*National Research Council of Canada postdoctoral fellow, now at Laboratoire de Physique Nucléaire, Université de Montréal, Montréal, Canada.

†Now at Département de Physique, Université Laval, Québec, Canada.

‡Now at Nuclear Physics Laboratory, University of Washington, Seattle Washington.

§Now at Department of Physics, Willamette University, Salem, Oregon.

[1]G. E. Brown, in *Proceedings of the International Conference on Nuclear Physics, Paris, France, 1964,* edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, France, 1964), Vol. 1, p. 129.

[2]H. G. Benson and J. M. Irvine, Proc. Phys. Soc. (London) 89, 249 (1966).

[3]P. Federman, Nucl. Phys. A95, 443 (1967); P. Federman and I. Talmi, Phys. Letters 19, 490 (1965).

[4]T. Engeland, Nucl. Phys. 72, 68 (1965).

[5]N. K. Glendenning, Phys. Rev. 137B, 102 (1965).

[6]R. Jaffe and W. J. Gerace, Nucl. Phys. A125, 1 (1969).

[7]J. Bonn *et al.*, Rutherford Laboratory Report No. RHEL/R136, 1966 (unpublished), p. 141.

[8]W. G. Davies *et al.*, Rutherford Laboratory Report No. RHEL/R156, 1967 (unpublished), p. 111.

[9]I. S. Towner and J. C. Hardy, Advan. Phys. 18, 401 (1969).

[10]A. A. Rush, E. J. Burge, V. E. Lewis, D. A. Smith, and N. K. Ganguly, Nucl. Phys. A107, 340 (1967).

[11]R. N. Glover and A. D. W. Jones, Nucl. Phys. 81, 268 (1966).

[12]P. E. Hodgson, Advan. Phys. 17, 563 (1968).

[13]M. Krick and G. J. F. Legge, Nucl. Phys. 89, 63 (1966).

[14]J. H. Towle and G. J. Wall, Nucl. Phys. A118, 500 (1968).

[15]B. C. Robertson, R. A. I. Bell, J. L'Ecuyer, R. D. Gill, and H. J. Rose, Nucl. Phys. A126, 431 (1969); R. D. Gill, B. C. Robertson, J. L'Ecuyer, R. A. I. Bell, and H. J. Rose, Phys. Letters 28B, 116 (1968).

[16]M. H. Shapiro *et al.*, Bull. Am. Phys. Soc. 13, 698 (1968).

[17]R. M. Drisko and F. Rybicki, Phys. Rev. Letters 16, 275 (1966).

[18]Gy. Bencze and J. Zimanyi, Nucl. Phys. 81, 76 (1966).

[19]J. K. Dickens, R. M. Drisko, F. G. Perey, and G. R. Satchler, Phys. Letters 15, 337 (1965).

[20]B. F. Bayman and N. M. Hintz, Phys. Rev. 172, 1113 (1968).

[21]G. Ripka, in *Lectures in Theoretical Physics,* edited by W. E. Brittin *et al.* (University of Colorado Press, Inc., Boulder, Colorado, 1965), p. 252.

[22]H. G. Benson and B. E. Flowers, Nucl. Phys. A126, 305 (1969); H. G. Benson, private communication.

[23]A. Arima, S. Cohen, R. D. Lawson, and M. H. Macfarlane, Nucl. Phys. A108, 94 (1968).

[24]F. Dönau, K. Hehl, C. Riedel, R. A. Broglia, and P. Federman, Nucl. Phys. A101, 495 (1967).

[25]J. M. Nelson and B. E. F. Macefield, Atlas Program Library Report No. 17, 1969 (unpublished).

PHYSICAL REVIEW C VOLUME 2, NUMBER 1 JULY 1970

Differential Scattering of Neutrons at Narrow Levels in ^{17}O†

J. L. Fowler and C. H. Johnson
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830

(Received 6 April 1970)

We measure the differential cross sections for scattering of neutrons from ^{16}O at or near four narrow resonances. The energy spread of the incident neutrons ranges from 5 to 13 keV full width at half maximum. We detect scattered neutrons as a function of angle with a stilbene crystal. Using phase shifts from previous work, we analyze the differential data and make resonant parameter assignments as follows (in order: excitation energy of ^{17}O in keV, J value, parity, and c.m. resonant width in keV: 5867 ($\frac{3}{2}$+, 6.6), 7164 ($\frac{5}{2}$−, 1.4), 7377 ($\frac{5}{2}$+, 0.5), and 7380 ($\frac{5}{2}$−, 1.1). In table form, we review level assignments of ^{17}O up to 7380-keV excitation energy and give estimates of reduced widths in single-particle units.

INTRODUCTION

Three years ago, in a paper on the scattering of neutrons[1] from ^{16}O, we listed the known levels of ^{17}O up to an excitation energy of 8.21 MeV together with their resonant energies, angular momenta, parities, widths, and reduced widths where these quantities were known. There were serious gaps in the experimental information. For approximately one third of the levels, we had to enter blanks for at least two of the parameter assignments. To fill in some of these gaps we have recently made extensive neutron measurements with good energy resolution of both the total cross sections[2,3] and the differential cross sections[4] for scattering neutrons from ^{16}O. Rose,[5] in studying the angular distribution of proton groups from the ^{14}N$(\alpha, p)^{17}$O* reaction from 13 to 18-MeV α-particle bombarding energy, has seen 18 levels in ^{17}O up to 7.56-MeV excitation energy, and has also made some resonant parameter assignments. From measurements of the polarization of neutrons from the $^{13}C(\alpha, n)^{16}O$ reaction, Schölermann[6] has made spin and parity assignments for seven relatively prominent (α, n) resonances.

