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Analysis of Nuclear Separation Energies

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It is observed that the proton separation energies S_p for even-even nuclei do not vary linearly with proton number in a strict sense. S_p values for even-even isotones with the same j, the angular momemtum of the last proton orbit, lie on a straight line. The systematics of nucleon separation energies are explained on the basis of the shell model for nuclei. It is assumed that the central potential well in which nucleons are supposed to move remains constant for a group of neighboring isotones with the last proton orbit characterized by the same j. This is also taken to be valid for isotopes with the last neutron orbit characterized by the same j. Furthermore, the interaction between the extracore neutrons, extracore protons, and their mutual interactions are simulated by average effective two-body interaction matrix elements.

1. INTRODUCTION

There is a good deal of empirical information which can be understood on the basis of the shell model for nuclei. As far as calculations of ground-state energies are concerned, we can use a single-particle picture provided we use a properly chosen effective two-body interaction. In the general single-particle approximation, the nuclear binding energy is expressed in terms of single neutron and proton binding energies in the central field. The residual interaction with closed shells and the orientation-independent part of the average n-n, p-p, and n-p interactions of the nucleons in their unfilled shells are included. If one neglects the configuration interaction, and assumes the validity of the pair-coupling scheme and the constancy of the effective central field in which the nucleons move, then the shell-model interaction parameters remain constant within a major-shell region. The nuclear binding energy will then be a quadratic function of the number of nucleons in the unfilled shells, and one would expect a linear variation of nucleon separation energies for the nuclei obtained from one another by adding pairs of nucleons. Linear dependence of nucleon separation energies on nucleon number obtained from the experimental values has been reported by several authors.[1-4] Linearity in double-nucleon separation energies has also been predicted by Zeldes, Gronau, and Lev (ZGL)[5] and Kravtsov and Skachov.[6] However, considerable discrepancies are found between the experimental values and Levy's[7] mass equation based on the above assumptions. We have also pointed out the departures of neutron separation energies from linearity in a previous communication.[8] One can adopt a different approach and try to obtain a mass equation in terms of variations of the central field itself when neutrons and protons are added to a given nucleus. The symmetry-energy effect can be explained by assuming such variation of the central potential well.[9,10] It is quite likely that the increase of nuclear radius with the addition of nucleons may result in a change of the nuclear potential well. The shell-model interaction parameters will then change from one nucleus to another. Better agreement with the experimental binding energies has been obtained by Thieberger[11] and ZGL[5] by allowing the interaction parameters to vary within a shell region. In that case, a linear variation of the nucleon separation energies, if it exists, is not expected. This paper deals with a critical study of the linearity of proton separation energies for even-even nuclei. Magic- and submagic-number effects on the orientation-independent parts of the p-p interaction are discussed. We also suggest a possible explanation of the systematics referred to here and in a previous paper.[8]

2. SYSTEMATICS OF PROTON SEPARATION ENERGIES FOR EVEN-EVEN NUCLEI

The proton separation energy S_p can be written as

$$S_p(Z,N)=E(Z,N)-E(Z-1,N)\,, \tag{1}$$

where $E(Z,N)$ is the binding energy of the nucleus (Z,N).

Using relation (1), we have calculated the proton separation energies for all possible even-even nuclei with the help of the 1964 mass table.[12] We have not considered those nuclei for which the errors are 1000 keV or larger. Nuclei with an odd proton or neutron are omitted to avoid the influence of the residual n-p interactions on the separation energies. Residual interactions may play an important role. Cohen[10] pointed out that they might have an influence such as the "self-binding effect" and the pairing energy. Figures 1, 2, and 3 show the dependence of proton separation energies on the proton number, where we have plotted the S_p values of (Z, N), $(Z+2, N)$, $(Z+4, N)$, etc., nuclei against proton number. The fraction near each point represents the angular momentum j of the preceding odd proton. The values of j are taken from a table of nuclear constants.[13,14] It is supposed that pairs are formed in orbits of angular momentum of the odd nucleons. Usually, points belonging to neighboring isotones with the same j values are connected by lines. There are, however, cases where we have only two points. These points are joined irrespective of their j values only to show that they belong to neighboring isotones. In cases where we have points all of different j values, a line is drawn arbitrarily through two points to show the deviations of other points from the line.

