Relativistic optical model analysis of medium energy p-^{4}He elastic scattering experiments

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The results of an optical model analysis of medium energy p-^{4}He elastic scattering cross section and analyzing power measurements are reported. The analysis is based on the use of the Dirac equation with a mixture of a Lorentz scalar potential and the timelike component of a four-vector potential.

[NUCLEAR REACTIONS Nucleon-nucleus scattering. Relativistic optical model analysis of p-^{4}He scattering. Calculated cross sections and analyzing powers.]

I. INTRODUCTION

In this paper we report the results of a relativistic optical model analysis of the recent p-^{4}He elastic scattering data of Klem *et al.*[1,2] These data, which cover an energy range from 561 to 1730 MeV and include both cross section and analyzing power measurements, are a significant addition to the available medium energy p-^{4}He elastic scattering data. In our earlier work we obtained good fits to p-^{4}He elastic scattering cross section measurements in this energy range.[3-5] The analyzing power measurements allow a more critical test of the model.

The relativistic optical model used here is discussed in Sec. II. The model is based on the Dirac equation with a mixture of a Lorentz scalar potential U_s and the timelike component of a four-vector potential U_0. We have observed that either a U_s- or a U_0-type potential could be used in an optical model analysis of medium energy proton-nucleus cross section data. The volume integral of the real part of the effective central potential obtained from these analyses has a systematic trend with energy that is almost identical to results obtained for heavier target nuclei[6] and, when extrapolated to low energies, is in agreement with results from low-energy p-^{4}He optical model analyses.[7-11] However, the polarizations calculated with these pure potentials are much too small. Calculated polarizations comparable in magnitude to measured values require a mixture of U_s and U_0 potentials in this model. In the present work we determine the appropriate mixture of U_s and U_0 which gives a mutual fit to the cross section and analyzing power measurements of Klem *et al.* We find that the volume integral of the real part of the effective central potential obtained from a mixture of U_s and U_0 is in agreement with the results for the pure U_s or U_0 potentials mentioned above.

An important reason for considering a relativistic optical model with a mixture of U_s and U_0 potentials is the possible interpretation of the results in terms of an effective nucleon-nucleus interaction derivable from meson exchange models of the two-nucleon interaction. A relativistic model with a mixture of U_s and U_0 potentials was examined by Duerr[12-14] more than 20 years ago in a reformulation of the Johnson-Teller model.[15] Within the past few years several groups have considered relativistic models like Duerr's in connection with the general problem of nuclear structure and stability. Miller and Green[16,17] and Brockmann and Weise[18,19] have examined the nuclear single particle bound state problem with a mixture of U_s and U_0 potentials in the Dirac equation. Walecka[20] has developed a model relativistic, many-body, quantum field theory composed of a baryon field, a neutral scalar meson field, and a neutral vector meson field. This model, which has been extended by Chin,[21] exhibits the saturation characteristic of nuclear matter. An extension of this model to finite nuclei is in progress.[22] It has been noted[18,23] that the coupling constants for the meson fields used in this model are in nominal agreement with the coupling constants for the corresponding fields used in one boson exchange models of the two-nucleon interaction, and that these coupling constants give the correct single particle level ordering in nuclei. In this study of the relativistic optical model at intermediate energies we find that a mixture of U_s and U_0 potentials similar to that used for the single particle bound state and nuclear matter problems gives a good representation of intermediate energy proton elastic scattering data.[24] Preliminary work on the relativistic optical model at low energies provides further evidence of the relevance of this model.[25] Thus there is a growing body of literature which suggests that a relativistic model with a mixture of U_s and U_0 potentials may be an adequate starting point for a description of nuclear single particle and optical properties over a three decade energy range. In addition, the importance of a Dirac equation based model of the nucleon channel and of the Lorentz transformation characteristics of model potentials has been emphasized in several recent papers on the problem of pion absorption and production.[26-29]

II. DISCUSSION OF THE MODEL

In relativistic optical model analyses of intermediate energy scattering experiments we use the Dirac equation given by

$$\{\vec{\alpha}\cdot\vec{p}+\beta[m+U_s(r)]+[U_0(r)+V_c(r)]\}\psi(\vec{r})=E\psi(\vec{r}), \tag{1}$$

where U_s is a Lorentz scalar potential, U_0 is the timelike component of a four-vector potential, and V_c is the Coulomb potential determined from the empirical nuclear charge distribution. A basic feature of this approach is the necessity of specifying the Lorentz transformation character of the potentials used in a relativistic wave equation. Consideration of this feature is absent in all nonrelativistic treatments. Although potentials which have any Lorentz character consistent with known conservation laws could be used, we have considered only U_0 and U_s. This choice is motivated by meson exchange considerations.[30-32] Major properties of the nucleon-nucleon interaction can be represented by one boson exchange potentials employing the known mesons ($\pi, \rho, \omega, \phi, \ldots$) plus the addition of the $J=0^+$, $T=0$ "σ" meson which is currently interpreted in terms of two-pion exchange processes. The dominant characteristics of the nucleon-nucleus interaction for a spin zero isospin zero target nucleus are expected to be represented by the exchange of neutral scalar and vector mesons.[16-19]

