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Validity of the closure approximation in multiple scattering theory

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The closure approximation is necessary for the evaluation of the double scattering term, for the reduction of the multiple scattering from a dynamic nuclear system to the scattering from fixed scattering centers, and for the derivation of the Glauber formalism from the Watson multiple scattering formalism. If closure is valid, there should be no double scattering term in the optical potential for an uncorrelated target nucleus. A simple Gaussian model, in which the intermediate nuclear states are scattering states, is used to calculate the binary scattering term in the optical potential for such a nucleus. The nonlocal binary potential is used, together with the local single-scattering potential, to predict the differential scattering cross section of 1 GeV protons incident on ^{4}He and ^{40}Ca. The results confirm the validity of the closure approximation for the heavier nucleus at all scattering angles. The results for the lighter nucleus confirm its use for small scattering angles but cast doubt on its use in the region of the first diffraction minimum. In the case of ^{4}He, the results also show that double scattering may improve the optical model fits to the scattering data.

[NUCLEAR REACTIONS Optical potential model, double scattering effects; relation to Watson, Glauber, theories.]

INTRODUCTION

The closure approximation (see, e.g., Ref. 1) is the assumption, in multiple-scattering theory,[2] that the only important states, in the completeness sum over intermediate nuclear states, have energies small compared to the kinetic energy of the incident projectile when such completeness sums occur between pairs of target nucleon scattering operators. Such an assumption enables the nuclear Green's function, which propagates the incident projectile between successive scatterings with nuclear nucleons, to be evaluated; the completeness sum can be carried out without reference to the different nuclear state energies which occur in the denominator of the Green's function.

The usual multiple-scattering formalism[2] for elastic scattering from nuclei includes all the dynamics of the composite target (the nucleus made up of moving, interacting nucleons) via the definition of a two-body (projectile-nucleus) optical potential. This potential is a sum of multiple-scattering terms: the single-scattering term in which the incident projectile scatters off a single target nucleon, the target nucleus remaining in its ground state, the double-scattering term in which the incident projectile excites the target nucleus from its ground state via one scattering with a nuclear nucleon and then returns the nucleus to its ground state via another nucleon scattering, etc.[3] It is necessary to use the closure approximation in order to evaluate the double-scattering term in the optical potential, the term binary in the projectile-nucleon scattering amplitudes; such evaluation is necessary in order to relate, e.g., nucleon-nucleus scattering to nucleon-nucleon scattering.[4]

The binary optical potential for scattering of a nucleon of mass M from a nucleus with A nucleons can be written as

$$\langle \vec{k}' | U_2(E) | \vec{k} \rangle = A(A-1) \sum_{m\neq 0} \int \frac{\langle O\vec{k}' | t_i^A(E) | m\vec{k}'' \rangle \langle m\vec{k}'' | t_j^A(E) | O\vec{k} \rangle}{E - E_m - [(A+1)/2AM] k''^2 + i\epsilon} d^3k'' \tag{1}$$

where E is the kinetic energy of the incident projectile, E is kinematically related to E,[3] $|m\vec{k}''\rangle$ represents a state in which the relative momentum of the projectile and target nucleus is $\vec{k}''$ and the internal energy state of the target is E_m (m can be a continuous index); t_i^A represents the scattering of the incident projectile by the ith target nucleon in the A nucleon nucleus. Note that nonrelativistic kinematics[3] is being used for the intermediate propagation of the nucleon in this evaluation of the binary part of the optical potential. If the closure approximation is made, Eq. (1) can be evaluated and the binary potential shown[3,5,6] to be proportional to

$$[\rho(\vec{r}_1, \vec{r}_2) - \rho(\vec{r}_1)\rho(\vec{r}_2)], \qquad (2)$$

where $\rho(\vec{r}_i)$ is the nuclear single-particle density and $\rho(\vec{r}_i, \vec{r}_j)$ is the nuclear two-particle density. Equation (2) vanishes if the target nucleons are uncorrelated. Thus, if the target nucleus is sufficiently massive to rule out center-of-mass recoil correlations[3] and if there are no dynamical correlations between the nuclear nucleons, *closure implies that the binary optical potential should vanish.* This would appear to explain the good fits to the heavy nucleus differential scattering data which have been found using the single-scattering optical potential.[7] Deviations from such good fits if found at larger angles for heavy nuclei would presumably be due to one or more of the following effects:

(a) the presence of off-energy-shell effects in the single-scattering potential which is linear in the nucleon-projectile scattering amplitude,[8]
(b) dynamical correlations in the target nucleus leading to a binary optical potential even for heavy nuclei,[6]
(c) corrections to the impulse approximation which lead to a binary potential proportional to A instead of A^2,[3]
(d) the presence, in the optical potential, of triple or higher order, scattering terms,
(e) a breakdown in the validity of the closure approximation.