It is observed that the proton separation energies S_p of even-even nuclei do not vary linearly with Z in a strict sense as predicted by de-Shalit[15] and others. For light nuclei, considerable discrepancies are noted. We find, however, that the S_p values for the set of even-even nuclei (Z, N), $(Z+2, N)$, $(Z+4, N)$, etc., with the same j values lie on a straight line. This characteristic is observed throughout. The same behavior has also been observed for the neutron separation energies.[8] Such conclusions cannot be drawn in the cases where we have only two points. The observed systematics of separation energies might possibly define the role of nuclear spin in the determination of interaction energies of the nucleons.

To study magic-number effects on the slopes $-m_p(j)$ of the isotonic S_p lines, we have drawn Fig. (4), where the values for the same j are connected by lines, and the neutron number is taken as the abscissa. $m_p(j)$ gives directly the orientation-independent part of the effective p-p interaction in the unfilled shells. Absolute values of the slopes decrease with the mass number. This has also been predicted by several authors,[16-18] who have considered average isotopic S_n and isotonic S_p lines. From Fig. (4) it is clear that comparatively lower values of the slopes are obtained near

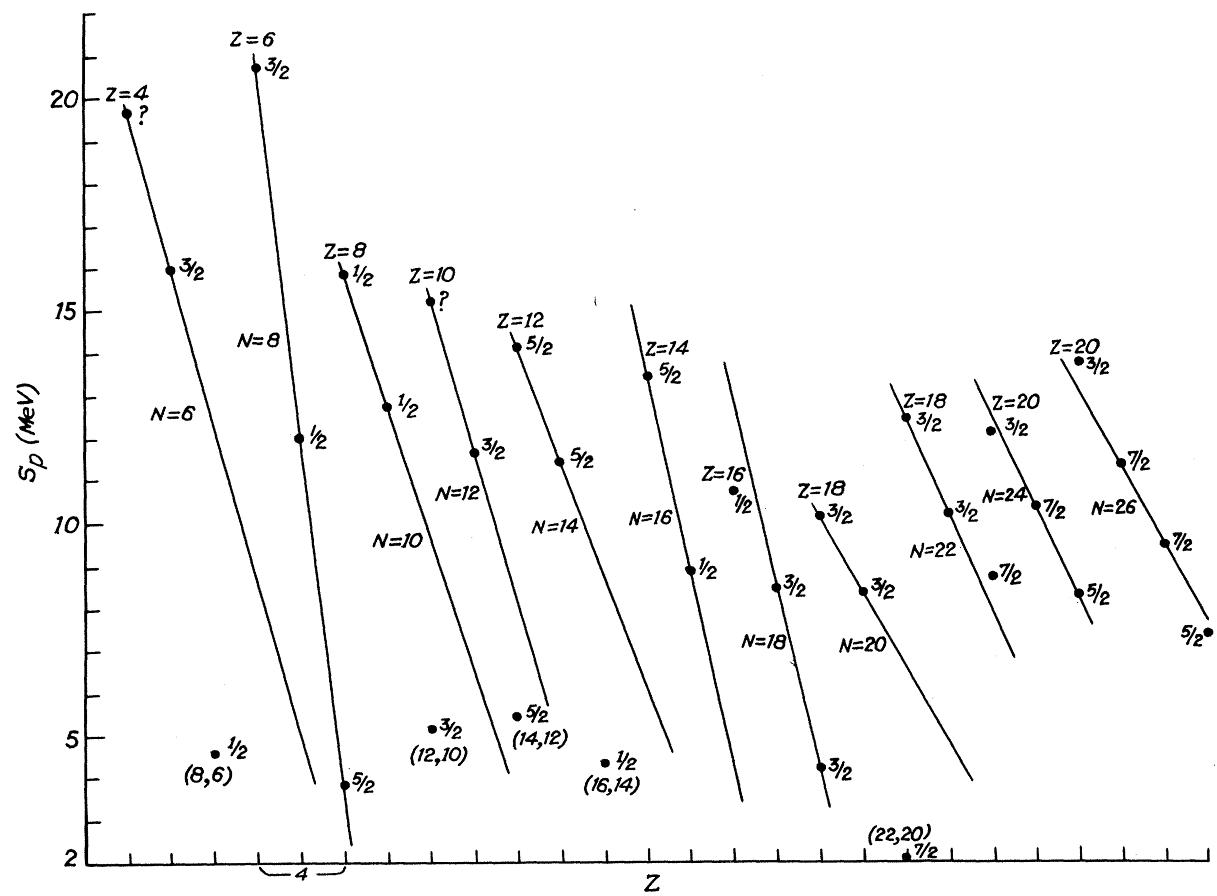

FIG. 1. Dependence of the proton separation energy S_p on proton number. The fraction near each point represents the spin of the preceding odd proton. $6 \leq N \leq 26$.

