Soft-photon theorem for the electromagnetic interaction of the relativistic two-nucleon system

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The low-energy theorem for emission of soft photons from two nucleons is derived in a three-dimensional relativistic theory. It is shown that if the free nucleon currents are defined in terms of the Foldy-Wouthuysen transformation, the exchange current in the soft-photon limit is exactly determined by the commutator of the nucleon-nucleon interaction with the electric dipole operator. The constraints imposed by this theorem are used to examine the conventional method of calculating the pair-excitation currents.

[NUCLEAR REACTIONS Low-energy theorem for the exchange current, relativistic effects.]

I. INTRODUCTION

The problem of the electromagnetic (em) interaction of nuclei is an old and yet a difficult one because we are concerned with the effect of the strong interaction, namely, the exchange current. Therefore Siegert's theorem,[1] which states that the electric multipole operators do not depend on the interaction between nucleons, is of special importance to phenomenology. Siegert's assumption is that the charge density is left unmodified by meson exchange between nucleons. It is well known that if this statement is valid, the exchange current in the soft-photon limit can be expressed in terms of the commutator of the nucleon-nucleon interaction with the electric dipole operator. In field theory, however, there exists the exchange charge density. Recently Hyuga and Ohtsubo[2] have found a nonvanishing two-body density even for static nucleons, which, as they argue, leads to the breakdown of Siegert's theorem.

On the other hand, in a more general context, Low[3] has shown that when the radiative amplitude is expanded in powers of k (energy of the radiated photon),

$$\epsilon_\mu^* M_\mu(k) = a/k + b + ck + O(k^2)\,, \tag{1.1}$$

where ϵ_μ^* is the polarization of the photon, the first two terms are calculated exactly in terms of the corresponding elastic amplitude and the em constants of the constituent particles. This result, known as Low's low-energy theorem, tells us that the exchange current contribution at $k=0$ is unambiguously defined whatever the exchange charge density.

The proof of the low-energy theorem (LET) for NN bremsstrahlung in a potential model was given, first by Feshbach and Yennie[4] and later by Heller,[5] and Liou and Sobel.[6-8] It is to be desired that LET is derived from the viewpoint of relativistic field theory. Ohtsubo, Fujita, and Takeda[9] obtain the exchange current from LET derived from field theory, but approximately for static nucleons. The purpose of this article is to show that the exchange current at $k=0$ is exactly determined by the NN interaction constructed from underlying field theory.

II. CURRENT CONSERVATION AND LOW-ENERGY THEOREM

A. Preliminaries

The Bethe-Salpeter equation for two nucleons mediated by quantum fields can be reduced to the relativistic three-dimensional equation.[10,11] The single-time bound state wave function projected onto the positive-energy states satisfies a Schrödinger-type equation with the Hamiltonian[12]

$$H = H_0 + V\,, \tag{2.1}$$

$$H_0 = (\vec{p}_1^{\,2} + m^2)^{1/2} + (\vec{p}_2^{\,2} + m^2)^{1/2}\,, \tag{2.2}$$

where $\vec{p}_i$ is the momentum of the ith nucleon and m is the nucleon mass. The NN interaction V must of necessity be nonlocal and dependent on the energy of the NN bound state.

When an external em field is acting, the matrix elements of the em current between the bound states can be expressed in terms of one-body and two-body vertex functions.[12,13] Translating these vertex functions into equivalent current operators in configuration space, we obtain the total current density

$$\vec{J}(\vec{x}) = \vec{J}^{(1)}(\vec{x}) + \vec{J}^{(2)}(\vec{x})\,, \tag{2.3}$$

and the total charge density

$$\rho(\vec{x}) = \rho^{(1)}(\vec{x}) + \rho^{(2)}(\vec{x}) . \quad (2.4)$$

From the equation of charge conservation[14]

$$\vec{\nabla}\cdot\vec{J}(\vec{x}) = ik\rho(\vec{x}) = -i[H, \rho(\vec{x})] , \quad (2.5)$$

it follows that

$$\vec{\nabla}\cdot\vec{J}^{(1)}(\vec{x}) = -i[H_0, \rho^{(1)}(\vec{x})] \quad (2.6)$$

and

$$\vec{\nabla}\cdot\vec{J}^{(2)}(\vec{x}) = -i[V, \rho^{(1)}(\vec{x})] + ik\rho^{(2)}(\vec{x}) . \quad (2.7)$$

