Semiclassical description of two-nucleon transfer between superfluid nuclei

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Two-nucleon transfer reactions between superfluid nuclei are treated in the formalism of Dietrich and Hara starting from a microscopic Hamiltonian in second quantization. Amplitudes for simultaneous and successive transfer of neutron pairs between tin isotopes are calculated in perturbation theory. Nonorthogonality effects are numerically estimated. The possibility of a nuclear Josephson effect is predicted just below the barrier.

[NUCLEAR REACTIONS ^{120}Sn(^{120}Sn, ^{118}Sn)^{122}Sn, semiclassical approx., calculated pair transfer amplitudes, nucl. Josephson effect.]

I. INTRODUCTION

In 1962 Josephson predicted a current of Cooper pairs between two weakly coupled superconductors due to the phase coherence of the superconducting ground state.[1] This current is driven by the relative phase of the two superconductors rather than by a finite voltage.

As many heavy nuclei exhibit properties similar to superconductivity, the question arises whether a nuclear Josephson effect could exist.[2] Situations analogous to the weak coupling of two superconductors occur in the fission process[3] or in heavy-ion collisions.[4] In each case the coupling is time dependent and one does not observe the current directly but observes its conjugate variable, the particle number distribution of the nucleons on the two fragments. Most of the models developed to treat this problem used phenomenological interactions, where the strength of the interaction is a free parameter.[4,5]

One of the results of these calculations is the fact that one finds a strong enhancement of the two nucleon tunneling amplitude between nuclei due to the coherence of the superconducting state. Now the question arises whether the coupling between two nuclei is strong enough and lasts so long that supercurrents may start to flow. If those supercurrents really exist, one should observe a multiple transfer of nucleon pairs, which now play the role of the Cooper pairs in solid-state physics.

In this paper, we consider a heavy-ion collision below the Coulomb barrier, so we are sure that other channels (except Coulomb excitation) do not disturb the Josephson effect, i.e., pair breaking processes are small. Furthermore, we can simplify the treatment by using the semiclassical approximation. We apply to this problem the formalism of Dietrich and Hara[6] (referred to as DH from now on) for the semiclassical description of heavy-ion reactions: One starts from the full many-body Hamiltonian of the total system in second quantization. Taking into account the Hilbert space of bound single-particle states in two nuclei moving on their classical trajectories one has to work with a nonorthogonal basis. We separate from the Hamiltonian the terms which transfer nucleons between the nuclei. As will be shown in Sec. II, two such operators exist with different structure: One allows transfer of a nucleon pair in one step, the other, however, transfers a pair in two steps. This second term is completely analogous to the tunneling operator of Gol'danski and Larkin.[2] Thus it is possible to distinguish uniquely between simultaneous and successive transfer contributions. Because in experiment we observe nuclei with definite particle numbers, we use particle number projected BCS states to separate the different transfer channels.

In Sec. III we apply perturbation theory up to second order to the calculation of the simultaneous and successive two-nucleon transfer amplitudes. These formal results are used to calculate pair-transfer properties in a sub-Coulomb collision of tin nuclei. Because the successive transfer mechanism turns out to be strongly dominant, a numerical estimate of nonorthogonality effects is given for this process. Finally it is discussed whether a nuclear Josephson effect is expected to exist or not. In Appendix A the formulas for nonorthogonality contributions are deduced. Appendix B contains a comparison of the DH formalism with another formulation of the two-nucleon transfer by Götz *et al.*,[7] which has been applied to collisions between tin nuclei in Ref. 8.

II. HAMILTONIAN

In this section we proceed in analogy to DH.[6] The dual basis is introduced in our treatment in

the same manner as in DH's work, so we refer the reader to the first two sections of Ref. 6. The decomposition of the Hamiltonian, however, will proceed in a different way due to the inclusion of superconductivity; so this will be treated explicitly in this section.

