascades; for general reference we quote examples given previously.[12,13]

1.43 and 1.85-MeV Levels

Previous experiments have resulted in spin-parity assignments of $\frac{3}{2}^+$ and $\frac{5}{2}^+$ for the levels at 1.43 and 1.85 MeV, and have also determined the $(L+1)/L$ mixing in the electromagnetic transitions which deexcite these levels.[3]

In the following discussion, however, we choose to examine the results of the present experiment independently, in order to determine the extent to which they may or may not confirm the previously quoted assignments.

We consider first the 1.85-MeV level, for which the measured correlation coefficients are shown in Table II. An attempt was made to fit the measured correlations for spins $J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$ for the 1.85-MeV level, with the spin parity of the ground state fixed as $J^{\pi}=\frac{1}{2}^{+}$. The data (Table II) could be fitted only for $J=\frac{5}{2}$ with the $(L=3)/(L=2)$ mixing ratio being restricted as $x=-(0.02\pm0.04)$, $x=+(3.0\pm0.5)$, or $-10.4\leq x\leq -0.47$. Only the first of these ratios is compatible with the previously measured mean lifetime $\tau=1.36\pm0.17$ psec of the 1.85-MeV state.[6] Either of the other two possibilities require $L=3$ components exceeding the most generous sum-rule limit, $\Gamma_{\gamma}\sim Z^2$,[14] and thus can be ruled out. The present result $x=-(0.02\pm0.04)$ is in reasonable agreement with the value $x=+(0.06\pm0.04)$ which is one of two given by Moss *et al.*,[3] and we take as an experimental restriction the average value $x=+(0.02\pm0.03)$.

It is also clear from the lifetime[6] that the transition is $E2$, rather than $M2$, since in the latter case the $M2$ strength would be greater than 140 Weisskopf units (W.u.), which is clearly unreasonable. Thus, the 1.85-MeV level is $J^{\pi}=\frac{5}{2}^{+}$, since it decays by $E2$ radiation to the $\frac{1}{2}^{+}$ ground state. The lifetime can also be used to rule against a significant $M3$ component, yielding the result $|x|<0.003$ where we have allowed for a factor of 100 enhancement of the $M3$ strength above the single-particle limit, and for two standard deviations in the measured lifetime.

The angular distribution of the $1.43\to0$ transition measured in coincidence with protons feeding the 1.43-MeV state can be fitted for both $J=\frac{3}{2}$ and $J=\frac{5}{2}$ for the 1.43-MeV level, while the possibilities $J=\frac{1}{2}$ and $\frac{7}{2}$ are excluded. If $J=\frac{3}{2}$ then we have the restriction $x\leq-0.5$ or $-0.1\leq x\leq 2.2$. For $J=\frac{5}{2}$ we find $x=-(0.16\pm0.22)$ or $|x|>1.7$. The latter possibility, which corresponds to a mixing of $L=3$ with $L=2$, radiation, is excluded by the lifetime of the state.[6] The former solution is excluded by the correlation data on the 3.63-MeV level, and also independently by that on the 4.05-MeV level.

As will be shown, the $3.63\to1.85\to0$ correlation results (Table II) lead to a $J=\frac{7}{2}$ assignment for the 3.63-MeV state. However, the $3.63\to1.43\to0$ correlations cannot be fitted under the assumption that the cascade is $\frac{7}{2}\to\frac{5}{2}\to\frac{1}{2}$. The $4.05\to1.43\to0$ correlations were also fitted under the assumption that the 4.05-MeV state is of $J=\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$, with the 1.43-MeV state assumed to be $J=\frac{5}{2}$ and the $1.43\to0$ mixing ratio taken as $x=-(0.16\pm0.22)$,

TABLE III. Summary of restrictions on the $(L+1)/L$ mixing ratios x for various transitions in P^{33}, as obtained from the present and also previous work.

Transition ($E_i\to E_f$)	J_i (assumed)	J_f	Restrictions on x: Present results	Previous results	Weighted average
$1.43\to0$	$\frac{3}{2}$	$\frac{1}{2}$	$x=+(0.58^{+0.35}_{-0.25})$	$x=+(0.59\pm0.10)$[a]	$x=+(0.59\pm0.10)$
$1.85\to0$	$\frac{5}{2}$	$\frac{1}{2}$	$x=-(0.02\pm0.04)$	$\|x\|<0.003$[b]	$\|x\|<0.003$
$2.54\to0$	$\frac{3}{2}$	$\frac{1}{2}$	$x=$ undefined	$x=-(0.16\pm0.04)$ or $x=+2.6\pm0.3$[a]	$x=-(0.16\pm0.04)$ or $x=+(2.6\pm0.3)$
$3.28\to0$	$\frac{3}{2}$	$\frac{1}{2}$	$x=$ undefined		$x=$ undefined
	$\frac{5}{2}$	$\frac{1}{2}$	$x=<-5$ or $x>0.57$		$x<-5$ or $x>-0.57$
$3.49\to1.43$	$\frac{3}{2}$	$\frac{3}{2}$	$-3.8\leq x\leq 0$ or $x=1.4\pm0.3$	$x=+(1.3\pm0.3)$[c]	$x=+(1.35\pm0.20)$
	$\frac{5}{2}$	$\frac{3}{2}$	$0\leq x\leq 2.8$	$x=+(0.14\pm0.10)$[c]	$x=+(0.14\pm0.10)$
$3.63\to1.85$	$\frac{7}{2}$	$\frac{5}{2}$	$x=+(0.05^{+0.11}_{-0.07})$	$x=+(0.21\pm0.10)$[c]	$x=+(0.1\pm0.1)$
$3.63\to1.43$	$\frac{7}{2}$	$\frac{3}{2}$	$x=+(0.14\pm0.12)$	$x=+(0.0\pm0.1)$[c]	$x=+(0.6\pm0.7)$
$4.05\to1.43$	$\frac{3}{2}$	$\frac{3}{2}$	$\|x\|>8$		$\|x\|>8$
	$\frac{5}{2}$	$\frac{3}{2}$	$x=-(0.19^{+0.05}_{-0.02})$		$x=-(0.19^{+0.05}_{-0.02})$
$4.19\to0$	$\frac{5}{2}$	$\frac{1}{2}$	$-\infty\leq x\leq-0.35$ or $-0.07\leq x\leq 3.73$		$-\infty\leq x\leq-0.35$ or $-0.07\leq x\leq 3.73$
$4.22\to1.85$	$\frac{7}{2}$	$\frac{5}{2}$	$x=0\pm0.1$		$x=0\pm0.1$