After the argument of Dalitz,[15] we can easily derive from (2.7) the expression for $\vec{J}^{(2)}(0) \equiv \vec{J}^{(2)}(\vec{k})|_{k=0}$ as follows[16]: Using Green's theorem, it is seen that

$$\vec{J}^{(2)}(0) = \int d^3x \vec{J}^{(2)}(\vec{x}) = -\int d^3x \vec{x}\vec{\nabla}\cdot\vec{J}^{(2)}(\vec{x}) . \quad (2.8)$$

Insertion of (2.7) into (2.8) yields

$$\vec{J}^{(2)}(0) = i\left[V, \int d^3x\, \vec{x}\rho^{(1)}(\vec{x})\right] . \quad (2.9)$$

Note that the two-body charge density term in (2.7) does not contribute to $\vec{J}^{(2)}(0)$ because we have put $k=0$.

If we assume pointlike nucleons and make a replacement

$$\rho^{(1)}(\vec{x}) \to \sum_{i=1}^{2} e_i \delta(\vec{x} - \vec{r}_i) , \quad (2.10)$$

where e_i and $\vec{r}_i$ are the charge and position operators, we get

$$\vec{J}^{(2)}(0) \to i\left[V, \sum_{i=1}^{2} e_i \vec{r}_i\right] , \quad (2.11)$$

which has been derived by Dalitz[15] in a nonrelativistic potential theory. Going to relativistic theory, we cannot make the replacement (2.10).

According to Liou and Sobel,[17] however, Eq. (2.11) is valid even when relativistic corrections of $O(m^{-2})$ are included. Superficially there seems to be a contradiction between the formalism developed in this subsection and that of Liou and Sobel.[8]

B. Definition of the exchange current

In order to solve the problem presented above, we examine the content of the one-body current density $\vec{J}^{(1)}(\vec{x})$. This contains the convection current $\vec{J}_i^{\text{conv}}(\vec{x})$ which is obtained from the kinetic operator $(\vec{p}_i^{\,2} + m^2)^{1/2}$ by the standard minimal substitution and obeys the equation

$$\vec{\nabla}\cdot\vec{J}_i^{\text{conv}}(\vec{x}) = -i\,[(\vec{p}_i^{\,2} + m^2)^{1/2} , \, e_i\delta(\vec{x} - \vec{r}_i)] , \quad (2.12)$$

i.e., $\vec{J}_i^{\text{conv}}(\vec{x})$ is brought about by the conduction of the pointlike nucleon charge. Actually, in addition to the point charge and the convection current, a charge density $\rho_i^{\text{rel}}(\vec{x})$ and a current flow $\vec{J}_i^{\text{rel}}(\vec{x})$ appear under the constraint

$$\vec{\nabla}\cdot\vec{J}_i^{\text{rel}}(\vec{x}) = ik\rho_i^{\text{rel}}(\vec{x}) , \quad (2.13)$$

whether the interaction between nucleons is present or not. The possible form of the four-current $J_{\mu i}^{\text{rel}}(\vec{x}) = (\vec{J}_i^{\text{rel}}(\vec{x}), i\rho_i^{\text{rel}}(\vec{x}))$, restricted by Eq. (2.13), is

$$\rho_i^{\text{rel}}(\vec{x}) = -\vec{\nabla}\cdot\vec{s}_i^{(1)}(\vec{x}) , \quad (2.14)$$

$$\vec{J}_i^{\text{rel}}(\vec{x}) = -ik\vec{s}_i^{(1)}(\vec{x}) + \vec{\nabla}\times\vec{\mu}_i^{(1)}(\vec{x}) , \quad (2.15)$$

where $\vec{s}_i^{(1)}(\vec{x})$ is any one-body operator and $\vec{\mu}_i^{(1)}(\vec{x})$ is a one-body magnetization density. It should be noted that from (2.14)

$$\int d^3x\, \rho_i^{\text{rel}}(\vec{x}) = 0 , \quad (2.16)$$

as is required. The explicit form of $\vec{s}_i^{(1)}(\vec{x})$ and $\vec{\mu}_i^{(1)}(\vec{x})$ is found from the Dirac equation for a nucleon in an external em field and to order m^{-2} it is given by Foldy and Wouthuysen (FW).[18] Complications due to the Wigner spin rotation[19] are simply taken into account by a modification of $\vec{s}_i^{(1)}(\vec{x})$ and do not alter our argument that follows.

