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Production Cross Section of Isotopes Emitted in 70-MeV α-Particle Interactions on C and N Targets

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The experimental cross sections for production of the Li, Be, and B isotopes and their isotopic ratios have been determined for interactions produced by 70-MeV α particles with the carbon and nitrogen nuclei of a nuclear emulsion. The cross-section values as well as the isotopic ratios have been fitted with the help of Rudstam's formula; there is good agreement for the carbon target and poor agreement for the nitrogen target.

INTRODUCTION

It is generally accepted that the elements Li, Be, and B are produced in spallation reactions induced by protons or α particles on C, N, and O nuclei, rather than originating in the normal processes of nucleosynthesis in stellar interiors.[1] Knowledge of the production cross sections of Li, Be, and B together with their isotopic ratios will be very interesting in order to confirm this hypothesis. Furthermore, these quantities are also useful for calculating the amount of matter traversed by cosmic rays. The quantitative analysis of the formation process by spallation could be carried out easily if measured values of cross sections could be assembled by means of a simple relation.

In this paper, we report the production cross sections for Li, Be, and B in the interaction of 70-MeV α particles with the carbon and nitrogen nuclei contained in a nuclear emulsion, and have tentatively fitted these values with the help of Rudstam's formula[2]

$$\sigma(A,Z)=\exp[PA-Q-R(Z-AS)^2],$$

where A and Z are the atomic mass and charge of an isotope, and P, Q, R, and S are parameters to be determined empirically.

EXPERIMENTAL TECHNIQUE

Several stacks of Ilford-K 600-μ-thick nuclear emulsion were exposed to a beam of α particles at the Karlsruhe accelerator. The flux was 2×10^5 particles per cm^2, and the mean energy of the particles was 98.2 ± 3.0 MeV; the beam made an angle of 45° with the plane surface of the plates. A more detailed description of the experimental arrangement was given in an earlier paper.[3]

The search for interaction stars was carried out on the second plate of each stack by systematically scanning from the top surface to the bottom. In that way, the energy interval of the interacting α particles extends from 60 to 80 MeV. We have analyzed 2843 stars having a number of prongs $n>2$. For every prong, we have a projected direction for the incident α particle and the emitted fragment. The set of experimental values for a given star provides the basic information to be subsequently treated by computer methods.

The energy and momentum conservation laws for nuclear reactions give the kinematical constraints to be imposed on each set of data by means of a program that takes into account all possible combinations of mass and charge for the reaction products in the case of C, N, and O targets; the energy of a fragment (Z,A) is obtained from the corresponding prong length with the help of Heckmann's relation.[4] Only reactions with zero or one emitted neutron have been considered.

Clearly, a reaction will be unambiguously identified by this method when a single solution is obtained for the interaction under analysis. We observed that in most of the interactions worked out by the program having multiple solutions, the target was oxygen; for that reason we could not obtain meaningful cross sections for fragments emitted in reactions on the oxygen target.

TABLE I. Cross section for production of d, t, and ^{3}He for carbon and nitrogen targets.

Fragment	Carbon σ (mb)	Nitrogen σ (mb)
^{2}H	85 ±10	209 ±29
^{3}H	23 ±4	146 ±23
^{3}He	48 ±7	209 ±29