For the neutron total-cross-section measurements[2,3] we used the $^7Li(p, n)^7Be$ neutrons produced by bombarding thin Li targets with protons from the Oak Ridge National Laboratory's 6-MV Van de Graaff machine. Our samples, BeO and Be, and the stilbene crystal neutron detector were those used with the open geometry described in Ref. 1. In a search for narrow resonances we covered, with a 2–3-keV energy spread, the neutron energy ranges from 1.116 to 1.162 MeV, from 1.64 to 1.70 MeV, and from 1.77 to 3.67 MeV (except for a small region of 10-keV steps near 1.9 MeV). We gave total cross section curves and results of a preliminary analysis in contributions to conferences.[2,3] In the following summary of the results we quote the resonant energies and widths, which we shall discuss in more detail in a future publication.[7]

We found no resonances at 1.142 and 3.633 MeV corresponding to known ^{17}O levels at 5.217 and 7.560-MeV excitation,[8] and no resonance corresponding to a reported 6.24-MeV state.[9] Rose,[5] using the $^{14}N(\alpha, p)^{17}O^*$ reaction, observed the 5.217-MeV state, and assigned it a high spin ($J=\frac{7}{2}$, $\frac{9}{2}$, or $\frac{11}{2}$); he also saw the 7.560-MeV state. He found no indication of a state at 6.24 MeV. We found the lab width of the $f_{7/2}$ resonance[10] at 1.651 MeV (5.696-MeV excitation energy) to be 3.6 ± 0.5 keV. At 1.689-MeV neutron energy we located, for the first time in neutron scattering, the known[8] state at 5.731-MeV excitation energy, and found it has a width less than 1 keV. The height of the peak at 1.833 MeV identified it as a $J=\frac{3}{2}$ resonance; the dip before the resonance suggested to us that it is interfering with the negative $d_{3/2}$ phase shift,[11] and thus is a $d_{3/2}$ resonance 7.0 ± 0.5 keV wide.[2] Our data indicated the 1.906- and the 2.353-MeV resonances correspond to a $p_{1/2}$ state and a $s_{1/2}$ state at 5.936 and 6.356 MeV, respectively. A 2.888-MeV resonance (seen also for the first time in neutron scattering) corresponds to the known[8] state at 6.859 MeV. It has a total width of less than 1 keV. On the basis of clues obtained by matching analog states of ^{17}F and ^{17}O, and by noting that there was some evidence[8] for a level near 6.99 MeV in ^{17}O, we looked for and found a narrow resonance at 3.006 MeV, corresponding to 6.970-MeV excitation in ^{17}O. Rose also found this level in the spectra of protons from the $^{14}N(\alpha, p)^{17}O$ reaction.[5] At 3.212 MeV we found, for the first time in neutron scattering, the $J=\frac{5}{2}$ resonance,[8] corresponding to a level at 7.164 MeV. We had sufficiently good energy resolution to assign its lab width as 1.4 ± 0.2 keV. We found the peak at 3.44 MeV, which had been assigned[8] $J=\frac{5}{2}$, to be a doublet separated by about 3 keV.

Since the high-resolution total-cross-section results revealed several resonances with sufficient widths to allow J-value assignments, we revised our experiment for measuring differential cross sections[1] in order to determine the neutron scattering at these resonances. In the theoretical expressions for differential cross sections near a resonance, the terms which give the interference of the resonant partial wave with the other partial waves vary much less rapidly with energy than is the case for the familiar Breit-Wigner resonance form for the total cross section.[11] Thus one obtains l values from differential cross sections measured with an energy spread several times the width of a resonance. Our differential cross sections near 1.833, 3.212, 3.438, and 3.441 MeV permit us to assign l values, hence parities, for the resonances at these energies.

DIFFERENTIAL CROSS SECTIONS

We have described the apparatus for measuring neutron differential cross sections of liquid oxygen in an earlier paper.[1] In the case of the present experiment, collimated neutrons from a $^7Li(p, n)^7Be$ source are incident upon liquid oxygen in a dewar flask supported in the center of a scattering chamber.[12] We detect scattered neutrons by means of a stilbene crystal and use pulse-shape discrimination against γ rays.[13] The dewar flask containing the oxygen and an identical empty container are mounted on a support so that either one or the other can be rotated into the neutron beam by remote control. We also move the neutron detector remotely and read the position of the sample and the detector from the control room. Since the experiment requires as small neutron energy spread as is feasible, we use a $^7Li(p, n)^7Be$ neutron source. We place the Li target in front of the tapered collimating slit as was done in the case of an earlier experiment on ^{208}Pb scattering.[12] In the present experiment, however, we evaporate Li metal onto a W backing in an evacuated bell jar, and carry the target in an argon-filled plastic container to the beam position. The W backing is soldered on the end of a 6.7-cm-long brass cylinder which can be inserted into the beam tube with the vacuum seal being made with an O ring. We shall describe this target and its characteristics more completely in a paper on total cross section measurements made with 2–3-keV energy resolution.[7]