Thus $\rho^{(1)}(\vec{x})$ and $\vec{J}^{(1)}(\vec{x})$ defined in the preceding subsection are explicitly written as

$$\rho^{(1)}(\vec{x}) = \sum_{i=1}^{2} [e_i\delta(\vec{x} - \vec{r}_i) + \rho_i^{\text{rel}}(\vec{x})] , \quad (2.17)$$

$$\vec{J}^{(1)}(\vec{x}) = \sum_{i=1}^{2} [\vec{J}_i^{\text{conv}}(\vec{x}) + \vec{J}_i^{\text{rel}}(\vec{x})] - \delta\vec{J}(\vec{x}) . \quad (2.18)$$

The reason for subtracting $\delta\vec{J}(\vec{x})$ from the one-body current is that, as was pointed out by Stichel and Werner,[20] $\vec{J}_i^{\text{rel}}(\vec{x})$ includes the two-body current

$$\delta\vec{J}(\vec{x}) = i\left[V, \sum_{i=1}^{2} \vec{s}_i^{(1)}(\vec{x})\right] , \quad (2.19)$$

because in Eq. (2.15),

$$-ik\vec{s}_i^{(1)}(\vec{x}) = i[H_0 + V, \vec{s}_i^{(1)}(\vec{x})] . \quad (2.20)$$

One can explicitly check that (2.17) and (2.18) satisfy Eq. (2.6).

However, the definition (2.18) of the one-body current is undesirable because it is different from the usual one in terms of the sum of the FW interactions of the constituents,

$$\vec{J}^{\text{I}}(\vec{x}) = \sum_{i=1}^{2} [\vec{J}_i^{\text{conv}}(\vec{x}) + \vec{J}_i^{\text{rel}}(\vec{x})] . \quad (2.21)$$

This form effectively contains a two-body part through (2.20), but we usually identify this with a one-body current. We accordingly define the two-body current $\vec{J}^{II}(\vec{x})$ as

$$\vec{J}(\vec{x})=\vec{J}^{I}(\vec{x})+\vec{J}^{II}(\vec{x})\,, \tag{2.22}$$

instead of $\vec{J}^{(2)}(\vec{x})$ in Eq. (2.3).

With Eqs. (2.12), (2.13), and (2.17) in mind, we have

$$\vec{\nabla}\cdot\vec{J}^{I}(\vec{x})=i\left[V,\ \sum_{i=1}^{2} e_i\delta(\vec{x}-\vec{r}_i)\right]+ik\rho^{(1)}(\vec{x})\,. \tag{2.23}$$

Therefore the continuity equation for $\vec{J}(\vec{x})$ implies

$$\vec{\nabla}\cdot\vec{J}^{II}(\vec{x})=-i\left[V,\ \sum_{i=1}^{2} e_i\delta(\vec{x}-\vec{r}_i)\right]+ik\rho^{(2)}(\vec{x})\,. \tag{2.24}$$

Comparing this equation with (2.7), we notice that $\rho^{(1)}(\vec{x})$ in (2.7) changes places with the point charge density. Thus Dalitz's assumption (2.10) is justified even in relativistic theory and the proof of LET by Liou and Sobel[8] is consistent with current conservation as they show explicitly.

C. Low-energy theorem

Equation (2.24) indicates that the exchange current $\vec{J}^{II}(\vec{x})$ consists of two parts, the interaction current $\vec{J}^{int}(\vec{x})$, where its divergence is given by

$$\vec{\nabla}\cdot\vec{J}^{int}=-i\left[V,\ \sum_{i=1}^{2} e_i\delta(\vec{x}-\vec{r}_i)\right], \tag{2.25}$$

and the genuine exchange current $\vec{J}^{ex}(\vec{x})$ which with $\rho^{(2)}(\vec{x})$ forms a conserved four-current $\vec{J}^{ex}_{\mu}(\vec{x})=(\vec{J}^{ex}(\vec{x}),\ i\rho^{(2)}(\vec{x}))$ and satisfies

$$\vec{\nabla}\cdot\vec{J}^{ex}(\vec{x})=ik\rho^{(2)}(\vec{x})\,. \tag{2.26}$$

The proof of Siegert's theorem in potential theory[21] was essentially based on (2.25) as clearly explained by Dalitz.[15] In the same manner as we derive $\vec{J}^{(2)}(0)$, we obtain

$$\vec{J}^{int}(0)=i\left[V,\ \sum_{i=1}^{2} e_i\vec{r}_i\right] \tag{2.27}$$

without any approximations.

