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Isobaric Spin Doublets in Be^8 Excited with the $B^{10}(d, \alpha)Be^8$ Reaction*

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The 2^+ levels in Be^8 at 16.6 and 16.9 MeV were investigated using the $B^{10}(d, \alpha)Be^8$ reaction at several angles and bombarding energies from 4.0 to 12.0 MeV. A pattern of interference between these two levels was observed at all angles and energies except at a deuteron energy of 4.0 MeV. Fits were made to the α-particle spectra using two different formulas for the differential cross section. The results are interpreted in terms of the isobaric spin purity of the levels and isobaric spin mixing in the reaction. The latter is found to vary from 4 to 99%. The effects of other 2^+ levels on the 16.6- and 16.9-MeV levels are also discussed. Differential-cross-section ratios of the 1^+ doublet at 17.6 and 18.1 MeV were simultaneously measured. This ratio depends upon deuteron bombarding energy in a way consistent with the change in isobaric spin mixing in the reaction. The 3^+ doublet at 19.05 and 19.22 MeV was investigated. Only one particle group is observed. Various explanations in terms of interference in this region are discussed. The excitation energy and width of the higher 1^+ level were found to be 18.146 ± 0.005 MeV and 138 ± 6 keV, respectively, based on an adopted value of 17.638 MeV for the excitation energy of the lower 1^+ level.

I. INTRODUCTION

The Be^8 nucleus furnishes a unique opportunity for observing isobaric spin mixing of close-lying states of the same spin and parity. In the region of excitation between 16.5 and 19.5 MeV there are pairs of levels having spin and parity 2^+, 1^+, and 3^+, in the order of increasing energy. It is in this region that one expects to find the $T=1$ analogs of the neighboring mass-8 nuclei. One member of each pair is thus expected to have an isobaric spin quantum number $T=1$, while the other member has $T=0$. There are then three isobaric spin "doublets" at about 17, 18, and 19 MeV with $J^\pi = 2^+$, 1^+, and 3^+, respectively. Furthermore, there is only one other level known in this region, and it has odd parity.

One expects nearby levels of similar configuration but different isobaric spin to be represented by wave functions of mixed isobaric spin, and indeed, earlier results show that neither member of the 2^+ doublet near 17 MeV has a dominant value of isobaric spin. When this doublet was observed through a reaction which should excite only the $T=0$ components of the wave function, an interesting interference between the two components of the doublet was observed. These results will be referred to below. The presence of three isobaric spin doublets in Be^8 seems to offer interesting comparisons, and the possibility of distinguishing isobaric spin mixing in the final states from that in the reaction forming the states.

The $Be^{10}(d, \alpha)Be^8$ reaction is a useful tool for probing these doublets. Previous work was done at deuteron bombarding energies of 3.5 and 4.0 MeV,[1] 7.5 MeV,[2] and 20.0 MeV.[3] These investigations indicated that isobaric spin is not strictly conserved. The yield ratio for the 2^+ doublet showed that both states were almost equally populated, whereas the 1^+ states appear to have quite unequal populations. Data obtained at a deuteron bombarding energy of 12.0 MeV[4] again indicate nearly equal population of the 2^+ doublet and an unequal population of the 1^+ pair of levels. The line shape of the α-particle groups from the $B^{10}(d, \alpha)Be^8$ reaction leading to the 2^+ doublet was unusual in that it did not consist of simple Breit-Wigner (BW) resonance shapes. The group shapes were not symmetric about their midpoints but appeared to have destructive interference between the groups and constructive interference on either side. The unusual group shape exhibited in these charged-particle spectra has only been seen before in the $N^{15}(p,\gamma)O^{16}$ and $N^{15}(p,\alpha)C^{12}$ reactions.[5]

The Be^8 nucleus has, of course, been extensively investigated, and there has been much work on the structure of the 16.6- and 16.9-MeV states. A good summary emphasizing isobaric spin effects and level structure was given by Paul.[6] The chief interests in the present work are, first, a more extensive study of the shape of the α-particle group leading to the 2^+ doublet, especially with reference to various proposed theoretical formulas; second, examination of the group shapes of the 1^+ doublet for a possible comparable interference effect; third, an attempt to observe the two members of the 3^+ doublet through the (d,α) reaction and an examination of group shape; and last, the study of changes of the doublet shapes with bombarding energy, with an eye to changes in isobaric spin mixing in the reaction itself, and hence, variation in the relative excitation of the $T=0$ and $T=1$ components of the doublets.

II. THEORY

The asymmetry observed in the α-particle groups leading to the 16.6- and 16.9-MeV levels in the $B^{10}(d,\alpha)Be^8$ reaction at a deuteron energy of 12.0 MeV was first described by Barker[7] with a simple expression for the differential cross section. Barker considered the "16.6-" and "16.9-" MeV states to be in the absence of any Coulomb interaction, very close-lying states of pure though different, isobaric spin. The Coulomb interaction then mixes the states, and the wave functions may be written symbolically as

$$\Psi^{16.6} \to 2^+[\alpha(T=0)+\beta(T=1)]\,,$$
$$\Psi^{16.9} \to 2^+[\beta(T=0)-\alpha(T=1)]\,.$$

Here $\alpha^2+\beta^2=1$, since it was assumed that only these two components of isobaric spin contribute to the wave functions. The actual separation is larger than that of the postulated pure states and depends on α, β, and the strength of the Coulomb interaction. The cross section for the (d,α) reaction has the form[7] of coupled BW terms

$$\sigma(\theta,Q)\approx\left|\frac{A_1}{(Q-Q_1)+i\frac{1}{2}\Gamma_1}+\frac{A_2}{(Q-Q_2)+i\frac{1}{2}\Gamma_2}\right|^2. \qquad (1)$$

Here Q_1, Q_2 and Γ_1, Γ_2 refer to the resonance energies and level widths, respectively, and the quantity Q is the energy in the system of the nucleus. The A's are proportional to $G_l^{1/2}\Gamma_l^{1/2}$, where the G_l and Γ_l are the widths for formation and decay, respectively.

Equation (1) shows that the asymmetry observed in the (d,α) data arises from the interference between levels 1 and 2. Since the decay of both levels is through their $T=0$ components, $\Gamma_1^{1/2}/\Gamma_2^{1/2}=\alpha/\beta$. If one assumes that the (d,α) reaction forms the states only through their $T=0$ components, then $G_1^{1/2}/G_2^{1/2}=\alpha/\beta$. This gives $A_1/A_2=\alpha^2/\beta^2>0$; a plus sign between the two terms in Eq. (1) and destructive interference between the two levels. If, however, the states are formed through their $T=1$ components, $G_1^{1/2}/G_2^{1/2}=-\beta/\alpha$, giving $A_1/A_2=-1<0$, and the group shape will show constructive interference between the levels.