Since for narrow resonances the knowledge of the energy and energy spread of the neutrons is important for the analysis of the results, we alter

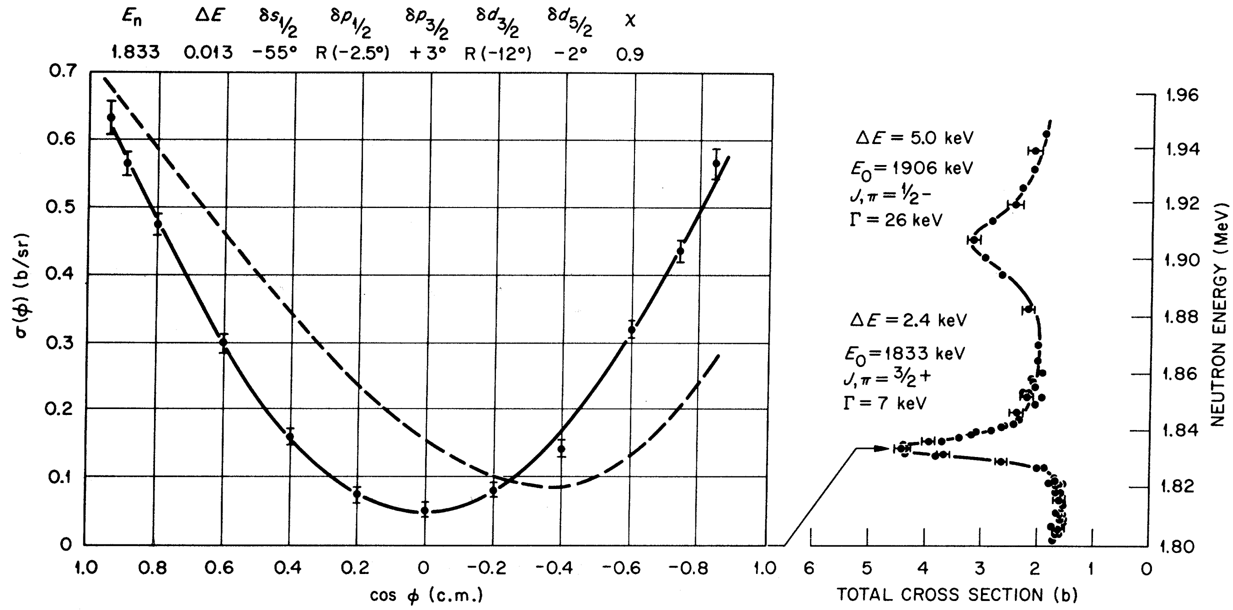

FIG. 1. Differential cross sections at 1.833 MeV. On the right-hand side is plotted the total cross section from 1.8 to 1.96 MeV as taken from Ref. 2. The solid curve is a fit to the data with the resonant parameters and energy spread listed beside each resonance. On the left-hand side is plotted $\sigma(\phi)$ versus $\cos\phi$ in the c.m. system as measured at the lab energy and energy spread listed above the figure. The energy for the angular distribution is also indicated by the arrow. The solid curve through the points is a least-squares fit to the data with the listed phase shifts. The notation, $R(-12°)$, for $d_{3/2}$ means that a potential scattering phase shift of −12° is added to the resonant phase shift calculated from the parameters of the 1833-keV resonance. The $p_{1/2}$ phase shift refers, in like manner, to the 1906-keV resonance. The symbol χ is the usual criterion of goodness of fit. The dashed curve gives the theoretical cross section for an assignment of opposite parity for the 1.833-MeV resonance.

somewhat the procedure we had used previously for measuring differential cross sections at nonresonant energies.[1] After evaporating a new target, we measure its thickness, as before, by measuring with a long counter[14] the rise at threshold in the yield of the ^{7}Li(p,n) reaction. This also gives us a calibration point for the proton energy scale.[15] In order to locate the narrow resonances as well as to measure the neutron energy spread at the resonant energy, we next locate the minimum in the neutron transmission of a BeO sample which is attached so that it can be moved into the neutron beam by the same apparatus that positions the scattering samples. We make a small correction to the neutron energy for the subsequent scattering measurements, because the scatterer subtends a finite angle, up to about 3° to the proton beam axis, whereas the detector in the above transmission measurements is nearly at 0°. Then we proceed with the differential-cross-section measurements as described in Ref. 1. In the present case, however, we find that our circuits are stable enough to permit us to calibrate our crystal detector in the 0° neutron beam after runs at three or four angles, rather than after each angle setting. We find we have to compensate for the energy drift of the 6-MV Van de Graaff by relocating the transmission minimum after the differential scattering runs at the three to four angles. We assign the energy resolution for each angular distribution by combining the energy spread due to the machine drift in quadrature with other effects such as the target thickness and the energy spread due to the angle subtended by the sample at the neutron source. The average rms energy drift of the 6-MV machine for a period of ~30 min is about 1 keV. We list beside each curve in Figs. 1 and 2, the full width at half maximum for the over-all energy spread.