The main effect of using a mixture of Lorentz scalar and timelike component of a four-vector potentials in the Dirac equation was originally discussed by Furry.[33] He pointed out that to lowest order in v/c the sum U_0+U_s contributes to the central potential, while the difference U_0-U_s appears in the spin-orbit potential. This feature of the potential mixtures plays an essential role in the description of both elastic scattering and polarization data at intermediate energies.

In order to compare with nonrelativistic optical models, it is convenient to write Eq. (1) in second-order form. The equation for the upper component is

$$(p^2+U_{\text{eff}}+U_{\text{so}}\vec{\sigma}\cdot\vec{L})\psi_u=[(E-V_c)^2-m^2]\,\psi_u, \tag{2}$$

where

$$U_{\text{eff}}=2EU_0+2mU_s-U_0^2+U_s^2-2V_cU_0+U_Di\,\vec{r}\cdot\vec{p}, \tag{3}$$

$$U_{\text{so}}=-\frac{1}{rA}\frac{\partial A}{\partial r}=-U_D, \tag{4}$$

and

$$A=E+m+U_s-U_0-V_c. \tag{5}$$

The effective central potential is defined by Eq. (3).[34] These equations show that if U_0 is repulsive (as it would be if it resulted from neutral vector meson exchange) and U_s is attractive (as it would be if it resulted from neutral scalar meson exchange), then U_0 and U_s tend to cancel in the effective central potential while they add in the spin orbit and Darwin potentials. Notice that there is no explicit spin dependence in the optical potentials U_0 and U_s; the effective spin-orbit potential is completely specified by the potential mixture.

III. ANALYSES AND RESULTS

In the analysis of medium energy nucleon-nucleus scattering data we use the simplest form for U_s and U_0 consistent with obtaining agreement with experiment. The scalar and vector optical potentials used in the p-^{4}He analysis are

$$U_0=(V_0+iW_0)f_0(r), \tag{6}$$

$$U_s=(V_s+iW_s)f_s(r), \tag{7}$$

where the shape factors have the form

$$f(r)=\frac{1+wr^2/c^2}{1+\exp[(r-c)/z]}. \tag{8}$$

We have taken the real and imaginary parts to have the same geometry; the imaginary potential is of the volume form, as would be expected at these energies. In this study we take the scalar and vector shape factors to be identical.

We began the analysis with a fit to the data at 1029 MeV. This was accomplished by allowing the individual potential strengths V_0, W_0, V_s, and W_s, as well as the geometry,[35] to vary until a fit was achieved. The solid curves in Figs. 1c and 2c show the fit obtained. The 1029 MeV potentials were then used as the starting point for analyses at the other energies. An interesting feature of the potential determined from the fit at 1029 MeV is that it reproduces the systematic behavior of both the cross section and analyzing power data over the entire energy range from 560 to 1730 MeV.[24] This behavior is shown by the dashed