As for the divergence-free current $J^{ex}_{\mu}(\vec{x})$, we cannot evaluate it without physical input, but charge conservation

$$\int d^3x\,\rho^{(2)}(\vec{x})=\rho^{(2)}(0)=0 \tag{2.28}$$

puts a restriction on its form. Equation (2.28) implies that $\rho^{(2)}(\vec{k})$ in $\vec{k}$ expansion starts with the linear term in $\vec{k}$. In other words

$$\rho^{(2)}(\vec{x})=-\vec{\nabla}\cdot\vec{s}^{(2)}(\vec{x}) \tag{2.29}$$

with $\vec{s}^{(2)}(\vec{x})$ an arbitrary two-body operator. The continuity equation (2.26) is satisfied only if

$$\vec{J}^{ex}(\vec{x})=-ik\,\vec{s}^{(2)}(\vec{x})+\vec{\nabla}\times\vec{\mu}^{(2)}(\vec{x})\,, \tag{2.30}$$

where $\vec{\mu}^{(2)}(\vec{x})$ is the two-body magnetization density. From Eq. (2.30), we find $\vec{J}^{ex}(\vec{k})=O(k)$ so that for the total exchange current

$$\vec{J}^{II}(\vec{x})=\vec{J}^{int}(\vec{x})+\vec{J}^{ex}(\vec{x})\,, \tag{2.31}$$

we have LET,

$$\vec{J}^{II}(\vec{k})=J^{int}(0)+O(k)\,, \tag{2.32}$$

while for the charge density,

$$\rho^{(2)}(\vec{k})=O(k)\,. \tag{2.33}$$

Namely, in the soft-photon limit, the exchange current is exactly determined by V and the exchange charge density must vanish.

III. LOW ENERGY THEOREM FOR *NN* BREMSSTRAHLUNG

Since LET (2.32) and (2.33) follows from current conservation alone, it must be valid for scattering problems as well as bound state ones. Here we prove LET for *NN* bremsstrahlung in field theory along the line of Low's prescription.

Field theory translated into the three-dimensional relativistic equation differs from potential theory in that the quasipotential V depends on energy. Therefore the proof of LET proceeds in the same manner as in potential theory if we pay due attention to the energy dependence of V. The method we employ is similar to Ref. 6, but m^{-1} expansion is not used.

The radiative amplitude M_μ in Eq. (1.1) is given by the matrix element of the current operator J_μ,

$$M_\mu(k)=\langle\psi'(E')\,|J_\mu|\,\psi(E)\rangle\,, \tag{3.1}$$

where $\psi(E)$ and $\psi'(E')$ are the initial and final wave functions, respectively, with energy E and $E'=E-k$. Using the *NN* transition operator determined by the Lippmann-Schwinger equation,

$$t(E)=V(E)+V(E)G_0(E)t(E)\,, \tag{3.2}$$

with $G_0(E)=(E+i\epsilon-H_0)^{-1}$, $\psi(E)$ is related to the free two-nucleon state $\psi_0(E)$ through

$$\psi(E)=[1+G_0(E)t(E)]\,\psi_0(E)\,. \tag{3.3}$$

We decompose J_μ into one-body and two-body parts, $J^{I}_{\mu}=(\vec{J}^{I},\ i\rho^{(1)})$ and $J^{II}_{\mu}=(\vec{J}^{II},\ i\rho^{(2)})$ as we did in Sec. II, and write (3.1) in the form of the matrix element between unperturbed states $\psi_0(E)$ and $\psi_0'(E')$,

$$M_\mu(k)=\langle\psi_0'\,|J^{I}_{\mu}+T^{E}_{\mu}+T^{I}_{\mu}\,|\psi_0\rangle\,. \tag{3.4}$$