Thus it is important to have an estimate of the correctness of the closure approximation before one can gain any physical insights from the comparison of scattering data with the predictions of multiple-scattering theory.

The use of closure is also required if one wishes to reduce the full multiple-scattering problem, where the individual members of the composite target participate in their own dynamical motions and interactions, to a problem of multiple scattering from fixed sources (a sum of potentials with fixed origins).[1] Since the derivation of the Glauber scattering theory[9] from the multiple-scattering theory of Watson[2] requires the assumption of fixed scattering centers,[10,11] it follows that closure is also necessary to relate the phenomenologically successful Glauber theory[12] to the more fundamental Watson approach.

Thus, whether one wishes to use the multiple-scattering formalism directly[4,7,8] or understand the relation between it and the Glauber formalism, it follows that it is important to check the validity of the closure approximation using some reasonable physical model.[1]

In order to check the validity of closure, we evaluate, without assuming closure, the binary potential for a completely uncorrelated nucleus by using a subset of the intermediate states $|m\rangle$ in Eq. (1), specifically, a set of continuum states representing one of the target nucleons scattering from the remaining $A-1$ nuclear nucleons. This continuum part of the binary optical potential should represent most of the second-order potential since the energy denominator, in Eq. (1), should be much larger for the discrete bound states $|m\rangle$ than for the scattering states $|m^+\rangle$, given a large incident energy E. Thus, *if closure is a valid approximation, we expect the calculated binary potential to be very small* [because of Eq. (2)] *compared to the single-scattering potential.* The evaluation of the binary term is carried out in the Calculation section of this paper using Gaussian product wave functions and a Gaussian nucleon scattering amplitude which should be valid in the 1 GeV incident energy range.[7,12]

The single-scattering optical potential U_1 is a local potential, i.e., it depends only upon the single variable $(\vec{k}' - \vec{k})^2$. The binary potential U_2 is nonlocal, depending upon the three variables k'^2, k^2, and $\vec{k}' \cdot \vec{k}$. It is thus difficult to find a meaningful numerical procedure which will answer the question: Is U_2/U_1 small? A meaningful question which can be directly answered numerically is: Does the inclusion or exclusion of the binary term in a calculation of scattering using the optical model potential make a significant change in the results? If closure is a good approximation, we would expect very little difference between a differential scattering cross section calculated with U_1 as the optical model potential and a cross section calculated with a potential $U_1 + U_2$. The physical example chosen is the 1 GeV scattering of protons from nuclei, specifically from ^{4}He and ^{40}Ca.

CALCULATION

Assuming A for the target nucleus to be very large, there will be no recoil and hence no kinematical correlations. Assuming product wave functions (no dynamical correlations), we expect the single-scattering optical potential to be well represented by the form

$$\langle \vec{k}' | U_1(E) | \vec{k} \rangle = A \langle O\vec{k}' | t_i^A(E) | O\vec{k} \rangle = -A\, F_{00}(\vec{q}) f(\vec{q}, E). \qquad (3)$$

Here we have made use of the form-factor approximation[3]; the momentum transfer is $\vec{q} = \vec{k}' - \vec{k}$; the nucleon-nucleon scattering amplitude is[3,7,12]

$$f(\vec{q}, E) = f_0 e^{-a q^2}, \quad f_0 = -(2\pi)^{-3}(i + \alpha)\sigma\eta/\epsilon, \qquad (3a)$$

where η and ϵ are the momentum and energy in the nucleon-nucleon center-of-mass system; $\eta/\epsilon=[(s-4M^2)/s]^{1/2}$ where s is the invariant square of the nucleon-nucleon energy. We fix s by assuming that the relative velocity in the nucleon-nucleon system is the same as the relative velocity in the nucleon-target system.[13] This implies that

$$\frac{s(s-4M^2)}{(s-2M^2)}=\frac{E(E+2M)}{E+M}, \tag{3b}$$

where E is the kinetic energy of the nucleon incident upon the heavy target. For $E=1$ GeV, we take $\sigma=4.35$ fm^2, $a=0.105$ fm^2, and $\alpha=0.1$ $[(1+\alpha^2)^{1/2}=1]$. The form factor $F_{00}(\vec{q})=\int\rho(\vec{r})e^{i\vec{q}\cdot\vec{r}}d\vec{r}$ equals one for zero momentum transfer. When evaluating this first-order potential, Eq. (3), the form factor will be that appropriate for the parabolic Fermi shape which leads to a reasonably good fit to the $p-{}^4$He, and $p-{}^{40}$Ca data at 1 GeV.[13-15]

