corresponding structure was observed in the $^{31}P(\alpha, \alpha_0)$ excitation curve, a $T=\frac{3}{2}$ assignment is favored for these levels. The parent states of these resonant levels in ^{35}Cl would occur at 7.4 and 8.4 MeV in ^{35}S. Since the level scheme of ^{35}S above 5-MeV excitation is not yet known, no comparison can be made at this time.

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$^{88}Sr(d, p)^{89}Sr$ Reaction in the Region of the $^{88}Sr(d, n)^{89}Y^A$ Threshold*

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$^{88}Sr(d,p)^{89}Sr$ excitation curves were measured at 90, 140, 160, and 170° for four states in ^{89}Sr: the $d_{5/2}$ ground state, 1.03-MeV $s_{1/2}$ state, 2.00-MeV d doublet, and 2.45-MeV $d_{3/2}$ state, at deuteron energies from 5.0 to 10.5 MeV. The excitation curves for ground state and 1.03-MeV $s_{1/2}$ state display cusps at the threshold of the charge-exchange-coupled channel, $^{88}Sr(d,n)^{89}Y^A$. The data are fit with coupled-channel Born-approximation calculations.

I. INTRODUCTION

The suggestion that charge exchange could couple analogous (d,p) and (d,n) channels was first given by Moore *et al.*,[1] who reported the experimental observation of an anomalous dip in the $^{90}Zr(d,p)^{91}Zr(d_{5/2}$ ground-state) excitation function at 170°, centered on the experimental $^{90}Zr(d,n)^{91}Nb^A(d_{5/2}$ analog resonance) threshold. A large number of subsequent experiments have provided examples of similar, usually somewhat weaker, anomalies in backward angle $(>130°)(d,p)$ or (p,d) excitation functions, for targets $^{91,92,94,96}Zr$,[2-4] $^{92,94}Mo$,[3] ^{89}Y,[5] ^{80}Se,[6] and also ^{40}Ar,[7] ^{48}Ca,[8] and ^{53}Cr.[9] In each case an apparent dip in the cross section, roughly an MeV broad, appears centered on the appropriate charge-exchange (d,n) threshold, providing a beautiful example of the long-predicted but rarely seen threshold cusp phenomenon. Various efforts to find similar effects using lighter- and heavier-mass nuclei have not succeeded.[10]

The most dramatic known example of the threshold effect is provided by $^{88}Sr(d,p)^{89}Sr(d_{5/2}$ g.s.) in the vicinity of the lab deuteron energy 7.4 MeV. The cusp, as the data presented here show in comparison with $^{90}Zr(d,p)^{91}Zr(d_{5/2}$ g.s.) data[11] at the same angles, is the same width and deeper by nearly a factor of 2. Thus, it provides a harsh test for theoretical descriptions of the threshold effect.

A theory of the (d,p) threshold effect was originally given in terms of the Lane model by Zaidi and Brentano.[12] Several more elaborate formulations have recently been presented by Zimanyi and Gyarmati,[13] Hooper,[14] and Bang and Zimanyi,[15] still in terms of the Lane model, but no calculations have been performed using such approaches. Any numerical calculation must overcome the convergence difficulties arising from the fact that stripping in the charge-exchange (d,n) channel occurs to an isobaric analog resonance, not a bound residual state.[16,17] General formulations exist to handle such problems,[18,19] but again these have not been applied in practical numerical calculations.

The simplest approach remains that of Ref. 12, which has been used by Tamura and co-workers[20,21] to calculate theoretical cross sections for ^{90}Zr$(d,p)^{91}$Zr$(d_{5/2}$ g.s.) and ^{92}Mo$(d,p)^{93}$Mo$(d_{5/2}$ g.s.). The agreement with the available data, using no arbitrary adjustable parameters, is excellent.[21] We have therefore adopted this approach in the analysis of the ^{88}Sr data, although, as will be clear from later discussion, the unavailability of ^{88}Sr-$(d,n\bar{p})^{88}$Sr data requires that our analysis involve one adjustable parameter.

II. EXPERIMENTAL METHODS

Self-supporting targets of natural strontium were fabricated by evaporating the metal in high vacuum onto detergent-coated glass slides. The slides were allowed to cool, and the bell jar was then filled with hydrogen gas. The slides were quickly transferred from the hydrogen atmosphere to liquid kerosene. In the kerosene bath the metal films were peeled from the slides and sandwiched between two perforated aluminum sheets which served as the target frame. The prepared targets were then quickly placed in a scattering chamber and stored there in vacuum.