Hereafter we suppress the energy dependence of wave functions for brevity. The transition operator T^{E}_{μ} is the contribution from the diagrams in which the photon is emitted before or after the interaction,

$$T^E_\mu = t(E')G_0(E')J^I_\mu + J^I_\mu G_0(E)t(E). \tag{3.5}$$

The last term in Eq. (3.4), which comes from internal photoemission, is

$$T^I_\mu = T^R_\mu + T^G_\mu, \tag{3.6}$$

where

$$T^R_\mu = t(E')G_0(E')J^I_\mu G_0(E)t(E), \tag{3.7}$$

$$T^G_\mu = [1 + t(E')G_0(E')]J^{II}_\mu[1 + G_0(E)t(E)]. \tag{3.8}$$

From its definition the four-divergence of J^I_μ is

$$\partial_\mu J^I_\mu(\vec{x}) = -i\left[H_0, \sum_{i=1}^{2} e_i\delta(\vec{x} - \vec{r}_i)\right] -ik\sum_{i=1}^{2} e_i\delta(\vec{x} - \vec{r}_i), \tag{3.9}$$

with $\partial_\mu = (\vec{\nabla}, -k)$. In the $\vec{k}$ space,

$$k_\mu J^I_\mu(\vec{k}) = -[H_0, Q_0] - kQ_0, \tag{3.10}$$

$$Q_0 = \sum_{i=1}^{2} e_i e^{-i\vec{k}\cdot\vec{r}_i}, \tag{3.11}$$

and $k_\mu = (\vec{k}, ik)$. We cast (3.10) in the form

$$k_\mu J^I_\mu(\vec{k}) = G_0^{-1}(E')Q_0 - Q_0G_0^{-1}(E), \tag{3.12}$$

which will be found useful in the following.

First note that owing to (3.12),

$$k_\mu\langle\psi_0'|J^I_\mu|\psi_0\rangle = 0. \tag{3.13}$$

We next calculate the divergence of T^E_μ by using (3.12) and find

$$k_\mu\langle\psi_0'|T^E_\mu|\psi_0\rangle = \langle\psi_0'|t(E')Q_0 - Q_0t(E)|\psi_0\rangle. \tag{3.14}$$

Finally, the divergence of T^R_μ becomes

$$k_\mu\langle\psi_0'|T^R_\mu|\psi_0\rangle = \langle\psi_0'|t(E')G_0(E')[G_0^{-1}(E')Q_0 - Q_0G_0^{-1}(E)] \times G_0(E)t(E)|\psi_0\rangle. \tag{3.15}$$

We now apply to Eq. (3.15) the operator identity derived by Liou,[6]

$$t(E')G_0(E')[G_0^{-1}(E')Q_0 - Q_0G_0^{-1}(E)]G_0(E)t(E) = -t(E')Q_0 + Q_0t(E) + [1 + t(E')G_0(E')] \times [V(E')Q_0 - Q_0V(E)][1 + G_0(E)t(E)], \tag{3.16}$$

and collect (3.13), (3.14), and (3.15) to get

$$k_\mu\langle\psi_0'|(J^I_\mu + T^E_\mu + T^R_\mu)|\psi_0\rangle = \langle\psi'|[V(E')Q_0 - Q_0V(E)]|\psi\rangle. \tag{3.17}$$

Since the total current must be conserved, $k_\mu M_\mu(k) = 0$ or

$$k_\mu\langle\psi_0'|T^G_\mu|\psi_0\rangle = -\langle\psi'|[V(E')Q_0 - Q_0V(E)]|\psi\rangle. \tag{3.18}$$

Recalling that from (3.8),

$$\langle\psi_0'|T^G_\mu|\psi_0\rangle = \langle\psi'|J^{II}_\mu|\psi\rangle, \tag{3.19}$$

we have

$$k_\mu J^{II}_\mu(\vec{k}) = -V(\overline{E} - \tfrac{1}{2}k)Q_0 + Q_0V(\overline{E} + \tfrac{1}{2}k), \tag{3.20}$$

where $\overline{E} = (E + E')/2$. In the $\vec{x}$ space,

$$\vec{\nabla}\cdot\vec{J}^{II}(\vec{x}) = -iV(\overline{E} - \tfrac{1}{2}k)\sum_{i=1}^{2} e_i\delta(\vec{x} - \vec{r}_i) + i\sum_{i=1}^{2} e_i\delta(\vec{x} - \tfrac{1}{2}\vec{r}_i)V(\overline{E} + \tfrac{1}{2}k) + ik\rho^{(2)}(\vec{x}). \tag{3.21}$$

It is interesting to compare (3.21) with (2.24). Since we suppressed the energy dependence of V in Sec. II, the meaning of (2.24) was ambiguous whether E or E' enters V. Equation (3.21) now puts a correct interpretation on (2.24).