[a]Weighted average value from Refs. 3 and 9.
[b]From the lifetime restriction of Ref. 6 (see text).
[c]From Ref. 9.

as given above. None of these possibilities permitted an acceptable fit to the data; i.e., the $J=\frac{5}{2}$ assignment to the 1.43-MeV level was excluded to a confidence of 99.9%. Thus we have on two counts excluded the possibility $J=\frac{5}{2}$, and therefore confirmed the spin of the 1.43-MeV level as $J=\frac{3}{2}$.

As will be discussed below, the correlation data on the $3.63\to1.43\to0$ and $4.05\to1.43\to0$ transitions determine the mixing in the $1.43\to0$ transition to be $x=+(0.58^{+0.35}_{-0.25})$. This agrees well with the value $x=+(0.63\pm0.17)$ obtained previously,[3] and we therefore adopt the average of these two values $x=(0.62\pm0.14)$ in the following discussion of the $1.43\to0$ correlation data.

If the mixing in the $1.43\to0$ transition were $M2/E1$, then the $M2$ strength computed for the above values of x and τ is $|M(2)|^2>200$ W.u., which is very unlikely.[14] Thus we conclude that the 1.43-MeV level has even parity, since the decay to the $\frac{1}{2}^+$ ground state must be an $E2/M1$ mixture in which both the $M1$ and $E2$ components are of similar strength.

In summary then, the present results confirm the assignments $J^\pi=\frac{3}{2}^+$ for the 1.43-MeV level, and $J^\pi=\frac{5}{2}^+$ for the 1.85-MeV level. The measured mixing ratios are also in good agreement with the results of Moss *et al.*[3] The best values for these mixing ratios are given in Table III.

2.54-MeV Level

The angular-correlation data on the $2.54\to0$ transition are well fitted for $J=\frac{3}{2}$, yielding a value $\chi^2=1.01$ in agreement with the best-fit value given in Table II. For assumed spins $J=\frac{5}{2}$ or $\frac{7}{2}$, the minimal values of χ^2 are, respectively, 25 and 100, which considerably exceed the 0.1% confidence limit at $\chi^2=5.5$, and thus may be rigorously rejected.

For the possibility $J=\frac{1}{2}$ the minimal value of χ^2 exceeds 4.5, which corresponds to a statistical probability of less than 1%. Thus $J=\frac{1}{2}$ is considered very unlikely. The results of Moss *et al.*[3] also express a preference for a $J=\frac{3}{2}$ assignment, with the minimal value of χ^2 for $J=\frac{1}{2}$ corresponding to a statistical probability of only ~2% (see Fig. 5 of Ref. 3). The results of both experiments when taken together rigorously exclude $J=\frac{1}{2}$, thus determining $J=\frac{3}{2}$ for the 2.54-MeV level. In the present experiment no restrictions were established for the mixing in the $2.54\to0$ transitions; the restrictions quoted in Table III are those of Ref. 3. The mean-life restriction given above that $\tau<0.3$ psec does not allow us to choose between either of the two possible solutions for x.

3.28-MeV Level

Figure 7 shows the experimental correlation data obtained on the 3.28-MeV level, as well as the least-squares Legendre-polynomial fits, summarized in Table II. Because of the energy degeneracy of possible $3.28\to1.85\to0$ and $3.28\to1.43\to0$ cascade γ rays, the correlation data on the 1.43- and 1.85-MeV γ rays could not be analyzed. The lower plot in Fig. 7 shows the results of an attempt to fit the correlation data on the $3.28\to0$ transition for assumed spins less than or equal to $\frac{7}{2}$ for the 3.28-MeV level. Here we have plotted the goodness-of-fit parameter χ^2 as a function of x, the $(L+1)/L$ mixing in the $3.28\to0$ transition. χ^2 is normalized to the degrees of freedom, and thus its expectation value is unity. As can be seen, only the $J=\frac{3}{2}$ and $J=\frac{5}{2}$ possibilities yield really acceptable fits to the data. The minimal value of χ^2 for $J=\frac{1}{2}$ corresponds