The procedure described is necessary, since when a thin film of strontium is exposed to air it oxidizes within 30 sec to brittle white flakes. The procedure used protects the targets, during transfer to the scattering chamber, with a thin film of kerosene.

Target thickness was obtained from Rutherford scattering at 30° with 3-MeV deuterons. The target used in this experiment was 670 μg/cm^2 thick.

Four Si(Li) detectors were placed at 170, 160, 140, and 90° and cooled to the temperature of dry ice in methanol to inhibit leakage current. The University of Texas EN Tandem Van de Graaff accelerator was used to provide a 0.5-μA deuteron beam over an energy range of 5.0 to 10.5 MeV. Data were accumulated on-line in a PDP-7 computer. An energy resolution of 50 keV was obtained for the proton groups from five low-lying levels in ^{89}Sr. Since the 1.93- and 2.00-MeV $l=2$ states were not resolved at all deuteron energies, they were summed together. Excitation curves were thus obtained for the $d_{5/2}$ g.s., 1.03-MeV $s_{1/2}$, 2.0-MeV $d_{5/2}$-$d_{3/2}$ doublet, 2.45-MeV $d_{3/2}$, and 2.75-MeV $g_{7/2}$ states of ^{89}Sr.

III. DISCUSSION OF RESULTS

The characteristics expected experimentally for the charge-exchange effect are thoroughly discussed in Refs. 11 and 21, and there is no need to repeat them here. The criterion $\Delta_C \leq 4Q$, where Δ_C is the Coulomb displacement energy of the residual nucleus and Q is the (d,p) Q value, is satisfied for both the $d_{5/2}$ g.s. transition and the 1.03-MeV $s_{1/2}$ transition. Hence both would be expected to show threshold dips. In Fig. 1 are plotted the excitation curves at 160° for the $d_{5/2}$ g.s., 1.03-MeV $s_{1/2}$ state, and 2-MeV $d_{5/2}$-$d_{3/2}$ doublet. The very strong cusp in the ground-state excitation curve is striking. It is interesting that the "point" of the cusp does not occur at 7.4 MeV, but rather closer to 7.5 MeV.

As usual in the $A\approx 90$ region, the threshold for the $s_{1/2}$ state occurs at an energy coinciding with a "natural" minimum in the cross section.[3] However, the cusp is still quite apparent, centered at 8.5 MeV, the expected threshold energy also being 8.5 MeV. No cusp would be expected at the threshold, about 9.4 MeV, for the 2.0-MeV doublet, since it violates the $\Delta_C \leq 4Q$ criterion, and no cusp is observed. Similarly no cusps are expected for the higher $d_{3/2}$ (2.45-MeV) and $g_{7/2}$ (2.75-MeV) states, and none are observed, as seen in Fig. 2.

Finally, a noticable "spike" is seen in the $s_{1/2}$ excitation curve at about 7.75 MeV, most prominently at 140°. A similar spike was observed by Heffner *et al.*,[3] in ^{92}Zr$(d,p)^{93}$Zr$(s_{1/2})$, at 7.4 MeV. An unpublished study by Michelman and Moore (summarized in Ref. 11) shows that a number of such resonancelike structures in the (d,p)-to-$s_{1/2}$ excitation curves in the $A\approx 90$ region line up with similar structures of opposite slope in the (d,p)-to-$d_{5/2}$ excitation curves. These "resonances" are probably related to the similar small dips seen at $E_p \approx \Delta_C$ in (p,d) reactions on various nuclei in the $A\simeq 90$ region by the Seattle group[22] and Michelman, Bonner, and Kulleck.[4] They certainly merit further investigation, but are very unlikely to be related to the threshold effect, because of their narrowness ($\lesssim$100 keV).