Differentiating both sides of (3.20) with respect to k_μ leads to

$$\vec{J}^{II}(\vec{k}) = i\left[V, \sum_{i=1}^{2} e_i\vec{r}_i\right] + O(k) \tag{3.22}$$

and

$$\rho^{(2)}(\vec{k}) = -\tfrac{1}{2}\{V', e_1 + e_2\} + O(k), \tag{3.23}$$

with $V' = \partial V(E)/\partial E$. The energy dependence of V thus produces the charge

$$Q^{(2)} = (e_1 + e_2)Z, \tag{3.24}$$

where $Z = -V'$, and we have used the fact that V or Z commutes with the total charge $e_1 + e_2$. Friar[22] has calculated the one-pion-exchange contribution to the nuclear charge and found Eq. (3.24). He has shown that the undesirable charge $Q^{(2)}$ cancels from the final result because the energy dependence of V necessitates the wave-function renormalization which modifies the total charge

$$e_1 + e_2 \to (e_1 + e_2)(1 - Z). \tag{3.25}$$

The Z term in (3.25) completely eliminates $Q^{(2)}$. Consequently, when we define the exchange charge density operator as the one inserted between the renormalized wave functions rather than the solutions of the Lippmann-Schwinger equation (3.3), we obtain

$$\rho^{(2)}(\vec{k}) = O(k) \tag{3.26}$$

in place of (3.23). In this way, LET (2.32) and (2.33) is again derived.

IV. ELECTRIC AND MAGNETIC DIPOLE OPERATORS

Now we consider some consequences of LET for radiative transitions. As discussed by Blatt and

Weisskopf,[23] the long wavelength reduction of the electric dipole operator $\vec{E}_1(k)$ leads to

$$\vec{E}_1(k)=\frac{i}{\sqrt{6\pi}}\vec{J}(0)+O(k^2)\,, \tag{4.1}$$

where

$$\vec{J}(0)=\int d^3x\,\vec{J}(\vec{x})=i\left[H,\ \int d^3x\,\vec{x}\rho(\vec{x})\right] \tag{4.2}$$

is the total current present in the whole space and proportional to k. Substituting Eqs. (2.17) and (2.29) into (4.2) gives

$$\vec{J}(0)=\sum_{i=1}^{2}\vec{J}_i^{\text{conv}}(0)+\vec{J}^{\text{int}}(0)+\vec{J}'\,, \tag{4.3}$$

where

$$\vec{J}'=-ik\sum_{i=1}^{2}\vec{s}_i^{(1)}(0)-ik\vec{s}^{(2)}(0) \tag{4.4}$$

represents the deviation from Siegert's prediction. In Eq. (4.3) the convection and interaction currents almost cancel each other so that there remains the quantity of order k which cannot be discriminated from the model-dependent $\vec{J}'$. In this sense Siegert's theorem is violated. However, we have seen that $\vec{J}^{\text{int}}(\vec{k})$ itself is finite at $k=0$ and quite distinct from $\vec{J}'$. When we evaluate the exchange current by adopting a strong interaction model, we can see whether the calculation preserves gauge invariance or not, by taking the soft-photon limit.