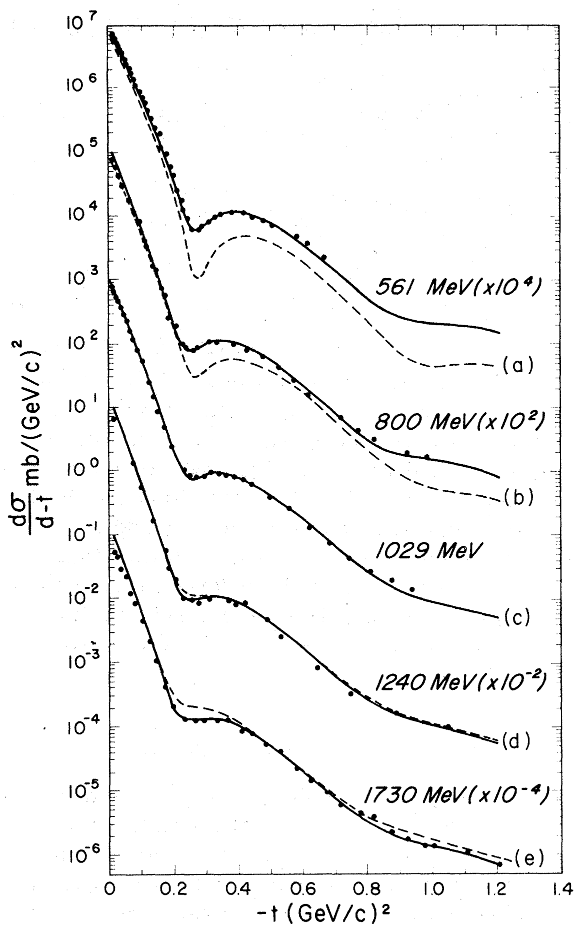

FIG. 1. Comparison of calculated and measured cross sections for p-^{4}He elastic scattering at 561, 800, 1029, 1240, and 1730 MeV. The solid lines are the fits at each energy. The dashed lines are the results obtained when the potential parameters from the fit to the 1029 MeV data are used in all of the calculations. The data are from Ref. (1).

curves in Figs. 1 and 2. The reduction in structure in both cross section and analyzing power between 560 and 1730 MeV is a natural consequence of kinematic effects present in a relativistic optical model a mixture of U_s and U_0 potentials.

Reasonable fits to the data at each energy were obtained by changing the strengths of the potentials keeping the shape factor unchanged from that determined at 1029 MeV. The results of the fits at each energy are shown by the solid curves in Figs. 1 and 2. The cross sections are of comparable quality to our previous results for ^{4}He as well as heavier target nuclei. The analyzing powers are represented as well by these calculations as they are in other treatments of these experiments.[36]

One of the advantages of the model used here is that it can be related to the extensive optical model analyses of low energy data. In particular, we have found in agreement with Greenlees *et al*.[37] that the volume integral per nucleon J_R/A of the real part of the effective central optical potential is well determined by the analysis of the p-^{4}He data. Van Oers *et al.*[6] have investigated the energy variation of J_R/A for proton scattering from heavier target nuclei. They found a logarithmic energy dependence given by

$$J_R/A=(J_R/A)_0+\beta \ln T_p\,, \tag{9}$$

where T_p is the proton kinetic energy in the laboratory. This type of energy dependence is in accord with Passatore's[38] application of Feshbach's dispersion relation.[39] Our results for J_R/A are shown in Fig. 3. The open circles are the results of the present analysis of the data of Klem *et al*; the closed circles are the results of our previous analyses of the cross section data from UCLA-LBL,[40,41] Saclay,[42] and CERN,[43] using a pure U_0 potential. The results show that the J_R/A values extracted from analyses with a pure U_0 potential are consistent with the mixed potential analyses. This confirms our hypothesis[3-5] that, at these energies, the presence of a sizable effective spin-orbit potential does not significantly affect the characteristics of the central potential. This has also been observed for heavier nuclei in the recent analysis[44] of 800 MeV LAMPF data.[45,46] The solid line in Fig. 3 is a least squares fit to Eq. (9) of the J_R/A values obtained from our analyses above 500 MeV. The dashed line in Fig. 3 shows the results of a recent analysis of Leung and Sherif[47] of data between 100 and 1154 MeV. They also find a logarithmic energy dependence for the real central volume integral. The extrapolation of our results to lower energies indicates that J_R/A goes through zero around 350 MeV, a value that is general agreement with although somewhat lower than the results of van Oers *et al.* for heavier target nuclei and Leung and Sherif for ^{4}He. At the lowest energies the extrapolated line in Fig. (3) falls between the results obtained by Satchler *et al.*[7] in their analysis of p-^{4}He cross sections and polarizations between 10 and 20 MeV. Thus the relativistic optical model described in this paper has the possibility of providing a systematic treatment of experimental data over a wide range of energies.

In our earlier analyses of p-^{4}He elastic cross sections we obtained reasonable fits to the data with either U_s- or U_0-type potentials. The availability of analyzing power data allows a mixture of U_s and U_0 potentials to be found. A characteristic parameter of this mixture is the volume in-