IV. CCBA ANALYSIS

Coupled-channel Born-approximation calculations (CCBA) were performed on the University of Texas CDC-6600 for ^{88}Sr(d,p) leading to the

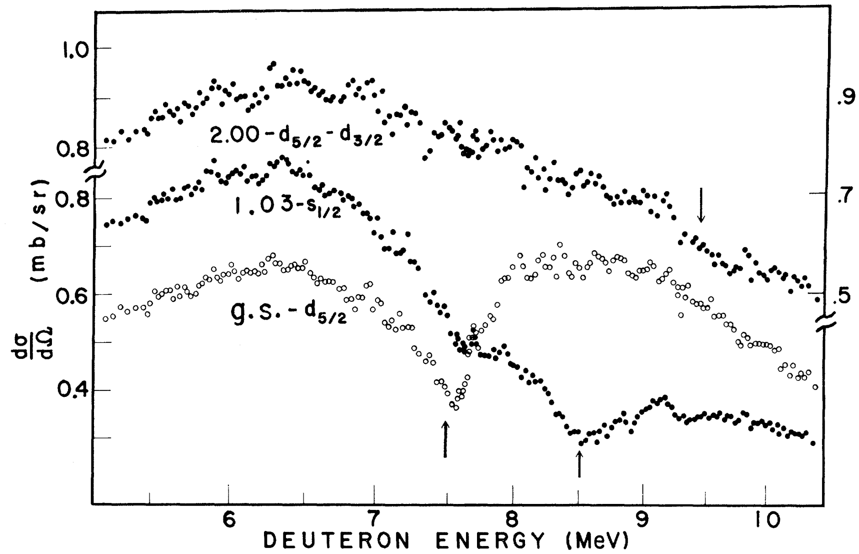

FIG. 1. Excitation curves at 160° as a function of lab deuteron energy, for the ^{88}Sr(d,p) reaction leading to the $d_{5/2}$ ground state, 1.03-MeV $s_{1/2}$ and 2.0 MeV $d_{5/2}$ - $d_{3/2}$ doublet states in ^{89}Sr. Vertical arrows mark the approximate location of the ^{88}Sr$(d,n)^{89}$Y^A threshold for each state, assuming $\Delta_C = 11.4$ MeV.

ground $d_{5/2}$, 1.03 MeV $s_{1/2}$, 2.0-MeV $d_{3/2}$-$d_{5/2}$ doublet, and 2.45-MeV $d_{3/2}$ states in ^{89}Sr, using the procedure suggested in Ref. 21, hereafter called CT.

The proton and deuteron optical potentials were obtained from the 7-MeV ^{88}Sr(d,p) study of Cosman, Enge, and Sperduto,[23] who performed distorted-wave Born-approximation (DWBA) analyses for 28 states in ^{89}Sr, up to 5.42 MeV in excitation. In the usual notation, their deuteron potential has $V = 96$ MeV, $W_D = 20$ MeV, $V_{so} = 0.0$, $r = 1.15$ fm, $r' = 1.34$ fm, $r_C = 1.3$ fm, $a = 0.81$ fm, and $a' = 0.68$ fm. The proton potential used has $V = 51.05$ MeV, $W_D = 13.5$ MeV, $V_{so} = 0.0$, $r = r' = r_C = 1.25$ fm, $a = 0.65$ fm, and $a' = 0.47$ fm. As in CT, the neutron potential was $V = 52.0$ MeV, $W_D = 8.1$ MeV (for $E_n > 0$), $V_{so} = 7.2$ MeV, $r = r' = r_{so} = r_C = 1.27$ fm, $a = a_{so} = 0.66$ fm, and $a' = 0.47$ fm. A real volume-type Lane potential was used, of strength $27(N-Z)/A$ MeV. Thus the actual real well depths in proton and neutron channels are $V_p = 55.0$ MeV and $V_n = 48.1$ MeV.

No part of any optical potential was varied with energy in calculating the excitation curves from