Barker[8] has more recently suggested a slightly different form of the cross section, which has a more fundamental basis in R-matrix theory. For levels such as the 16.6- and 16.9-MeV states in Be^8, the differential cross section can be written as

$$\sigma(\theta,Q)\approx\sum_x\left|\frac{\displaystyle\sum_{\lambda=1}^{2}\frac{A_{\lambda x}}{Q-Q_\lambda}}{\displaystyle 1+\frac{i}{2}\sum_{\lambda=1}^{2}\frac{\Gamma_\lambda}{Q-Q_\lambda}}\right|^2. \qquad (2)$$

The $A_{\lambda x}$ are again proportional to the formation and decay widths $G_{\lambda x}^{1/2}\Gamma_\lambda^{1/2}$. The sum over λ is the sum over the levels 1 and 2 (16.6- and 16.9-MeV states), whereas the sum over x accounts for the incoherent contributions to the formation process. We will later consider these incoherent processes to be attributed solely to the isobaric-spin process. Therefore x will take on values of 1 and 2 referring to $T=0$ and $T=1$ formation processes. The significant difference between Eqs. (1) and (2) is that in case a given level is not fed, Eq. (1) reduces to a simple BW single resonance, whereas Eq. (2) gives a differential cross section still affected by the presence of that resonance, and the resulting cross section may not have a simple BW form.

Finally, the effect on the doublet of other levels having the same spin and parity should be considered. Generally such other levels do not lie close and are not very wide. In Be^8, however, the situation now appears to be different. In particular, there are other 2^+ levels above and below the 2^+ doublet at 17 MeV. These include the well-known levels at 2.9 and 19.9 MeV, both having widths of the order of 1 MeV; and possibly other levels are present. Barker[9] suggested a level at 9 MeV in excitation having $J^\pi=2^+$ with a width of about 10 MeV. The effects of these other 2^+ levels are not represented by Eq. (2). Barker suggested the inclusion of a "third level" in the differential cross section. Here the effects of all other 2^+ levels are accounted for by the "third level" in a three-level approximation, and the cross section is written as

$$\sigma(\theta, Q) \approx \sum_x \left| \frac{\sum_{\lambda=1}^{2} \frac{A_{\lambda x}}{Q - Q_\lambda} + A_x}{1 + \frac{i}{2}\left(\sum_{\lambda=1}^{2} \frac{\Gamma_\lambda}{Q - Q_\lambda} + B\right)} \right|^2 . \quad (3)$$

Now Q_λ and Γ_λ are the resonance energies and widths, and A_x and B represent the effects of the third level. One may make the energy dependence of Eq. (3) independent of B by adjusting Q_λ, Γ_λ, $A_{\lambda x}$, and A_x appropriately.[9] The resulting form of the cross section is

$$\sigma(\theta, Q) \approx \frac{\sum_{\lambda,\mu=1}^{2} \frac{\overline{A}_{\lambda\mu}}{(Q - \overline{Q}_\lambda)(Q - \overline{Q}_\mu)} + 2\sum_{\lambda=1}^{2} \frac{\overline{A}_\lambda}{Q - \overline{Q}_\lambda} + \overline{A}}{1 + \frac{1}{4}\left[\sum_{\lambda=1}^{2} \frac{\overline{\Gamma}_\lambda}{Q - \overline{Q}_\lambda}\right]^2},$$

where (4)

$$\overline{A}_{\lambda\mu} = \sum_x \overline{A}_{\lambda x}\overline{A}_{\mu x},$$

$$\overline{A}_\lambda = \sum_x \overline{A}_{\lambda x}\overline{A}_x,$$

$$\overline{A} = \sum_x \overline{A}_x^2 .$$

Now $\overline{A}_{\lambda\mu}$, $\overline{A}_\lambda$, and $\overline{A}$ may depend on bombarding energy and observation angle, but $\overline{Q}_\lambda$ and $\overline{\Gamma}_\lambda$ should be independent of energy and angle.

One implication of the presence of other interfering levels is that the observed resonance energies and widths obtained from Eq. (2) may be incorrect and may depend upon bombarding energy and observation angle.

A precedent for using a cross section involving several interfering levels has been set by Barker.[10] The application of a three-level form of Eq. (2) has been used to analyze the 0^+ states in Be^8 involved in the famous "ground-state ghost."[11] This "ghost" appears as an anomaly in the Be^9(p, d)Be^8 reaction at about 750 keV above the ground state. If one uses the $\alpha - \alpha$ scattering phase shifts and simultaneously fits the Be^9(p, d)Be^8 data and uses a three-level cross section of the form of Eq. (2), the "ghost" can be explained as resulting from an interference between the 0^+ ground state, a 0^+ state at 6.0 MeV ($\Gamma \approx 10.0$ MeV), and any other "third" 0^+ levels.

It should be pointed out that Eqs. (2) through (4) are all one-channel approximations (the decays of the 2^+ levels are through the $\alpha - \alpha$ channel), and hence cannot properly be used to analyze the 1^+ and 3^+ doublets at 18 and 19 MeV. These doublets are above the threshold for proton decay.

III. MEASUREMENT, ANALYSIS, AND RESULTS

The Tandem Van de Graaff and the broad-range magnetic spectrograph of the Argonne National Laboratory were used to obtain most of the data. Ilford nuclear emulsion plates, type K0, were used to detect the α particles.

Exposures were made at 12.0-MeV deuteron bombarding energy at lab observation angles of 10, 30, 50, 80, and 114°. Exposures were also made at an observation angle of 30° and bombarding energies of 11, 10, 9, 8, 7, 6, and 5 MeV. Finally, a run at 4 MeV and at a lab angle of 35° was made using the Notre Dame electrostatic accelerator and broad-range magnetic spectrograph.

The plates were scanned in $\frac{1}{2}\times 10$-mm strips, and the number of counts in each strip was recorded. The data were then reduced by a computer which performed the following calculations: a conversion from position on the plate to a Q value (in Be^8); a laboratory-to-c.m. intensity correction; and a correction for the variation of solid angle along the plate. The corrected data points were plotted by a Calcomp plotter as the number of counts per increment in Q versus the Q value in Be^8.

The plots were examined for the presence of groups from other reactions, and corresponding points were removed. The following were the common contaminant reactions: C^{12}(d, α)B^{10} (g.s.); C^{13}(d, α)B^{11} (4.45, 5.03); N^{14}(d, α)C^{12} (12.7, 13.29, 14.08); O^{16}(d, α)N^{14} (2.33, 3.945); B^{10}(d, t)B^9 (g.s.). The numbers in parentheses correspond to the excitation energy in MeV of the residual nucleus.

After removal of points arising from contaminant reactions, the particle groups were fitted with appropriate functions. Where interference effects appeared, functions of the type of Eqs. (1) through (4) were used. In other cases expressions based on the single-level BW formula were used. All of the fitting was done with a VARIABLE METRIC MINIMIZATION code,[12] which minimized the χ^2 function by simultaneously varying the intensities, level widths, and resonance energies. A background function, generally taken to be a straight line, was also simultaneously varied.