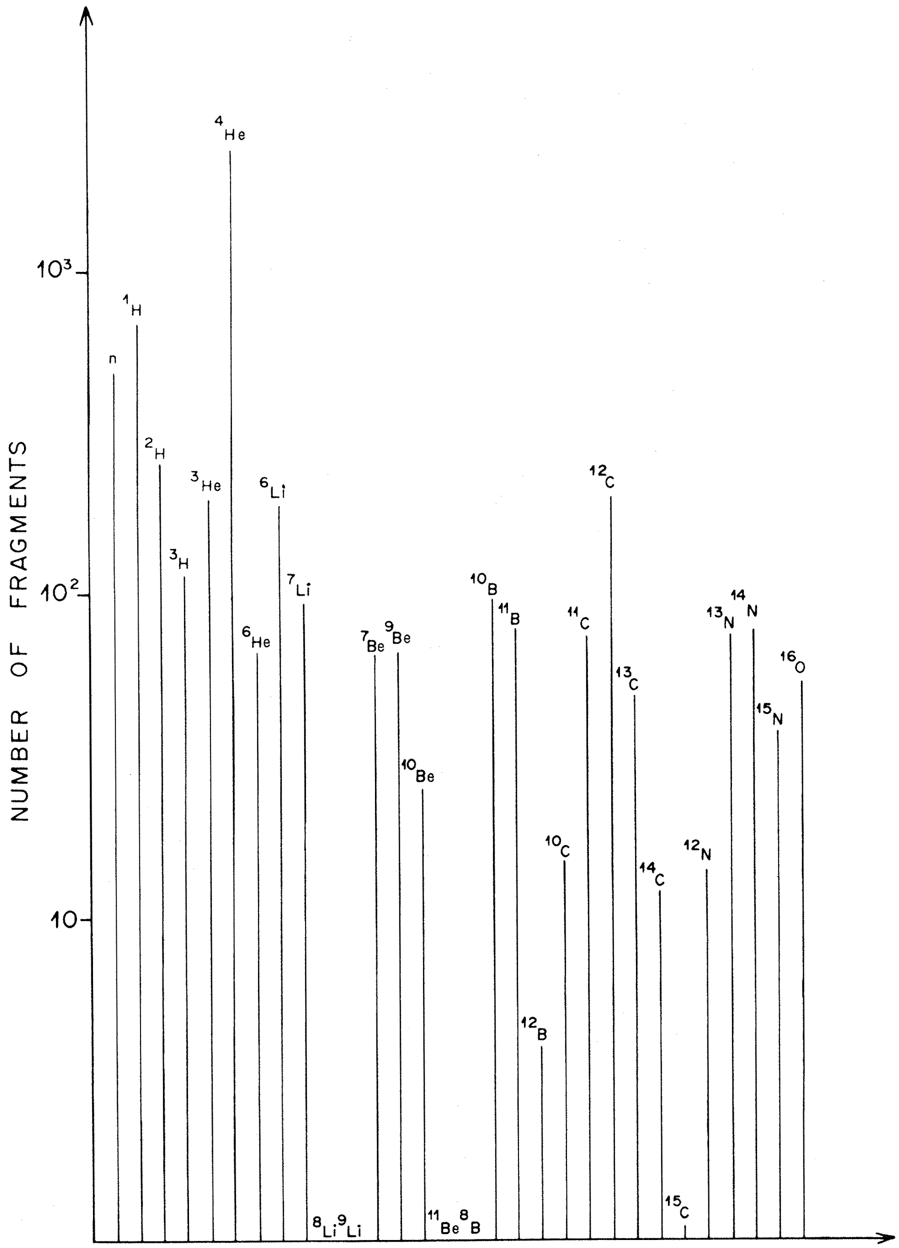

FIG. 1. Total number of fragments emitted in the interactions on C, N, and O.

RESULTS

The complete isotopic spectrum of the emitted fragments in the interactions of α particles with C, N, and O is shown in Fig. 1; as expected, the production is higher for the more stable isotopes.

Table I gives the experimental cross sections for ^{2}H, ^{3}H, and ^{3}He fragments obtained for C and N targets. These values can be used to illustrate the contribution to the formation of deuterons, tritons, and ^{3}He by interactions of medium-Z elements with interstellar helium.

The cross sections for production of isotopes with $A \geqslant 6$ are listed in Table II; the errors take into account the statistics and the scanning efficiency. As we did not find any ^{14}C for the carbon target, an upper limit was estimated within the limits of our statistical fluctuations. We remark that for 70-MeV incident α particles the cross-section values are higher than the corresponding ones for 90 MeV,[5] particularly for some isotopes such as ^{6}Li and ^{11}C in the case of the nitrogen target.

The experimental energy spectra are shown in Figs. 2 to 5.

RUDSTAM'S RELATION

The isospin rule for the cross sections was introduced by Bernas *et al.*[6] in their analysis of proton interactions with light nuclei. This dependence of the cross-section values on ΔT_3 (the isotopic-spin difference between the initial and final nuclei) suggested to Audouze *et al.*[7] that the experimental

TABLE II. Cross sections for fragments with $A \geqslant 6$ emitted in $\alpha + {}^{12}$C and $\alpha + {}^{14}$N at 70 MeV.