Following Sec. 2 of Ref. 6 we introduce as a "working space" the direct sum of bound single-particle wave functions in two isolated nuclei on their classical trajectories. The transformation into the moving frames is performed by generalized time-dependent Galilean transformations [Eqs. (2.3) and (2.4) in Ref. 6]. It is immediately clear that these single-particle states, even if they form orthogonal sets in the individual nuclei, form a nonorthogonal basis in the direct sum because the overlap of wave functions in different nuclei does not vanish. This complication is remedied in DH's treatment by the introduction of the so-called dual (or biorthogonal) basis. In this basis the Hamiltonian of the total system may be written in second quantization by introducing particle creation and destruction operators in normal (c^+, c) or dual states $(\tilde{c}^+, \tilde{c})$. One notes that the Hamiltonian contains only creation operators c^+ in normal states and destruction operators $\tilde{c}$ in dual states. These operators satisfy simple generalized fermion anticommutation rules. They are time dependent because the particles are bound to nuclei which move on classical trajectories. Considering the time-dependent Schrödinger equation DH have shown that the time dependence of the creation and destruction operators gives rise to an extra term in the Hamiltonian. Once this so-called acceleration term is included in the Hamiltonian, any operator $\hat{\Omega}=\sum_{3,4}(\tilde{\psi}_3|\hat{\Omega}|\psi_4)c_3^\dagger\tilde{c}_4$ must be considered as time dependent or not if the matrix elements $(\tilde{\psi}_3|\hat{\Omega}|\psi_4)$ are time dependent or not. The creation and annihilation operators can be treated as if they were time independent; the effects of their time dependence are entirely contained in the above-mentioned acceleration term, which describes the effect of inertial forces due to classical motion on the intrinsic single-particle motion. The final Hamiltonian is semiclassical because of two facts:

(i) It depends on the position vectors R and velocities $\dot{R}$ of the two nuclei (we omit vector notation).

(ii) The matrix elements depend implicitely on R and $\dot{R}$ because the single-particle basis depends on these quantities.

Subtracting from the final expression for the Hamiltonian [Eq. (2.28) in Ref. 6], the Fermi energies λ of the nuclei, we arrive at the Hamiltonian we want to decompose:

$$\hat{H}(t)=\sum_{3,4}\left(\tilde{\psi}_3\left|\frac{[\hat{p}-m\dot{R}_4(t)]^2}{2m}+m\ddot{R}_4(t)[\hat{x}-R_4(t)]-\lambda_4\right|\psi_4\right)c_3^\dagger\tilde{c}_4+\frac{1}{4}\sum_{3,4,5,6}(\tilde{\psi}_3\tilde{\psi}_4|\bar{v}|\psi_5\psi_6)c_3^\dagger c_4^\dagger\tilde{c}_6\tilde{c}_5\,. \tag{2.1}$$

The conventions are those of DH, i.e., numbers 1 (1′...) and 2 (2′...) refer to nuclei 1 or 2 whereas 3 (4...) refers to both. The tildes refer to dual states, and $(|\bar{v}|)$ represents an antisymmetrized two-body matrix element,

$$(\tilde{\psi}_3\tilde{\psi}_4|\bar{v}|\psi_5\psi_6)\equiv(\tilde{\psi}_3\tilde{\psi}_4|v|\psi_5\psi_6)-(\tilde{\psi}_3\tilde{\psi}_4|v|\psi_6\psi_5)\,. \tag{2.2}$$

As we want to describe transfer reactions between superfluid nuclei, we must now introduce Bogoliubov quasiparticles. By defining the quasiparticle operators by

$$\begin{aligned}\gamma_3^\dagger&=u_3c_3^\dagger-v_3\tilde{c}_{-3}\,,\\ \tilde{\gamma}_3&=u_3\tilde{c}_3-v_3c_{-3}^\dagger\,,\end{aligned} \tag{2.3}$$

one automatically satisfies the generalized fermion anticommutation rules

$$\begin{aligned}\{\tilde{\gamma}_3,\tilde{\gamma}_4\}&=\{\gamma_3^\dagger,\gamma_4^\dagger\}=0\,,\\ \{\tilde{\gamma}_3,\gamma_4^\dagger\}&=\delta_{34}\,,\end{aligned} \tag{2.4}$$

under the same conditions for the u, v coefficients as usual. The state with quantum number -3 is the conjugate to the state 3 taken in the rest frame of the respective nucleus. From definition (2.3) one obtains two vacuum states, a normal and a dual one [$|0)$ is the normal particle vacuum]:

$$\begin{aligned}|\omega)&=\prod_{3>0}(u_3+v_3c_3^\dagger c_{-3}^\dagger)|0)\,,\quad \tilde{\gamma}_4|\omega)=0\\ (\tilde{\omega}|&=(0|\prod_{3>0}(u_3+v_3\tilde{c}_{-3}\tilde{c}_3)\,,\quad (\tilde{\omega}|\gamma_4^\dagger=0\,.\end{aligned} \tag{2.5}$$