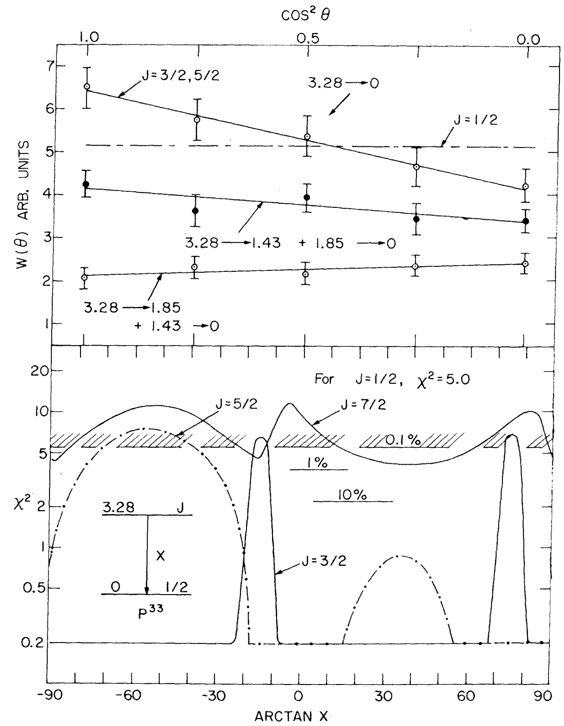

FIG. 7. Proton-γ angular-correlation results for the P^{33} 3.28-MeV level. The measured correlations are shown in the upper plot. The lower plot shows the results of an attempt to fit the $3.28\to0$ correlation for various assumed spins for the initial level. Here we have plotted the goodness-of-fit parameter χ^2 as a function of arctan$x$, where x is the $(L+1)/L$ mixing in the $3.28\to0$ transition. The parameters of this fit are the substate populations $P(\frac{1}{2})$ and $P(\frac{3}{2})$. χ^2 is normalized so that its expectation value (for the correct J and x) is unity. The values of χ^2 corresponding to statistical probabilities of 10, 1, and 0.1% are marked. Possible assignments for the 3.28-MeV level of $J=\frac{3}{2}$ or $\frac{5}{2}$ are clearly allowed for most values of x, while the possibility $J=\frac{7}{2}$ is ruled against very strongly, since the minimal value of χ^2 in this case corresponds to a statistical probability of only 1%.

to a statistical probability of less than 1%.

For $J = \frac{7}{2}$, χ^2 dips below the 0.1% confidence limit only for significant mixing of $L = 4$ with $L = 3$ radiation. This possibility is rigorously rejected from the quoted mean-lifetime restriction $\tau < 0.3$ psec, since the $L = 4$ component would have a strength exceeding the single-particle estimate[14] by several orders of magnitude.

Thus, we conclude that for the 3.28-MeV level $J = \frac{3}{2}$ or $\frac{5}{2}$, with the possibility $J = \frac{1}{2}$ ruled against very strongly.

3.49-MeV Level

For this level, the only transition whose angular correlation could be reliably extracted from the data was the $3.49 \rightarrow 1.43$ cascade transition. The observed anisotropy (Table II) immediately rules out the possibility $J = \frac{1}{2}$ for the 3.49-MeV level. Satisfactory fits to the data were achieved for assumed spins $J = \frac{3}{2}$ or $\frac{5}{2}$ for the initial level with the mixing in the $3.49 \rightarrow 1.43$ transition as specified in Table III. The minimal values of χ^2 for assumed spins of $\frac{7}{2}$ and $\frac{9}{2}$ were, respectively, 9 and 16, which considerably exceeds the 0.1% confidence limit corresponding to $\chi^2 = 5.5$. In summary, the 3.49-MeV state is found to be $J = \frac{3}{2}$ or $\frac{5}{2}$, and for either case the $3.49 \rightarrow 1.43$ transition *may* be pure dipole in character, although significant mixing of a quadrupole component is not disallowed.

3.63-MeV Level

Figure 8 shows a portion of the results of an analysis of the correlation data on the 3.63-MeV level. In the upper plot [Fig. 8(a)] are shown the results of a χ^2 analysis of the correlation information on the unresolved members of the $3.63 \rightarrow 1.85 \rightarrow 0$ cascade. Plotted here are the minimal values of χ^2, as computed from a least-squares fit to the data for discrete values of the $(L+1)/L$ mixing ratio x in the first member of the cascade. The mixing ratio for the $\frac{5}{2} \rightarrow \frac{1}{2}$ second member is fixed at $x_2 = 0$, in accordance with the results summarized above on the $1.85 \rightarrow 0$ transition. Results are shown for assumed values $J = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}$ for the spin of the initial state. For each case the possible variations due to finite-size effects were estimated, and for cases where the effect may be nontrivial, are shown by the dashed curves.

We conclude from Fig. 8(a) that only the possibilities $J = \frac{7}{2}$ or $\frac{9}{2}$ gives acceptable agreement with the experimental data. The value of χ^2 for $J = \frac{3}{2}$ exceeds the 0.1% confidence limit for all values of x, while for $J = \frac{5}{2}$ the minimal value of χ^2 corresponds to a statistical probability for occurrence of only 0.3%.

If $J = \frac{7}{2}$ for the 3.63-MeV level, then the $3.63 \rightarrow 1.85$ transition must essentially be either pure

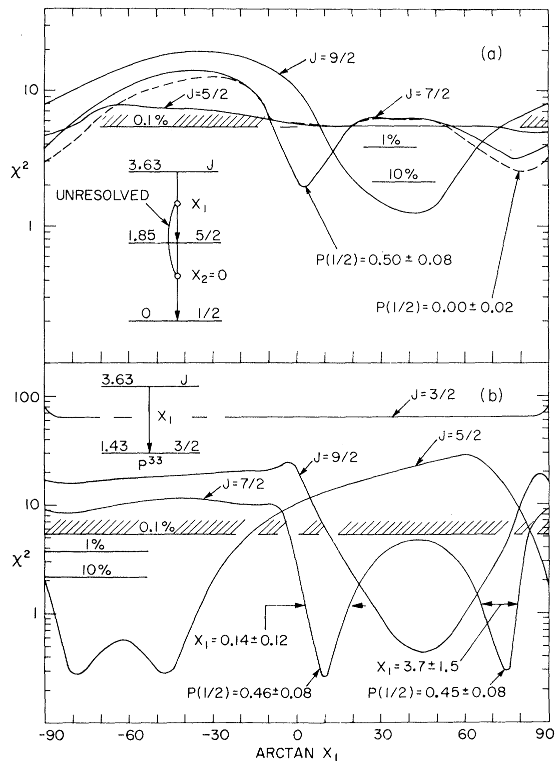

FIG. 8. Partial results of an analysis of proton-γ correlation measurements for the 3.63-MeV level of P^{33}. The upper plot shows the results of an attempt to fit the correlation data on the unresolved members of the $3.63 \rightarrow 1.85 \rightarrow 0$ cascade, while the lower plot shows a similar attempt for the $3.63 \rightarrow 1.43$ transition. An acceptable fit to all of the correlation data can be obtained only for the $J = \frac{7}{2}$ possibility. Values of the population parameter $P(\frac{1}{2})$ are shown for $J = \frac{7}{2}$; the normalization is such that $2P(\frac{1}{2}) + 2P(\frac{3}{2}) = 1$.

