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# Assessment of Two-Step Processes in $(p, t)$ Reactions*

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Multiple-step processes (core excitation) are evaluated for the $(p,t)$ reaction at a typical energy (30 MeV) and for a typical moderately collective spherical nucleus (nickel). Our model nucleus has three kinds of states; the ground and collective $2^+$, which have a strong direct transition, other typical noncollective states which also can be produced in a single step, and a two-phonon triplet, which can be produced only through multiple-step processes. We find that these latter are produced as strongly as the other noncollective states and that the angular distributions and polarizations are characterized by the multipolarity of the over-all transition and not by the multiplicity of the reaction mechanism. Comparison is made with the distorted-wave Born approximation. We find that it underestimates the $(p,t)$ cross sections to all three types of states, and that even relative cross sections are in error by up to a factor 2, and absolute cross sections by as much as a factor 5.

## 1. INTRODUCTION

Particle transfer reactions have been treated customarily as proceeding directly from the target ground state to the final state by a simple deposit or pickup of the transferred particle (simple or composite) which leaves the other nucleons undistributed.[1] Undoubtedly this is the dominant mechanism by which most low-lying levels are produced. However some nuclear levels, even at low excitation, will have a parentage that is based more on an excited state of the target rather than on the ground state. If this excited state is produced with appreciable cross section in inelastic collisions, then alternate modes of producing the final state in the transfer reaction are possible. These are the two-step modes that proceed through the intermediate state produced by inelastic collisions with the incoming or outgoing particle. In Fig. 1, two extreme cases of pure parentage are illustrated for the $(t,p)$ reaction. One state of $(A+2)$ is assumed to have a structure in which $A$ nucleons (which we shall refer to as the core) are in a state of motion corresponding to the ground state of nucleus $(A)$. In the other, the state of these $A$ nucleons corresponds to an excited state of nucleus $(A)$. The former can be reached by a direct transition which deposits a

pair of neutrons onto the target ground state. The latter can be reached only through the higher-order processes that involve the inelastic production of the parent state. Of course in real nuclei, the parentage of any state will not be pure. Instead each state of $(A+2)$ will have many parents. In principle, therefore, the final state can be reached both directly through that component of its wave function that has the ground state of $(A)$ as parent, and indirectly through the other components. Whether, in fact, the indirect modes compete with the direct will depend both on the fraction of parentage that is based on excited states of the core, and on the strength with which these states are produced in inelastic collisions.

The primary purpose of this investigation is to assess whether higher-order processes are likely to be significant in two-nucleon transfer reactions, and if so, whether there are any special characteristics by which they can be distinguished from direct transitions. While this question would seem largely academic as long as attention was focused on the lowest-lying states, it acquires ever more importance as improving experimental techniques allow detailed investigations of higher-lying levels whose parentage is expected to be more complicated. It is already evident that a number of interesting new phenomena involving
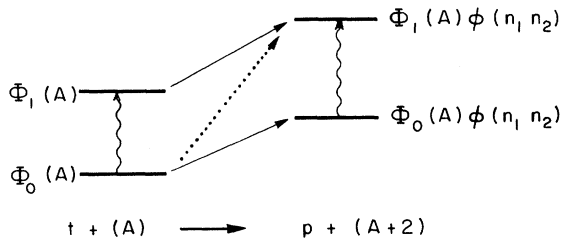
$$\Phi_1 (A) \phi (n_1 n_2)$$

$$\Phi_0 (A) \phi (n_1 n_2)$$

$$\Phi_1 (A)$$

$$\Phi_0 (A)$$

t + (A) ⟶ p + (A+2)

FIG. 1. An idealized example of two states in the nucleus $(A+2)$, the lower one having a pure parentage based on the ground state of nucleus $(A)$ and the other on an excited state. The higher state in $(A+2)$ is sometimes said to be "core excited." It clearly can be reached only by multiple-step processes that involve the inelastic excitation of the core by either the triton or proton.

second-order processes will be uncovered.[2]

It has been customary to compute by use of the distorted-wave Born approximation (DWBA) the cross section for those states that can be produced directly.[1,3] In this approximation, the wave function for the relative motion between the nucleus and free particle in the entrance and exit channels is generated by a one-channel optical potential. However when inelastic processes are very strong, the usual optical potential may not provide a sufficiently accurate representation for these wave functions *within* the nuclear interior, just where they have their largest overlap with the nuclear wave functions appearing in the DWBA integrals. Since the optical parameters are chosen so as to reproduce the observed elastic cross section, this assures that the wave function is correct in the *external* region. However if a particular inelastic transition is very strong, the population of the excited state becomes large enough that deexcitation back to the ground state becomes significant. This process cannot be described by a one-channel optical potential and it produces changes in the wave function in the nuclear interior, just where it is needed in the reaction calculation. In this circumstance the inelastic processes play a role even for states that are produced directly in the transfer reaction. Therefore, as a secondary purpose of this investigation, we examine at what point the strength of inelastic transitions leads to significant errors in the usual DWBA for direct single-step transitions.

The method that we use to include the multiple-step processes (sometimes referred to as core excitation) that proceed through excited states of either target or residual nucleus has been described in previous publications.[4,5] If the inelastic processes are neglected, the results correspond precisely to the usual DWBA. In other words, we treat the particle transfer reaction in the usual

way,[6] and in first order only. The inelastic transitions are treated to all orders among the states considered. We refer to it as the source-term method, and it is equivalent in its results to the procedure described by Penny and Satchler,[7] though apparently it is more amenable to numerical calculation.