Defining the contraction as

$$\overline{\alpha_1\,\alpha_2}\equiv(\tilde{\omega}|\,\alpha_1\alpha_2|\omega)\,, \tag{2.6}$$

one arrives immediately at the Wick-theorem for biorthogonal states,

$$\alpha_1\alpha_2=:\alpha_1\alpha_2:+\overline{\alpha_1\,\alpha_2}\,, \tag{2.7}$$

where α_1, α_2 may be any linear combination of $c^\dagger$, $\tilde{c}$ or $\gamma^\dagger, \tilde{\gamma}$ operators and : : defines the normal product with respect to the γ operators. Wick's-theorem is easily extended to products of more than two operators as given in textbooks except for the different definition of the contraction.

The operators $c^\dagger, \tilde{c}$ and $\gamma^\dagger, \tilde{\gamma}$ approach the usual ones for an orthonormal basis if the nuclei are infinitely apart (and the overlap vanishes). The u, v coefficients, by definition, describe the "superfluid" states of isolated nuclei and are thus time independent.

Now we decompose the Hamiltonian of Eq. (2.1),

$$\hat{H}(t)=\hat{H}_1+\hat{H}_2+\hat{H}_{\rm int}(t)\,, \tag{2.8}$$

where the Hamiltonians $\hat{H}_1$ and $\hat{H}_2$ are time independent and describe the two nuclei in their respective rest systems. The operator $\hat{H}_{\rm int}(t)$ results from the interaction between the nuclei and also contains inertial forces as well as nonorthogonality contributions. It vanishes if the nuclei are far apart. Furthermore one may decompose each term of (2.8) according to Wick's theorem to extract fully contracted as well as two- and four-operator parts [$\hat{h}$ is one of the operators on the right-hand side of (2.8)],

$$\hat{h}=\hat{H}^{(0)}+\hat{H}^{(2)}+\hat{H}^{(4)}\,. \tag{2.9}$$

It is immediately clear from these considerations that only $\hat{H}^{(2)}_{\rm int}(t)$ and $\hat{H}^{(4)}_{\rm int}(t)$ are able to transfer particles between nuclei. So it is our aim to extract these terms. Before stating the result of this somewhat lengthy but straightforward procedure (analogous to DH) we have to write down some definitions. The generalized Hartree-Fock and pairing potentials are given by

$$V_{3,4}=\sum_5 (\tilde{\psi}_3\tilde{\psi}_5|\bar{v}|\psi_4\psi_5)v_5{}^2\,, \tag{2.10}$$

$$\begin{aligned} W_{3,4}&=\frac{1}{2}\sum_5 (\tilde{\psi}_3\tilde{\psi}_4|\bar{v}|\psi_5\psi_{-5})u_5v_5\,,\\ \tilde{W}_{3,4}&=\frac{1}{2}\sum_5 (\tilde{\psi}_{-5}\tilde{\psi}_5|\bar{v}|\psi_3\psi_4)v_5u_5\,. \end{aligned} \tag{2.11}$$

As we want to separate time-dependent parts from time-independent ones, and since the nonorthogonality terms produce a part of the time dependence of the matrix elements we also have to single out the effects of nonorthogonality. According to Eq. (2.13) of DH we put {M is the overlap matrix $M=[(\psi_3|\psi_4)]$}

$$M^{-1}=1+\tilde{K}\,, \tag{2.12}$$

and write the dual states in the form

$$(\tilde{\psi}_3|=(\psi_3|+\sum_4 \tilde{K}_{3,4}(\psi_4|\,. \tag{2.13}$$

This leads in a natural way to a decomposition of the matrix elements

$$V_{3,4}=V^0_{3,4}+\bar{V}_{3,4}\,, \tag{2.14}$$

where the first term $V^0_{3,4}$ is obtained from $V_{3,4}$ by replacing $\bar{\psi}$ by ψ,

$$V^0_{3,4}=\sum_5 (\psi_3\psi_5|\bar{v}|\psi_4\psi_5)\,v_5{}^2\,, \tag{2.15}$$

and where the second term represents the nonorthogonality corrections contained in the matrix $\tilde{K}$:

$$\begin{aligned} \bar{V}_{3,4}=\sum_{5,6}\Big[&\tilde{K}_{3,6}(\psi_6\psi_5|\bar{v}|\psi_4\psi_5)+\tilde{K}_{5,6}(\psi_3\psi_6|\bar{v}|\psi_4\psi_5)\\ &+\sum_7 \tilde{K}_{3,6}\tilde{K}_{5,7}(\psi_6\psi_7|\bar{v}|\psi_4\psi_5)\Big]\,v_5{}^2\,. \end{aligned} \tag{2.15'}$$