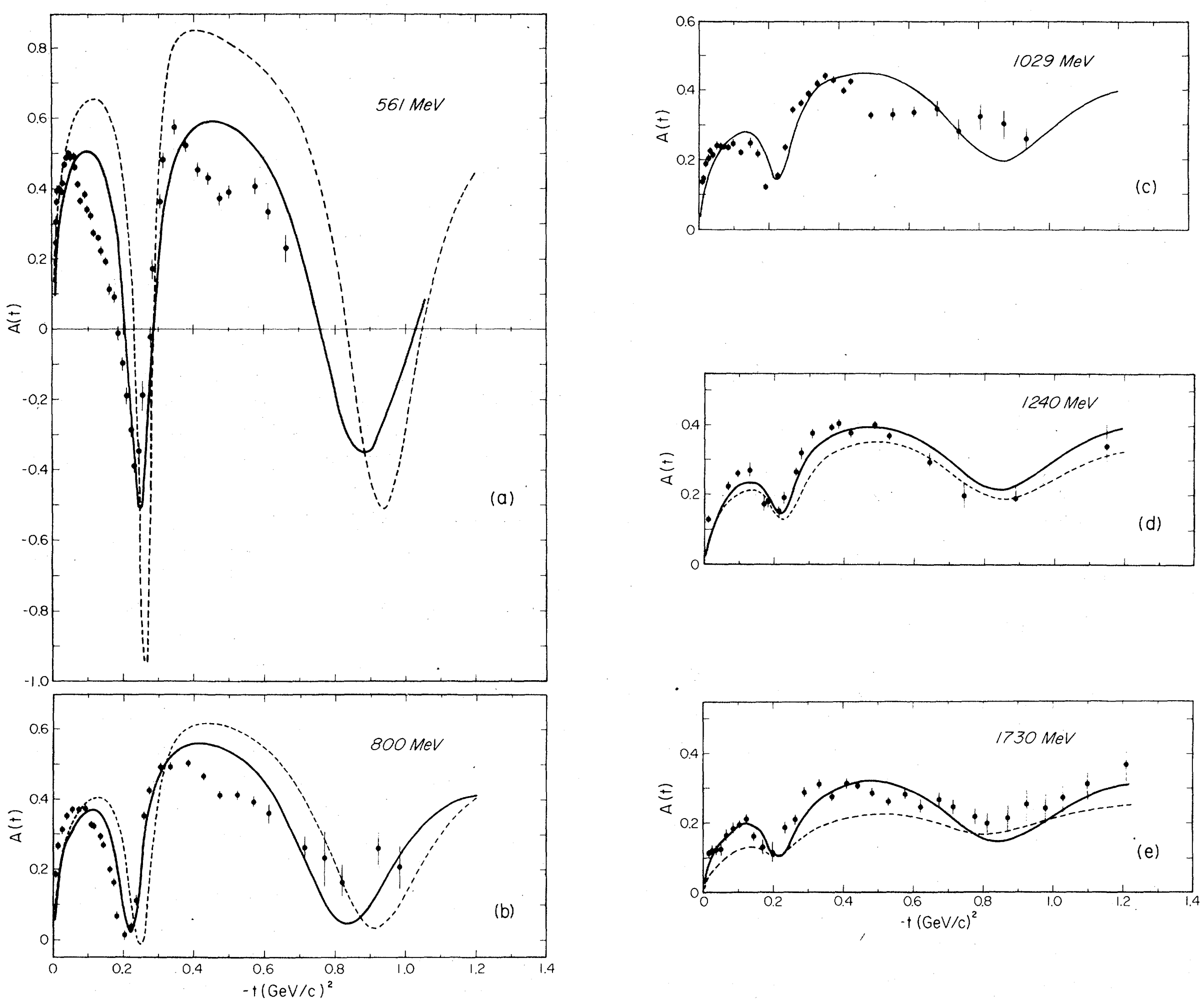

FIG. 2. Comparison of calculated and measured analyzing powers for p-^{4}He elastic scattering at 561, 800, 1029, 1240, and 1730 MeV. The solid lines are the fits at each energy. The dashed lines are the results obtained when the potential parameters from the fit to the 1029 MeV data are used in all of the calculations. The data are from Ref. (2).