Isotope	Carbon σ (mb)	Nitrogen σ (mb)
^{6}He	11 ±3	77 ±16
^{6}Li	55 ±7	220 ±31
^{7}Li	30 ±5	102 ±19
^{7}Be	26 ±5	52 ±13
^{9}Be	26 ±5	58 ±14
^{10}Be	6 ±2	33 ±10
^{10}B	31 ±5	105 ±19
^{11}B	16 ±3	99 ±18
^{12}B	1.3 ±0.9	2.7 ± 2.7
^{10}C	6 ±2	8 ± 5
^{11}C	18 ±4	102 ±19
^{12}C	42 ±6	289 ±37
^{13}C	16 ±3	28 ± 9
^{14}C	<0.6	14 ± 6
^{12}N	1.9 ±1.1	19 ± 7
^{13}N	5 ±2	129 ±22
^{14}N	2.5 ±1.3	110 ±20
^{15}N		63 ±14
^{16}O		27 ± 9

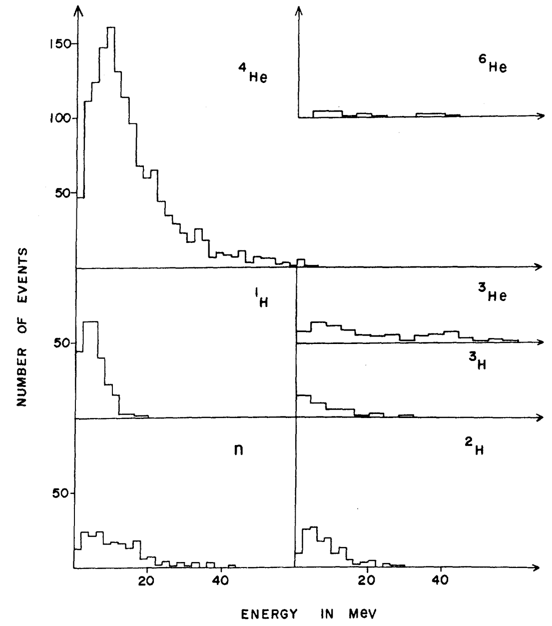

FIG. 2. Energy spectra of fragments emitted in $\alpha + {}^{12}$C reactions.

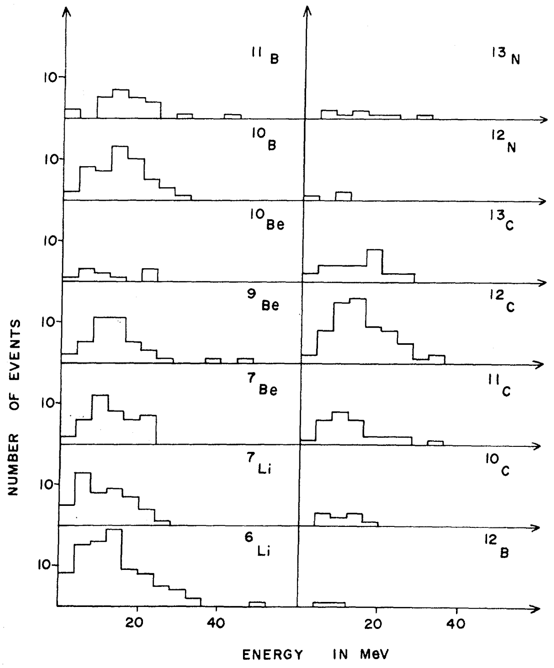

FIG. 3. Energy spectra of fragments emitted in $\alpha + {}^{12}$C reactions.