In complete analogy we decompose the pairing potentials

$$\begin{aligned} W_{3,4}&=W^0_{3,4}+\bar{W}_{3,4}\,,\\ \tilde{W}_{3,4}&=\tilde{W}^0_{3,4}+\bar{\tilde{W}}_{3,4}\,, \end{aligned} \tag{2.16}$$

and the two-body matrix elements

$$(\tilde{\psi}_3\tilde{\psi}_4|\bar{v}|\psi_5\psi_6)=(\psi_3\psi_4|\bar{v}|\psi_5\psi_6)+\bar{\bar{v}}_{3,4,5,6}\,. \tag{2.17}$$

The quantities with the superscript 0's are defined as in the usual pairing theory and only quantities with a bar contain nonorthogonality contributions. The potentials V^0, W^0, $\tilde{W}^0$ contain summations over the single-particle states of both nuclei. Thus we divide these potentials into two parts referring to the two nuclei,

$$\begin{aligned} V^0_{3,4}&=V^0_{3,4}(1)+V^0_{3,4}(2)\,,\\ V^0_{3,4}(i)&=\sum_i (\psi_3\psi_i|\bar{v}|\psi_4\psi_i)v_i{}^2\,,\quad i=1,2\,. \end{aligned} \tag{2.18}$$

Analogous decompositions hold for the pairing potentials.

After the application of Wick's theorem, the Hamiltonian $\hat{H}_1$ of Eq. (2.8) has the form

$$\begin{aligned} \hat{H}_1=&\sum_1\left\{\left[\left(\psi_1\left|\frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m}\right|\psi_1\right)-\lambda_1+\tfrac{1}{2}V^0_{11}(1)\right]v_1{}^2+\tfrac{1}{2}W^0_{1,-1}(1)v_1u_1\right\}\\ &+\sum_{1,1'}\left\{\left[\left(\psi_1\left|\frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m}\right|\psi_{1'}\right)-\lambda_1\delta_{11'}+V^0_{11'}(1)\right]:c^\dagger_1\tilde{c}_{1'}:+\tfrac{1}{2}\left[W^0_{11'}(1):c^\dagger_1c^\dagger_{1'}:+\tilde{W}^0_{11'}(1):\tilde{c}_1\tilde{c}_{1'}:\right]\right\}\\ &+\frac{1}{4}\sum_{1,1',1'',1'''}(\psi_1\psi_{1'}|\bar{v}|\psi_{1''}\psi_{1'''}):c^\dagger_1c^\dagger_{1'}\tilde{c}_{1'''}\tilde{c}_{1''}:\,. \end{aligned} \tag{2.19}$$

An analogous expression holds for $\hat{H}_2$. As the wave functions in the matrix elements of (2.19) refer only to one nucleus, the operators $\hat{H}_1$ and $\hat{H}_2$ are time independent. If one takes into account

$$\left(\psi_1 \left| \frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m} \right| \psi_{1'}\right) = \left(\varphi_1 \left| \frac{\hat{p}^2}{2m} \right| \varphi_{1'}\right) \tag{2.20}$$

(the functions φ_1 are defined in the rest frame of nucleus 1), one sees that $\hat{H}_1$ and $\hat{H}_2$ describe two isolated nuclei in their rest systems. The determination of the basis is now clear. As in a usual BCS calculation one requires that the terms of $\hat{H}_1$ containing operators $\gamma^\dagger\gamma^\dagger$ and $\tilde{\gamma}\tilde{\gamma}$ must vanish. From this condition one determines self-consistently the single particle functions and u, v coefficients.[9] This is equivalent to the BCS variational ansatz, where the chemical potential λ is the Lagrange parameter which fixes the particle number.