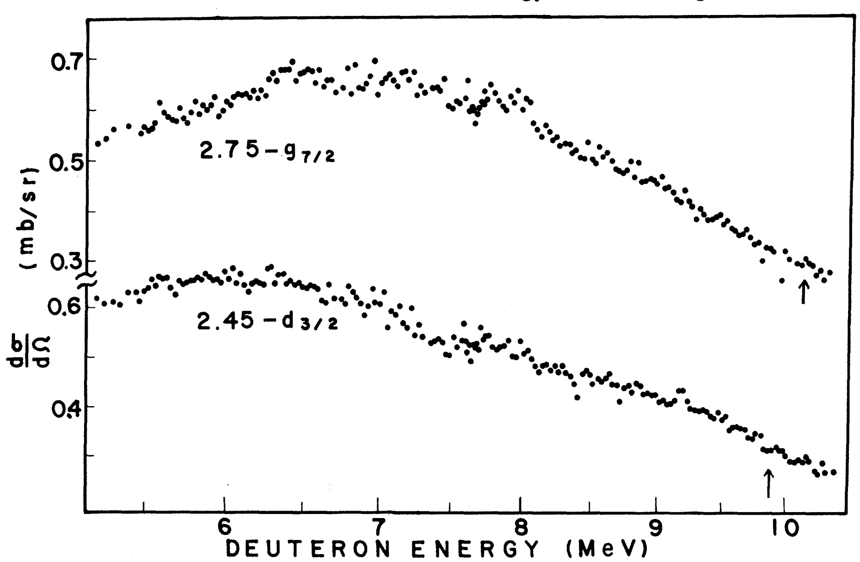

FIG. 2. Excitation curves at 160° as a function of lab deuteron energy, for the ^{88}Sr(d,p) reaction leading to the 2.45-MeV $d_{3/2}$ and 2.75-MeV $g_{7/2}$ states in ^{89}Sr. Vertical arrows mark the approximate locations of the ^{88}Sr$(d,n)^{89}$Y^A threshold for the two states, assuming $\Delta_C = 11.4$ MeV.

5.0 to 11.0 MeV, except for W_D of the ^{88}Sr$(d,n)^{89}$Y^A neutrons. For $E_d > 7.4$ MeV, W_D was fixed at 8.1 MeV. Between 6 and 7.4 MeV, W_D was allowed to increase smoothly from 0.0 to 8.1 MeV, as in CT.

The CCBA calculations have a single adjustable parameter. Following CT, this is R_C, the radius at which a propagating Coulomb wave is matched onto the usual bound-state neutron function in order to construct the wave function of the proton in the isobaric analog resonance (IAR) in ^{89}Y^A.[20] One could fix R_C independently, as CT did, by fitting ^{88}Sr$(d,n\tilde{p})$ data simultaneously, leaving no adjustable parameters in the model. However, such data are not available. Thus we were guided in our choice of R_C by its systematic behavior as stressed in CT.

CT found $R_C = 6.5$ fm for ^{90}Zr$(d,n\tilde{p})$ through the $d_{5/2}$ IAR, and $R_C = 9.5$ fm for ^{92}Mo$(d,n\tilde{p})$ through the $d_{5/2}$ IAR. The ^{93}Tc^A $d_{5/2}$ IAR lies nearly an MeV lower in the Coulomb barrier than the ^{91}Nb^A $d_{5/2}$ IAR, making this difference plausible.

The ground-state $\tilde{p}$ decay c.m. energy is 4.7 MeV for ^{91}Nb^A and 5.0 MeV for ^{89}Y^A. The Coulomb displacement energy is 11.9 MeV for ^{91}Zr-^{91}Nb^A and 11.4 MeV for ^{89}Sr-^{89}Y^A. Thus the first $d_{5/2}$ IAR in ^{89}Y^A is ~0.8 MeV closer to the top of the Coulomb barrier than the $d_{5/2}$ IAR in ^{91}Nb^A, at once seeming to explain the more striking cusp seen for ^{88}Sr$(d,p)^{89}$Sr. However, the results of ^{88}Sr(p,p)[24-26] and ^{90}Zr(p,p)[27] IAR analyses give, respectively, $\Gamma_p = 4$–8, $\Gamma = 12$–16 keV, and $\Gamma_p = 4$, $\Gamma = 22$ keV, for the two resonances. Since a compound-elastic correction was made in the ^{90}Zr analysis but not in the ^{88}Sr analyses, it is likely that $\Gamma_p(^{91}\text{Nb}^A\ d_{5/2}$, 4.71 MeV$) \approx \Gamma_p(^{89}\text{Y}^A\ d_{5/2}$, 5.02 MeV$)$.

Hence we would expect $R_C(^{89}\text{Y}^A) \lesssim R_C(^{91}\text{Nb}^A)$ and a stronger cusp. In fact we find that $R_C = 6.4$ fm gives a theoretical cusp very similar to the experimental ^{88}Sr(d,p) cusp. Comparison of ^{90}Zr(d,p) and ^{88}Sr(d,p) shows that at 170° the cusp is a departure of 50% from the expected DWBA cross section for ^{89}Sr$(d_{5/2}$ g.s.) compared with 36% for ^{91}Zr$(d_{5/2}$ g.s.).

In Figs. 3–6 are shown the CCBA calculations compared with the available data at various angles. For the excited states, a value of R_C was chosen to give a reasonable fit. Since the states with $4Q < \Delta_C$ are not very sensitive to R_C, its value is not a very important consideration in these calculations.