We apply corrections to the differential scattering data for spurious background, self-attenuation in the sample, finite angular resolution, energy dependence of the detector efficiency, and multiple scattering, as discussed in Ref. 1. We compare the efficiency of the crystal with that of the long counter before and after each of two sets of angular distribution runs. In addition to these other corrections, we also have to correct for the effect of the second group of neutrons from the ^{7}Li(p,n) source. This correction[12] depends on the ratio of the number of neutrons in the second group to that in the primary group,[14,16] on the ratio of the effi-

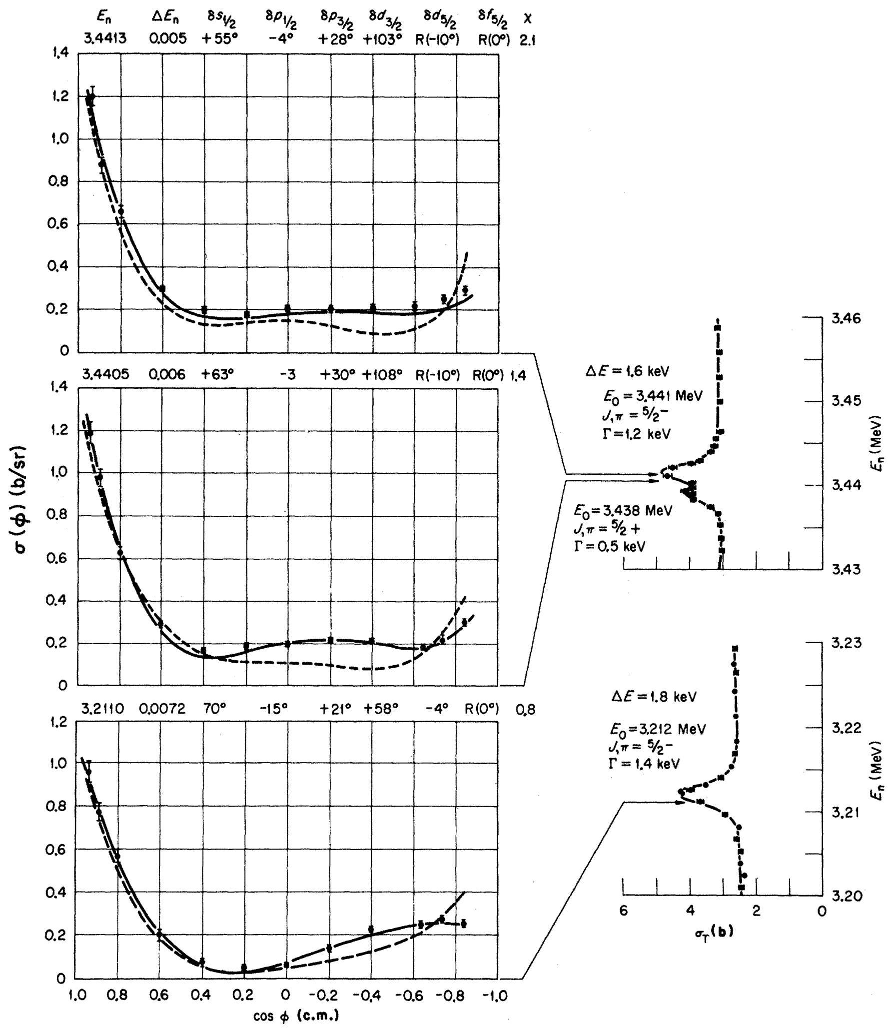

FIG. 2. Differential cross sections at 3.211, 3.4405, and 3.4413 MeV. For details see caption for Fig. 1.

ciency of the stilbene crystal at the neutron energy of the second group to that of the energy of the primary group, and on the ratio of the ^{16}O differential cross sections at the two energies. We estimate the error of all of these corrections as being one third the magnitude of the correction, and have added these errors in quadrature with the standard counting errors. The vertical flags in Figs. 1 and 2 represent the combined errors.

ANALYSIS

The solid lines through the experimental differential cross-section points in Figs. 1 and 2 result from a least-squares phase-shift fit to the data, and the phase shifts and resonant parameter which give these fits are listed in each figure. For the phase-shift analysis we use program ANNA, altered as discussed in Ref. 1. A subroutine of the program calculates differential cross sections for neutron scattering from 0-spin nuclei from a set of phase shifts, two of which can be taken from parameters of nearby resonances. The main program then averages these theoretical cross sections over the energy spread of the measurements and uses matrix operations to adjust preselected nonresonant phase shifts in order to arrive at a minimum in the weighted squares of the deviations between the experimental points and the calculated cross sections. As starting points for the nonresonant phase shifts, we use values from the previously published[1] curves of phase shift versus energy. The resonant parameters, such as the widths of the resonances and the J value, are best taken from the total-cross-section data, for which it is possible to use better energy resolution.[2,3] In Figs. 1 and 2, the total cross sections and the parameters deduced from them are given on the right-hand side of the figures. In the previous section, we have explained how we find the average energy at which the differential measurements are made, as well as the energy spread given as ΔE in the figures.