neutron magic numbers. Thus a decrease in the value at $N=40$ leads to the presence of a subshell at that region. Coryell[19] has also observed a subshell effect at $N=40$ on the β-stability line. Recently, Wing[20] has made a comprehensive comparison of nuclidic mass formulas, and has pointed out the indications of a shell effect at $N=41$. Neutron pairing energies,[21] however, do not display any subshell effect at $N=40$. Absence of a subshell at this region has also been predicted by Zeldes[4] and ZGL.[5] Dewdney,[22] in his elaborate study of the minima and curvatures of isobaric sections of the mass surfaces using the 1961 mass table,[23] did not find any definite submagic-number effect at $N=40$. We see from the figures 1, 2, and 3 that several slope values are strongly dependent on only one experimental value. This weakens our arguments.

3. DISCUSSION

The deviation from the linearity observed in nucleon separation energies for the even-even nuclei can not be explained on the basis of the liquid-drop model. The trends in the separation energies are attributed to the detailed behaviour of the shell-model potential well and the effective interaction of the nucleons in the unfilled shells.

We shall keep the basic assumptions of the simple shell model unchanged; namely, that the nuclear states can be approximately described by jj-coupling wave functions of nucleons in a central field. The interaction of the particles in closed shells contributes the same amount to the binding energies of nuclei with the same closed shells. Since nucleon separation energy is the difference of nuclear binding energies, we are mainly interested in the interactions of extra particles with those in closed shells, and the interactions between the extra particles outside closed shells. Thus the proton separation energy can be written as