## 2. NUCLEAR MODEL

To carry out the investigation described above, we adopt a model for a nucleus having a collectivity typical of spherical nuclei, which we refer to nominally as Ni. The ground state is taken to be a BCS vacuum. There are excited states of two-quasiparticle configurations, the lowest of which is the collective $2_1^+$ state.[8] In addition we construct a triplet of two-phonon states by using the operator that creates the collective $2^+$ state. This state, which we sometimes call the one-phonon state, has the structure,

$$|2_1^+\rangle = B_2{}^\dagger |BCS\rangle, \tag{1}$$

$$B_{2, M}{}^\dagger = \tfrac{1}{2} \sum_{a, b} \eta_{ab} [\alpha_a{}^\dagger \alpha_b{}^\dagger]_{2, M}, \tag{2}$$

where $a \equiv nlj$, and $\alpha_a{}^\dagger$ creates a quasiparticle in the state $a$. The $\eta_{ab}$ are the configuration mixing amplitudes, and the square bracket denotes vector coupling. The two-phonon states are

$$|J\rangle = \frac{1}{\sqrt{2}} [B_2{}^\dagger B_2{}^\dagger]_J |BCS\rangle, \quad J = 0, 2, 4. \tag{3}$$

They have the special significance that they cannot be produced directly by the two-nucleon transfer reaction. This follows because they are four-quasiparticle states and therefore cannot be connected to the ground-state vacuum by a two-particle process. Presumably, in real nuclei ideal two-phonon states do not exist. In terms of quasiparticles, they would also have two quasiparticle components (and others). We shall comment on this again later after presenting our results for the idealized model described here. The main parent of the two-phonon states is, *in any case*, the collective $2^+$, and the two-nucleon transfer reaction connecting them to this state is enhanced in the same sense as the transition from the ground to the one-phonon state is enhanced. (The parentage amplitudes in the two cases are equal within a statistical factor.)

Our model therefore possesses three kinds of states: the ground and $2_1^+$, which have a strong direct transition; the remaining two-*quasiparticle* states $(0^+, 2^+, 4^+)$, which have weaker direct transitions (however, the ground state is their main parent); and finally the two-*phonon* states,

which have a strong transition from the collective $2_1^+$ but which are not fed directly at all.

## 3. PARAMETERS

To assess the effect of inelastic processes on the two-nucleon transfer reaction, we consider the $(p, t)$ reaction on a moderately collective nucleus, nickel, at a typical bombarding energy of $E_p = 30$ MeV. A complete body of data would consist of the elastic and inelastic cross sections of the $2^+$ collective state (since it is the most important intermediate state for those included in our model) in both the initial and final nuclei, at the appropriate energies of the $(p, t)$ reaction. This body of data does not exist, but fortunately there is data on neighboring nuclei which is sufficient to define the parameters realistically for our model calculations.

The reaction we consider is

$$^{62}\text{Ni}(p, t)^{60}\text{Ni}, \quad E_p = 30 \text{ MeV}. \qquad (4)$$
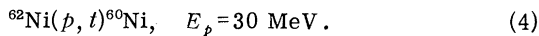
The $Q$ of the reaction is $-10$ MeV. There exist proton data[9] at 30 MeV on $^{60}$Ni and triton data[10] at 20 MeV on $^{62}$Ni. These data we use to define the optical-model parameters for protons and tritons in Eq. (4), and it is shown in Fig. 2. The elastic cross sections were computed by solving the coupled equations for the set of states described in Sec. 2. The corresponding optical-model parameters are labeled C.C. in Table I. (The parametrization of the Oak Ridge group is employed.[11]) We also used an optical-model search routine to obtain parameters of the one-channel optical potential that reproduce the *same* elastic cross section as obtained in the coupled-channel calculation. These are needed for comparison of our results for the $(p, t)$ reaction with those predicted by the usual DWBA. The solid line in the Fig. 2 represents *both* these calculations. The dashed line corresponds to the usual one-channel optical potential using the same parameters as in the cou-
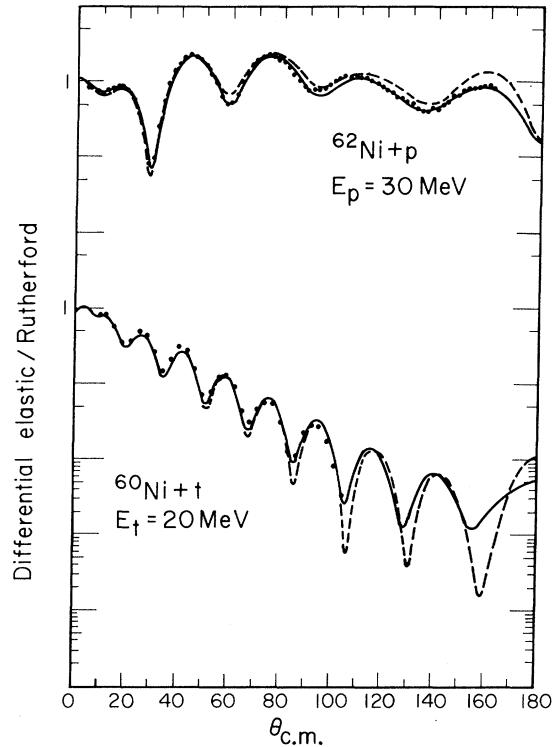


FIG. 2. Elastic cross sections are shown. The solid line represents *two* calculations: (1) a coupled-channel one involving all the states described in Sec. 2 and using the optical parameters labeled C.C. in Table I; and (2) a usual one-channel optical-model calculation using *adjusted* parameters labelled "elastic" in Table I which reproduce the same elastic cross section as the coupled-channel calculation. The dashed lines are also one-channel optical-model calculations but use the *same* parameters as the coupled-channel calculation. The differences show the effect of coupling to other states. Proton data is for $^{60}$Ni (Ref. 9) and triton data for $^{62}$Ni (Ref. 10).

pled-channel calculation. The difference shows the effect of the coupling of other channels of which the collective $2^+$ is the most important. Figure 2 illustrates the importance of backward-angle scattering for determining the effect of the

TABLE I. Optical-model parameters. The "elastic" ones yield the same elastic cross section as obtained in the coupled-channel calculation which uses the C.C. parameters.

|  | $V$ | $W$ | $W_D$ | $r_V$ | $r_W$ | $r_C$ | $a_V$ | $a_W$ |
|---|---|---|---|---|---|---|---|---|
|  |  |  |  | triton |  |  |  |  |
| C.C. | $-158.35$ | $-22.9$ | 1.094 | 1.22 | 1.506 | 1.25 | 0.695 | 0.8 |
| elastic | $-149.04$ | $-27.$ | 4.313 | 1.274 | 1.576 | 1.25 | 0.6559 | 0.8828 |
|  |  |  |  | proton |  |  |  |  |
| C.C. | $-54.$ | $-2.$ | $-5.2$ | 1.09 | 1.3 | 1.2 | 0.772 | 0.64 |
| elastic | $-54.087$ | $-3.239$ | $-5.367$ | 1.099 | 1.295 | 1.2 | 0.772 | 0.601 |
|  |  |  | $V_{SO}$ | $r_{SO}$ | $a_{SO}$ |  |  |  |
|  |  |  | $-5.74$ | 1.022 | 0.688 |  |  |  |

coupling on the optical-model parameters.