As another example of the consequences of LET, consider the magnetic dipole operator defined by

$$\vec{\mu}=\tfrac{1}{2}\int d^3x\,\vec{x}\times\vec{J}(\vec{x})\,. \tag{4.5}$$

Bringing together all the formulas for $\vec{J}^{\text{I}}(\vec{x})$ and $\vec{J}^{\text{II}}(\vec{x})$ in Sec. II, we can write

$$\vec{\mu}=\sum_{i=1}^{2}\left[\tfrac{1}{2}\vec{r}_i\times\vec{J}_i^{\text{conv}}(0)+\vec{\mu}_i^{(1)}(0)\right]+\tfrac{1}{2}\vec{R}\times\vec{J}^{\text{int}}(0)+\vec{\mu}^{(2)}(0)\,. \tag{4.6}$$

The first term in the square brackets is the orbital magnetic moment, while $\frac{1}{2}\vec{R}\times\vec{J}^{\text{int}}(0)$ with $\vec{R}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$ is the Sachs exchange magnetic moment.[24,15] The Sachs moment is now completely canceled by a part of the orbital moment, namely, by

$$\tfrac{1}{2}\vec{R}\times\sum_{i=1}^{2}\vec{J}_i^{\text{conv}}(0)\,,$$

due to LET. Eventually the $\vec{R}$ dependent terms in $\vec{\mu}$ disappear entirely.

It may be summarized that both of the electric and magnetic dipole operators cannot be uniquely determined by the requirement of gauge invariance alone, but LET imposes restrictions on their forms in a model-independent way.

V. PAIR EXCITATION CURRENTS

As an illustration of the usefulness of LET for calculating exchange currents in a field theoretic way, let us consider the ω-meson exchange.

In the standard treatment of exchange effects, we calculate the pair excitation diagrams. If we take the effective Lagrangian for $\omega N\bar{N}$ coupling,

$$\mathcal{L}_{\omega NN}=ig\bar{N}\gamma_\mu N\phi_\mu\,, \tag{5.1}$$

with N as the nucleon field and ϕ_μ as the meson field, the effective Hamiltonian for the $\omega N\bar{N}$ interaction is given by the FW transformation to $O(m^{-2})$,

$$H_\omega^{\text{FW}}=g\phi_0-\frac{g}{m}\vec{\phi}\cdot\vec{p}-\frac{g}{2m}\vec{\sigma}\times\vec{\nabla}\cdot\vec{\phi}+\frac{g}{8m^2}\vec{\nabla}\cdot(\dot{\vec{\phi}}+\vec{\nabla}\phi_0)+\frac{g}{4m^2}\vec{\sigma}\cdot(\dot{\vec{\phi}}+\vec{\nabla}\phi_0)\times\vec{p}\,. \tag{5.2}$$

Symmetrization is necessary when the nucleon momentum $\vec{p}$ appears. By $\vec{\phi}\cdot\vec{p}$, for instance, we mean $\frac{1}{2}(\vec{\phi}\cdot\vec{p}+\vec{p}\cdot\vec{\phi})$. The effective Hamiltonian for the process $\omega N\to\gamma N$ through $N\bar{N}$ pair excitation has the form

$$H_{\gamma\omega}^{\text{pair}}=\frac{eg}{m}\vec{A}\cdot\vec{\phi}+\frac{eg}{4m^2}\vec{\sigma}\cdot\vec{A}\times(\dot{\vec{\phi}}+2\vec{\nabla}\phi_0)-\frac{eg}{4m^2}\vec{\sigma}\cdot(\dot{\vec{A}}+2\vec{\nabla}A_0)\times\vec{\phi}\,, \tag{5.3}$$

where A_μ is the em vector potential.

With these Hamiltonians, we compute the one-meson exchange contribution to the em interaction,

$$H_{\text{em}}^{\text{pair}}=\frac{e_1}{m^2}\vec{A}(\vec{r}_1)\cdot\vec{p}_2v(r)-\frac{e_1}{2m^2}\vec{A}(\vec{r}_1)\cdot\vec{\sigma}_1\times\vec{\nabla}_rv(r)-\frac{e_1}{2m^2}\vec{A}(\vec{r}_1)\cdot\vec{\sigma}_2\times\vec{\nabla}_rv(r)+(1\longrightarrow 2)\,. \tag{5.4}$$