$$S_p(Z,N)=\epsilon_p+(p-1)m_p+nI_{np}+\tfrac{1}{2}\pi_p\ , \qquad (2)$$

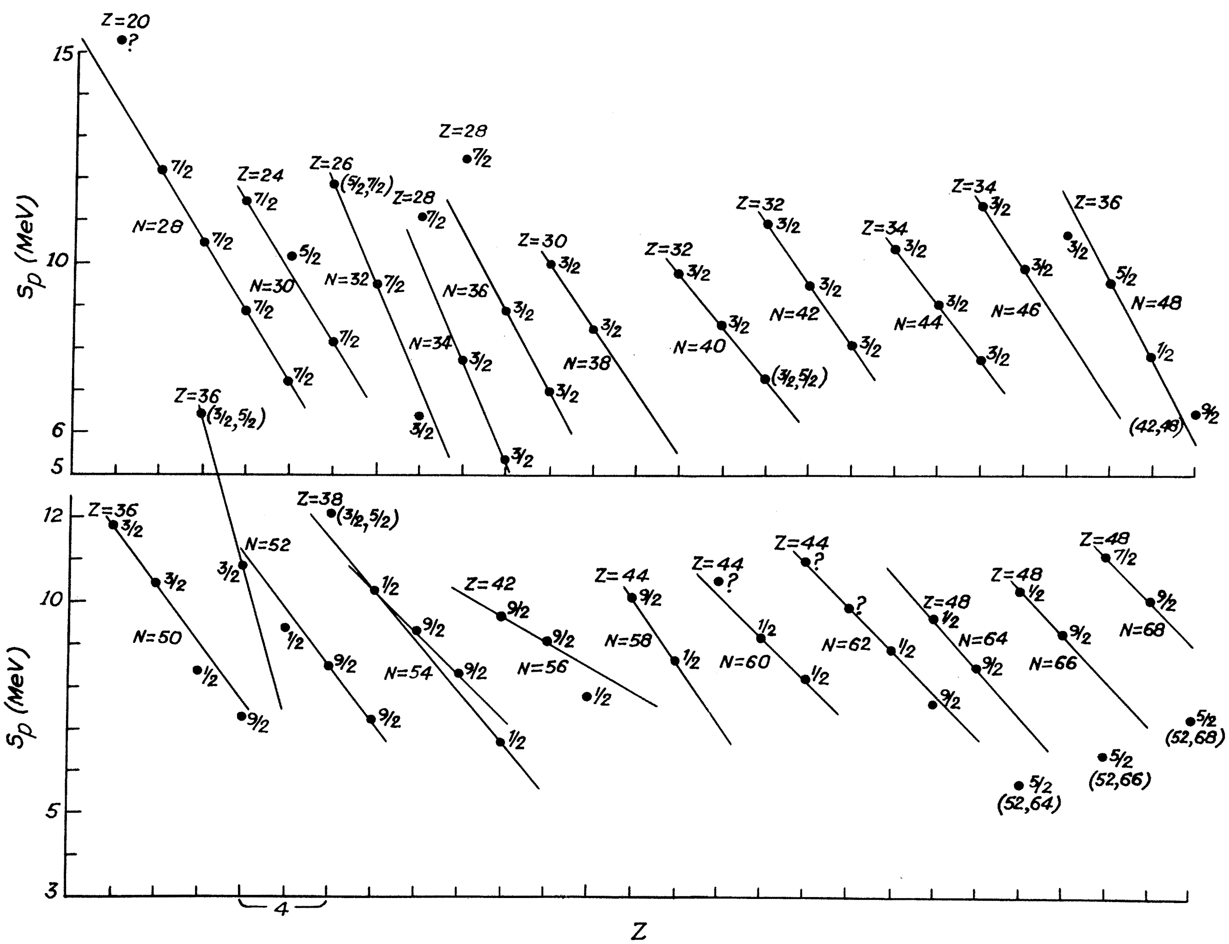

FIG. 2. Dependence of the proton separation energy S_p on proton number. The fraction near each point represents the spin of the preceding odd proton. $28\leq N\leq 68$.

where n and p denote the number of neutrons and protons in the unfilled shell, ϵ_p is the single-proton binding energy in the central field including its residual interaction with the closed shells, m_p and I_{np} characterize the orientation-independent part of the average effective interaction of two protons and a neutron-proton pair, and π_p is the proton pairing energy in the unfilled shell. A similar expression is valid for the neutron separation energies.

Now it is an experimental fact that all even-even nuclei have zero angular momentum. This can be explained easily on the basis that individual nucleons with equal and opposite j form pairs of zero angular momentum. This, of course, is due to the residual interaction between the nucleons. Thus, we may think that when a proton is taken out of an even-even nucleus (Z, N) to get the corresponding S_p value, it comes out from an orbit which is characterized by the spin of the preceding odd proton, which is also the ground-state spin of a $(Z-1, N)$ nucleus. On this basis, one would expect a strong dependence of S_p values for even-even nuclei on the angular momentum of the preceding odd proton. This agrees with our findings. A similar reasoning is also valid for the neutron separation energies.

Yamada and Matumoto,[24] in their analysis of S_n and S_p systematics, ignored the possible differences in the n-p interaction parameters for different subshells belonging to the same major shell. However, for a given configuration j_n and j_p, I_{np} depends on J, the total angular momentum, and splits the levels. The order and the magnitude of the splitting depends on the nature of the two-body interaction potential V_{np}. An appreciable spread in the n-p interaction parameter has been obtained by Ferguson.[25]

To explain the observed linearity of nucleon separation energies for even-even nuclei, we propose that the central potential well in which the nucleons are supposed to move remains constant for a group of neighboring isotones with the last proton orbit characterized by the same j, and for isotopes with the last neutron orbit characterized by the same j. We also assume that the n-n, p-p, and n-p inter-