In the calculations reported here, we did not include a spin-orbit term in the triton optical potential. The strength of this term is expected, on theoretical grounds, to be about one third that of the nucleon spin-orbit strength. We did include it in several calculations, but its effect on the $(p, t)$ reaction was very small.

The strength of the inelastic transition to the $2^+$ collective state in both target and final nucleus is vital in our analysis because it is the parent of the two-phonon states. Again we have to rely on an extrapolation from other situations, but this is quite adequate for our purposes. For tritons,[12] the cross section to the $2^+$ state has been measured in $^{64}$Ni at 20 MeV. For protons,[13] we do not have data at the appropriate energy, but the reac-

TABLE II. Parameters of the direct interaction causing inelastic transitions [see Eq. (5)].

|  | $V_0$ | $r_0$ |
|---|---|---|
| Proton | −55 | 1.85 |
| Triton | −70 | 2.3 |

tion has been studied at 18 and 40 MeV, and we use the strength of the direct interaction obtained there. We are able to handle a direct interaction of the form

$$V(r) = [V_0 + V_1(\sigma_1 \cdot \sigma_2)]e^{-(r/r_0)^2}. \tag{5}$$

However, since the $V_1$ part is so unimportant for the excitation of the collective state, we have set it equal to zero in these calculations. The re-
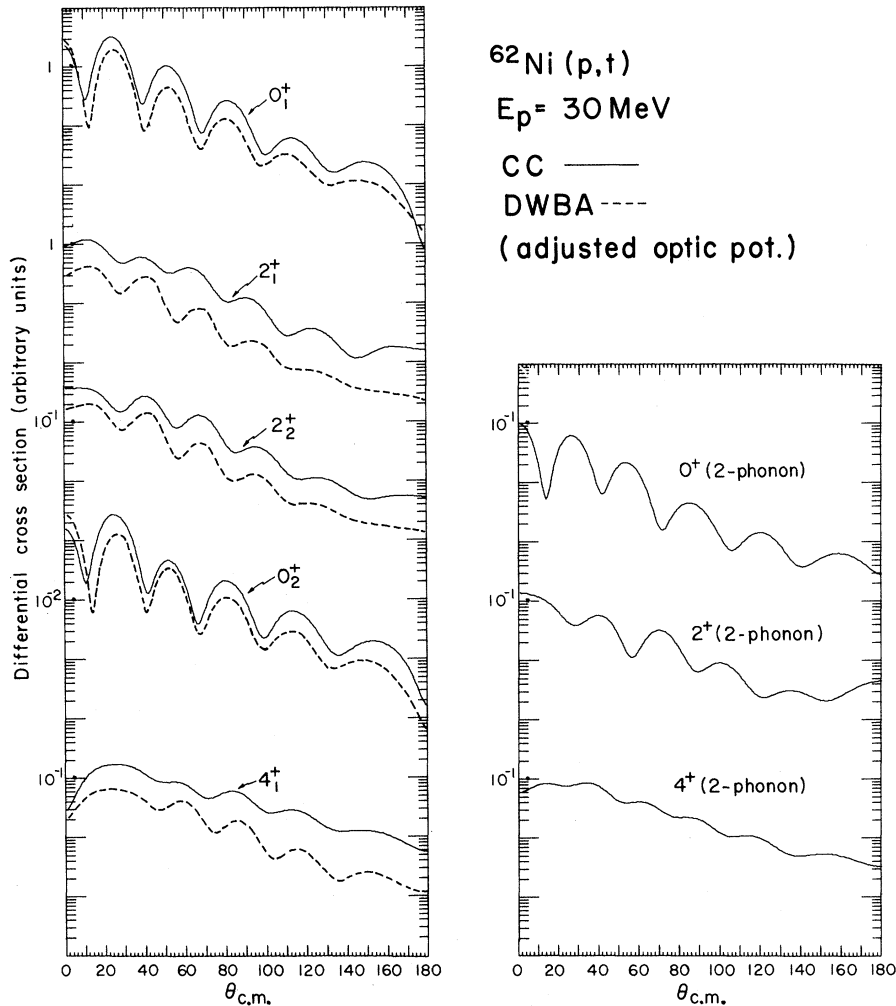


$^{62}$Ni $(p,t)$
$E_p = 30$ MeV
CC ———
DWBA – – – –
( adjusted optic pot.)

FIG. 3. Calculated cross sections of the $^{62}$Ni$(p,t)$ reaction to ground and two-quasiparticle states are shown on the left, and "microscopic" two-phonon states on the right. Solid lines include multiple-step processes. Optical parameters are labeled C.C. in Table I. Microscopic structure of the states is described in Sec. 2. Dashed lines show DWBA calculations, using optical parameters, labeled "elastic" in Table I, which reproduce the same triton and proton elastic cross sections as the coupled-channel calculation.

maining two parameters $V_0$ and $r_0$ for protons and
tritons are shown in Table II. The range param-
eter for tritons bears the kind of relationship to
that for protons as described for other composite
particles elsewhere.[14] It is larger because of the
finite extension of the triton.

The strength of the interaction causing the $(p, t)$
reaction is unimportant in our calculations be-
cause we treat the reaction in first order. Thus
all our calculated $(p, t)$ cross sections scale as
the square of this strength. However, we believe
our arbitrary units are approximately millibarns.