Since we are interested in transfer processes, we determine $\hat{H}_{\rm int}(t)$ of Eq. (2.8) and arrive at the following expression:

$$\hat{H}_{\rm int}(t) = \hat{H}^{(0)}_{\rm int}(t) + \hat{H}^{(2)}_{\rm int}(t) + H^{(4)}_{\rm int}(t)\,, \tag{2.21}$$

where

$$\hat{H}^{(0)}_{\rm int}(t) = \sum_1 \left\{ \left[\sum_3 \tilde{K}_{1,3}(\psi_3 \left| \frac{[p-m\dot{R}_1(t)]^2}{2m} + m\ddot{R}_1(t)[\hat{x}-R_1(t)] \right| \psi_1) + \tfrac{1}{2} V^0_{11}(2) + \tfrac{1}{2}\overline{V}_{11} \right] v_1^{\,2} + \tfrac{1}{2}(W_{1-1}(2) + \overline{W}_{1-1}) v_1 u_1 \right\} + 1 \longleftrightarrow 2\,, \tag{2.22}$$

$$\begin{aligned}\hat{H}^{(2)}_{\rm int}(t) = {} & \sum_{1,1'} \left\{ \left[\sum_3 \tilde{K}_{1,3} \left(\psi_3 \left| \frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m} \right| \psi_{1'}\right) + V_{11'}(2) + \overline{V}_{11'} \right] :c_1^\dagger \tilde{c}_{1'}: \right. \\ & \left. + \tfrac{1}{2}\{[W^0_{11'}(2) + \overline{W}_{11'}]:c_1^\dagger c_{1'}^\dagger: + [W^{0\dagger}_{11'}(2) + \overline{\tilde{W}}_{11'}]:\tilde{c}_1\tilde{c}_{1'}:\} \right\} \\ & + \sum_{1,3} \left\{ \left[\left(\tilde{\psi}_1 \left| \frac{[\hat{p}-m\dot{R}_2(t)]^2}{2m} \right| \psi_2\right) + V_{1,2} \right] :c_1^\dagger \tilde{c}_2: + \tfrac{1}{2}\,[W_{1,2}:c_1^\dagger c_2^\dagger: + \tilde{W}_{12}:\tilde{c}_1\tilde{c}_2:] \right\} \\ & + 1 \longleftrightarrow 2 + \sum_{3,4} (\tilde{\psi}_3 | m\ddot{R}_4(t)[\hat{x}-R_4(t)] | \psi_4):c_3^\dagger \tilde{c}_4:\,,\end{aligned} \tag{2.23}$$

$$\begin{aligned}H^{(4)}_{\rm int}(t) = {} & \frac{1}{4} \sum_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} (\tilde{\psi}_{\alpha_1}\tilde{\psi}_{\alpha_2} | \bar{v} | \psi_{\alpha_3}\psi_{\alpha_4}):c^\dagger_{\alpha_1} c^\dagger_{\alpha_2} \tilde{c}_{\alpha_4}\tilde{c}_{\alpha_3}: \\ & + \frac{1}{4} \sum_{1,1',1'',1'''} \overline{v}_{11'1''1'''}:c_1^\dagger c_{1'}^\dagger \tilde{c}_{1'''}\tilde{c}_{1''}: + \frac{1}{4} \sum_{2,2',2'',2'''} \overline{v}_{22'2''2'''}:c_2^\dagger c_{2'}^\dagger \tilde{c}_{2'''}\tilde{c}_{2''}:\end{aligned} \tag{2.24}$$

The symbol $1 \longleftrightarrow 2$ means that one has to add all preceding lines with 1 and 2 interchanged. The sum over $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ is defined as a summation over all possible quadruplets of quantum numbers, where not all of them belong to the same nucleus. In deriving $\hat{H}_{\rm int}(t)$ the property

$$(\psi_3 | m\ddot{R}_3(t)[\hat{x}-R_3(t)] | \psi_3) = m\ddot{R}_3(t)(\varphi_3|\hat{x}|\varphi_3) = 0 \tag{2.25}$$

was used (φ_3 is the single-particle functions in the rest system).