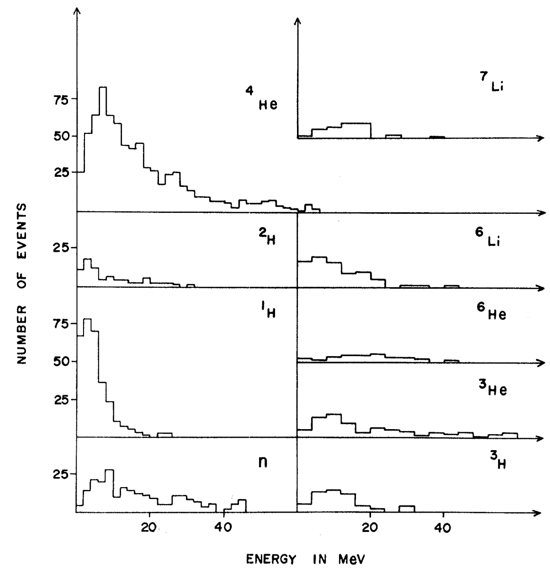

FIG. 4. Energy spectra of fragments emitted in $\alpha + {}^{14}N$ reactions.

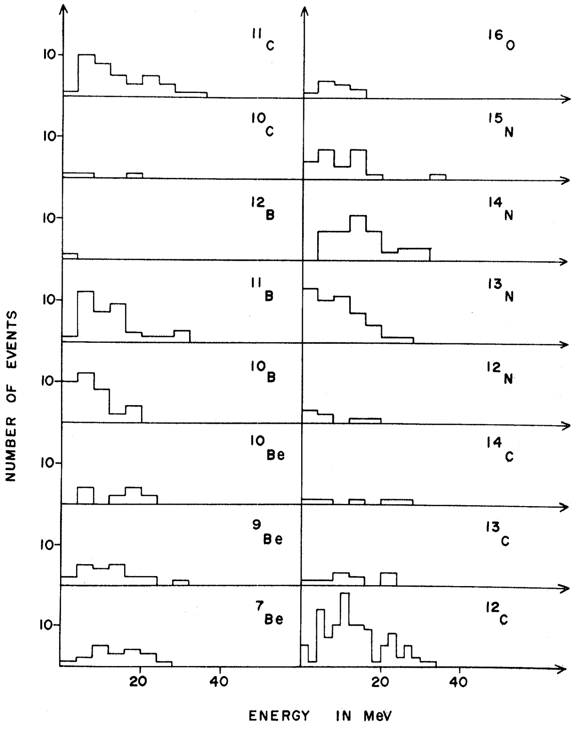

FIG. 5. Energy spectra of fragments emitted in $\alpha + {}^{14}N$ reactions.

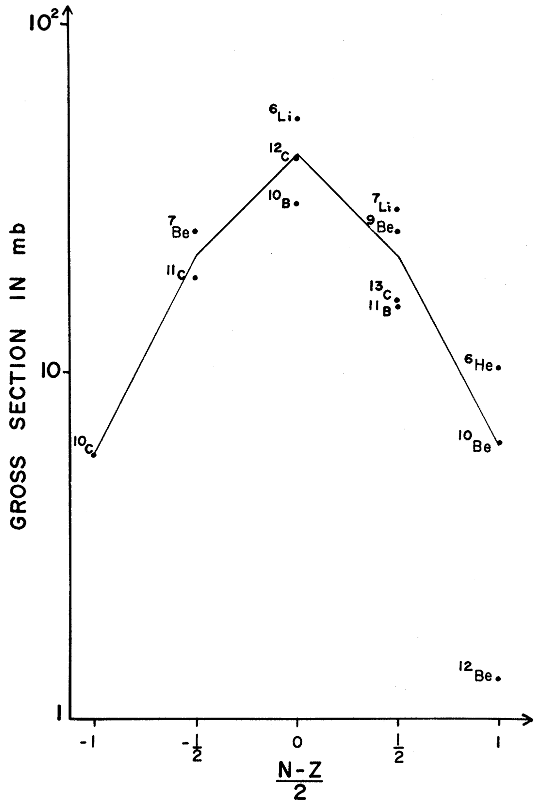

FIG. 6. Cross sections of fragments emitted in $\alpha + {}^{12}C$ reactions plotted versus ΔT_3.

TABLE III. Estimated values of the parameters P, Q, R, and S for carbon and nitrogen targets.

Parameters	Carbon	Nitrogen
P	-0.139 ± 0.005	-0.065 ± 0.005
Q	-4.78 ± 0.01	-5.55 ± 0.05
R	1.65 ± 0.12	2.77 ± 0.17
S	0.498 ± 0.003	0.503 ± 0.003

TABLE IV. Cross sections for fragments emitted in $\alpha + {}^{12}C$ reactions.