Finally the single-particle bound-state wave
functions, in terms of which the nuclear wave func-
tions are expressed, are harmonic-oscillator
functions having a constant $\nu = 0.25$ F$^{-2}$ $(\psi \sim e^{-\frac{1}{2}\nu r^2})$.
We confirmed (see later) that as concerns an eval-
uation of the role of inelastic processes, correc-
tion of the asymptotic behavior of the oscillator
functions is not necessary.

## 4. RESULTS

The complete calculation for the $^{62}$Ni$(p, t)^{60}$Ni
reaction at 30 MeV is shown in Fig. 3. All the in-
elastic couplings between the eight states in each

nucleus that are implied by their microscopic
structure are included to all orders.[15] All the par-
ticle-transfer couplings leading from all target
states to all final states are included with the
strengths prescribed by their structure.[5,6] The
features that determine the strength of this cou-
pling between a pair of states are (1) the fraction
of parentage of the state in the heavier nucleus
that is based on the other nucleus, and (2) the de-
gree to which the extra pair of neutrons are cor-
related in the way they exist in the triton. As ex-
pected, the ground and collective $2^+$ states have
the largest cross sections. Two very surprising
facts can be learned from Fig. 3. First, the two-
phonon states, which, as discussed in Sec. 2, can
be excited only through higher-order processes,
nonetheless have cross sections just as strong as
the other noncollective states, running about one
tenth that of the collective $2^+$ state. Second, the
angular distributions are largely characterized by
the multipolarity of the transition, *independent* of
whether the state was produced directly, or
through an intermediate state. The polarizations
of outgoing tritons from this reaction are shown
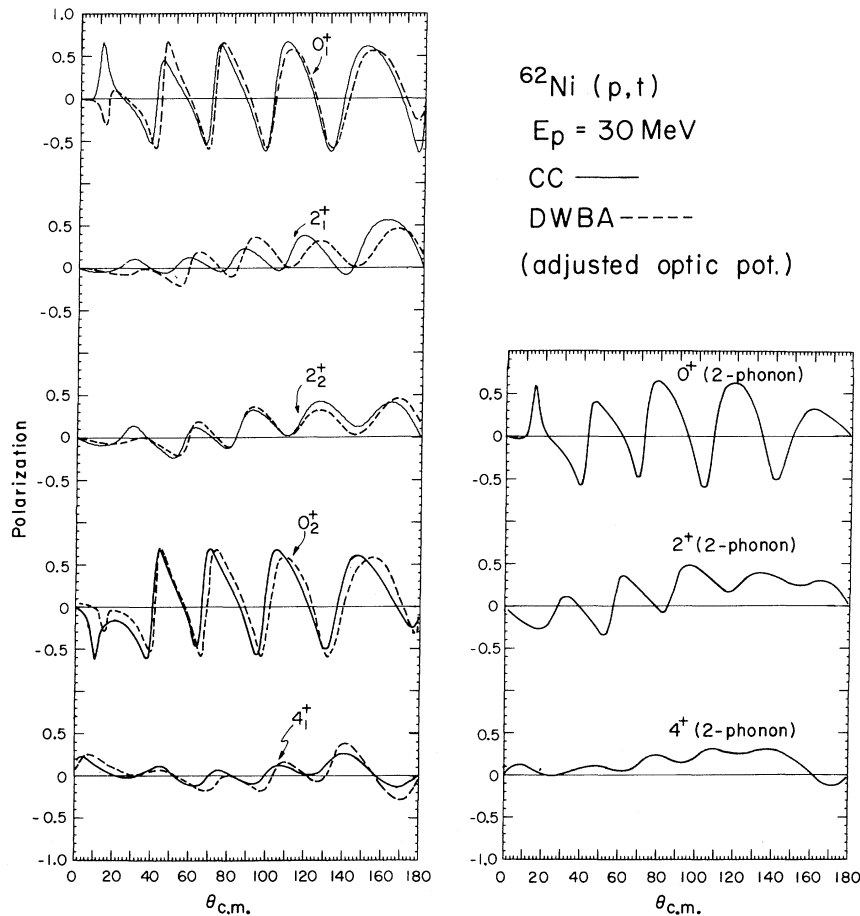in Fig. 4 and, like the angular distributions, are



$^{62}$Ni (p,t)

$E_p$ = 30 MeV

CC ——

DWBA -----

(adjusted optic pot.)

FIG. 4. Polarization of
outgoing tritons of the $^{62}$Ni-
$(p,t)$ reaction correspond-
ing to the cross sections
of Fig. 3.

characterized by the multipolarity of the over-all transition. Thus we find that under the typical circumstances of this calculation there is nothing about the angular distributions or polarizations that can be used to distinguish multiple processes from direct ones, and the probability of the higher-order transitions is as large as typical noncollective ones.

In Fig. 3 we also compare a DWBA calculation of those states that *can* be produced in a single step with the full calculation. The DWBA cross sections were computed according to the usual prescription. For this comparison, that means that the optical parameters were chosen to reproduce the *same* proton and triton elastic cross sections as emerged from the coupled-channel calculation as shown in Fig. 2. One sees in Fig. 3 discrepancies of up to a *factor* 5 in absolute cross sections and up to nearly 2 in relative cross sections. The DWBA in every case underestimates cross sections. Thus, even for a nucleus no more strongly collective than nickel, application of the usual DWBA leads to large errors in relative cross sections. The higher-order processes do, in fact, play a very important role. This is shown in more detail in Fig. 5, where the cross sections of individual paths leading to the final states are shown. One sees that the direct route, if it alone were present, accounts for about half the cross section of the collective $2^+$ state. The other major contributions are the transition through the collective $2^+$ state of the target (not shown), and through the ground state of the final nucleus. Although the *cross section* of this latter process is only about $\frac{1}{6}$ that of the direct, its *amplitude* is about 40% of the direct process. Of course the cross section for exciting a state which can be reached in several distinct ways is obtained by squaring the sum of the amplitudes for the individual ways. In our model, the two-phonon states are not produced directly. The two main paths go through the collective $2^+$ states of the target and final nucleus. The corresponding cross sections are shown, and it is seen that the one involving the inelastic transition in the triton channel, as compared with the proton channel, differs in two ways: (1) The frequency of oscillation is slightly faster, which probably corresponds to a more surface-dominated reaction; and (2) the oscillations are deeper. The second point can be understood in terms of the much stronger forward peaking in the triton inelastic scattering compared with the proton, as can be seen by comparing Figs. 6 and 7. Thus the structure of the reaction itself (see the direct processes) is preserved.

