How Realistic Is the Tabakin Nucleon-Nucleon Potential?*

Peter Signell
Physics Department, Michigan State University, East Lansing, Michigan 48823
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It is shown that the Tabakin potential apparently predicts a deuteron quadrupole moment which is smaller than that previously assumed by a factor of 3. This discrepancy arises from the original use of an inaccurate formula relating the quadrupole moment of the deuteron to the threshold slope of the ${}^3S_1$-${}^3D_1$ coupling parameter ϵ_1. The new value indicates a corresponding Tabakin tensor force which is considerably weaker than is required. Also, the small D-wave spin-orbit splitting given by the Tabakin potential at 142 MeV is found to be different from the small experimental value.

Some time ago, Tabakin[1] devised a nonlocal separable nucleon-nucleon potential for investigating saturation and convergence properties of this type of model interaction in many-particle calculations. In line with these rather modest aims, Tabakin did not try to obtain a precise fit to the nucleon-nucleon data, but did try to reproduce the main features of the phase shifts. Following the publication of the Tabakin potential, it has been used in a variety of nuclear calculations,[2] since its separable character has enabled users to write down its matrix elements in closed form. It has come to be referred to as a realistic nucleon-nucleon interaction.

The criteria for judging whether a particular nucleon-nucleon potential is to be considered as "realistic" certainly depends upon the intended application. Thus, for determining the model dependence of nucleon-nucleon bremsstrahlung the Tabakin potential was found to be insufficiently precise.[3] However, for nuclear-structure calculations one is generally interested in more general and qualitative questions such as those posed by Tabakin. For these, it is generally believed[4] that the low-energy nucleon-nucleon data should be fit rather well but that the higher-energy data, in the region of the change of sign of the S-wave phase shifts, need be fit only qualitatively. In particular, it is well known[5] that the quadrupole moment of the deuteron implies that there must be a strong tensor force. Kerman, Svenne, and Villars[2] stated succinctly the uncertainty as to whether disturbing results of nuclear calculations might possibly be attributed to the Tabakin potential's lack of fit to the two-body data.

It has previously been noticed[6] that some results of multiparticle calculations can be quite sensitive to the central-to-tensor-potential ratio in the ${}^3S_1$-${}^3D_1$ state, which ratio is intimately connected to the quadrupole moment of the deuteron through the ${}^3S_1$-${}^3D_1$ coupling parameter ϵ_1. In fact, Tabakin obtained his model prediction for the quadrupole moment through the use of an approximate formula for the analytic continuation of ϵ_1 from the low-energy scattering region to the deuteron's negative energy.

The analytic-continuation formula used by Tabakin was due to Biedenharn and Blatt,[7] and is based on the linearity of ϵ_1 with energy at threshold:

$$\epsilon_1(k^2)=\frac{d\epsilon_1}{dk^2}\bigg|_{k^2=0} k^2, \quad \text{for small } k.$$

Here k denotes the momentum of either nucleon in the c.m. frame, and $k^2=ME_L/2\hbar^2$. One assumes that this threshold form can be analytically continued to the deuteron position at an equivalent lab-frame energy, which is the negative of twice the binding energy of the deuteron. There, ϵ_1 becomes the negative of the deuteron asymptotic ${}^3D_1/{}^3S_1$ wave function ratio, η:

$$\eta=-\epsilon_1(-k_d^{\,2})=\frac{d\epsilon_1}{dk^2}\bigg|_{k^2=0} k_d^{\,2},$$

where $k_d=0.232\text{F}^{-1}$. One now uses Biedenharn and Blatt's[7] approximate formula for η:

$$\eta\simeq(1-k_d r_t)^2\sqrt{2}\,Q_d k_d^{\,2},$$

where Q_d is the quadrupole moment of the deuteron, and r_t is the effective range for scattering in the ${}^3S_1$ state. *This formula was intended*[7] *to simulate the actual* η *which would be produced by potentials* to within an accuracy of 10%. Comparing the last two formulas listed above, one can solve for Q_d:

$$Q_d=\frac{d\epsilon_1}{dk^2}\bigg|_{k^2=0}[(1-k_d r_t)^2\sqrt{2}\,]^{-1}.$$

Tabakin adjusted his potential parameters in order to produce a value of $d\epsilon_1/dk^2$ at threshold which, when inserted into this formula, would yield the experimentally known value of Q_d.