Isotope	Experimental σ (mb)	Calculated σ (mb)
^{6}He	11 ± 3	10
^{6}Li	55 ± 7	52
^{7}Li	30 ± 5	31
^{7}Be	26 ± 5	30
^{9}Be	26 ± 5	23
^{10}Be	6 ± 2	6
^{10}B	31 ± 5	30
^{11}B	16 ± 3	18
^{10}C	6 ± 2	5
^{11}C	18 ± 4	16

cross sections induced by proton interactions on C, N, and O could be fitted with the help of Rudstam's formula, in which the term $\exp[-R(Z-SA)^2]$ describes the ΔT_3 dependence. Audouze *et al.* used parameters different from Rudstam's to check the results.

We applied Rudstam's relation in the case of reactions induced by 90-MeV **α** particles on C and N targets.[5] We found fair agreement between the experimental and calculated cross-section values. To estimate the parameters P, Q, R, and S, we adapted the program MALIK[8] to a minimum-χ^2 test.

As Figs. 6 and 7 show, our cross-section values at 70 MeV depend on ΔT_3. For this reason, we decided to use Rudstam's formula to fit our values, and following a procedure similar to that used at 90 MeV we calculated the cross sections with the help of the estimated parameters P, Q, R, and S presented in Table III. For each parameter the given error corresponds to the limits of the interval in which the χ^2_{min} remains constant.

Tables IV and V show the calculated cross sections compared with the experimental values obtained for fragments emitted in the interactions on C and N targets. We find fair agreement for C, but rather poor agreement for N.

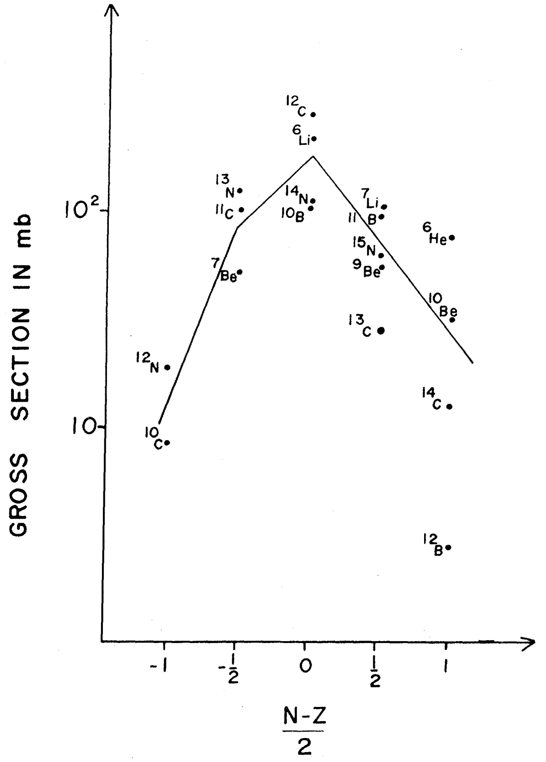

FIG. 7. Cross sections of fragments emitted in $\alpha + {}^{14}N$ reactions plotted versus ΔT_3.

TABLE V. Cross sections for fragments emitted in $\alpha + {}^{14}N$ reactions.

Isotope	Experimental σ (mb)	Calculated σ (mb)
^{6}He	77 ±16	10
^{6}Li	220 ±31	174
^{7}Li	102 ±19	76
^{7}Be	52 ±13	87
^{9}Be	58 ±14	66
^{10}Be	33 ±10	10
^{10}B	105 ±19	134
^{11}B	99 ±18	57
^{12}B	2.7 ± 2.7	6
^{10}C	8 ± 5	10
^{11}C	102 ±19	70
^{12}C	289 ±37	118
^{13}C	28 ± 9	49
^{14}C	14 ± 6	5
^{12}N	19 ± 7	9

TABLE VI. Experimental and calculated values for isotopic ratios of fragments emitted in $\alpha + {}^{12}C$ reactions. The three last ratios are determined assuming ^{10}Be decay.