A. 17.6- and 18.1-MeV Levels

The primary interest in this pair of 1^+ levels was a determination of the ratio of the cross section for formation of the $T = 1$ member to that for formation of the $T = 0$ member. As the width of the 17.6-MeV, $T=1$ level is only 10.7 ± 0.5 keV,[13] the corresponding α-particle group is narrow and well isolated. The level is relatively weakly excited. No interference effects were observed, and it was assumed that the group shape would be given by the BW formula if experimental widths were negligible.

This particle group was used to determine target stopping and input energy; the excitation energy being taken as 17.638 ± 0.002 MeV.[13] Since the broadening due to target stopping was always larger than the natural level width, an integral over the target thickness of the BW equation was used to fit the particle groups. The integrated Breit-Wigner equation (IBW) has the form

$$\sigma(\theta, Q)=\int_0^\tau \frac{A^2}{(Q-Q_R+t)^2+\frac{1}{4}\Gamma^2}dt =\frac{A^2}{\frac{1}{2}\Gamma}\tan^{-1}\left[\frac{(\Gamma/2)\tau}{(Q-Q_R)(Q-Q_R+\tau)+\frac{1}{4}\Gamma^2}\right]. \quad (5)$$

The integration is made over the target thickness t. The width Γ was fixed at a value of 10.7 keV, and the amplitude A, the resonance Q value Q_R, and the total target stopping τ were varied to determine a χ^2 minimum. The resulting value of Q_R was compared with that based on the assumed excitation energy, and the nominal value used in the computation was adjusted to produce agreement. The total yield for the IBW is given by

$$Y=2\pi A^2\tau/\Gamma. \quad (6)$$

Target stopping in terms of equivalent spread in Q value of the 17.6-MeV group from the $B^{10}(d, \alpha)Be^8$ reaction ranged from 14 to 65 keV. By using the change in stopping with change in α-particle energy, it was possible to calculate an equivalent spread in Q value for the 18.1-MeV group and for an excitation energy of 16.75 MeV (an average value used for both the 16.6- and 16.9-MeV groups). In three cases the 17.6-MeV group was obscured by contaminant groups. In these cases the stoppings were calculated from other runs on the same targets.

Parameters and yields for the 18.1-MeV level could be determined from the simple BW resonance formula because target stopping was a negligible fraction of the natural level width. The yield is expressed by

$$Y=2\pi A^2/\Gamma. \quad (7)$$

Ratios of the differential cross section for formation of the 18.1-MeV level to that for the 17.6-MeV level are shown in Table I. Listed in Table I are the nominal deuteron bombarding energies, the laboratory observation angle, the target stopping, the yield ratios, and the associated errors. The quoted errors were calculated from the errors given by the fitting program for the intensity, target thickness, and level width. The effects on the yield ratio of these errors [found by differentiating Eqs. (6) and (7)] were combined as the square root of the sum of the squares. It should be noted that in this table and the others following, target stopping and level widths are given in terms of the Q values of the $B^{10}(d, \alpha)Be^8$ reaction.

In obtaining a yield ratio at a bombarding energy of 9.0 MeV, the 18.1-MeV level was fitted in the usual manner. The spectrum had an unusual shape, which indicated the possible presence of an unknown contaminant group. As a result, the yield for this level is more uncertain than the listed fitting error indicates.

The yield ratio at a deuteron energy of 4.0 MeV was very difficult to obtain, since the yield to the 18.1-MeV level was small. As a result, this group was hand-fitted, and the ratio quoted can only be considered a reasonable estimate. Also at this lower bombarding energy, the output α-particle energy is just above the Coulomb barrier. The effect on the ratio, however, is less than 10%. A plot and an interpretation of the cross-section ratios will be given below.

It was also possible to obtain an accurate excitation energy and width for the 18.1-MeV level. The Q value and widths are listed in Table II, along with the nominal bombarding energy and observation angle for each of the seven runs used in the average. The average Q value listed corresponds to an excitation energy of 18.146 ± 0.005 MeV. In each case the 17.638-MeV level was used as a standard from which the input energy and target effects were obtained. The uncertainties result from the errors associated with the Q value for the 17.638-MeV level,[13] the target stopping, and instrumental errors.

TABLE I. Differential-cross-section ratios of the 1^+ doublet.

Input energy (MeV)	Onservation angle (deg)	Target stopping ΔQ (keV)	Yield ratios $d\sigma_{18.1}/d\sigma_{17.6}$
12.0	10	64.6	11.1 ± 3.3
11.0	30	37.6	10.3 ± 3.8
10.0	30	39.7	11.8 ± 2.7
9.0	30	43.1	6.9 ± 1.2[a]
8.0	30	47.4	8.2 ± 1.3
6.0	30	26.0	5.5 ± 2.5
4.0	35	20.5	4.75[b]

[a]Bad spectrum for 18.1-MeV level (see text).
[b]Best estimate (see text).

TABLE II. Parameters of the 18.1-MeV level.

Input energy (MeV)	Observation angle (deg)	Q value (MeV)	Level width (keV)
12.0	10	-0.339 ± 0.009	138 ± 17
12.0	30	-0.336 ± 0.013	140 ± 39
10.9	30	-0.324 ± 0.006	112 ± 33
10.0	30	-0.345 ± 0.006	158 ± 20
8.0	30	-0.324 ± 0.003	132 ± 9
6.0	30	-0.326 ± 0.008	155 ± 15
9.0	50	-0.317 ± 0.006	132 ± 13
Weighted mean		-0.327 ± 0.002	138 ± 6

B. 16.6- and 16.9-MeV Levels

The major portion of this work was the analysis of the 2^+ doublet. The unusual asymmetries exhibited in the α-particle spectra posed problems of fitting the group shape and deducing the yield ratios. The only applicable theory available at the start of this investigation was Barker's coupled BW formula, Eq. (1). The effects of target stopping and the presence of contaminants are of unusual importance if one is to accurately fit the group shapes.

A method similar to the one used in obtaining target stopping from the 17.6-MeV group was tried again by integrating the coupled BW expressions [Eq. (1)] over this target thickness (τ). Fits were obtained using the target thickness determined from the 17.6-MeV level and varying the parameters A_1, A_2, Q_1, Q_2, Γ_1, and Γ_2. The resulting level widths (L.W.) were, however, the same as those obtained using the simple expression

$$\text{L.W.} = [(\text{O.W.})^2 - (\text{T.W.})^2]^{1/2}, \quad (8)$$

where O.W. is the observed width found using Eq. (1), and T.W. is the target width. Hence, all level widths were calculated by using Eq. (8) to correct the widths produced by fitting the groups with one or more of Eqs. (1) through (4).