One sees that the over-all agreement in shape is good, except at 140°. Undoubtedly the fits for all states could be improved by variation of deuteron and proton optical parameters. However, we feel it is important to demonstrate here that a reasonable fit can be obtained to the threshold cusp using only standard DWBA parameters.

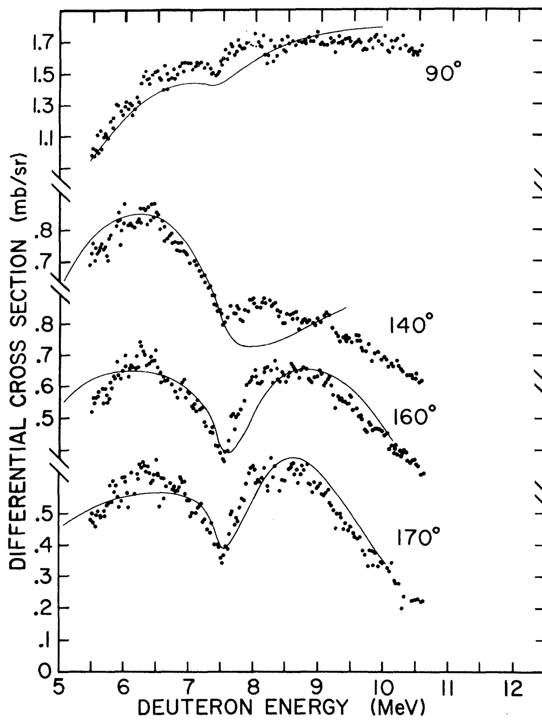

FIG. 3. Excitation curves at 170, 160, 140, and 90° as a function of lab deuteron energy, for ^{88}Sr$(d,p)^{89}$Sr$(d_{5/2}$-g.s.). The solid curves are CCBA calculations with $R_C = 6.4$ fm.

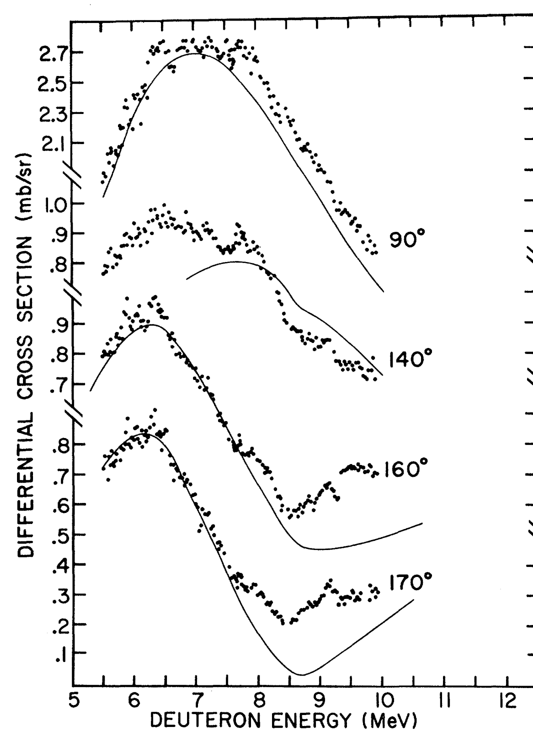

FIG. 4. Excitation curves at 170, 160, 140, and 90° as a function of lab deuteron energy, for ^{88}Sr$(d,p)^{89}$Sr(1.03-MeV $s_{1/2}$). The solid curves are CCBA calculations with $R_C = 6.2$ fm.