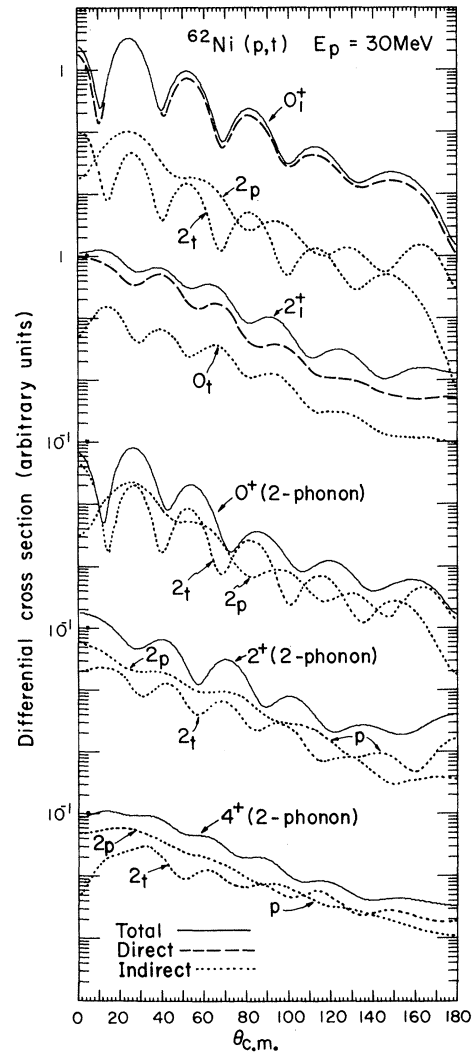Concerning the noncollective two-quasiparticle states $(0_2, 2_2, 4_1)$ shown in Fig. 3, their main par-



FIG. 5. Dashed lines represent cross sections for ground and collective $2^+$ states corresponding to the direct $(p,t)$ transition from the ground to final state, if it alone were possible, while dotted lines represent two-step processes going through the collective $2^+$ state in either the proton or triton system or through the ground state of the triton system as marked by subscripts. The idealized two-phonon states do not have a direct transition, as explained in Sec. 2. Solid lines represent cross sections in which all inelastic and transfer processes allowed by the structure of the states take place. Amplitudes of the individual processes add.

ent is the ground state, and they are produced almost exclusively through the direct transition from the ground state. Even so, we see that the DWBA underestimates their cross sections. The cause for this, as discussed in the introduction, is the modification of the wave function in the elastic channel due to coupling to the $2^+$ collective state. This is an important effect even in such moderately collective nuclei as these.
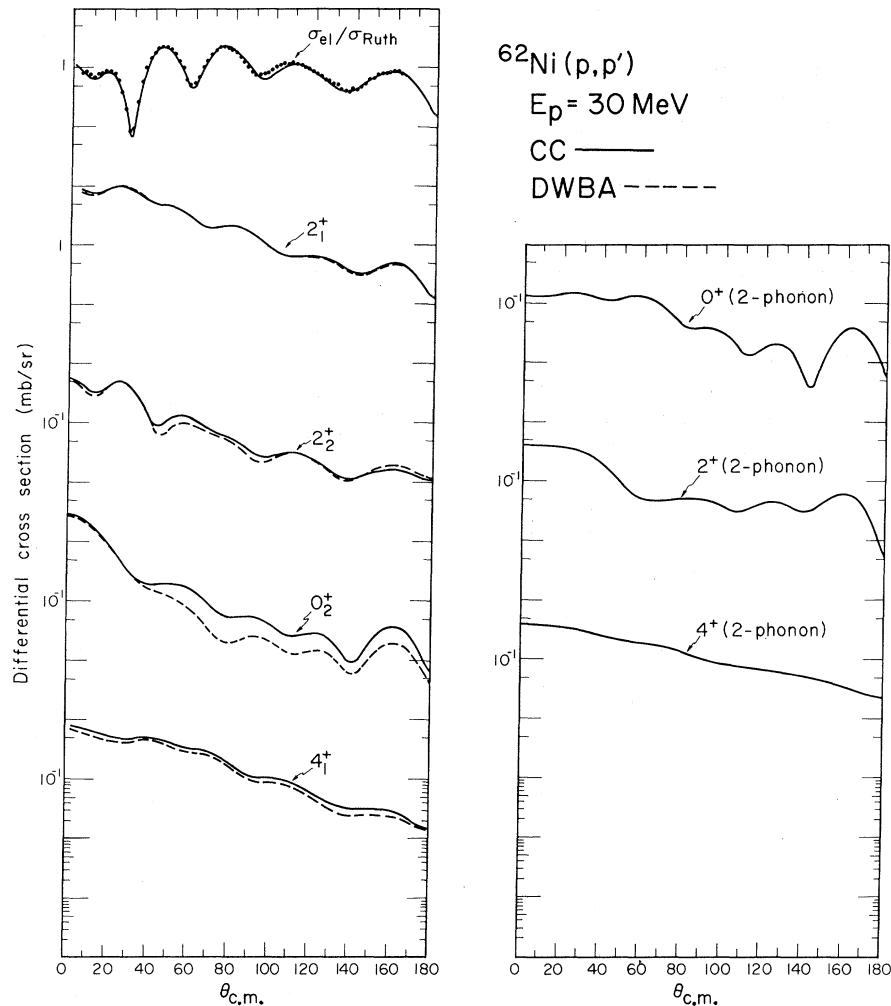
FIG. 6. Corresponding to the reaction of Fig. 3, the proton scattering from the $^{62}$Ni calculated by the coupled-channel method using the microscopic description of Sec. 2 is compared with the DWBA. The optic parameters of the former are labeled C.C. in Table I while those of the latter, which have been adjusted to yield the same elastic cross sections, are labeled "elastic." The experimental elastic data (Ref. 9) is for $^{60}$Ni.