dipole or pure quadrupole, consistent with the two minima shown in Fig. 8(a). The solutions for the population parameters are indicated by giving (for each) the values of $P(\frac{1}{2})$ determined from the least-squares fit, noting that the normalization is such that

$$2P(\tfrac{1}{2}) + 2P(\tfrac{3}{2}) = 1 .$$

For the $J = \frac{9}{2}$ possibility we conclude that the transition may not be pure quadrupole, but must rather have a significant octupole admixture, i.e., $x > 0.14$. The most favorable case for mixing occurs for $E3/M2$. However, the single-particle estimates for the lifetimes of $E3$ and $M2$ transitions of this energy are, respectively, 4.5×10^{-7} and 2.3×10^{-12} sec. The experimental restriction on the mean lifetime of the 3.63-MeV level, $\tau < 0.3$ psec, therefore implies an enhancement of *both* the $E2$ and $M2$ components of ~100, which we consider very unlikely.

The results of a similar analysis of the data on the $3.63 \to 1.43$ transition are shown in Fig. 8(b). Acceptable fits to the data are found only for $J=\frac{5}{2}$, $\frac{7}{2}$, $\frac{9}{2}$. For $J=\frac{9}{2}$, the experimental angular-correlation data can be fitted only for $x > 0.12$, which in this case corresponds to the mixing of $L=4$ with $L=3$ radiation. This result implies, in conjunction with the previously noted lifetime restriction, enhancements for both the $L=3$ and $L=4$ components that are several orders of magnitude greater than that expected from the sum-rule restriction $\Gamma_\gamma < Z^2$.[14] The net information on both the $3.63 \to 1.43$ and $3.63 \to 1.85 \to 0$ correlations thus allows a firm rejection of the possibility $J=\frac{9}{2}$, as well as the possibility $J=\frac{3}{2}$. We now consider in greater detail the results for $J=\frac{5}{2}$ and $\frac{7}{2}$. If $J=\frac{5}{2}$, the quadrupole/dipole mixing in the $3.63 \to 1.43$ transition is restricted to be in the range $x < -0.3$ or $x \cong \infty$. Both members of the $3.63 \to 1.43 \to 0$ cascade were fitted simultaneously, where the mixing in the first member of the cascade was restricted as given above, while possible mixing (x_2) in the second member was allowed to vary between $-\infty$ and $+\infty$. The minimal value of χ^2 thus obtained was $\chi^2 = 5.7$, which exceeds the 0.1% confidence limit of $\chi^2 = 3.3$. Thus the possibility $J=\frac{5}{2}$ for the 3.63-MeV level, which was ruled against also by the $3.63 \to 1.85 \to 0$ correlation data, may now be firmly excluded from further consideration.

Thus we conclude that the spin of the 3.63-MeV level is $J=\frac{7}{2}$. We now consider both members of the $3.63 \to 1.43 \to 0$ cascade in order to fix the mixing in the second member. This was done by computing a simultaneous fit to both sets of angular distribution data, with the mixing in the $3.63 \to 1.43$ transition restricted to vary within the range $-0.5 \le x_1 \le 8.1$ [as determined from Fig. 8(b)] and with the mixing x_2 in the $1.43 \to 0$ transition allowed to vary between $-\infty$ and $+\infty$. The resultant three-dimensional plot of χ^2 versus the two parameters x_1 and x_2 shows a well-defined minimum for $\arctan x_2 \cong 30°$, corresponding to a solution $x_2 = +(0.58^{+0.58}_{-0.38})$. Only one minimum is found for x_1, namely $x_1 = 0.14 \pm 0.12$, in agreement with the lesser solution for x_1 shown in Fig. 8(b); the possibility $x_1 = 3.7 \pm 1.5$ is now excluded. Thus, the correlation data on the $3.63 \to 1.43 \to 0$ transitions determine uniquely the mixing in both members of the cascade.

We now return to a consideration of Fig. 8(a). For $J=\frac{7}{2}$, there are two solutions for the mixing in the $3.63 \to 1.85$ transition. For $x \sim 0$ the population parameters are $P(\frac{1}{2}) = (0.50 \pm 0.08)$ and $P(\frac{3}{2}) = (0.00 \pm 0.08)$. For $x \sim \infty$, the solution requires $P(\frac{1}{2}) = (0.00 \pm 0.02)$ and $P(\frac{3}{2}) = (0.50 \pm 0.02)$. The latter case is clearly in disagreement with the alignment evident in Fig. 8(b) namely $P(\frac{1}{2}) = (0.46 \pm 0.08)$, $P(\frac{3}{2}) = (0.04 \pm 0.08)$. Thus, only the possibility $x \cong 0$ for the $3.63 \to 1.85$ transition is consistent with both sets of data; the overlap between the population parameters deduced from each is satisfactory.

If the parity of the 3.63-MeV level were odd, the $3.63 \to 1.43$ transition would be $M2$ in character, and for this transition energy the single-particle mean lifetime is $\tau = 82 \times 10^{-12}$ sec. Given the experimental limit of $\tau < 0.3 \times 10^{-12}$ sec and allowing for two standard deviations in the measured branching ratio leads to a lower limit for the $M2$ strength $|M(M2)|^2$ of 156 W.u. This is extremely unlikely and we thus conclude that the $3.63 \to 1.43$ transition has $E2$ character and the 3.63-MeV level has positive parity.