Turning now to the binary part of the optical potential, the sum over intermediate nuclear states in Eq. (1) includes bound states—other than the ground state—of the A target nucleons, continuum two-body scattering states, etc. The energy denominators $E-E_m$ are assumed to be very large for the bound states when $E\simeq1$ GeV so that the discrete part of the intermediate sum over states will be assumed small enough to be ignored. The subset of states which will be kept in the sum have the form $|m^+\rangle=\Omega^+_{1,A-1}|A-1\rangle|1\rangle$ where $|A-1\rangle$ is a bound state of $A-1$ nucleons, $|1\rangle$ represents the relative motion of this $A-1$ nucleus with respect to the remaining nucleon, and where $\Omega^+_{1,A-1}$ is the Möller scattering operator[16] representing the scattering of the Ath target nucleon by the remaining $A-1$ nucleon nucleus. We assume that a multiple-scattering optical potential can be used to describe this scattering, i.e.,

$$\Omega^+_{1,A-1}=1+G\,V_{1,A-1}\Omega^+_{1,A-1}, \tag{4}$$

$$V_{1,A-1}=(A-1)t+\cdots, \tag{4a}$$

where G is an appropriate Green's function. Thus $\Omega^+_{1,A-1}-1$ is at least first order in the nucleon-nucleon scattering operator t; keeping these terms in $|m^+\rangle$ means that the binary potential calculated, via Eq. (1), with $|m^+\rangle$ would contain terms cubic and higher in t. Restricting the binary potential to quadratic powers of t means keeping only the lowest-order contribution from $\Omega^+_{1,A-1}$, i.e., $|m^+\rangle=|A-1\rangle|1\rangle$.

Using the form-factor approximation for the terms of the binary potential implies that

$$\langle\vec{k}',n|t_i^A(E)|\vec{k},m\rangle=-F_{i,nm}(\vec{k}'-\vec{k})f_i(\vec{k}'-\vec{k},E), \tag{5}$$

where

$$F_{i,nm}(\vec{q})=\int\psi_n^*(\vec{r}_1\cdots\vec{r}_A)\psi_m(\vec{r}_1\cdots\vec{r}_A)\times e^{i\vec{q}\cdot\vec{r}_i}d\vec{r}_1\cdots d\vec{r}_A. \tag{6}$$

Assuming the existence of a complete orthonormal set of single particle wave functions for the target nucleons, the bound states of the target nucleus can be written as

$$\psi_n(\vec{r}_1\cdots\vec{r}_A)=\prod_{i=1}^{A}\psi_i(\vec{r}_i), \tag{7}$$

whereas the intermediate scattering states of the target nucleus with relative momentum $\vec{p}$ are

$$\psi_{m+i}(\vec{r}_1\cdots\vec{r}_A)=|1\rangle|A-1\rangle=\frac{e^{i\vec{p}\cdot\vec{r}_i}}{(2\pi)^{3/2}}\prod_{\substack{j=1\\ j\neq i}}^{A}\psi_j(\vec{r}_j). \tag{8}$$

Thus, for the intermediate states of our model

$$F_{i,nm^+}(\vec{q})=\frac{1}{(2\pi)^{3/2}}\int\psi_1^*(\vec{r})e^{i(\vec{p}+\vec{q})\cdot\vec{r}}d\vec{r}, \tag{9a}$$

and

$$E_m=\frac{A}{A-1}\frac{P^2}{2M}. \tag{9b}$$

We may now write the matrix elements of (1) as

$$\begin{aligned}\langle m\vec{k}''|t_j|O\vec{k}\rangle&=-F_{j,m0}(\vec{k}''-\vec{k})f(\vec{k}''-\vec{k}),\\ \langle O\vec{k}'|t_i|m\vec{k}\rangle&=-F^*_{i,m0}(\vec{k}''-\vec{k}')f^*(\vec{k}''-\vec{k}'),\end{aligned} \tag{10}$$

where

$$F_{j,m0}(\vec{q})=J_j(\vec{q}-\vec{p}) \tag{11a}$$

and

$$J_i(\vec{p})=\frac{1}{(2\pi)^{3/2}}\int e^{i\vec{p}\cdot\vec{r}}\psi_i(\vec{r})d\vec{r}. \tag{11b}$$