These expressions for $\hat{H}_{\rm int}(t)$ seem to be rather complicated, but they have a simple physical meaning. First of all we have achieved the correct separation into time-dependent and time-independent parts as required by Eq. (2.8). Nonorthogonality contributions, as well as interactions between the nuclei and acceleration terms vanish for large distances. So $\hat{H}_{\rm int}(t)$ describes the perturbation exerted on the unperturbed single-particle states by the reaction partner in a nucleus-nucleus collision and therefore must vanish under the same condition. The c-number part $\hat{H}^{(0)}_{\rm int}(t)$ gives the change of the energy of the system if both nuclei remain in their ground states during the collision. The operator $\hat{H}^{(2)}_{\rm int}(t)$ describes quasiparticle excitations in the nuclei induced by nonorthogonality and by the Hartree-Fock potential of the reaction partner. It also contains transfer terms which will be discussed in the next section. Furthermore, the acceleration term contributes to quasiparticle excitation and transfer. The first

term of the operator $\hat{H}^{(4)}_{\rm int}(t)$ is a pure transfer operator because the wave functions in the matrix element must not belong all to the same nucleus. The nonorthogonality terms in $\hat{H}^{(4)}_{\rm int}(t)$ induce only intrinsic excitations. The transfer operators are investigated in the next section.

III. TRANSFER

In principle one should solve the full time-dependent Schrödinger equation [Eq. (4.9) of DH] in the space of eigenstates of $\hat{H}_1+\hat{H}_2$ to calculate the two-nucleon transfer amplitudes. We solve this equation only up to second order as we want to distinguish between successive and simultaneous transfer amplitudes. The amplitudes in first and second order for a transition from state $|\Psi_\alpha\rangle$ to $\langle\tilde{\Psi}_\delta|$ are

$$f^{(1)}_\delta(t)=-\frac{i}{\hbar}\int_{-\infty}^{t}dt'\,e^{(i/\hbar)(E_\delta-E_\alpha)t'}\langle\tilde{\Psi}_\delta|\hat{H}_{\rm int}(t')|\Psi_\alpha\rangle \tag{3.1}$$

$$f^{(2)}_\delta(t)=-\frac{1}{\hbar^2}\sum_\beta\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''\langle\tilde{\Psi}_\delta|\hat{H}_{\rm int}(t')|\Psi_\beta\rangle \times e^{(i/\hbar)(E_\delta-E_\beta)t'}e^{(i/\hbar)(E_\beta-E_\alpha)t''}\langle\tilde{\Psi}_\beta|\hat{H}_{\rm int}(t'')|\Psi_\alpha\rangle. \tag{3.2}$$

The states $|\Psi_\alpha\rangle,\ldots$ are full many-body eigenstates of $\hat{H}_1+\hat{H}_2$ with eigenenergies $E_\alpha,\ldots$. The states $\langle\tilde{\Psi}_\alpha|,\ldots$ are the dual states. In the case of two-nucleon transfer these states will be defined in (3.7), (3.8), and (3.29). The probability of finding the system in channel δ if it had been in α at the beginning is up to second order given by

$$P_{\delta\alpha}=\lim_{t\to\infty}|f^{(1)}_\delta(t)+f^{(2)}_\delta(t)|^2. \tag{3.3}$$

We have to specify the states of the unperturbed system. As our channel wave functions must have good particle number to treat the transfer properly and since correlations due to superfluidity should also be taken into account, we must take particle number projected BCS states for the description of the nuclei in their ground states. Using these channel functions and their duals the probability of transfer is given immediately by Eq. (3.3).

Following Refs. 10 and 11 the particle number projected ground state of nucleus 1 and its dual is given by

$$|\omega_1,n_1)=C_{n_1}\oint dz\,z^{-n_1-1}\prod_{l>0}(u_l+v_l z c^\dagger_l c^\dagger_{-l})|0),$$
$$(\tilde{\omega}_1,n_1|=-C^*_{n_1}\oint dz\,z^{-n_1-1}\times(0|\prod_{l>0}(u_l+v_l z\tilde{c}_{-l}\tilde{c}_l). \tag{3.4}$$

The number of pairs is n_1 and C_{n_1} is defined by the normalization condition $(\tilde{\omega}_1,n_1|\omega_1,n_1)=1$, which yields

$$|C_{n_1}|^2=\frac{1}{4\pi^2\,{}_1R^0_0}. \tag{3.5}$$

The so called residuum integrals are given by[10]

$${}_1R^N_n(l_1,\ldots,l_N)=\frac{1}{2\pi i}\oint dz\,z^{-(n_1-n)-1}\times\prod_{\substack{l>0\\ l\neq|l_1|,\ldots,|l_N|}}(u_l^2+v_l^2 z) \tag{3.6}$$

Similar expressions hold for nucleus 2. The free phase of the coefficients C does not raise any difficulty because it affects the different transfer amplitudes in the same way as will be seen later