In summary, the 3.63-MeV level is found to have $J^\pi = \frac{7}{2}^+$. The $3.63 \to 1.85$ transition is almost purely dipole, with possible quadrupole mixing restricted by $x = +(0.05^{+0.11}_{-0.07})$. The $3.63 \to 1.43$ transition may have a small octupole admixture, although the mixing ratio $x = 0.14 \pm 0.12$ is not inconsistent with a pure quadrupole character for this transition.

4.05-MeV Level

As a first step in the analysis, the angular distribution of the $4.05 \to 1.43$ transition was fitted for assumed spins $J=\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, or $\frac{9}{2}$ for the initial level. (The possibility $J=\frac{1}{2}$ is directly excluded by the size of the nonzero $P_2(\cos\theta)$ coefficient given in Table II.) For the first three possibilities acceptable fits to the data were achieved for certain ranges of the $(L+1)/L$ mixing parameter x. For $J=\frac{9}{2}$ however, the best fit to the data yields a value $\chi^2 = 15$, which considerably exceeds the 0.1% limit at $\chi^2 = 5.5$, and thus we reject this from further consideration. The restrictions on x for the acceptable possibilities are as follows:

$$J=\tfrac{3}{2}:\ -\infty \le x \le +\infty \quad \text{(i.e., no restriction)},$$

$$J=\tfrac{5}{2}:\ |x| > 8.0 \ \text{ or } x = -(0.38^{+0.20}_{-0.13}),$$

and

$$J=\tfrac{7}{2}:\ +0.27 \le x \le +2.7\,.$$

We next investigated the net correlation information in the $4.05 \to 1.43 \to 0$ cascade, where for each spin possibility the mixing x_1 in the $4.05 \to 1.43$ cascade was restricted to vary only within the range indicated above, and the mixing x_2 in the $1.43 \to 0$ transition was taken as variable. These results exclude $J=\frac{7}{2}$ as a possibility for the spin of the 4.05-MeV state and determine for the $1.43 \to 0$ transition that $x_2 = +(0.6^{+0.5}_{-0.4})$.

We now have from the present experiment two independent determinations of the mixing in the $1.43 \to 0$ transition, which agree well with each other and also with the previously quoted result of Moss, Poore, Roberson, and Tilley.[3] We therefore take a weighted average of these values, x

$= 0.62 \pm 0.14$, for the final analysis of the 4.05 $\rightarrow$ 1.43 $\rightarrow$ 0 correlation data.

Figure 9 shows the results of a simultaneous fit to the distribution data on each member of the 4.05 $\rightarrow$ 1.43 $\rightarrow$ 0 cascade. As indicated in the inset level diagram, x_1 is taken as the variable, while the mixing in the 1.43 $\rightarrow$ 0 transition is fixed within the range $x_2 = +(0.62 \pm 0.14)$. Only $J = \frac{3}{2}$ or $\frac{5}{2}$ assignments for the 4.05-MeV level yield acceptable fits. For $J = \frac{3}{2}$ we have the restriction $|x| > 8.1$ (taken at the 1% confidence limit), while for $J = \frac{5}{2}$, $x_1 = -(0.19^{+0.05}_{-0.02})$. These results were all obtained from analysis of the correlation data obtained for a bombarding energy $E_t = 2.45$ MeV. An independent analysis of the data obtained for $E_t = 3.1$ MeV yields almost identical results, confirming the conclusions of the analysis summarized schematically in Fig. 9. Unfortunately, the two measurements do not in any way distinguish between the two possibilities $J = \frac{3}{2}$ or $J = \frac{5}{2}$ for the spin of the 4.05 MeV level. The reason for this is that the population parameters $P(\frac{1}{2})$ and $P(\frac{3}{2})$ describing the alignment of the 4.05-MeV level are essentially the same for both bombarding energies, as is evidenced in part by the similarity of the correlations shown in Table II measured at the two bombarding energies.

In conclusion, the spin of the 4.05-MeV level is either $J = \frac{3}{2}$ or $J = \frac{5}{2}$. The character of the 4.05 $\rightarrow$ 1.43 transition is found to be almost pure quadrupole if $J = \frac{3}{2}$, while for the $J = \frac{5}{2}$ possibility there must be significant quadrupole/dipole mixing, as is summarized in Table III.

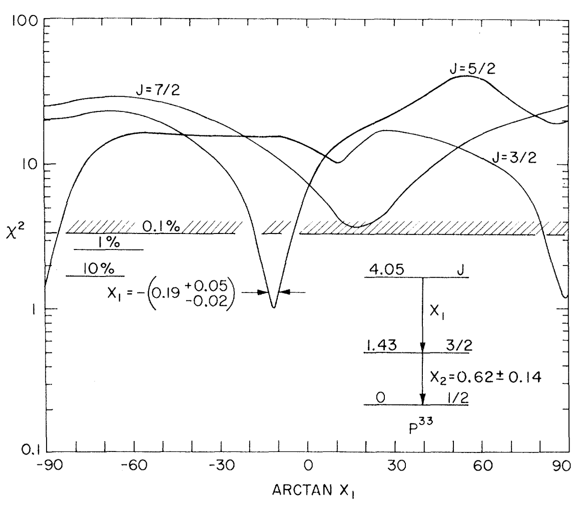

FIG. 9. Results of a simultaneous fit to correlation data for both members of the 4.05 $\rightarrow$ 1.43 $\rightarrow$ 0 cascade transitions. These results rule out the possibility $J = \frac{7}{2}$ for the spin of the initial state, and determine that for either of the remaining possibilities, $J = \frac{3}{2}$ or $\frac{5}{2}$, there must be a nonzero quadrupole component in the 4.05 $\rightarrow$ 1.43 transition.