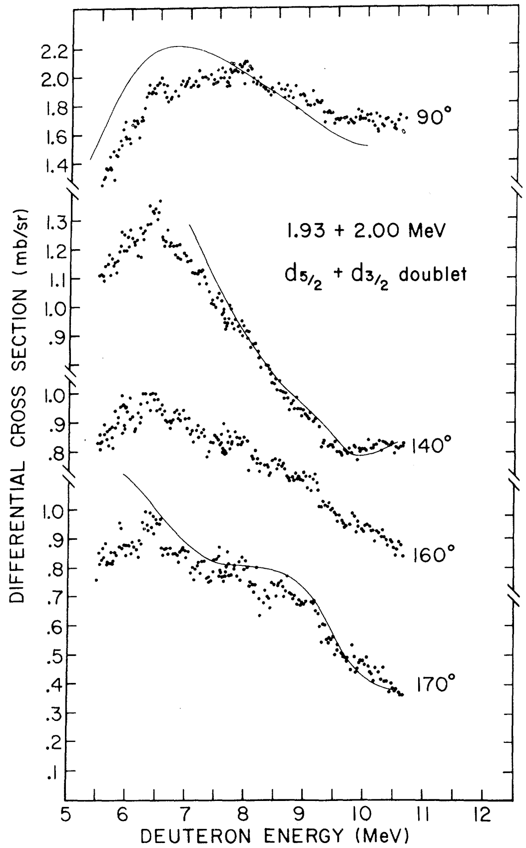

FIG. 5. Excitation curves at 170, 160, 140, and 90° as a function of lab deuteron energy, for ^{88}Sr$(d,p)^{89}$Sr(2.0-MeV doublet). The solid curves are CCBA calculations with $R_C = 6.0$ fm.

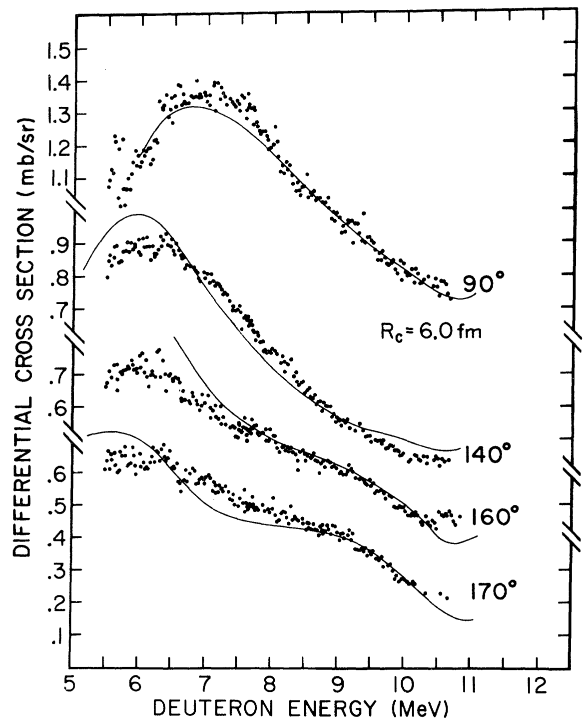

FIG. 6. Excitation curves at 170, 160, 140, and 90° as a function of lab deuteron energy, for ^{88}Sr$(d,p)^{89}$Sr(2.45 MeV $d_{3/2}$). The solid curves are CCBA calculations with $R_C = 6.0$ fm.

The normalizations required for the fits shown are, for $d_{5/2}$ g.s., 0.8; for 1.03 MeV $s_{1/2}$, 0.8; for 2.0 MeV d doublet, 1.0; for 2.45 MeV $d_{3/2}$, 0.5. Since these factors are extracted considering only angles greater than 90°, they should not necessarily agree with spectroscopic factors extracted from normalizing to the first stripping peak. However, for purposes of comparison, Cosman, Enge, and Sperduto[23] find for $d_{5/2}$ g.s., 0.79; for 1.03 MeV $s_{1/2}$, 0.90; for 2.0-MeV $d_{5/2}$-$d_{3/2}$ doublet 0.1 + 0.6; and, for 2.45 MeV $d_{3/2}$, 0.45. The agreement is seen to be satisfactory (i.e., within 30%).

One may conclude from the analysis presented here that it is easier to explain the cusp's appearance at 170° where it is largest, or at 90° where it is vanishingly small, than at intermediate angles where its precise appearance is very sensitive to all optical parameters. This fact is implicit, if not explicit, in the results of earlier analyses, and is worth stressing.[21,28] As is apparent from Fig. 3, the ground-state cusp is still visible at 90°, theoretically and experimentally. This is the only case known to the authors in which the cusp persists to angles forward of ~120° (see for example Fig. 4 of Ref. 21).

The small (~100 keV) structure at 7.75 MeV in the $s_{1/2}$ excitation curve is of course not predicted by these calculations. It is seen from Fig. 4 that this "resonance" is very prominent at 140° and may be associated with a "shoulder" running from 7.75 MeV to the threshold at 8.5 MeV. If, as seems possible, such anomalies are due to isobaric analog resonances in the outgoing proton channel[4] near $E_p \approx \Delta_C$, one could use the methods recently developed by Tamura and Coker[29] and by de Toledo Piza[30] to describe them.

The fits for the $d_{3/2}$-$d_{5/2}$ doublet, shown in Fig. 5, are made assuming pure $d_{3/2}$, since there is no apparent j dependence in the excitation curves.

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