Here $\vec{A}(\vec{r}_1)$ is the vector potential at the position $\vec{r}_1$,

$$v(r)=\frac{g^2}{4\pi}m_\omega Y_0(m_\omega r)\,, \tag{5.5}$$

$Y_0(x)=e^{-x}/x$, m_ω is the mass of ω and $\vec{r}=\vec{r}_1-\vec{r}_2$. We have neglected the effect of retardation of the ω meson. The exchange current at $k=0$ now becomes

$$\vec{J}^{\text{pair}}(0) = -\frac{e_1}{m^2}\vec{p}_2 v(r) + \frac{e_1}{2m^2}\vec{\sigma}_1 \times \vec{\nabla}_r v(r) + \frac{e_1}{2m^2}\vec{\sigma}_2 \times \vec{\nabla}_r v(r) + (1 \leftrightarrow 2)\,, \tag{5.6}$$

$$\rho^{\text{pair}}(0) = 0\,. \tag{5.7}$$

In order to see whether LET is satisfied or not, we derive the one-meson exchange potential, ignoring retardation,

$$V_\omega = \frac{1}{4m^2}\vec{\sigma}_1 \cdot \vec{\nabla}_r \times \vec{p}_1 v(r) + \frac{1}{2m^2}\vec{\sigma}_2 \cdot \vec{\nabla}_r \times \vec{p}_1 v(r) + (1 \leftrightarrow 2) + \left(1 + \frac{1}{4m^2}\vec{\nabla}_r{}^2\right) v(r) - \frac{1}{m^2}\vec{p}_1 \cdot \vec{p}_2 v(r) + \frac{1}{4m^2}(\vec{\sigma}_1 \cdot \vec{\sigma}_2 \vec{\nabla}_r{}^2 - \vec{\sigma}_1 \cdot \vec{\nabla}_r \vec{\sigma}_2 \cdot \vec{\nabla}_r) v(r)\,, \tag{5.8}$$

and calculate the commutator of V_ω with the electric dipole operator to find

$$i\left[V_\omega, \sum_{i=1}^{2} e_i \vec{r}_i\right] = -\frac{e_1}{m^2}\vec{p}_2 v(r) + \frac{e_1}{4m^2}\vec{\sigma}_1 \times \vec{\nabla}_r v(r) + \frac{e_1}{2m^2}\vec{\sigma}_2 \times \vec{\nabla}_r v(r) + (1 \leftrightarrow 2)\,, \tag{5.9}$$

in disagreement with (5.6). Thus the conventional treatment of the pair currents leads to the violation of gauge invariance.

This fact was expected beforehand since $H^{\text{pair}}_{\gamma\omega}$ does not coincide with the minimal interaction obtained from H^{FW}_{ω} in Eq. (5.2) through the gauge invariant substitution $\vec{p} \to \vec{p} - e\vec{A}$. In fact, the interaction

$$H^{\text{minimal}}_{\gamma\omega} = \frac{eg}{m}\vec{A} \cdot \vec{\phi} + \frac{eg}{4m^2}\vec{\sigma} \cdot \vec{A} \times (\dot{\vec{\phi}} + \vec{\nabla}\phi_0) \tag{5.10}$$

differs from $H^{\text{pair}}_{\gamma\omega}$ by the amount which is not gauge invariant separately.

A consistent method of reducing the original relativistic Hamiltonian in the presence of the em and meson fields to the nonrelativistic one is the FW transformation which furnishes us with

$$H^{\text{FW}}_{\gamma\omega} = H^{\text{minimal}}_{\gamma\omega} - \frac{eg}{4m^2}\vec{\sigma} \cdot (\dot{\vec{A}} + \vec{\nabla}A_0) \times \vec{\phi}\,. \tag{5.11}$$

Since the second term in (5.11) is a current-conserved interaction, gauge invariance of $H^{\text{FW}}_{\omega} + H^{\text{FW}}_{\gamma\omega}$ is obvious. By using $H^{\text{FW}}_{\gamma\omega}$ or $H^{\text{minimal}}_{\gamma\omega}$ instead of $H^{\text{pair}}_{\gamma\omega}$, we can ascertain that the exchange current at $k=0$ is equal to (5.9) and that LET is recovered.

The above example clearly shows that if we define the one-body em interaction in terms of the FW transformation, the calculations of the exchange currents must also be made based on the effective Hamiltonians obtained by the FW transformation. Conversely speaking, any elaborate estimates of exchange effects cannot be conclusive unless we calculate the one-body term consistently, since they are intimately connected with each other.

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$$\vec{\nabla} \cdot \vec{J}^{(2)}(\vec{x}) = -i\left[V, \sum_{i=1}^{2} e_i \delta(\vec{x} - \vec{r}_i)\right],$$

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