The difficulty posed by the presence of contaminant groups in the region of the 16.6- and 16.9-MeV levels was surmounted by the simple removal of parts of the spectra arising from contaminant groups. Because the line shape was given by the functions used, and the computer was programmed to ignore the deleted points, this posed no serious problem in fitting the data. The two spectra seen in Fig. 1 illustrate the problem. The upper portion of Fig. 1 shows a spectrum with no contaminant problem, whereas the lower portion shows a serious one. In the lower plot of Fig. 1 the contaminant group lies directly "under" the 16.6-MeV group. In this case greater or lesser amounts of the spectrum were deleted, the fitting computation

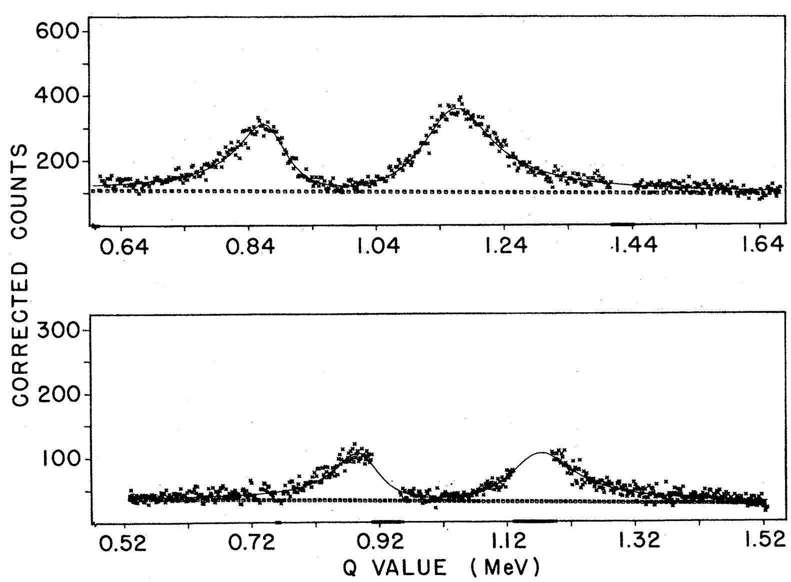

FIG. 1. Plots of the α-particle groups leading to the 16.6- and 16.9-MeV levels of Be^8 which illustrate the procedure used to eliminate contaminant groups. The crosses are the corrected data points and the squares are the background produced by the fitting computation. Where groups of crosses lie along the axis, contaminant groups have been deleted from the spectra by substituting zeros for the data points. The computer is programmed to ignore these points. In the upper plot, contaminant groups have negligible effect. In the lower plot, they obscure major portions of the spectrum. Equation (1) of the text gives the form of the curves shown as the fits. The upper plot comes from a run at 10.0 MeV and 50°, and the lower plot from a run at 6.00 MeV and 30°.

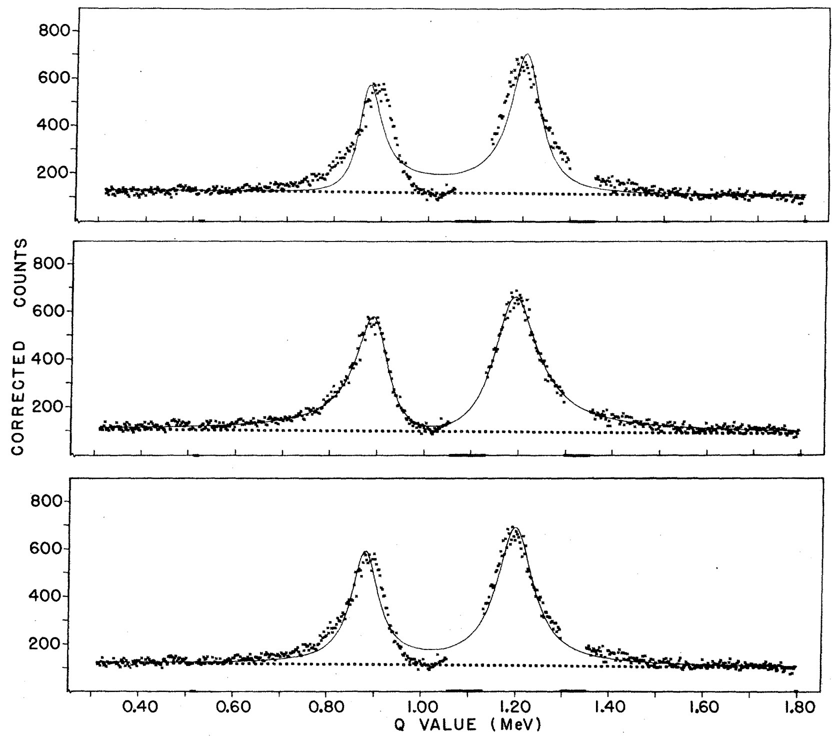

FIG. 2. Fits to the 16.6- and 16.9-MeV group using different forms for the differential cross section. The upper plot shows the best fit obtained with an equation of the form of Eq. (1) of the text but with a minus sign between the resonance terms. The middle plot shows the fit with Eq. (1) ("coupled Breit-Wigner") and the lower plot illustrates the fit with two noninterfering Breit-Wigner terms.

made, and the parameters from the different fits compared. The final acceptable fit was that shown; the removal of more of the spectrum made almost no change in the resulting level parameters. The values of χ^2 per degree of freedom (χ^2/N) for the upper and lower spectra were 1.28 and 1.15, respectively.

All of the spectra were fitted with the coupled BW differential cross section, Eq. (1). The quality of fit for all except the 4.0-MeV data was excellent. Selective fits were also made using the coupled BW equation with a minus sign between the two BW amplitudes, and also with two uncoupled BW resonances. The results can be seen in Fig. 2. The upper plot shows the data fitted with an equation similar to Eq. (1) but with a minus sign between the two terms. This equation would represent the case in which only the $T=1$ part of each level was populated; the fit is not acceptable. The middle plot shows the same data fitted with Eq. (1), which implies that only the $T=0$ parts of the wave function are excited. The fit is excellent and is representative, in quality, of all the fits except the 4.0-MeV data. In the lower plot of Fig. 2, the 2^+ doublet was fitted with two noninterfering BW resonances; the fit is again unacceptable. It might be noted at this point that the only data successfully fitted with the *un*coupled BW resonance were the 4.0-MeV data.

Figure 3 shows the 4.0-MeV data fitted with the *un*coupled BW resonance shape (solid curve) and the coupled BW equation (dashed curve). It can be seen that the uncoupled BW equation results in a more acceptable fit.

The quality of fit was, in general, determined by the value of χ^2/N, but the final judgment was somewhat subjective. It was felt that the region between the two 2^+ resonances was very sensitive to the functional form for the cross section, and the "quality of fit" in this region was weighted very heavily by the authors.