As can be seen at the right-hand side of Fig. 1, the total cross section shows a dip before the 1.833-MeV resonance. This dip, which results from the interference between potential and resonance scattering, suggests that the resonance is $d_{3/2}$ rather than $p_{3/2}$, because the $d_{3/2}$ potential phase shift at these energies is about $-12°$, whereas the $p_{3/2}$ phase shift is nearly zero.[1] The differential cross section shown on the left-hand side clearly verifies the 1.833-MeV peak is a $d_{3/2}$ resonance. For the theoretical fit we include the effect of the tail of the $p_{1/2}$, 1.906-MeV resonance, which our total-cross-section results show has a width of 26 keV. The angles listed in parentheses, after the R's, are the potential phase shifts which, together with the resonant parameters listed beside the total-cross-section curve, give the resonant phase shifts. These nonresonant phase shifts are consistent with those found previously at neighboring energies (see Fig. 7 of Ref. 1). With these same nonresonant phase shifts, a $p_{3/2}$ resonance would have the differential cross section given by the dashed curve. The last entry in the row of parameters above the differential-cross-section curve is χ, i.e., the square root of the weighted squares of the deviations between the experimental points and the theoretical curves divided by the differences between the number of points and the number of adjustable phase shifts.[12] For a good fit, χ should be around unity. The χ for the dashed $p_{3/2}$ curve is 12.7, which is considerably larger than the $\chi = 0.9$ for the $d_{3/2}$ fit. The average value of χ for all the fitted data in Figs. 1 and 2 is 1.3.

The resonance at 3.212 MeV, corresponding to 7.164-MeV excitation, has been assigned $J=\frac{5}{2}$ on the basis of angular distribution of neutrons from the $^{13}C(\alpha, n)^{16}O$ reaction.[17,18] The total cross section at 3.212 MeV (Fig. 2) indicates it is a $\frac{5}{2}$ resonance 1.4 ± 0.2 keV wide. The differential cross sections are well-fitted ($\chi = 0.8$) as a $f_{5/2}$ resonance 1.4 keV wide. The dashed curve, which is for a $d_{5/2}$ assignment, is an appreciably worse fit ($\chi = 3.8$).

Our recent high-resolution total-cross-section measurements show that the peak around 3.44 MeV (Fig. 2) arises from two resonances.[2,3] The solid line through the total-cross-section data in the upper right-hand corner of Fig. 2 is the calculated theoretical cross-section curve given by the resonance parameters listed beside the total-cross-section data and averaged over the experimental energy spread of 1.6 keV. The nonresonant phase shifts ($\delta s_{1/2} = +70°$, $\delta p_{1/2} = -14°$, $\delta p_{3/2} = +37°$, and $\delta d_{3/2} = +103°$) for this fit are taken from the previous phase shift analysis,[1] but are adjusted among themselves slightly to give the off-resonant cross section. A recent unpublished measurement of the total neutron yield from the $^{13}C(\alpha, n)^{16}O$ reaction, resulting from bombarding a thick ^{13}C target with monoenergetic α particles, confirms the presence of the two resonances separated by about 3 keV[19] at about 3.44 MeV in the n-^{16}O lab system. This measurement shows the yield of neutrons at the lower-energy resonance is approximately two times that at the upper resonance. The widths of the resonances given in Fig. 2 are consistent with those observed in the $^{13}C(\alpha, n)^{16}O$ reaction.

Angular distributions of neutrons from the $^{13}C(\alpha, n)^{16}O$ reaction, at energies corresponding to the 3.44-MeV peak measured with the 22-keV[17] and 20-keV[18] α-particle energy spread, indicate the (α, n) yield is from a $\frac{5}{2}$ resonance. Since two thirds of the $^{13}C(\alpha, n)^{16}O$ yield is from the lower-energy resonance,[19] the angular distributions suggest that at least the lower-energy resonance has $J=\frac{5}{2}$. But the height and width of the peak in the total cross section at the upper resonance indicates it is a $J=\frac{5}{2}$ resonance also. Since the separation of the resonances, 3 keV, is somewhat larger than their widths, 0.5 and 1.2 keV, there cannot be appreciable interference between them; thus the angular distributions of the (α, n) neutrons averaged over the two resonances is consistent with these both having $J=\frac{5}{2}$.

Even though the evidence from the $^{13}C(\alpha, n)^{16}O$ reaction suggests the lower resonance has $J=\frac{5}{2}$, we also have tried many other assignments for this narrow resonance. The background $s_{1/2}$, $p_{3/2}$, and $d_{3/2}$ phase shifts are such[1] that a $s_{1/2}$, $p_{3/2}$, or $d_{3/2}$ resonance would show up with pronounced interference effects. In the $s_{1/2}$ and $d_{3/2}$ cases, which have background phase shifts of ~90°, such a resonance would appear as a dip in the total cross section; such a dip is not observed. Since the $p_{3/2}$ phase shift is ~30°, the interference effect in scattering would result in a more slowly rising cross section before the resonance than is observed. On the basis of this lack of interference in the total cross section, we eliminate all resonant assignments below $d_{5/2}$ and $f_{5/2}$ except $p_{1/2}$. One can, in fact, fit both the total and differential neutron cross sections under the assumptions that at 3.438 MeV there is a $p_{1/2}$ resonance ~1.5 keV wide, and that at 3.441 MeV there is a $d_{5/2}$ resonance ~1 keV wide. This fit, however, is inconsistent with the recent $^{13}C(\alpha, n)^{16}O$ neutron yield experiment[19] which shows the first resonance has a width of less than 1 keV. Thus, the only satisfactory fit to both the present total and differential neutron cross-section data, as well as to the $^{13}C(\alpha, n)^{16}O$ reaction data[19] including the neutron angular distributions from this reaction,[18] is the set of resonant parameters listed in Fig. 2. We conclude that there is a $d_{5/2}$ resonance 0.5 ± 0.2 keV wide at