4.19- and 4.22-MeV Levels

Although these two levels were unresolved in the particle spectrum, we have determined (as explained previously) that the 4.2-MeV γ radiation observed at $E_t = 3.1$ MeV can be attributed solely to the decay of the 4.19-MeV level. Similarly, the cascade radiation through the 1.85-MeV level observed for $E_t = 2.45$ MeV can be attributed to the decay of the 4.22-MeV level. Figure 10(a) shows the results of an analysis of the angular correlation of the 4.19-MeV γ ray observed at $E_t = 3.1$ MeV, and Fig. 10(b) shows the results of a similar analysis for the cascade radiation observed for $E_t = 2.45$ MeV.

The lower plot in Fig. 10 shows the results of a simultaneous fit to the 4.22 $\rightarrow$ 1.85 $\rightarrow$ 0 correlation data obtained at $E_t = 2.45$ MeV for assumed spins $\frac{3}{2} \leq J \leq \frac{11}{2}$ for the 4.22-MeV level. The variable in this fit was the mixing x_1 in the 4.22 $\rightarrow$ 1.85 transition, the mixing in the 1.85 $(\frac{5}{2}^+) \rightarrow 0(\frac{1}{2}^+)$ transition being fixed at $x_2 = 0$ in keeping with the previously discussed results on this transition. Only the $J = \frac{7}{2}$ assumption leads to an acceptable fit to the data, which also requires a quadrupole/dipole mixing ratio of about zero. All other possibilities (includ-

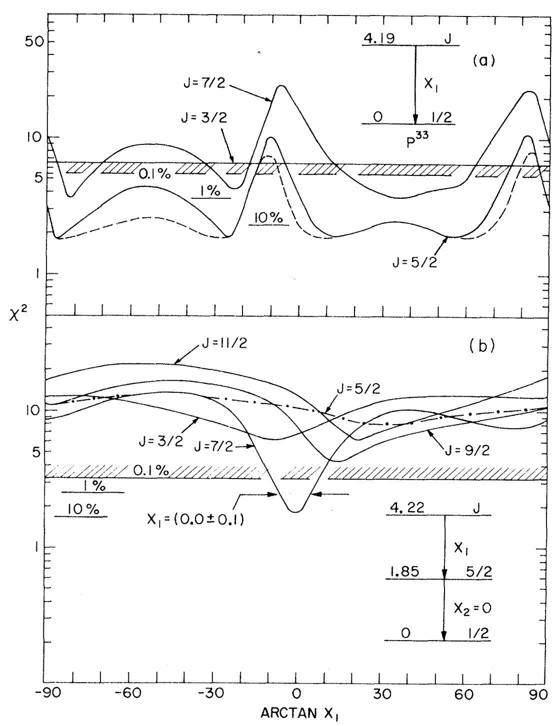

FIG. 10. Partial results of an analysis of proton-γ correlation data for the 4.19-MeV level (upper plot) and 4.22-MeV level (lower plot) of P^{33}. The transitions whose correlation patterns were fitted are indicated in the insert. These results determine $J = \frac{7}{2}$ for the 4.22-MeV level, and $J = \frac{5}{2}$ or $\frac{7}{2}$ for the 4.19-MeV level.

ing $J=\frac{1}{2}$) are rigorously excluded.

The upper plot Fig. 10(a) shows the result of a fit to the $4.19\rightarrow0$ correlation data obtained at $E_t=3.1$ MeV for assumed spins $\frac{3}{2}\leq J\leq\frac{7}{2}$ for the 4.19-MeV level. As can be seen, the $J=\frac{3}{2}$ possibility is rigorously excluded, as might have been expected from the $P_4(\cos\theta)$ term evident in the $1.85\rightarrow0$ distribution pattern. $J=\frac{1}{2}$ is similarly excluded.

For $J=\frac{7}{2}$, we have the restriction (taken at the 0.1% confidence limit) that $|x|>0.25$, corresponding to an $(L=4)/(L=3)$ mixing. The $4.19\rightarrow0$ transition was not positively identified in the Ge(Li) spectra. However, the resolving time used for the p-γ coincidence circuitry imposes a restriction on the mean life of the 4.19-MeV level of $\tau<80$ nsec. This in turn implys that the $L=4$ component must be enhanced by a factor of at least 10 above the single-particle estimate.[14] Thus we consider the $J=\frac{7}{2}$ possibility as quite unlikely, although it cannot be rigorously excluded. A better restriction on the lifetime of the 4.19-MeV level would clearly be useful here.

The parity of the 4.22-MeV level has been previously established as odd from the results of Davies, Harvey, and Darcey[7] who determined $l_n=3$ and thus $J^\pi=\frac{5}{2}^-$ or $\frac{7}{2}^-$. Our results eliminate the $\frac{5}{2}^-$ possibility, and thus $J^\pi=\frac{7}{2}$ for the 4.22-MeV level, as indicated in Fig. 6.

IV. SUMMARY AND DISCUSSION

Our conclusions on the P^{33} level structure have been summarized schematically in Fig. 6, which shows the spin-parity assignments and γ-branching ratios for states of $E_{ex}<4.3$ MeV. Table III summarizes our conclusions on possible multipole mixing in the γ-ray transitions we have studied, and for comparison gives an average of all previously reported[3,6,9] results.

As noted in Sec. III, the present results provide independent confirmation of the spin-parity assignments previously reported for[3,5-7] the ground state and first three excited states. Our determination $J=\frac{3}{2}$, $\frac{5}{2}$ for the 3.28-MeV level is also in agreement with the results of Davies, Harvey, and Darcey, who conclude that the level is (most probably) populated in the $P^{31}(t,p)P^{33}$ reaction by $l_n=2$ transfer, and thus $J^\pi=(\frac{3}{2}^+, \frac{5}{2}^+)$. We have confirmed the previously reported $1.85\rightarrow1.43$ γ-ray branch reported by Moss *et al.*[3] Our value of (8 ± 3)% for the branching ratio is in good agreement with the value (6 ± 2)% reported by them.