All of the data were also fitted using Barker's two-level cross section, Eq. (2). The quality of fit obtained with the two-level formula was also extremely good but showed, in general, improvement over the coupled BW formula only for the 4-MeV run. For this run the two-level equations yielded essentially the same curve as the *un*coupled BW formula.

It should be noted that Eq. (2) involves the sum over the incoherent contributions to the formation process. The sum over x was first assumed to have only one term, namely, the contribution from the $T=0$ part of each level. These fits were of a good quality, but the region between the two levels was not fitted satisfactorily, since the one contribution implies complete destructive interference in this region. Therefore, the sum was always taken to include two values for x to represent a contribution from both the $T=0$ and $T=1$ part of each level.

The results of the fits using the coupled BW

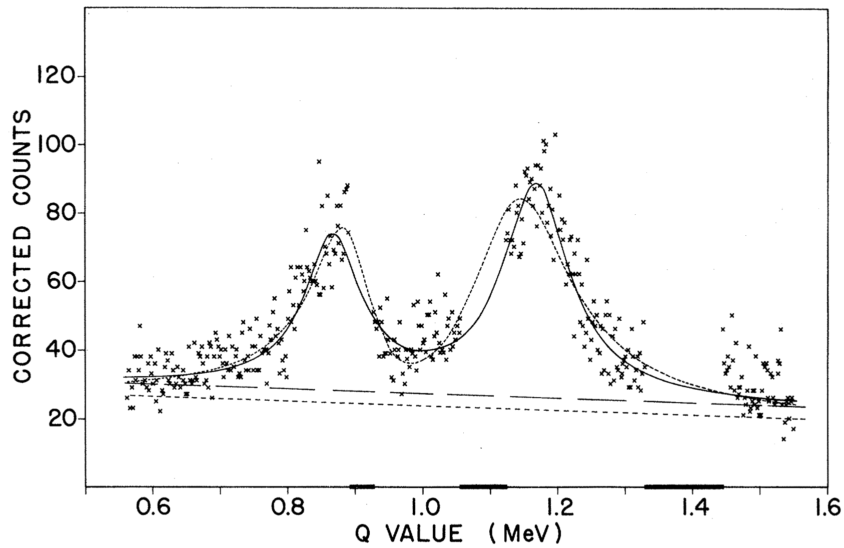

FIG. 3. Spectrum from a run at 4.00 MeV and 35° showing the 16.6- and 16.9-MeV groups. As in Fig. 1, crosses along the abscissa axis represent data points deleted because of contaminant groups. The solid curve is a fit using the uncoupled Breit-Wigner (BW) formula. This curve is also obtained using Eq. (2) of the text (two-level equation). The dashed curve is a fit using Eq. (1) of the text (coupled BW).

cross section and the two-level cross section are listed in Table III. Shown in this table are the nominal input energies (T_I); the observation angles (θ); the level-width parameters (Γ_1, Γ_2) with errors, obtained with the coupled BW equation; the separation energies (ΔQ) obtained with the coupled BW, with associated errors; the level-width parameters (Γ_1, Γ_2), and the separation energies (ΔQ) with their errors, obtained from the two-level cross section. A plot of the separation energies will be presented later in the Discussion Section.

The errors associated with the parameters listed in Table III are strictly fitting errors obtained from the VARIABLE METRIC code and do not indicate the reproducibility, because they come from only a single observation. It is felt, however, that the errors associated with the separation energies are more representative of the true errors than those for the other parameters.

It can be seen from Table III that the level widths obtained with the two forms of the cross section are essentially the same. The discrepancy at a deuteron energy of 4.0 MeV is misleading, since the coupled BW cross section did not give a satisfactory fit. It also should be remembered that the widths and separation energies obtained from Eqs. (1) and (2) represent different quantities, and hence should not necessarily agree with each other.

Table IV lists the intensity factors $A_{\lambda x}$ obtained

TABLE III. Level parameters for the 2^+ doublet obtained with the coupled Breit-Wigner formula, Eq. (1), and two-level formula, Eq. (2).

		Coupled Breit-Wigner			Two-level		
Input energy (MeV)	Observation angle (deg)	Width of 16.6-MeV level (keV)	Width of 16.9-MeV level (keV)	Difference in excitation energy (keV)	Width of 16.6-MeV level (keV)	Width of 16.9-MeV level (keV)	Difference in excitation energy (keV)
12.0	10	114 ± 11	81 ± 15	290 ± 2	120 ± 12	85 ± 12	303 ± 3
12.0	30	109 ± 4	77 ± 3	289 ± 1	111 ± 3	76 ± 3	297 ± 1
12.0	50	106 ± 5	80 ± 6	285 ± 2	107 ± 6	79 ± 6	294 ± 2
12.0	80	107 ± 23	74 ± 13	300 ± 7	109 ± 13	76 ± 8	309 ± 4
12.0	114	120 ± 8	76 ± 10	291 ± 2	114 ± 10	75 ± 11	301 ± 3
11.0	30	106 ± 2	75 ± 4	289 ± 1	106 ± 4	78 ± 3	301 ± 1
10.0	30	115 ± 2	74 ± 3	290 ± 1	115 ± 4	76 ± 4	305 ± 1
10.0	50	129 ± 5	87 ± 6	286 ± 2	127 ± 6	89 ± 8	308 ± 2
9.0	30	112 ± 3	71 ± 4	284 ± 1	110 ± 4	72 ± 4	303 ± 2
9.0	50	113 ± 4	81 ± 6	285 ± 1	113 ± 5	83 ± 6	302 ± 2
8.0	30	104 ± 3	77 ± 4	287 ± 1	105 ± 3	78 ± 4	296 ± 1
6.0	30	122 ± 13	83 ± 8	272 ± 3	119 ± 12	84 ± 8	293 ± 4
5.0	30	119 ± 10	92 ± 7	287 ± 4	113 ± 9	92 ± 8	317 ± 4
4.0	35	183 ± 11	106 ± 12	243 ± 5	121 ± 12	102 ± 9	321 ± 6

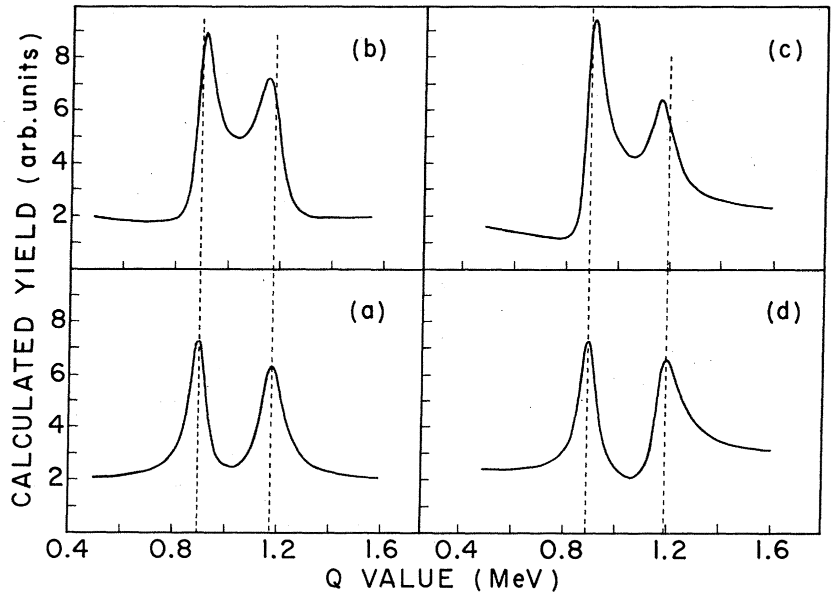

FIG. 4. A few of the shapes that can be reasonably generated from the "three"-level formula, Eq. (3) of the text. The vertical dashed lines indicate the midpoints of the "groups" in (a) and (d).