For simplicity we assume Gaussian wave functions when evaluating the binary potential:

$$\psi(\vec{r})=\psi_0e^{-(r/R)^2/2},\quad \psi_0^{-2}=\pi^{3/2}R^3, \tag{12}$$

so that

$$J(\vec{p})=J_0e^{-bp^2},\quad b=\tfrac{1}{2}R^2. \tag{13}$$

We normalize R at 1.37 fm for ^{4}He,[17] so that for arbitrary A, $R=0.863A^{1/3}$ fm; note that $b=0.372A^{2/3}$ fm^2 is larger than a, as expected. The nucleon-nucleon amplitudes $f(\vec{q})$ are taken from Eq. (3a); thus we ignore the variation of the nucleon-nucleon energy E with momentum transfer[3] during the multiple-scattering process.

The binary optical potential Eq. (1) may be written $\langle \vec{k}' | U_2 | \vec{k} \rangle = A(A-1)(V_2 - W_2)$ where V_2 includes the ground state in its sum over intermediate states whereas W_2 only consists of the ground state term. The subtraction of W_2 is necessary since the term with $m=0$ is omitted from the sum in Eq. (1). The sum over intermediate states is now $\int d^3p$ in our model so that

$$V_2 = \int \frac{d^3p\, d^3k'' f_i^*(\vec{k}'' - \vec{k}') f_j(\vec{k}'' - \vec{k}') J_j(\vec{k}'' - \vec{k} - \vec{p}) J_i^*(\vec{k}'' - \vec{k}' - \vec{p})}{E - [A/(A-1)](P^2/2M) - [(A+1)/A](k''^2/2M) + i\epsilon} . \tag{14}$$

Setting $\vec{k}'' = \vec{u}$, $\vec{p} = \vec{v}$, and A very large, we get

$$V_2 = 2M |J_0 f_0|^2 e^{-(a+b)(k'^2+k^2)} \int d^3u\, e^{-2(a+b)[u^2 - (\vec{k}'+\vec{k})\cdot\vec{u}]} \int d^3v \frac{e^{-2b(v^2 - \vec{u}\cdot\vec{v})}}{2ME - u^2 - v^2 + i\epsilon} . \tag{15}$$

the $\vec{v}$ integral can be evaluated in terms of Dawson's integral[18] D; after doing the angular part of the $\vec{u}$ integration, the result is

$$V_2 = \pi^2 i \frac{M}{b} |J_0 f_0|^2 e^{-(a+b)(k'^2+k^2)} 2\pi \int_0^\infty u^2 du\, e^{-2(a+b)u^2} \frac{1}{u|\vec{k}'+\vec{k}|} (e^{|\vec{k}'+\vec{k}|u} - e^{-|\vec{k}'+\vec{k}|u})$$

$$\times \frac{\theta(2ME - u^2)}{u} \Big\{ e^{-2b(2ME-u^2)} [e^{-2bu(2ME-u^2)^{1/2}} - e^{2bu(2ME-u^2)^{1/2}}]$$

$$+ \frac{2i}{\pi^{1/2}} e^{(1/2)bu^2} [D(X_+) - D(X_-)] \Big\} , \tag{16}$$

where $X_\pm \equiv (2b)^{1/2}[\frac{1}{2}u \pm (2mE - u^2)^{1/2}]$ and θ is the unit step function. Thus V_2 is reduced to a single quadrature which can be evaluated numerically as a subroutine of the computer program which integrates the scattering equation.

The remaining part of the binary potential is

$$W_2 = \int d^3k'' \frac{F_{i00}^*(\vec{k}'' - \vec{k}') f_i^*(\vec{k}'' - \vec{k}') F_{j00}(\vec{k}'' - \vec{k}) f_j(\vec{k}' - \vec{k})}{E - [(A+1)/2AM]k''^2 + i\epsilon} , \tag{17}$$

where

$$F_{i00}(q) = \int \psi_0^*(\vec{r}_1 \cdots \vec{r}_A) \psi_0(\vec{r}_1 \cdots \vec{r}_A) e^{i\vec{q}\cdot\vec{r}_1} d\vec{r}_1 \cdots d\vec{r}_A$$

$$= \int |\psi_i(r)|^2 e^{i\vec{q}\cdot\vec{r}} dr = |\psi_0|^2 \int e^{-(r/R)^2} e^{i\vec{q}\cdot\vec{r}} d\vec{r}$$

$$= |\psi_0| (2m)^{3/2} J_0 e^{-b' q^2} \tag{18}$$

and $b' = \frac{1}{2}b$. Thus

$$W_2 = \frac{8\pi^{3/2}}{R^3} |f_0 J_0|^2 2M e^{-(a+b')(k'^2+k^2)} \int d^3k'' \frac{e^{-2(a+b')k''^2} e^{2(a+b')(\vec{k}'+\vec{k})\cdot\vec{k}''}}{2ME - [(A+1)/A]k''^2 + i\epsilon} . \tag{19}$$