Until this point, we have refrained from a discussion of the results of Hardie *et al.*[9] in order that our results, arrived at independently, might be compared on an equal basis. Hardie and coworkers have measured p-γ correlations in the $Si^{30}(\alpha,p)P^{33}$ reaction utilizing a Ge(Li) detector for γ-ray detection. Specifically, they have thus observed the γ-ray decay of the states at 1.43, 1.85, 3.28, 3.49, 3.63, and 4.22 MeV. Their conclusions on level excitation energies have been included in Table I (column three). For the 1.43-MeV level they determine a quadrupole/dipole mixing $x=0.58\pm0.11$, in good agreement with present results and that of Moss *et al.*[3] For the higher-lying levels they determine spin assignments as follows: 3.49-MeV level, $J=\frac{3}{2}$, $\frac{5}{2}$; 3.63-MeV level, $J=\frac{7}{2}$. Their restrictions on possible multipole mixings are given in Table III (column four).

In summary, we find excellent agreement between the two sets of conclusions on allowed spin-parity assignments and on the corresponding multipole mixings, and accordingly present the averaged values for x given in Table III (column five). Note that for the $3.49\rightarrow1.43$ transition, the solution $x=1.4\pm0.3$ allowed by the present experiment for the possibility $J=\frac{3}{2}$ can now be excluded.

Similarly, there is with one exception good agreement on the γ branching of these states. For the 3.49-MeV level they report branching ratios of (40 ± 15)% and (60 ± 15)% for transitions to the 1.43- and 1.85-MeV levels, respectively, while the corresponding branches for the 3.63-MeV level are, respectively, (70 ± 20)% and (30 ± 20)%. Although less accurate, the values are in good agreement with those summarized in Fig. 6. The exception is for the 3.28-MeV level. Hardie *et al.* set a 12% limit on a possible ground-state branch, in contradiction to the (49 ± 8)% branch observed in the present experiment. The presence of the ground-state branch is quite evident in the decay spectrum of Fig. 3, and therefore we tend to rely on the branching ratios quoted in Fig. 6, although an independent check on this point is clearly desirable. Again, the relative branching to the 1.43- and 1.85-MeV states could not be determined in these coincidence studies even with the improved resolution of the Ge(Li) detector. Finally, they were able to place an upper limit of 15% on a possible $4.22\rightarrow0$ transition. It is thus clear that in the (α,p) reaction it is the 4.22-MeV state which is populated, rather than the 4.19-MeV level. This point is further substantiated by the measured[9] energies.

In conclusion, the level diagram given in Fig. 6 adequately summarizes the presently available information on the lower-lying states of P^{33}. We now consider briefly a comparison of this experimental spectrum of levels with that predicted theoretically.

The intermediate-coupling calculations reported by Bouten, Elliot, and Pullen[2] included the first three $T=\frac{3}{2}$ states of the $A=33$ system. We note that while their predicted order of spins and parities for the first three $T = \frac{3}{2}$ levels ($\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$) agrees with those found experimentally, they predict that the $\frac{3}{2}^+$ first excited state should lie at about 100 to 200 keV above the $\frac{1}{2}^+$ (ground) state and that the $\frac{5}{2}^+$ (second excited) state should be at an excitation energy of about 800–1000 keV. This relative spacing certainly does not agree with the experimental situation.

Figure 11 shows the results of the shell-model calculation given in Ref. 1 and the present experimental situation in P^{33}. In performing the calculation, Glaudemans, Wiechers, and Brussaard[1] assumed that the $d_{5/2}$ shell was closed and that levels in P^{33} arose from all possible configurations involving a proton and two neutrons in the $2s_{1/2}$ and $d_{3/2}$ shells. The dashed lines connect experimentally known and theoretically predicted states that can be reasonably associated with one another. The theoretically predicted position and spin of the first excited state agrees well with the experimental results. Qualitatively, at least, the lifetime and multipole mixing of the $1.43 \to 0$ γ ray are consistent with the shell-model picture for the ground and first excited states. The $E2/M1$ mixing ratio of 0.59 ± 0.10 implies some dipole retardation relative to the quadrupole strength, and the measured lifetime[6] of the 1.43-MeV level corresponds to an $M1$ strength of only 0.01 W.u. The major components of the ground- and first-excited-state wave functions given in Ref. 1 (and expected from simple shell-model considerations) are

$$[(s_{1/2}{}^3)_{1/2,\,1/2}(d_{3/2}{}^2)_{0,\,1}]_{1/2,\,3/2}$$

and

$$[(s_{1/2}{}^2)_{0,\,1}(d_{3/2}{}^3)_{3/2,\,1/2}]_{3/2,\,3/2}\,,$$

respectively. For a transition between these components a nucleon must make a $d \to s$ transition which cannot occur via a dipole operator. Hence, the dipole strength in the transition must arise from smaller components in the wave functions which lead to a retardation in the $M1$ strength, as is seen experimentally. The spin and parity of the second excited state in both the experimental and theoretical pictures of P^{33} is $\frac{5}{2}^+$. As noted in Refs. 3 and 5 it would, however, be misleading to suggest that the experimental 1.85-MeV level is the 2.50-MeV level predicted by the theory, since the two states have somewhat different character. The large spectroscopic factor for this level measured in the $S^{34}(d, He^3)P^{33}$ reaction[5] evidently indicates an appreciable excitation of the $d_{5/2}$ shell which is not accounted for in the calculation of Glaudemans, Wiechers, and Brussaard.[1] As shown in Fig. 11 we have associated both of the remaining positive-parity levels at 2.54 and 3.63 MeV for which the spin and parity is definitely established with levels predicted by the theory. The calculation does not, of course, predict odd-parity levels, since the nucleons external to the closed $d_{5/2}$ shell are restricted in this case to the $2s_{1/2}$-$1d_{3/2}$ shell.