In contrast to the above DWBA calculations, we did another calculation in which the *same* optical parameters were used as for the coupled-channel calculation. In this case the elastic cross section (see Fig. 2) is not well reproduced, especially at large angles. However, the DWBA now agrees better with the complete calculation at forward angles, for those states that can be produced in a single step, as seen in Fig. 8. One can understand why these DWBA cross sections are larger than those of Fig. 3, since the optical parameters here do not contain absorption corresponding to the low-lying levels. In particular, the imaginary part is smaller. However, it is hard to see how this would be a reliable prescription in general.

If it is true that in a nucleus as mildly collective as nickel the DWBA underestimates cross sections by as much as a factor of 5, and fails in relative cross sections by a factor of 2 or so, one may ask how weak must the collectivity of the intermediate state in the two-step process be before the errors are suitably reduced. In Fig. 9 we show a calculation analogous to that of Fig. 3 except that the strength of the direct reaction causing inelastic transitions has been reduced by $\sqrt{2}$. This cuts the inelastic cross section to the collective state by about 2, so that two-step processes are correspondingly weaker. The DWBA calculation was again carried out by adjusting the optical parameters so that the elastic cross sections of the coupled-channel calculations for both protons and tritons were reproduced. The DWBA still underestimates the $(p, t)$ cross sections but does considerably better than for the case of stronger collectivi-
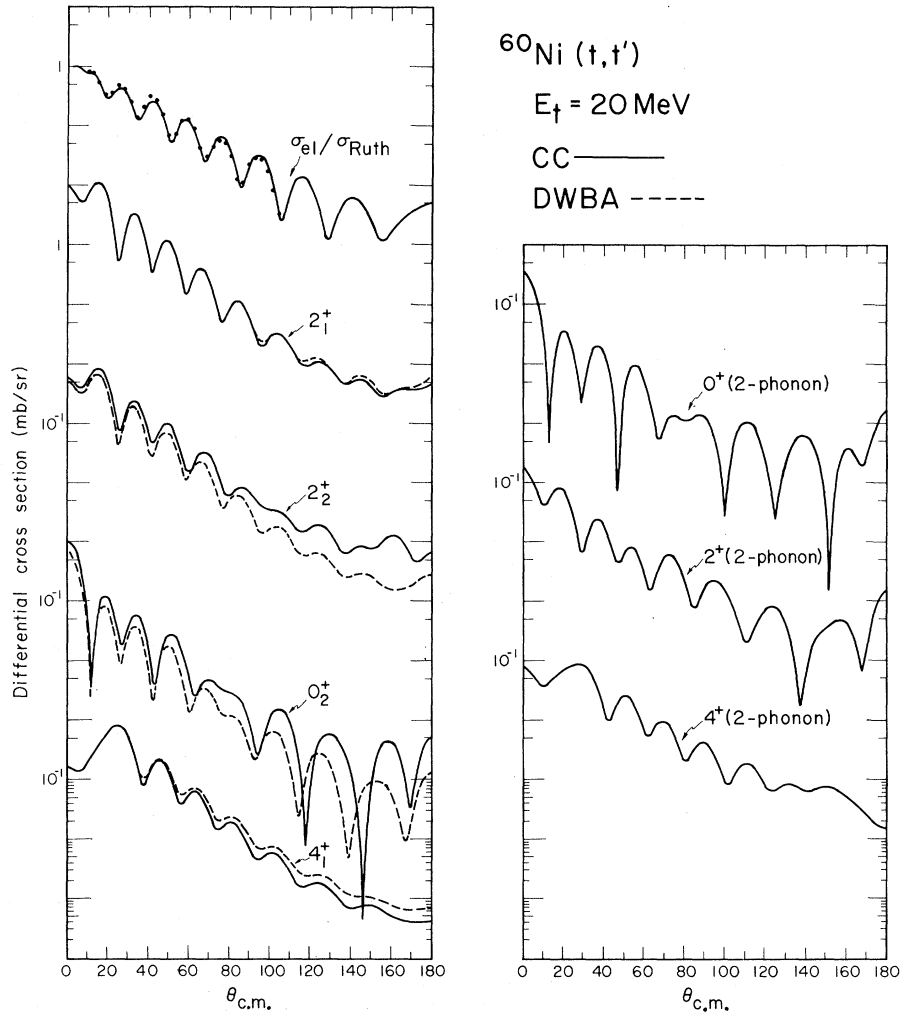
FIG. 7. Triton scattering from $^{60}$Ni at the energy of the outgoing tritons in the reaction shown in Fig. 3. Description is the same as in Fig. 6. Triton data is for $^{62}$Ni (Ref. 10).

ty in Fig. 3. The relative cross sections are, however, in very good agreement with the full calculation, so that we may conclude that if the collectivity of an intermediate state in a two-step $(p, t)$ reaction is about half as strong as it is in nickel, then the DWBA works very well for those states that have the target ground state as their main parent. The cross sections to the two-phonon states, which can be produced only by multiple processes, are reduced by almost a factor of 2. In other words they scale, as expected, with the collectivity of the parent intermediate state.

Although the DWBA underestimates the $(p, t)$ reaction calculation, it does better, especially at forward angles, for inelastic scattering, as seen in Figs. 6 and 7. There the inelastic cross sections corresponding to the reaction calculation of Fig. 3 are shown. The optic parameters of the DWBA were adjusted to yield the same elastic cross section as the coupled-channel calculation.