Again letting A be very large, this integral can be directly evaluated in terms of Dawson's integral to give

$$W_2 = \frac{\pi^2 i}{(a+\frac{1}{2}b)} \frac{1}{|\vec{k}'+\vec{k}|} \left(\frac{2\pi^{1/2}}{R}\right)^3 M |f_0 J_0|^2 e^{-[a+(1/2)b](k'^2+k^2)}$$

$$\times \theta(2ME) \Big\{ e^{-4[a+(1/2)b]ME} [e^{-2[a+(1/2)b]|\vec{k}'+\vec{k}|(2ME)^{1/2}} - e^{2[a+(1/2)b]|\vec{k}'+\vec{k}|(2ME)^{1/2}}]$$

$$+ \frac{2i}{\pi^{1/2}} e^{(1/2)[a+(1/2)b](k'^2+k^2)} [D(y_+) - D(y_-)] \Big\} , \tag{20}$$

where

$$y\pm = [2(a+\tfrac{1}{2}b)]^{1/2}[\tfrac{1}{2}|\vec{k}'+\vec{k}| \pm (2ME)^{1/2}].$$

The binary potential given by Eqs. (16) and (20) is nonlocal in contrast with the local single-scattering potential (primary potential) given by Eq. (3). It is thus difficult to determine numerically whether U_2 is very small compared to U_1, as required by the closure approximation, or not. An alternative, indirect method is to use the potentials in a Lippman-Schwinger scattering equation,[16] solve for the differential scattering cross section for some interesting projectile and target, and to see how much effect the inclusion or exclusion of U_2 has on the predicted cross section.

The numerical calculations were done for proton projectiles with a laboratory kinetic energy of 1 GeV incident on ^{4}He and ^{40}Ca targets. The kinetic energy term in the relativistic Schrödinger scattering equation was taken to have the form

$$H_0 = (k^2+M^2)^{1/2} + (k^2+M_{\rm Nuc})^{1/2}; \tag{21}$$

the parameters appearing in the potential are those of Eqs. (3a), (3b), *et seq*. The results are given in Figs. 1 and 2, the dashed curve being the scattering due to U_1 alone; the solid curve represents the differential scattering cross section to be expected when the optical potential is U_1+U_2 [Eqs. (3), (16), and (20)]. The data[14,15] are shown just for comparison's sake; no attempt has been made to adjust the parameters to improve the fit to the data.

DISCUSSION

The numerical results for $p-{}^4$He (Fig. 1) indicate that the inclusion of U_2 raises the cross section by 10% for small angle scattering. It follows that closure is a reasonable approximation for these small momentum transfers and hence the Glauber approach is a useful tool for analyzing data in this region (provided that due caution is taken to avoid the pretense of "precision" fits to the data). However, change is much more marked in the vicinity of the diffraction structure, the value of the first diffraction minimum being raised al-

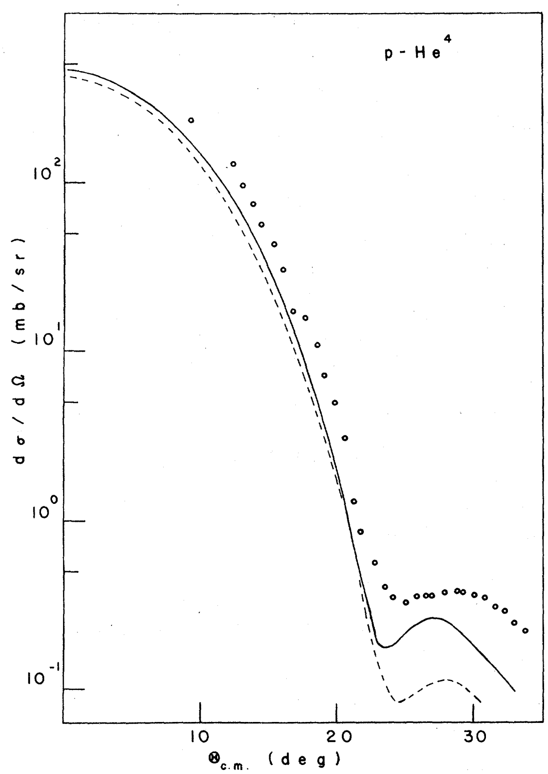

FIG. 1. Differential scattering cross section fo 1 GeV protons incident upon ^{4}He. The dashed curve is the cross section calculated with U_1. The solid line represents the cross section calculated with U_1+U_2. The data are from Ref. 14.