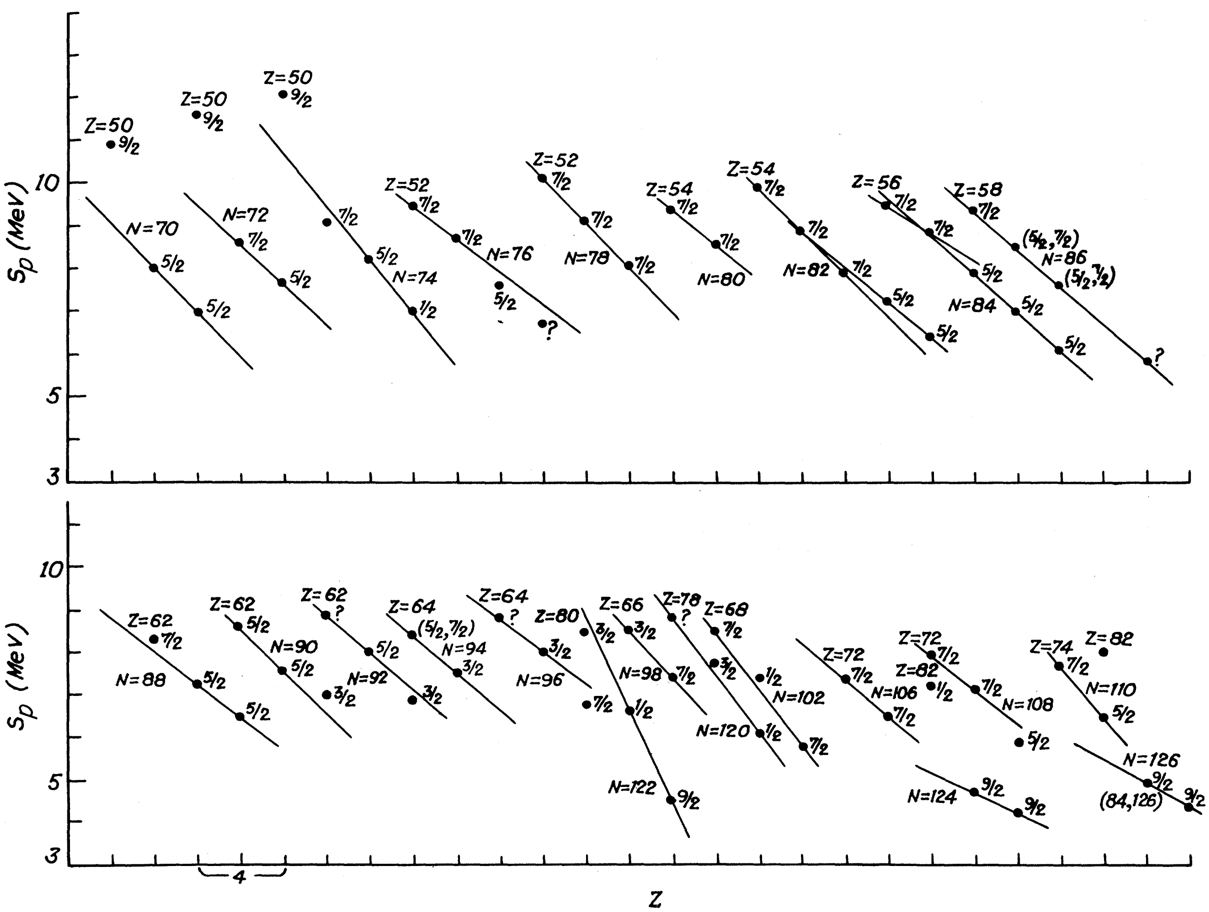

FIG. 3. Dependence of the proton separation energy S_p on proton number. The fraction near each point represents the spin of the preceding odd proton. $70 \leq N \leq 126$.

action energies of the nucleons in the unfilled shell of the isotones can be simulated by an average effective interaction. A similar assumption is also made for the isotopes. We make no attempt to obtain these interaction parameters in terms of any basic two-body interaction potential. However, even for an arbitrary interaction potential, the interaction energy will depend explicitly on j. Since the binding energy of closed shells forms a considerable part of the total binding energy of a nucleus, a slight change in the central potential well will have considerable effect on the interaction parameters.

Hence, the expression for the proton separation energies takes the form

$$S_p(Z,N)=\epsilon_p(j)+(p-1)m_p(j)+nI_{np}(j)+\tfrac{1}{2}\pi_p. \quad (3)$$

Obviously, the S_p values for a group of isotones of the same parity type and same j lie on a straight line. Systematics of neutron separation energies can also be explained in a similar way.

The large deviations observed in the case of light nuclei are possibly due to the extra binding energy for nuclei[26] with $Z=N$.

The dependence of the separation energies on the state of the last nucleon orbit shows that the binding energy of a nucleus depends on the way and order it has been built up from its constituents. In other words

$$E(N,Z)=\sum_{\text{protons}} S_p+\sum_{\text{neutrons}} S_n \quad (4)$$

is dependent on the order of the summation. Kümmel *et al.*,[27] however, ignored this effect in the development of their mass formula.