from the two-level formula, Eq. (2). Listed in Table IV are the nominal input energies, the observation angles, the values of $A_{\lambda x}$, and a quantity R which represents the relative $T=1$ to $T=0$ population.

The ratio, R, is given by

$$R=\frac{H_{11}H_{22}-H_{12}^2}{(H_{11}+H_{12})(H_{12}+H_{22})}, \quad (9)$$

where

$$H_{11}=A_{11}^2+A_{12}^2,$$

$$H_{12}=A_{11}A_{21}+A_{12}A_{22},$$

$$H_{22}=A_{21}^2+A_{22}^2.$$

This factor, which was pointed out by Barker,[9] describes the ratio of population of $T=1$ components to $T=0$ components. It is to be noted that this ratio goes from a few percent at 12 MeV to 99% at 4 MeV. The significance of this will be discussed below.

Finally, the fact that there are possibly other very broad 2^+ levels in Be^8 implies that the "three"-level formula should not be ignored. The "three"-level cross section requires searching on 12 parameters with the VARIABLE METRIC routine. This number of parameters requires a large amount of computer time. It was, therefore, very instructive to simulate results and determine the effects of the different "third-level" parameters

TABLE IV. Intensity factors $A_{\lambda x}$ and the ratio (R) representing the relative $T=1$ to $T=0$ population.

Input energy (MeV)	Observation angle (deg)	A_{11}	A_{21}	A_{12}	A_{22}	R
12.0	10	1.76	1.81	0.270	−0.466	0.079
12.0	30	1.34	0.884	0.180	−0.244	0.040
12.0	50	0.75	0.557	0.191	−0.136	0.071
12.0[a]	80	···	···	···	···	···
12.0[a]	114	···	···	···	···	···
11.0	30	1.29	0.911	−0.311	0.212	0.054
10.0	30	1.45	0.882	0.257	−0.366	0.082
10.0	50	0.956	0.677	0.484	−0.175	0.14
9.0	30	1.27	0.694	−0.196	0.439	0.13
9.0	50	1.04	0.737	0.339	−0.235	0.10
8.0	30	1.41	0.950	0.316	−0.421	0.11
6.0	30	0.489	0.352	0.205	−0.124	0.14
5.0	30	0.473	0.377	0.229	−0.180	0.23
4.0	35	0.366	0.215	0.297	−0.262	0.99

[a] At 12.0 MeV 80°, and 12.0 MeV 114°, reliable fits were not obtained because there were too few tracks. The fits do, however, indicate very little $T=1$ component.

on the line shapes. Shown in Fig. 4 are a few of the artificial spectra that were generated. This figure represents extreme cases, and is only intended to help the reader visualize the effect of a "third" level interfering with the 2^+ states. All the curves in Fig. 4 were generated assuming the level widths of the 16.6- and 16.9-MeV levels to be 110.0 and 78.0 keV, respectively, and their separation to be 290.0 keV. The relative intensities [$A_{\lambda x}$ of Eq. (3)] were the same in all four cases. The parameter [A_x of Eq. (3)] which describes the contribution of a "third level" was varied in order to produce a significant distortion of the spectrum. It is seen that the shapes in Fig. 4(a), 4(b), 4(c), and 4(d) are quite different, and the apparent widths (full width at half maximum) and separations vary. The shapes in Fig. 4(a) and 4(d) most closely represent the majority of data from the present experiment. The curve in 4(a) represents a destructive interference between the 16.6- and 16.9-MeV levels with a small contribution from the "third level," whereas 4(d) depicts the same conditions except with a much larger "third-level" contribution. The apparent width of the 16.6-MeV level was changed approximately 40% and the level separation approximately 10%, simply by changing the contribution of the "third level." The curves in 4(b) and 4(c) again demonstrate the effect of a small and large contribution, respectively, of the "third level." In these cases, however, there is constructive interference in the 16-MeV doublet. Remembering that for all the curves in Fig. 4 the basic level parameters ($A_{\lambda x}$, Γ_λ, and Q_λ) are the same, one sees the possible importance of the "third-level" contribution to the spectral shape and apparent level parameters.

No attempt was made to fit all of the data with Barker's "three"-level cross section. The fits that were obtained did not differ in quality from the two-level cross section. As good fits were already obtained with Eq. (2), one could not determine all the parameters in Eq. (3) from these data.

C. 19-MeV Levels

Analysis in this excitation region was extremely difficult for several reasons. The first reason is an apparently discontinuous background starting at an excitation energy of about 19 MeV. This can be seen in Fig. 5. Figure 5 shows groups leading to the 17.6-, 18.1-, and 18.9-MeV levels and a large group in the region of the 19-MeV levels. These groups are indicated by arrows. The curves drawn through this spectrum are simply to guide the reader's eye.