3.438 MeV, and a $f_{5/2}$ resonance 1.2 ± 0.2 keV wide at 3.441 MeV. Interchanging the parities of the two resonances gives the dotted curves for the fits to the differential cross sections at 3.4405 and 3.4413 MeV. These are clearly not satisfactory fits. In fact, this interchange of parity assignments increases χ by a factor of 4 for the angular distribution at 3.4405 MeV, and by a factor of 2.4 at 3.4413 MeV. Similar misfits result if the parity of both levels is taken to be the same, either plus or minus.

Since the even-parity or $\frac{5}{2}^+$ member of this doulet has the larger yield in the $^{13}C(\alpha, n)$ reaction, one anticipates that measurements with poor resolution on the $^{13}C(\alpha, n)$ reaction will be dominated by this member. Indeed, Schölermann obtained a $\frac{5}{2}^+$ assignment from his analysis of the polarization of the neutrons from the $^{13}C(\alpha, n)^{16}O$ reaction.[6] However, his α-particle energy spread was so large, ~600 keV, that he had to include in his calculations the effect of other resonances as far removed as 300 keV in excitation energy. Thus for the assignment of the parameters for the (α, n) resonance, which we find to be a doublet, he had to consider the effect of three other resonances. Furthermore, some of the assignments he took for the distant resonances differ from recent assignments in the literature.[1,20]

DISCUSSION

The table summarizes the information on levels of ^{17}O through an excitation energy of 7.38 MeV, the highest excitation energy of levels investigated in this present work. In the first column we have obtained the excitation energies from the neutron energies for the scattering resonances by use of the latest value (4.1426 MeV) of the difference of mass[21] between ^{16}O plus a neutron and ^{17}O. The corrected values of the neutron resonant energies in the second column, designated by Refs. 2 and 3, are mentioned in the Introduction, and will be discussed more fully in a subsequent publication.[7] In cases where the resonances are narrow enough so that their peaks can be accurately located, that is $\Gamma_n < 10$ keV, we estimate our resonant energies to be accurate to ±3 keV. For the broader resonances, one has to fold in the uncertainty in the location of the resonant peak, which is in the neighborhood of 5% of the resonant width.

In the next to the last column, labeled S, we list spectroscopic factors for bound states in ^{17}O. For the ground state and the first excited state, Naqib

TABLE I. Level parameters for ^{17}O states. All entries not otherwise indicated are from Ajzenberg-Selove and Lauritsen (Ref. 8).

E_x[a] (MeV)	E_n (MeV)	Γ c.m. (keV)	l	J	S	$\frac{\gamma^2}{\hbar^2/\mu a^2}$
0			2	5/2	0.8–1.0[b]	
0.871			0	1/2	0.8–1.0[b]	
3.058			1	(1/2)	0.02[c]	
3.846			3	(5/2)[d]	0.02[c]	
4.558	0.442[e]	45[e]	1	3/2		0.06[e]
5.083	1.000[f]	94[f]	2	3/2		0.45[f]
5.215[h]		<8		7/2, 9/2, or 11/2[h]		
5.377	1.312[f]	41[f]	1	3/2		0.02[f]
5.696	1.651[i]	3.4[i]	3[j]	7/2[j]		0.09[i]
5.731	1.689[i]	<1[i]				
5.867	1.833[i]	6.6[i]	2[k]	3/2[k]		0.009[k]
5.936	1.906[i]	24.5[i]	1	1/2		0.01[i]
(6.24)						
6.356	2.353[i]	135[i]	0	1/2		0.02[i]
6.859	2.888[i]	<1[i]				
6.970	3.006[i]	<1[i]				
7.164	3.212[i]	1.4[i]	3[k]	5/2[k]		0.005[k]
7.294	3.350[l]	500[l]	2	3/2		0.23[l]
7.377	3.438[i]	0.5[k]	2[k]	5/2[k]		0.0002[k]
7.380	3.441[i]	1.1[k]	3[k]	5/2[k]		0.003[k]

[a]See Ref. 21.
[b]See Ref. 22.
[c]See Ref. 23.
[d]T. K. Alexander, C. Broude, and A. E. Litherland, Nucl. Phys. 53, 593 (1964).
[e]A. Okazaki, Phys. Rev. 99, 55 (1955).
[f]H. R. Striebel, S. E. Darden, and W. Haeberli, Nucl. Phys. 6, 188 (1958).
[g]C. P. Browne, Phys. Rev. 108, 1007 (1957).
[h]See Ref. 5.
[i]See Refs. 2 and 3.
[j]See Ref. 10.
[k]Present data.
[l]See Ref. 1.

and Green,[22] in a recent analysis of latest absolute-cross-section measurements of the $^{16}O(d, p)^{17}O$ reaction, give spectroscopic factors close to unity with an uncertainty of 20%. For an estimate of the spectroscopic factors for the bound 3.058 and 3.846 states, we normalize the spectroscopic factors of Keller[23] to those of Naqib and Green.