One can directly test the accuracy of the Biedenharn-Blatt formula for η by comparing to the ex-

TABLE I. Values for η, the asymptotic D/S ratio in the deuteron, calculated from approximations to potential scattering and from potential scattering directly. The corresponding values of Q_D for the potentials of the last three lines were all within 3% of the experimental value of Q_D (see Table II) used for the η values of the first two lines.

	Ref.	Year	η
Blatt-Weisskopf formula (approx.)	8	1952	0.021
Biedenharn-Blatt formula (approx.)	7	1954	0.0075
Yamagouchi-Yamagouchi potential (exact)	10	1954	0.028
Hamada-Johnston potential (exact)	11	1961	0.027
Nine Glendenning-Kramer potentials (exact)	9	1962	0.026–0.027

act results for published potentials, and this is done in Table I. The first line lists the prediction of Blatt and Weisskopf's[8] older and simpler formula for η:

$$\eta \simeq \sqrt{2}\, Q_d k_d^{\,2}.$$

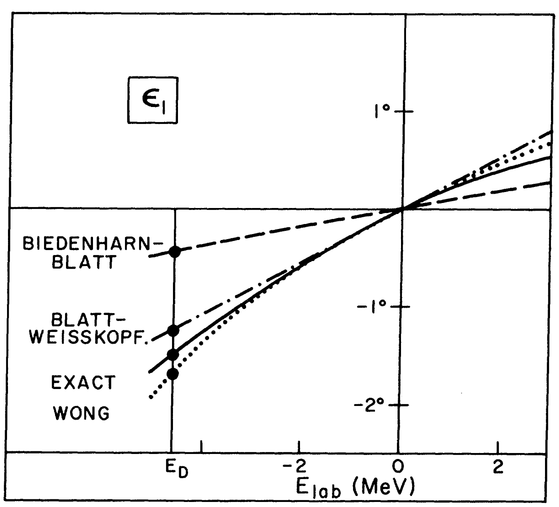

FIG. 1. The Blatt-Biedenharn coupling parameter ϵ_1, and its analytic continuation to negative energies by several methods. The broken lines indicate curves calculated from the approximation formulas (see text). The dotted line was calculated directly from the Wong formula (see Ref. 12). The solid line was calculated by direct solution of the Schrödinger equation at E_D and for $E_{1ab} > 0$. The solid line between E_D and $E_{1ab} = 0$ is an interpolation. All parameters used in computing the various lines were those of the Hamada-Johnston potential. Values of ϵ_1 computed by us directly from the Tabakin potential for $E_{1ab} > 0$ were in such close agreement with the dashed Biedenharn-Blatt line that they could not be plotted as a distinguishable line.

It is interesting to note the very small range of η displayed by the nine different Glendenning-Kramer[9] potentials, despite the fact that those potentials produced a rather wide range of ϵ_1 and ${}^3S_1$ values at higher energies. The Yamagouchi-Yamagouchi[10] potential is nonlocal and separable like the Tabakin potential, while the Hamada-Johnston potential[11] is a local one with a hard core. Also of interest is Wong's[12] low-energy formula for ϵ_1, which does not involve Q_d but does include one-pion exchange and the ${}^3S_1$ scattering length and effective range. Wong found $\eta = 0.029$ in his approximation.

One deduces from the above discussion and from Table I that the Biedenharn-Blatt formula is an exceedingly poor one. The effect of its use on Tabakin's potential is illustrated in Fig. 1, where one sees that the threshold slope of ϵ_1 for the Tabakin potential appears to be too small by a factor of 3. Furthermore, using Tabakin's threshold slope in the Blatt-Weisskopf formula, one obtains an estimate of Q_d for Tabakin's potential with greatly improved accuracy. This value is quoted in the first line of Table II. It would seem to indicate a much

TABLE II. Experimental values of the ${}^3S_1$ scattering length (see Ref. a) and effective range (see Ref. a) parameters, quadrupole moment of the deuteron (see Ref. b), and D-wave spin-orbit splitting at 142 MeV (see Ref. c) compared with those predicted by the Tabakin potential. The scattering length was computed by Tabakin for his model. The value of Q_d for the Tabakin potential is our estimated one (see text). If anything, it should be high, since the Tabakin ϵ_1 curve for $E_{1ab} > 0$ has the opposite curvature from the Hamada-Johnston and Wong ϵ_1 curves. Values predicted by the Hamada-Johnston potential are shown for comparison. Note that the deuteron binding energies produced by extrapolation at the Tabakin, Hamada-Johnston, and experimental values of a_t and r_t are, respectively, 1.18, 2.29, and 2.23 MeV.

	a_t (F)	r_t (F)	Q_d (F^2)	${}^3D_{LS}$ (deg)
Tabakin pot.	7.08	1.95	0.104(est.)	2.9
Hamada-Johnston pot.	5.38	1.77	0.285	1.2
Experiment	5.38	1.71	0.282	0.8
	±0.03	±0.03	±0.001	±0.4

[a]M. A. Preston, *Physics of the Nucleus* (Addison-Welsley Publishing Company, Inc., Reading, Massachusetts, 1962).