The nearby presence of the Be^7+n threshold is

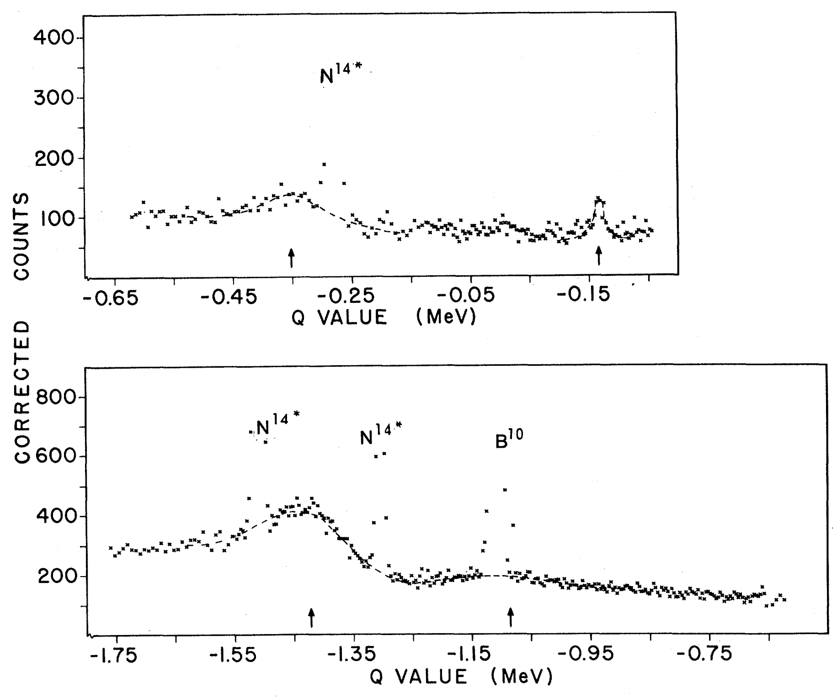

FIG. 5. α-particle spectrum from runs at 11.97 MeV and 30°. The region of excitation covered by the upper curve is from 17.5 to 18.4 MeV, and in the lower curve, from 18.4 to 19.5 MeV. Arrows mark the 17.6- and 18.1-MeV levels in the top plot (1^+ doublet), and they mark the 18.9-MeV level and the region of the assumed 3^+ doublet in the lower plot. Curves are drawn for guidance only. The contaminant groups are labelled with the symbol of the residual nucleus.

the source of the discontinuity observed in the background between Q values of -1.25 and -1.75 MeV. The second difficulty arises from the density of contaminant groups present in the 19-MeV region (Fig. 5 shows a run with a minimal contaminant problem). Thirdly, the wide (≈500-keV) level at 18.9 MeV contributes strongly to this region.

Analysis of this excitation region was initially started on the assumption of two 3^+ levels. Inspection of all the spectra revealed the presence of only one pronounced particle group corresponding to an excitation in Be^8 around 19.2 MeV. No other particle group besides that from the 18.9-MeV level could be detected above background in this region.

Attempts were made to fit this observed group with the single-level BW formula. The resultant level parameters, however, varied over a wide range, depending upon the background initially put in the fitting code. Attempts were made to fit the 18.9-MeV group and then subtract its contribution from the spectrum. The major difficulty, however, was the question of background under the 19.2-MeV level.

If one assumes that there exist two 3^+ levels in this energy region, it is possible to produce a constructive interference between them such that only one apparent group results. In Fig. 6, the dashed curve represents two noninterfering levels: the dot-dash curve, two levels interfering to produce destructive interference [Eq. (1)]; and the solid curve, two levels interfering to produce a constructive interference between the peaks [Eq. (1) with a minus sign between the terms]. The three curves were obtained using the same level parameters. This represents a rather simple explanation of the idea of interfering levels, which deserves a much more detailed theoretical approach.

It can be seen from Fig. 6 that if a sloping background were added to the solid curve, the presence of two levels would go undetected. This type of constructive interference was not used to fit the data, since it is not clear whether one or two levels exist in this region. This ambiguity might be resolved by the use of a charged-particle reaction other than $B^{10}(d, \alpha)Be^8$. Meanwhile, the results in this energy region in Be^8 are inconclusive.

IV. DISCUSSION

The differential-cross-section ratios of the 1^+ doublet at high bombarding energies demonstrate that there is little isobaric spin violation in this region. There appears to be very little isobaric spin mixing in the final state, because in contrast

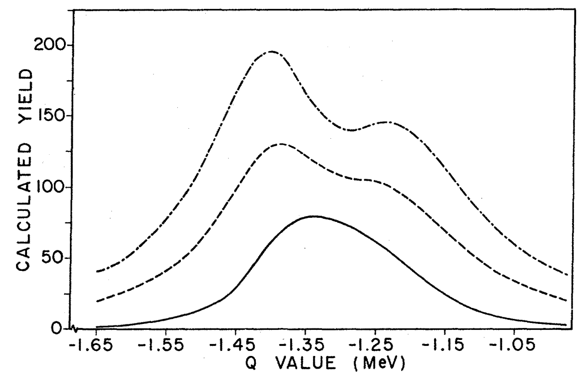

FIG. 6. Different possibilities for interference of two levels using Eq. (1). The dot-dashed curve results from assuming destructive interference between the levels, the dashed curve from assuming no interference, and the solid curve from assuming constructive interference.

to the 2^+ doublet, the α-particle spectrum is symmetric. This may be expected, because the level overlap is small. Between 10 and 12 MeV, the 18.1-MeV state is excited 10 to 12 times as strongly as the 17.6-MeV state. The differential-cross-section ratios, however, do depend upon bombarding energy, as can be seen from Fig. 7, which is a plot of data listed in Table I. This plot shows that as the deuteron bombarding energy is decreased, the ratio of the differential yield of the 18.1-MeV level to that of the 17.6-MeV level decreases. In the discussion to follow, it will be assumed that the yield ratios at the higher energies set an upper limit on the isobaric spin mixing of the two final states, and that the decreasing ratio at lower energies arises from mixing during the reaction.

The results for the 2^+ doublet can be summarized as follows: First, and probably most signif-

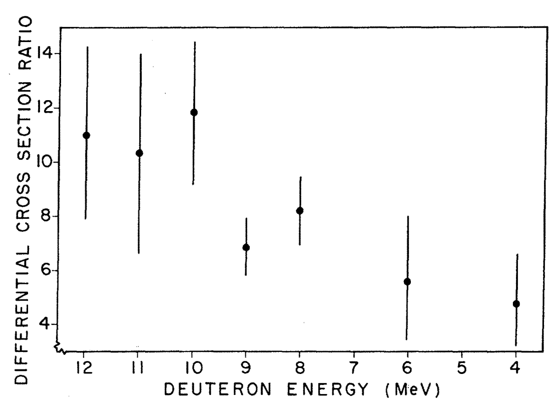

FIG. 7. Ratio of the differential cross section for formation of the 18.1-MeV level to that for formation of the 17.6-MeV level as a function of deuteron bombarding energy. The 12-MeV observation was at 10°; at 4 MeV it was at 35°, and at other energies it was at 30°.

icant, is the change in shape of the α-particle spectrum of this doublet from a strong interference pattern at 12.0 MeV to an apparent noninterference pattern at 4.0 MeV. Second is the fact that the coupled BW equation does not fit the 4-MeV data, but the two-level formula, Eq. (2) does fit both the 4-MeV and higher-energy data. An attempt was made to generalize Eq. (1) over $T=0$ and $T=1$ components, but the effort was abandoned. Third, as can be seen from Table III, the level parameters (Γ_1, Γ_2, ΔQ) obtained from the two-level equation vary somewhat as the input energy and observation angle varies. The variations in separation energies (ΔQ) are the most pronounced. Figure 8 is a plot of these separation energies versus deuteron energy.