In the last column, we list the reduced widths in single-particle units of the unbound states of ^{17}O. In calculating the neutron penetrabilities to obtain these reduced widths, we use the procedure recommended by Vogt[24] and choose a nuclear radius $a = 4.2$ F such that a square well of this radius will give the experimental bound-s-state energy and the s-wave phase shifts near zero energy.[1,25] In such a well a single-particle s state has a reduced width of $\hbar^2/\mu a^2$, where μ is the reduced neutron mass.[24] There is some evidence that this procedure may give an underestimate of the reduced widths. Naqib and Green,[22] in extrapolating their (d, p) stripping analysis to the 5.08-MeV state, suggest this state might have a reduced width between 0.8–1.0 of the single-particle limit. If this is correct, it would appear that the procedure we have adopted may be in error, at least for this state, by ~50%.

As we pointed out in our previous paper[1] on scattering of neutrons from ^{16}O, there have been at least two theoretical approaches to the explanation of the spectra of mass-17 nuclei. On one hand, there has been the approach based on shell-model states,[26-28] and on the other, there has been the approach in terms of the cluster model.[29,30] Since 1967 additional work along both lines of attack has been reported. Zuker, Buck, and McGrory[31,32] have made an exact shell-model calculation including all possible states of five particles in the $1p_{1/2}$, $1d_{5/2}$, and $2s_{1/2}$ orbits in a spherical nucleus. Since the $1p_{3/2}$ orbits were not considered, the calculations corresponded to the particles moving in the field of a ^{12}C core. Neglect of the $d_{3/2}$ orbits limited the applicability of the model to low-lying states. The calculations gave the $\frac{5}{2}^+$ and the $\frac{1}{2}^+$ states as essentially single-particle states coupled to a correlated ^{16}O core. The next higher states were multiple particle-hole configurations, mostly of odd parity. Bobker,[33] using essentially the same shell model for calculating configuration mixing in the $1p_{1/2}$, $1d_{5/2}$, and $2s_{1/2}$ subshells, has also given the character and energies of the odd-parity states of ^{17}O. These theorists (Bobker and Zuker *et al.*) obtained remarkable agreement with the energies of the observed odd-parity states of ^{17}O up to ~7-MeV excitation energy. They predicted several levels in the ^{17}O spectrum which may be the unassigned narrow levels we see[2,3] in neutron scattering from ^{16}O. Zuker *et al.*[32] predicted a five-particle four-hole $d_{3/2}$ state at ~5.3 MeV. Since they neglected the $1d_{3/2}$ orbits in their calculations, they cannot identify this level with the 5.083-MeV state (Table I), which is largely a single-particle state. Their $1d_{3/2}$ state could be the 7-keV-wide $\frac{3}{2}^+$ state at 5.867 MeV which we discuss in this paper (Fig. 1). Zuker *et al.*[32] predicted other even-parity states around 7-MeV excitation energy which possibly can be identified with the observed even-parity states in this energy region (Table I).

Recent theoretical investigations have indicated that the first excited 0^+ state of ^{16}O at 6.056 MeV may involve α-like clusters.[34] In ^{17}O the relatively broad $s_{1/2}$ and $d_{3/2}$ states and the narrow $d_{5/2}$ state, grouped together around 7 MeV (Table I), suggest the coupling of the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ essentially single-particle states of ^{17}O with the excited 0^+ state of the ^{16}O core. A calculation based on the concept of the compound nucleus, in which a $2s_{1/2}$, $1d_{3/2}$, or $1d_{5/2}$ neutron is bound to the 0^+ α-cluster breathing mode of the ^{16}O core, gave the widths and approximate energies of the even-parity resonances around 7 MeV.[35] The parameters defining the breathing mode also gave two other excited 0^+ states of ^{16}O energies at which such states are observed.[8,36]

ACKNOWLEDGMENT

We are indebted to F. H. Ward for constructing and testing the oxygen dewar, to W. T. Newton and W. C. H. White for maintaining the accelerator in excellent condition for our use, and to R. M. Feezel for assisting during the experiment.

---

*Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

[1]C. H. Johnson and J. L. Fowler, Phys. Rev. 162, 890 (1967).

[2]C. H. Johnson, F. X. Haas, J. L. Fowler, F. D. Martin, R. L. Kernell, and H. O. Cohn, in *Proceedings of the Second Conference on Neutron Cross Sections and Technology, Washington, D. C., March, 1968,* edited by D. T. Goldman (U. S. Government Printing Office, Washington, D. C., 1968), National Bureau of Standards Special Publication No. 299, Vol. II, p. 851.

[3]J. L. Fowler, C. H. Johnson, and F. X. Haas, in *Proceedings of International Symposium on Nuclear Structure, Dubna, 1968,* (International Atomic Energy Agency, Vienna, Austria, 1969), p. 1.