[b]J. P. Auffray, Phys. Rev. Letters 6, 120 (1961); P. Signell and P. M. Parker, Phys. Letters 27B, 264 (1968).

[c]See, for example, L. Heller and M. S. Sher, Phys. Rev. 182, 1031 (1969); P. Signell, in *Advances in Nuclear Physics*, edited by M. Baranger and E. Vogt (Plenum Press, Inc., New York, 1969), Vol. 2. The experimental value quoted in Table II is from a phase-shift analysis by the present authors, to be published.

weaker low-energy tensor force than that demanded by experiment. One should note, however, that the Tabakin ϵ_1 at higher energies is larger than from other models and experiment. Thus, in some calculations the low-energy and higher-energy defects may tend to cancel.

It has previously been noted[13] that the Tabakin potential provides only a qualitatively good fit to the proton-proton scattering data in the energy range 0–330 MeV. In the $^1S_0$ state, adjustments of up to 20% in the model parameters are required[14] in order to substantially improve the fit. In addition, the small D-wave spin-orbit splitting given by the Tabakin potential is rather different from that given, for example, by the Hamada-Johnston potential. Comparison of both to experiment is shown in the last column of Table II. However, the discrepancy noted above for the low-energy central/tensor ratio in the $^3S_1$-$^3D_1$ state should be regarded as much more serious.

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Note added in proof: It has been brought to the author's attention by Levinger that low-energy parameters for the Tabakin potential have been published by Clement, Serduke, and Afan.[15]

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[1]F. Tabakin, Ann. Phys. (N.Y.) 30, 51 (1964).

[2]Among many interesting calculations are these:
(a) nuclear Hartree-Fock; A. K. Kerman, J. P. Svenne, and F. M. H. Villars, Phys. Rev. 147, 710 (1966); W. H. Bassichis and M. R. Strayer, Phys. Rev. Letters 23, 30 (1969); W. H. Bassichis and A. K. Kerman, Massachusetts Institute of Technology Report No. CTP-124 (unpublished);
(b) nuclear shell model; Y. K. Gambhir and Ram Raj, Phys. Rev. 161, 1125 (1967); and T. T. S. Kuo, E. Baranger, and M. Baranger, Nucl. Phys. 81, 241 (1966);
(c) nuclear matter; J. J. MacKenzie, Phys. Rev. 179, 1002 (1969);
(d) optical model; A. K. Kerman, A. D. MacKellar, and J. F. Reading, "Low Energy Neutron-Oxygen Scattering Derived from Two-Body Forces," Massachusetts Institute of Technology CTP Report (unpublished).

[3]D. Marker and P. Signell, Phys. Rev. 185, 1286 (1969).

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[5]M. Baranger, Massachusetts Institute of Technology Report No. CTP-109; in "Proceedings of the Eighth Eastern Theoretical Physics Conference, September, 1969," edited by F. Rohrlich (Physics Department, Syracuse, New York, to be published).

[6]B. L. Scott and S. A. Moszkowsky, Ann. Phys. (N.Y.) 14, 107 (1961).

[7]L. C. Biedenharn and J. M. Blatt, Phys. Rev. 93, 1387 (1954); see also P. Signell, Michigan State University Report No. COO-2061-1; in "Proceedings of the Midwest Theory Conference, 1969, University of Iowa" (unpublished).

[8]J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), especially see pp. 104–106.

[9]N. K. Glendenning and G. Kramer, Phys. Rev. 126, 2159 (1962).

[10]Y. Yamagouchi and Y. Yamagouchi, Phys. Rev. 95, 1635 (1954).

[11]T. Hamada and I. D. Johnston, Nucl. Phys. 34, 382 (1961).

[12]D. Y. Wong, Phys. Rev. Letters 2, 406 (1959).

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[14]P. Signell and B. VerWest, to be published.

[15]D. M. Clement, F. J. D. Serduke, and I. R. Afnan, Nucl. Phys. A139, 407 (1969).