An attempt was made to see if an average value for the two widths and the separation energy would give satisfactory fits to all the data from 12 to 6 MeV. The average values taken from Table III for the two-level formula were $\Gamma_{16.6}=113$ keV, $\Gamma_{16.9}=80$ keV, and $\Delta Q=302$ keV. It was concluded that the quality of fits as determined by χ^2/N was essentially the same as that when the parameters Γ_1, Γ_2, and ΔQ were varied. A careful inspection of the curves, however led one to judge that the fits with the "two-level" parameters given in Tables III and IV were superior. This judgement of fit was hindered where contaminant groups obscured a crucial part of the spectrum. Good fits at 4- and 5-MeV bombarding energy require ΔQ to be somewhat larger than the average value.

We shall now consider simultaneously the results obtained for the 1^+ and 2^+ doublets. One can symbolically write the wave functions for these doublets as:

$$\psi^{16.6} \rightarrow 2^+[\alpha(T=0)+\beta(T=1)],$$
$$\psi^{16.9} \rightarrow 2^+[\beta(T=0)-\alpha(T=1)],$$
$$\psi^{17.6} \rightarrow 1^+[\delta(T=0)+\gamma(T=1)],$$
$$\psi^{18.1} \rightarrow 1^+[\gamma(T=0)-\delta(T=1)],$$

and note that the ratio[7] δ^2/γ^2 is about 0.06, whereas α^2/β^2 is about unity. It is then possible to interpret most of the results for the yield ratios, the shape of the 2^+ doublet, and the formula giving the best fit in terms of *reaction* isobaric spin purity.

If we assume that the *reaction* completely conserves isobaric spin, then the 2^+ doublet should have almost equal yields, whereas the 1^+ doublet will have very different yields. Also the 2^+ doublet will show a pronounced interference pattern in the α-particle spectrum. If, as the deuteron bombarding energy is lowered, compound nuclear effects introduce isobaric spin violations into the

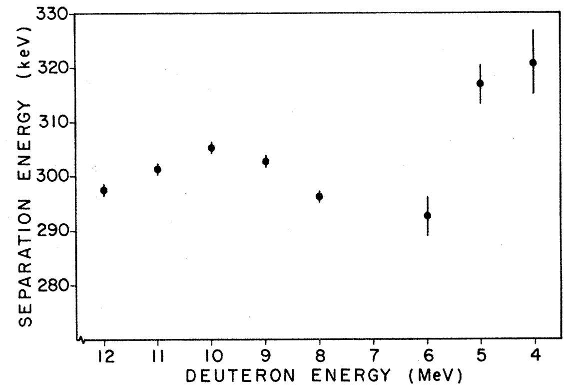

FIG. 8. Separation energy (ΔQ) of the 2^+ doublet as a function of bombarding energy. At 4 MeV the observation angle was 35°, and at all other energies it was 30°. The values plotted are those obtained from fitting the data with the text Eq. (2) and are listed in Table III under "two-level" formula. Error bars show uncertainties arising only from the fitting procedure.

reaction, then the yield ratio of the 1^+ doublet will decrease, whereas the equal population of the 2^+ doublet will remain. The shape of the α-particle spectrum, however, will show a "decrease" in the interference pattern, since more contributions from the $T=1$ parts are present.

Since the coupled BW formula, Eq. (1), assumes that only the $T=0$ part of each level is formed, it is not surprising that the two-level formula, Eq. (2), is required to fit all the data. The two-level formula does not impose the restriction of isobaric spin purity *in the reaction*, as does Eq. (1).

Consider, therefore, that the two-level formula does describe the $B^{10}(d,\alpha)Be^8$ reaction, and consider the fact that if we assume population of only the $T=0$ component of the 2^+ doublet, then the observed α-particle spectrum will have a destructive interference between the groups. If some $T=1$ part of each 2^+ member is populated, then the shape of the spectrum will be different, depending upon the relative amount of $T=1$. If the $T=0$ and $T=1$ components of each level are almost equally populated, then the observed spectrum will show little interference. The shape will not in general be represented by simple BW expressions and the level parameters obtained from the BW formula will not be correct.

It is felt that this is the correct interpretation of the results. At 12.0-MeV deuteron bombarding energy, the data cannot be fitted with acceptable quality with the two-level cross section, Eq (2), having only $T=0$ formation contributions, but small amounts (4 to 8%) of $T=1$ are needed. As the deuteron energy is decreased, the $T=1$ contributions become more significant, as seen in Table IV. In fact, at 4.0-MeV deuteron energy, the $T=1$ population is 99% of the $T=0$ population. Therefore, at 4.0 MeV the apparent interference disappears.

The point now arises as to the explanation of the possible variation of level parameters observed as bombarding energy and observation angle were changed. One would like a functional relation for the cross section that would eliminate the observed variations and extract the true level paremeters. This relation must include all other 2^+ levels, and allow for the population of the $T=0$ and $T=1$ parts of the levels as a function of bombarding energy and observation angle.

Barker has suggested such a cross section; however, there exists an ambiguity of the "interfering background," caused by the presence of other 2^+ levels, being present with a "natural"-type background resulting from multiparticle breakup arising from deuterons incident on B^{10}.

The conclusions that can be drawn at this point are that the two-level formula probably does not yield the true level parameters associated with the 2^+ levels, but the two-level cross section does indicate the change of isobaric spin purity in the $B^{10}(d, \alpha)Be^8$ reaction as the reaction parameters are varied. Whether it is possbile to obtain the true level parameters from other reactions, such as $Li^6(He^3, p)Be^8$ and $Li^7(He^3, d)Be^8$, is questionable. Isobaric spin may not enter, but the presence of other levels still presents a problem.

A clear example of this effect can be seen by comparing the separation energies obtained from this present work with that obtained by Marion[14] using the $Li^7(He^3, d)Be^8$ reaction and neglecting interference. The separation energy for the 2^+ states obtained from the (He^3, d) reaction is 274 ± 3 keV, whereas the values obtained from the (d, α) reaction are all over 290 keV. Barker[9] has reported privately that at a He^3 energy of 15 MeV, a distinct interference pattern was seen with this reaction in work done at The Australian National University.

If there does exist a 3^+ doublet, the (d, α) reaction is an extremely poor tool with which to investigate it, since only one α-particle group was seen in all of the data. There are several possible explanations. The first is that these two states do not have the same spin and parity, and that the lower level (19.05 MeV) has a pure $T=1$ character. This is probably the most reasonable conclusion. The second possibility is that the states do have the same spin and parity, but that their isobaric spins are not mixed. This seems highly unlikely in view of the strong mixing in the 2^+ doublet. Finally, it would be possible to have the same spin and parity, and interference such that the resulting spectrum would appear as only one group (see Fig. 6). It is, therefore, important to investigate this energy region in order to determine the relative spins and parities. Such investigations should be carried out with reactions other than the $B^{10}(d, \alpha)Be^8$ reaction. Investigations in this region are planned using the reactions $Li^7(He^3, d)Be^8$ and $Li^6(He^3, p)Be^8$.

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