To be specific, we now consider a collision of two nuclei, where one nucleon pair is transferred from nucleus 1 to 2. The outgoing nuclei should also be in their ground states; so the channel wave functions for initial and final configurations are

$$|\Psi_\alpha\rangle=|\omega_1,n_1)|\omega_2,n_2), \tag{3.7}$$
$$\langle\tilde{\Psi}_\delta|=(\tilde{\omega}_2,n_2+1|(\tilde{\omega}_1;n_1-1|. \tag{3.8}$$

Consider the one-step amplitude from Eq. (3.1). Only the $\hat{H}^{(4)}_{\rm int}(t)$ part of the complete interaction Hamiltonian (2.21) may give contributions to simultaneous transfer. As we restrict our considerations to ground-state transfers only the following operator can contribute:

$$\hat{O}(t)=\frac{1}{4}\sum_{1,2}(\tilde{\psi}_2\tilde{\psi}_{-2}|\bar{v}|\psi_1\psi_{-1}):c^\dagger_2c^\dagger_{-2}\tilde{c}_{-1}\tilde{c}_1:. \tag{3.9}$$

Using particle number projected states, real particles are transferred, so we apply in (3.9) Wick's theorem backwards,

$$\hat{O}(t)=\frac{1}{4}\sum_{1,2}(\tilde{\psi}_2\tilde{\psi}_{-2}|\bar{v}|\psi_1\psi_{-1})c^\dagger_2c^\dagger_{-2}\tilde{c}_{-1}\tilde{c}_1 -\frac{1}{2}\sum_1 W_{1-1}(2):\tilde{c}_{-1}\tilde{c}_1:-\frac{1}{2}\sum_2 W_{2-2}(1):c^\dagger_2c^\dagger_{-2}: -\frac{1}{4}\sum_{1,2}(\tilde{\psi}_2\tilde{\psi}_{-2}|\bar{v}|\psi_1\psi_{-1})\overbrace{c^\dagger_2c^\dagger_{-2}\tilde{c}_{-1}\tilde{c}_1}. \tag{3.10}$$

The first term of this equation is that part of $\hat{H}^{(4)}_{\rm int}(t)$ which includes simultaneous transfer of a nucleon pair. So we calculate the tranfer matrix element of the first order amplitude [see Eq. (3.1)]

$$\langle\tilde{\Psi}_\delta|\hat{H}_{\rm int}(t)|\Psi_\alpha\rangle=\sum_{1,2>0}(\tilde{\psi}_2\tilde{\psi}_{-2}|\bar{v}|\psi_1\psi_{-1}) \times(\bar{\omega}_2,n_2+1|c_2^\dagger c_{-2}^\dagger|\omega_2,n_2) \times(\bar{\omega}_1,n_1-1|\bar{c}_{-1}\bar{c}_1|\omega_1,n_2) \tag{3.11}$$

After some algebra one arrives at the expression

$$\langle\tilde{\Psi}_\delta|\hat{H}_{\rm int}(t)|\Psi_\alpha\rangle=\sum_{1,2>0}(\tilde{\psi}_2\tilde{\psi}_{-2}|\bar{v}|\psi_1\psi_{-1})u_1v_1u_2v_2 \times16\pi^4C^*_{n_1-1}C_{n_1}C^*_{n_2+1} \times C_{n_2 1}R_1^1(1)_2R_0^1(2). \tag{3.12}$$

Within the framework of the formalism this expression is still exact. [The approximations used up to now are (1) the semiclassical approach, (2) the restriction to bound single-particle states.] We apply now two other approximations. Using the saddle point method for the evaluation of the residuum integrals one finds that all of them are equal.[10] This is equivalent to a change from an FBCS to a usual BCS calculation. One has then to take also the BCS values of the u,v coefficients in (3.12). As shown in Ref. 11 this leads only to very negligible changes in the sums over u,v pairs of Eq. (3.12). We get therefore,

$$16\pi^4C^*_{n_1-1}C_{n_1}C^*_{n_2+1}C_{n_2 1}R_1^1(1)_2R_0^1(2)\approx1. \tag{3.13}$$

It should be noted that this approximation does not imply that we omit particle number conservation.