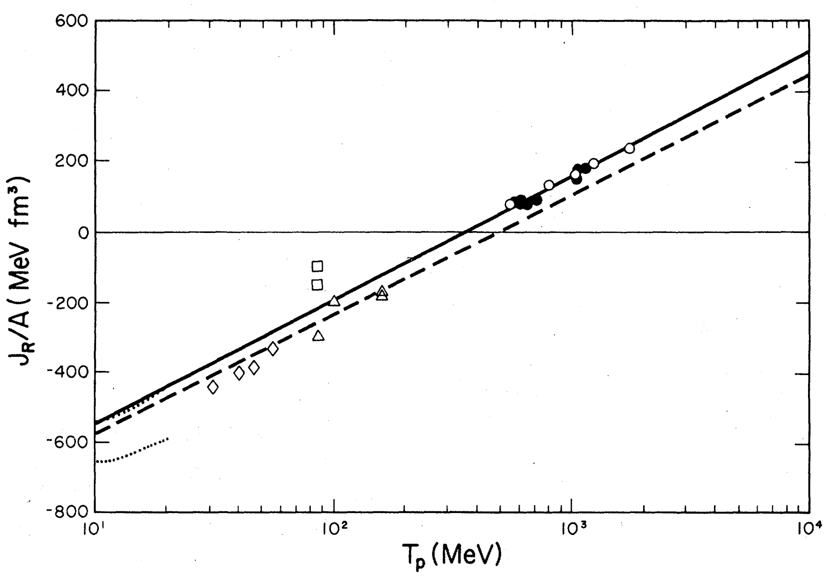

FIG. 3. The values of the J_R/A for the real central effective potential from p-^{4}He elastic scattering. T_p is the laboratory kinetic energy of the proton. The closed circles are from pure U_0 analyses; the open circles are from the present work. The values shown by the dotted lines are from Ref. (7), the diamonds from Ref. (8), the boxes from Ref. (10), and the triangles from Ref. (11). The solid line is a fit to Eq. (9) of our analyses above 500 MeV. The dashed line is a fit to Eq. (9) of the results given in Ref. (47).

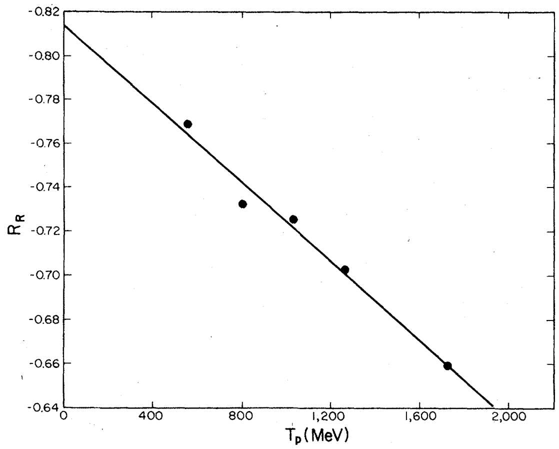

FIG. 4. Values of the ratio R_R given by Eq. (10) obtained from the present analyses using a mixture of U_s and U_0 potentials.

tegral ratio

$$R_R = \int V_0(r)d\vec{r} \Big/ \int V_s(r)d\vec{r} \,. \tag{10}$$

We find this ratio to be well determined from an analyses of both cross section and analyzing power data. The values of R_R obtained at each energy are shown in Fig. 4. There is a slight, approximately linear, variation of this ratio with energy. The solid line shown in Fig. 4 is the least squares fit to the values of R_R obtained at each energy.

An interesting feature of the ratio R_R is that it can be related to a similar ratio extracted from the relativistic nuclear matter theory. For example, in Walecka's relativistic mean field theory[20] the ratio is given by

$$R_R = -\frac{c_v^2}{c_s^2}\frac{\rho_b}{\rho_s} \,, \tag{11}$$

where c_s and c_v are the dimensionless scalar and vector coupling constants in Walecka's theory;

TABLE I. Selected values of the ratio R_R obtained from relativistic mean field theory of nuclear matter.

k_f (fm^{-1})	c_v^2	c_s^2	ρ_s/ρ_b	R_R
1.42	195.7	266.9	0.925	−0.793
1.31	266.2	348.7	0.932	−0.819

ρ_b is the baryon density of nuclear matter which is related to the fermi wave number k_f by $\rho_b = \frac{2}{3}k_f^3/\pi^2$; ρ_s is the scalar density which is determined self-consistently at a given baryon density. The coupling constants are the only free parameters in Walecka's theory. They are determined by solving the nuclear matter equations such that the binding energy per nucleon is a minimum for specified values of the binding energy and k_f. Typical results for a binding energy of 15.75 MeV are shown in Table I. The ratio R_R given the last column of Table I is insensitive to reasonable changes in k_f; it is also insensitive to reasonable changes in the binding energy. In contrast to the coupling constants shown in Table I, the quantity R_R is quite stable with respect to changes in the nuclear matter input.

The values of R_R given in Table I may be compared with an extrapolation of the least squares fit shown in Fig. 4. The extrapolated value of R_R at zero kinetic energy, -0.814 ± 0.006, is in good agreement with the results from Walecka's theory. Given the simplicity of the model used in our analyses the quantitive agreement with Walecka's theory should probably not be taken too seriously. Nevertheless the comparison illustrates a common feature of relativistic optical model analyses of medium energy experiments and relativistic nuclear matter theory.

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$$J/A=\frac{1}{2EA}\int U_{\rm eff}\,d\vec{r}.$$

The contribution to J/A from the Darwin term U_D is small and may be neglected for energies considered in this study.
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