[4]J. L. Fowler, C. H. Johnson, Bull. Am. Phys. Soc. 14, 494 (1969).
[5]F. A. Rose, Nucl. Phys. A124, 305 (1969).
[6]H. Schölermann, Z. Physik 220, 211 (1969).
[7]C. H. Johnson, J. L. Fowler, F. X. Haas, and R. M. Feezel, to be published.
[8]F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. 11, 1 (1959).
[9]H. B. Burrows, C. F. Powell, and J. Rotblat, Proc. Roy. Soc. (London), A209, 478 (1951).
[10]R. O. Lane, A. S. Langsdorf, J. E. Monahan, and A. J. Elwyn, Ann. Phys. (N. Y.) 12, 135 (1961).
[11]H. B. Willard, L. C. Biedenharn, P. Huber, E. Baumgartner, *Fast Neutron Physics,* edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1963), Vol. II, pp. 1217–1316.
[12]J. L. Fowler, Phys. Rev. 147, 870 (1966).
[13]R. W. Peelle and T. A. Love, National Academy of Sciences – National Research Council Publication No. 1184, p. 146 (1964).
[14]J. H. Gibbons and H. W. Newson, *Fast Neutron Physics,* edited by J. B. Marion and J. L. Fowler (Interscience Publishers, Inc., New York, 1960), Vol. I, pp. 133–174.
[15]C. H. Johnson, C. C. Trail, and A. Galonsky, Phys. Rev. 136, B1719 (1964).
[16]P. R. Bevington, W. W. Rolland, and H. W. Lewis, Phys. Rev. 121, 871 (1961).
[17]M. G. Rusbridge, Proc. Phys. Soc. (London) A69, 830 (1956).
[18]R. B. Walton, J. D. Clement, and F. Boreli, Phys. Rev. 107, 1065 (1957).
[19]J. K. Bair and F. X. Haas, private communication.
[20]D. Lister and A. Sayres, Phys. Rev. 143, 745 (1966).
[21]C. Maples, G. W. Goth, and J. Cerny, Nucl. Data 2, 429 (1966).
[22]I. M. Naqib and L. L. Green, Nucl. Phys. A112, 76 (1968).
[23]E. L. Keller, Phys. Rev. 121, 820 (1960).
[24]E. Vogt, Rev. Mod. Phys. 34, 723 (1962).
[25]J. L. Fowler and H. O. Cohn, Phys. Rev. 109, 89 (1958).
[26]G. E. Brown and A. M. Green, Nucl. Phys. 75, 401 (1965).
[27]B. Margolis and N. de Takacsy, Can. J. Phys. 44, 1431 (1966).
[28]Q. Ho-Kim, Nuovo Cimento LIB, 492 (1967).
[29]S. Matthies, V. G. Neudachin, and Yu F. Smirnov, Nucl. Phys. 38, 63 (1962).
[30]K. Wildermuth and W. McClure, *Cluster Representations of Nuclei,* (Springer-Verlag, Berlin, 1966), Vol. 41.
[31]A. P. Zuker, B. Buck, and J. B. McGrory, Phys. Rev. Letters 21, 39 (1968).
[32]A. P. Zuker, B. Buck, and J. B. McGrory, in *Proceedings of International Conference on Properties of Nuclear States, Montreal, Canada, 1969,* edited by M. Harvey *et al.* (Presses de l'Université de Montréal, Montréal, Canada, 1969), p. 193.
[33]J. Bobker, Phys. Rev. 185, 1294 (1969).
[34]J. Eichler, T. Marumori, and K. Takada, Progr. Theoret. Phys. (Kyoto), 40, 60 (1968).
[35]J. L. Fowler, in *Proceedings of the International Conference on Properties of Nuclear States, Montreal, Canada, 1969,* edited by M. Harvey *et al.* (Presses de l'Universite de Montréal, Montréal, Canada, 1969), p. 334.
[36]E. B. Carter, G. E. Mitchell, and R. H. Davis, Phys. Rev. 133, B1421 (1964).

PHYSICAL REVIEW C VOLUME 2, NUMBER 1 JULY 1970

Low-Energy Theorem for Nucleon-Nucleon Bremsstrahlung*

M. K. Liou
Department of Physics, University of Manitoba, Winnipeg, Canada
(Received 10 December 1969; revised manuscript received 12 March 1970)

The low-energy theorem is derived for nucleon-nucleon bremsstrahlung. We assume that the two nucleons interact through a potential which can be any model, nonlocal as well as local. For those potentials which depend explicitly upon momentum and/or angular momentum operators, the gauge terms arising from these operators are included in the derivation. These gauge terms are always important in the study of the off-energy-shell effects of the two-nucleon interaction, and for the $np\gamma$ process they are essential in the derivation of the low-energy theorem. It is found that the gauge terms are canceled precisely by parts of the terms which represent the photons emitted by the internal nucleon lines.

I. INTRODUCTION

Recently, Heller[1-3] has derived the low-energy theorem for bremsstrahlung in a potential model. He has shown that when the bremsstrahlung amplitude $\vec{M}$ is expanded in powers of the photon momentum K,

$$\vec{M}=\vec{A}/K+\vec{B}+\vec{C}K+\ldots, \tag{1}$$

the coefficients $\vec{A}$ and $\vec{B}$ are independent of the off-energy-shell effects. Without introducing the concept of potential, the theorem was first derived by Low[4] using the formalism of quantum field theory.

In Heller's derivation the potential was assumed