It was noted in connection with the $(p, t)$ reaction that the multipolarity of the transition determined, to a large degree, the angular distribution. This is still true for tritons but less so for the protons, which are more sensitive to the interior conditions. This can be understood easily in terms of a discussion given elsewhere.[16]

All our reaction calculations discussed so far used the same (harmonic-oscillator) single-particle wave functions as were employed in the original structure calculation. It has been recognized for a long time that concerning reactions, it is important to correct the tail of these functions, since they decay too rapidly at large radius.[6, 17] To confirm that our conclusions concerning the strength with which the two-phonon states are produced is not materially effected by the treatment of the tails, Fig. 10 compares two calculations, in one of which the harmonic-oscillator function describing the c.m. of the transferred neutron pair
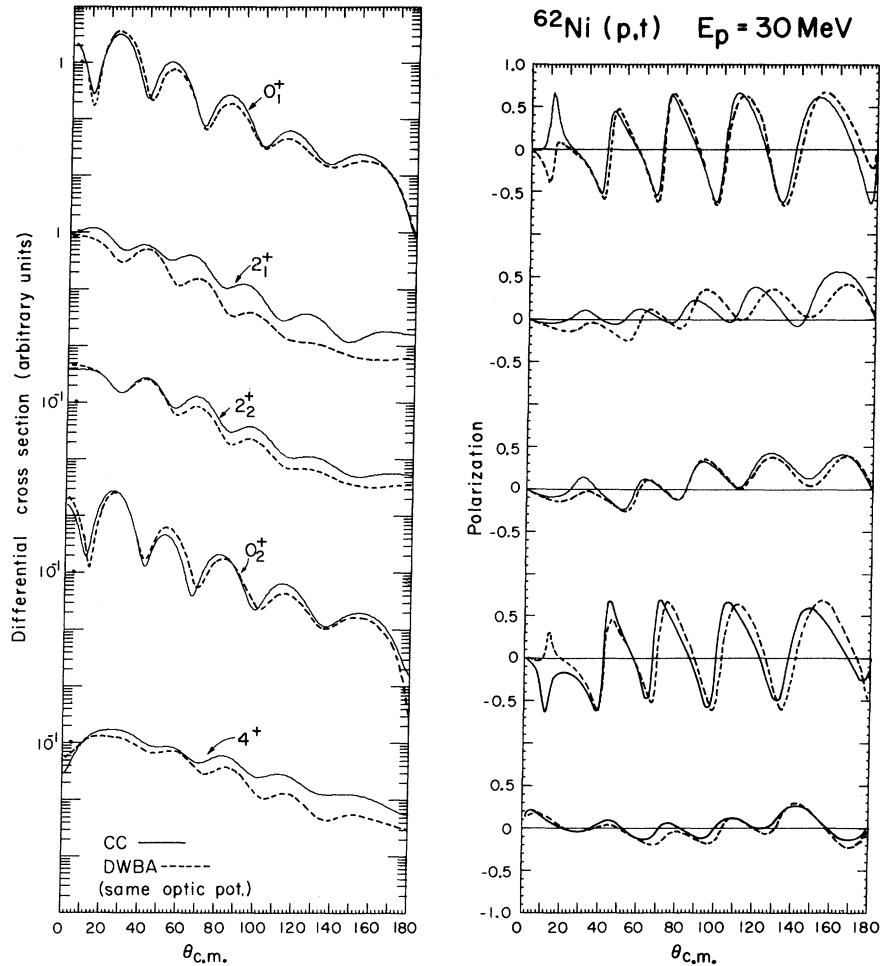
FIG. 8. Coupled–channel and DWBA calculation using the same optical parameters in both calculations. Elastic cross sections therefore do not agree (see Fig. 2). Two-phonon states are not shown, because they cannot be reached directly (therefore, have zero DWBA cross section).

is truncated and matched to a Hankel function of the appropriate separation energy.

## 5. SUMMARY AND CONCLUSIONS

We have studied two-step processes in $(p, t)$ reactions in a model which contains some states (a triplet) which have as their parent a state which has an enhanced inelastic transition from the ground state, namely the collective $2^+$ state. We found that although these states can be produced only indirectly by two-step transitions through the parent state, their cross sections are comparable in magnitude to other noncollective states which can be produced directly. This finding holds for a nucleus, namely nickel, in which the collectivity of the intermediate state is only moderate. The two-step processes scale as the strength of the inelastic transition from the ground to the inter-

mediate parent state. Therefore, since most nuclei are more collective than nickel, our result probably indicates a widespread phenomenon. Moreover, both angular distributions and polarizations are characterized mainly by the multipolarity of the over-all transition and not by the multiplicity of the reaction process. The two-step processes therefore do not have a special fingerprint, at least not under the typical conditions of our calculation.

Presumably, in real nuclei *ideal* two-phonon states such as we studied do not exist. Some of their character persists, since the radiative selection rules hold approximately, but they most likely possess admixtures in their wave function other than the two-phonon component of Eq. (3). Conversely, other near-lying states share the two-phonon character. Suppose that a noncollective state has a 10% admixture of two-phonon character.
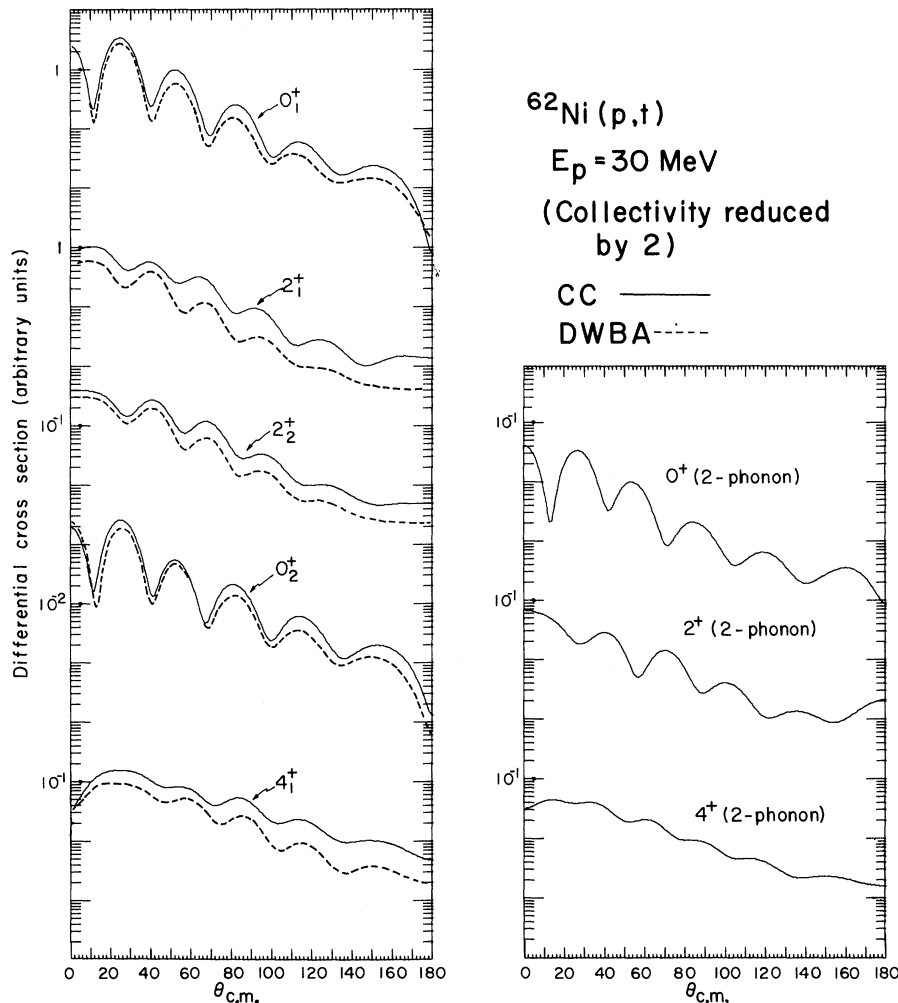
FIG. 9. The same calculation as Fig. 3 except that the strength of the interaction causing inelastic transitions is reduced by $\sqrt{2}$. For our purposes this corresponds to a collectivity of the $2_1^+$ state of about $\frac{1}{2}$ that of Fig. 3.