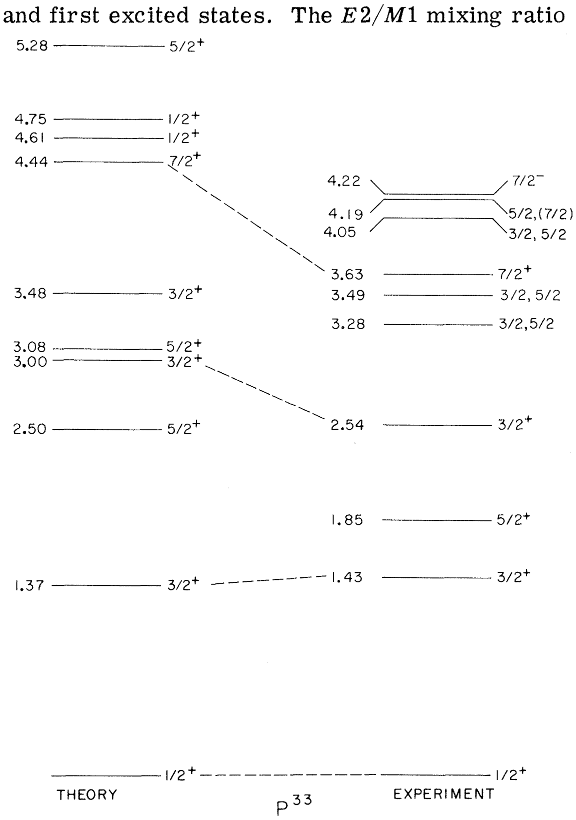

FIG. 11. Comparison of the experimental P^{33} level scheme, as obtained in this and all previous work, with the scheme predicted theoretically (see Ref. 1) by Glaudemans, Wiechers, and Brussaard. The correspondence of various levels are indicated by the connecting dashed lines.

Since the second excited state of P^{33} evidently involves excitation of the $d_{5/2}$ shell,[5] one expects such contributions to higher states; and it appears that a more detailed comparison between the experimental and theoretical level schemes should await a calculation including excitation of the $d_{5/2}$ shell. The level diagram resulting from such a calculation by Wildenthal *et al.* (allowing excitations of the $d_{5/2}$ shell) is given in Ref. 9. This calculation puts the first $J^\pi = \frac{5}{2}^+$ level at 1.72 MeV, and this level could correspond to the experimental 1.85-MeV level. The level diagrams given in Ref. 9 also show that excitations of the $d_{5/2}$ shell cause an increase in the density of levels below 4.5 MeV, and like the inert Si^{28} core calculation,[1] predicts an excited $J^\pi = \frac{1}{2}^+$ level within the first seven excited states which has not been found experimentally.

Some attempts have been reported to identify $T = \frac{3}{2}$ states in S^{33} and Cl^{33}.[6,15] Dubois has, on the basis of his $S^{34}(He^3, \alpha\gamma)S^{33}$ data, assigned $T=\frac{3}{2}$ to the 5.48-, 6.90-, and 7.35-MeV levels in S^{33} which would make them the analog of the ground and first two excited states of P^{33}, respectively. The assignments $l_n=0$ and 2 for the two lowest $T=\frac{3}{2}$ levels[15] in S^{33} are consistent with the $J^\pi=\frac{1}{2}^+$ and $\frac{3}{2}^+$ assignments to the ground and first excited states of P^{33}. The definite positive-parity assignment to the 1.85-MeV level in P^{33} means, however, that the 7.35-MeV level in S^{33} either does not have $T=\frac{3}{2}$, or the tentative $l_n=3$ assignment to transferred neutrons leaving S^{33} in this state from Ref. 15 is incorrect. As Dubois[15] pointed out, his data suggest a $T=\frac{3}{2}$ level at 6.36-MeV in S^{33} which corresponds to 0.88-MeV excitation in P^{33}. The energy discrepancy of 0.55 MeV rules out the possibility that this is the analog of the 1.43-MeV level. Studies of the reactions $Si^{30}(\alpha,p)P^{33}$,[3-5] $S^{34}(d, He^3)P^{33}$,[5] and $P^{31}(t,p)P^{33}$ have given no evidence for a level at $E_x \simeq 0.88$ MeV in P^{33}. In the present work, we studied the γ-ray decay of nine levels which would lie at higher excitation energy in P^{33} and found no evidence for decay through a level at 0.88 MeV. Hence, we find the evidence against the existence of a level at 0.88 MeV in P^{33} quite definite and conclude that the 6.36-MeV level in S^{33} is not $T=\frac{3}{2}$. It would be interesting to extend Dubois's work[15] to higher excitation energy in S^{33} since Davies, Harvey, and Darcey[7] have suggested that the 8.15-MeV level in Cl^{33} is the isobaric analog of the 2.54-MeV level in P^{33}. An extension of the $S^{34}(He^3, \alpha\gamma)$-$S^{33}$ work to this region of S^{33} could aid in identifying the corresponding level in S^{33}.

Finally, we note that while a $J^\pi=\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$ spin progression is observed for the first three states of both P^{31} and P^{33}, the third excited state of P^{31} is $J^\pi=\frac{1}{2}^+$. From the experimental evidence presently available (Fig. 6) the second $\frac{1}{2}^+$ state in P^{33} would appear to be at $E_{ex}>4.2$ MeV.

ACKNOWLEDGMENTS

The authors are grateful to C. Z. Nawrocki and F. A. Mahnken for preparing all targets used in the present work.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

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