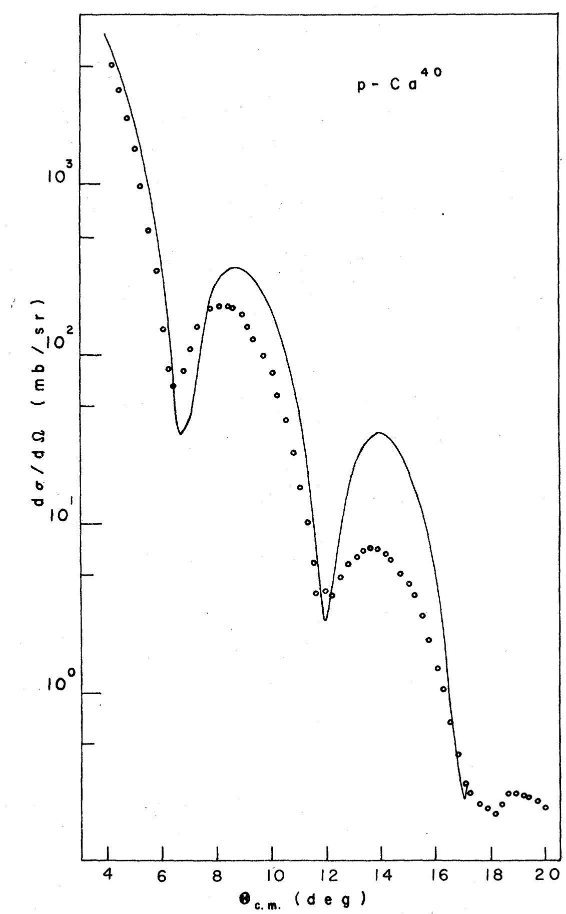

FIG. 2. Differential scattering cross section for 1 GeV protons incident upon ^{40}Ca. The solid curve represents the cross section calculated with U_1 or with U_1+U_2. The data are from Ref. 15.

most by a factor of 2 via the inclusion of U_2 and the rate of recovery to the diffraction peak being somewhat diminished. It thus appears that the validity of the closure approximation becomes doubtful just in the region where its use is most important, that is, the region in which target nucleon correlations are sought via analysis of multiple-scattering terms.[6]

Instead of looking at this calculation as a check of the closure approximation, it may, with equal validity, be viewed as a partial evaluation of the double-scattering contribution to the optical model potential. In this guise it is interesting to note that even this incomplete double scattering makes a major contribution toward improving the fit to the p-^{4}He scattering data[14] in the region of the first diffraction minimum and the subsequent maximum. Although no attempt has been made to choose parameters on the basis of quality of fit to the nucleon-nucleus data, the quality of the fit to the data in Fig. 1 suggests that the physical model represented by this calculation contains many of the important elements needed to successfully link elastic nucleon-nucleon scattering to elastic nucleon-nucleus scattering in the energy and momentum transfer regions shown. If the improvement to the data fit resulting from the inclusion of our model for U_2 is not fortuitous, it follows that corrections to the closure approximation will have to be an important part of any future attempts at a precision fit to nucleon-nucleus data. That is, the relative success of this simple model in improving fits to the data adds validity to the doubts it casts on the closure approximation for light nuclei.

In the case of p-^{40}Ca, the calculations with and without U_2 are indistinguishable. Figure 2 shows the calculation with U_1 plotted against the data of Alkhazov *et al.*[15] The inclusion of U_2 makes no difference in the calculated cross section. This result indicates that by $A=40$, the closure approximation is extremely accurate, and may be used with complete confidence.

Given the importance of the closure approximation, it can be concluded from the simple model of this paper that closure is valid for all but the lightest nuclei. In the case of light nuclei, however, doubt has been cast on its utility—a doubt which must be removed before we can be confident about our understanding of multiple-scattering phenomena and our ability to fit data using either the Glauber formalism or the optical potential.

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