Previously one would have ignored this component and calculated the cross section to this state on the basis of those components that can be produced directly. Let us denote by $F$ the amplitude that can be produced directly. Then in this approximation the cross section is $F^2$. However, since we now know that the two-phonon state is excited as strongly as a typical noncollective state and that its angular distribution is nearly the same, then corresponding to the 10% admixture we should add $F/\sqrt{10}$ to the above amplitude. The phase could vary anywhere from $-1$ to $+1$ with the result that the cross section for the admixed state could be anywhere from 0.4 to 1.6 times the result based on the neglect of the 10% two-phonon admixture.

Concerning those states which are produced dominantly by the direct transition from the ground, the DWBA does underestimate their cross sections also, and seriously. This is due to

changes in the wave function in the elastic channel induced by inelastic coupling to the collective $2^+$ state. We see no reason to believe that this effect is exclusive to the $(p, t)$ reaction and suggest therefore that the DWBA also underestimates $(d, p)$ cross sections more or less, depending on the degree of collectivity. Thus it appears that single-particle spectroscopic factors may be overestimated in the usual DWBA analysis.

Although our findings give cause for pessimism, we do point out that the so-called source-term method that we have used to include higher-order processes in particle transfer reactions does provide a feasible means of incorporating these previously neglected processes. Moreover, if one suspects that special parentage relations exist among certain states, one can now explicitly compute the results of such relations. Two interesting examples have already been studied at this laboratory.[2]
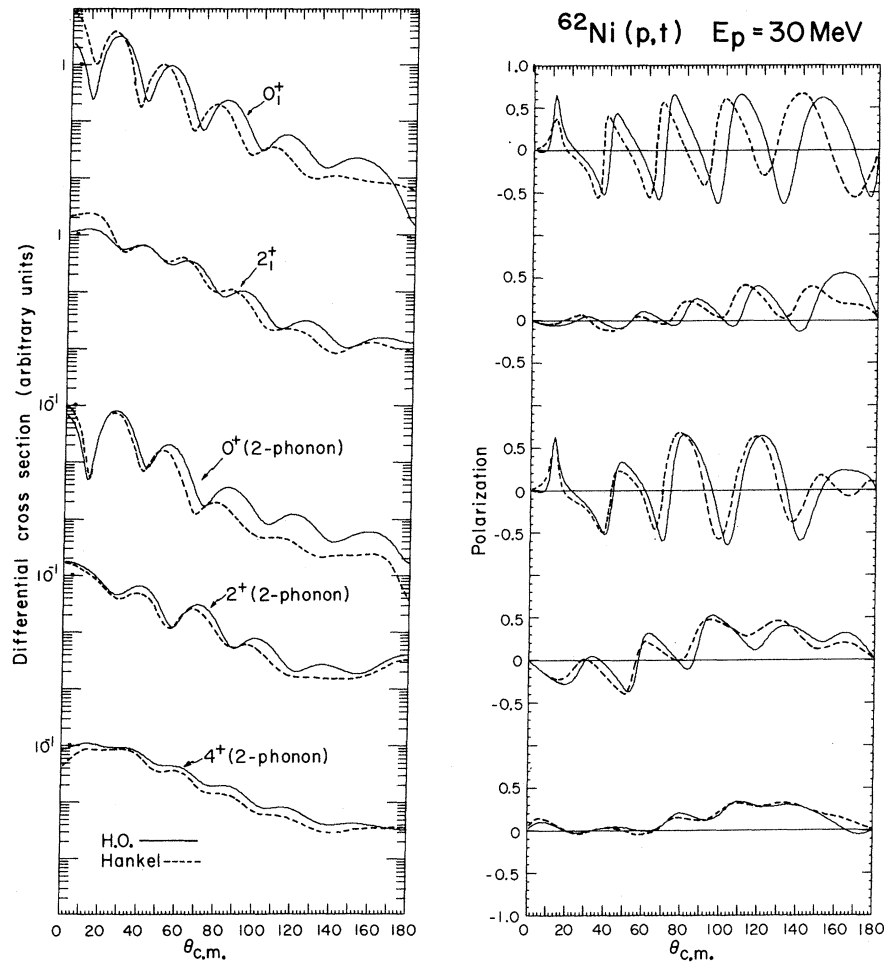
FIG. 10. Cross sections for the $^{62}$Ni$(p,t)$ reaction computed using oscillator bound-state wave functions compared with the matching of these to a Hankel function tail.

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