It is to be noted that the absolute values of the slopes decrease with the mass number. This is qualitatively obvious, since in large heavy nuclei the average distance between any two outer nucle-

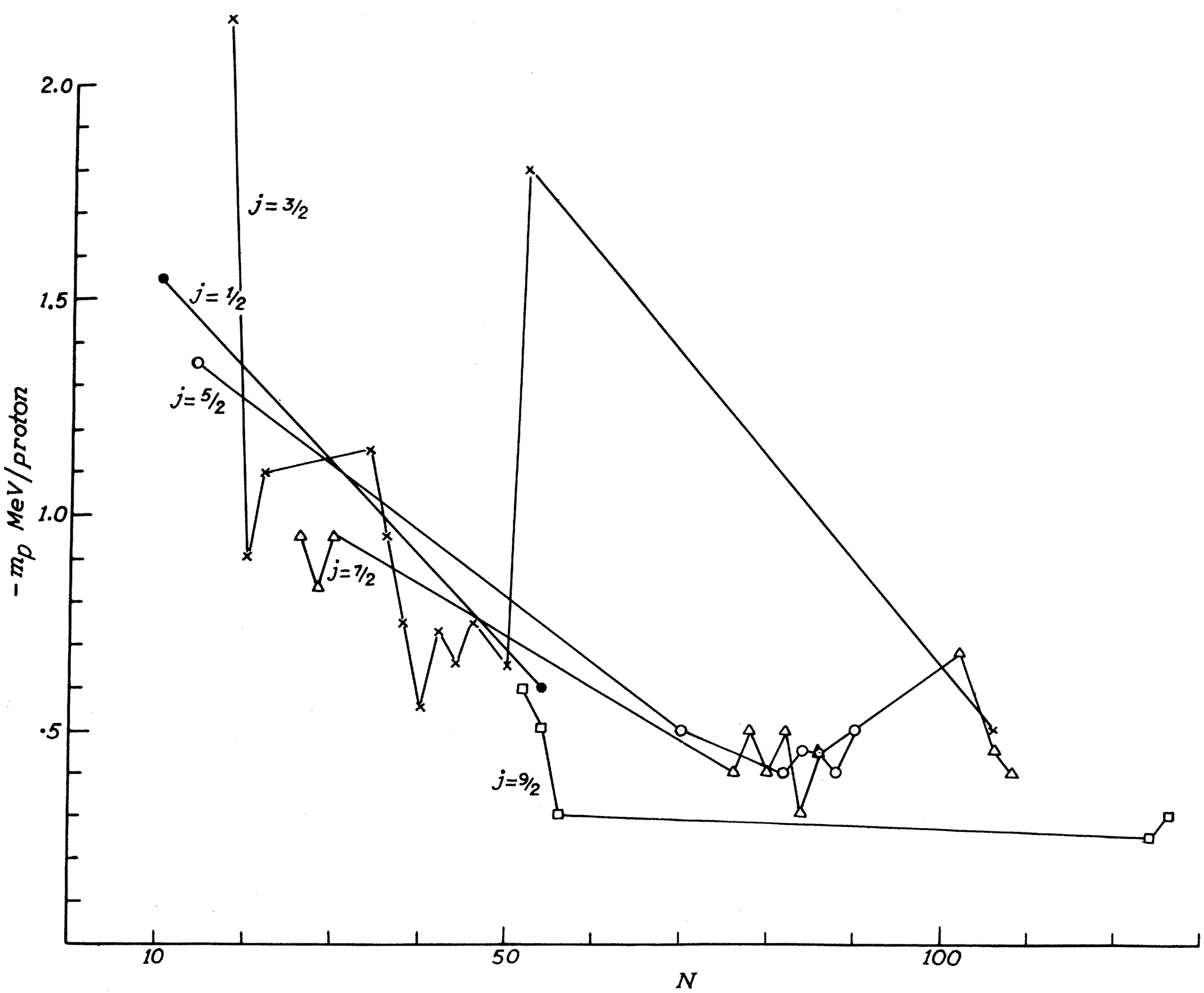

FIG. 4. Variation of the slope $-m_p(j)$ of the isotonic S_p line with neutron number. Points for a given j are connected by lines.

ons is larger, and therefore, their average interaction energy will be smaller. It is also worth mentioning that the slopes of the isotonic S_p lines are larger in absolute values than those of the isotopic S_n lines.[8] This is due to Coulomb repulsion between the protons.

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