Ratio	Experimental value	Calculated value
^{7}Li/^{6}Li	0.54 ±0.10	0.59
(^{7}Li + ^{7}Be)/(^{6}He + ^{6}Li)	0.85 ±0.12	0.96
^{11}B/^{10}B	0.51 ±0.12	0.59
(^{11}B + ^{11}C)/(^{10}B + ^{10}B + ^{10}C)	0.78 ±0.14	0.83
Be/Li	0.26 ±0.04	0.24
Be/B	0.45 ±0.08	0.43
Li/B	1.70 ±0.20	1.79
^{10}Be decay		
Be/Li	0.21 ±0.04	0.19
Be/B	0.33 ±0.06	0.31
Li/B	1.56 ±0.17	1.64

TABLE VII. Experimental and calculated values for isotopic ratios of fragments emitted in $\alpha + {}^{14}N$ reactions. The three last ratios are determined assuming ^{10}Be decay.

Ratio	Experimental value	Calculated value
^{7}Li/^{6}Li	0.46 ±0.09	0.44
(^{7}Li + ^{7}Be)/(^{6}He + ^{6}Li)	0.52 ±0.09	0.88
^{11}B/^{10}B	0.95 ±0.24	0.42
(^{11}B + ^{11}C)/(^{10}Be + ^{10}B + ^{10}C)	1.38 ±0.25	0.82
Be/Li	0.20 ±0.04	0.28
Be/B	0.29 ±0.06	0.27
Li/B	1.43 ±0.17	0.98
^{10}Be decay		
Be/Li	0.13 ±0.03	0.24
Be/B	0.16 ±0.04	0.23
Li/B	1.29 ±0.15	0.94

The isotopic ratios ^{7}Li/^{6}Li and ^{11}B/^{10}B calculated using Rudstam's relation are shown in Tables VI and VII, together with the experimental values for those ratios. In fact, since ^{6}He is a β emitter and ^{7}Be decays by electron capture to ^{7}Li, we have determined also the isotopic ratio (^{7}Li + ^{7}Be)/(^{6}He + ^{6}Li); such ratios are of interest for the study of nucleosynthesis of the Li, Be, and B elements. In a like manner, by assuming ^{10}B to be the decay product of the ^{10}Be isotope on a cosmic time scale ($\tau \sim 10^6$ yr), and since ^{10}C and ^{11}C are β emitters, we have determined the isotopic ratio (^{11}B + ^{11}C)/(^{10}Be + ^{10}B + ^{10}C) in the case of boron.

CONCLUSION

Since the cross-section values determined for spallation reactions with α particles in medium-mass nuclei are greater than those values for protons, we point out that the contribution of the α particles to the synthesis of Li, Be, and B elements is not insignificant compared with that of protons, particularly near the threshold of formation, even though the universal abundance of helium is about 10%. For that reason, it is interesting to study the cross sections for the production of Li, Be, and B elements by α particles at different energies.

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Discussion of a New Technique for Solving the Bethe-Goldstone Equation

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The new method of Truelove and Nicholls for obtaining reaction matrix elements for nuclear-structure calculations is discussed. In this method, the Bethe-Goldstone wave function is expanded in terms of eigenfunctions of two interacting nucleons bound in a common potential well. The Bethe-Goldstone equation, which is written in terms of an expansion over noninteracting two-particle states, is then solved iteratively. In practice, the method is most easily applied when a harmonic-oscillator basis is used; the Pauli operator Q can then be treated exactly. The convergence of the Truelove-Nicholls iteration scheme and of the above two expansions is investigated. It is shown that the original method is incorrect for nucleon-nucleon potentials with an infinite hard core. A simple way of correcting the method is presented.

I. INTRODUCTION

Nuclear-structure calculations based on realistic nucleon-nucleon potentials are, in general, perturbation expansions involving the nuclear reaction matrix G. The precise form of the expansion can vary depending upon the particular nuclear properties being calculated and the model space in which one chooses to work, but in all cases a central task is the evaluation of the appropriate reaction matrix elements to use in the expansion.

The defining equation for G can always be cast in the form

$$G(\omega) = V + V \frac{Q}{\omega - H_0} G(\omega) . \tag{1}$$

Here V is the nucleon-nucleon potential, the Pauli