For the moment we neglect nonorthogonality contributions

$$(\tilde{\psi}|\rightarrow(\psi|. \tag{3.14}$$

In Sec. IV it will be shown that below the interaction barrier the transfer amplitudes are not drastically changed by nonorthogonality effects. Introducing the generalized Galilean transformation into the moving frame of the nucleus [see Eq. (2.3) of DH]

$$\hat{G}_3=e^{(i/\hbar)\gamma_3(t)}e^{(i/\hbar)m\dot{R}_3(t)\hat{x}}e^{-(i/\hbar)R_3(t)\hat{p}}, \tag{3.15}$$

we write the single-particle states in BCS phase convention[12]

$$\begin{aligned}|\psi_1\rangle&=\hat{G}_1|j_1m_1\rangle_{\rm BCS}=\hat{G}_1|j_1m_1\rangle_{\rm CS}\\ |\psi_{-1}\rangle&=\hat{G}_1|j_1-m_1\rangle_{\rm BCS}\\ &=\hat{G}_1(-)^{j_1-m_1+l_1}|j_1-m_1\rangle_{\rm CS},\quad m_1>0\end{aligned} \tag{3.16}$$

where $|\ \rangle_{\rm CS}$ means a state in Condon-Shortley phases. After some algebra Eq. (3.12) can be written

$$\langle\tilde{\Psi}_\delta|H_{\rm int}(t)|\Psi_\alpha\rangle=\sum_{j_1j_2}u_{j_1}v_{j_1}u_{j_2}v_{j_2}\frac{2j_1+1}{2}\frac{2j_2+1}{2}L_{j_2j_1}(t),$$

$$L_{j_2j_1}(t)=\frac{2(-)^{l_1+l_2}}{[(2j_1+1)(2j_2+1)]^{1/2}} \times{}_{\rm CS}\langle(j_2j_2)0|\hat{G}_2^\dagger\hat{G}_2^\dagger v\hat{G}_1\hat{G}_1|(j_1j_1)0\rangle_{\rm CS}. \tag{3.17}$$

The two-particle states $|(j_1j_1)0\rangle_{\rm CS}$ are coupled to $J=0,M=0$. The pairs of $\hat{G}$ operators act on the coordinates of the two nucleons. The weighting factors are selected in analogy to the gap equation of ordinary pairing theory:

$$\Delta=G\sum_j u_jv_j\frac{2j+1}{2}. \tag{3.18}$$

Here the G denotes the pairing constant.

If the matrix elements $L_{j_2j_1}(t)$ have all the same phase, the transfer amplitude gains an enhancement of order $(\Delta_1/G_1)\cdot(\Delta_2/G_2)$ due to the coherence of the sum (3.17). Whether this is true will be investigated numerically. This enhancement would be a simple extension of (p,t) and (t,p) results.[13]

The final result for the first-order transfer amplitude is

$$f_\delta^{(1)}=-i\sum_{j_1j_2}u_{j_1}(n_1-1)u_{j_2}(n_2)v_{j_1}(n_1) \times v_{j_2}(n_2+1)\frac{2j_1+1}{2}\frac{2j_2+1}{2}N_{j_2j_1}, \tag{3.19}$$

$$N_{j_1j_2}=\int_{-\infty}^{+\infty}\frac{dt}{\hbar}e^{(i/\hbar)(E_\delta-E_\alpha)t}L_{j_2j_1}(t).$$

Here we show explicitly to which nucleus with n_1,n_2 pairs the coefficients u,v refer. Finally it should be noted that the time dependence of the L matrix comes from the motion of the nuclei on the trajectories.

Now we calculate the second-order amplitude of Eq. (3.2). We include in the perturbing Hamiltonian only terms from $H^{(2)}_{\rm int}(t)$ [see Eq. (2.23)] and omit second-order terms from $H^{(4)}_{\rm int}(t)$ which means that we omit pair breaking processes in the intermediate channel. The only terms which may contribute to this process are now collected in the "active Hamiltonian":

$$H^{(2)}_{\rm act}(t)=\hat{H}_a(t)+\hat{H}_b(t)+\hat{H}_c(t), \tag{3.20}$$

with the definitions

$$\hat{H}_a(t)=\sum_{1,2}\left[\left(\tilde{\psi}_2\left|\frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m}\right|\psi_1\right)+V_{2,1}\right]:c_2^\dagger\bar{c}_1:, \tag{3.21}$$

$$H_b(t)=\frac{1}{2}\sum_{1,1'}[W^0_{11'}(2)+\bar{\bar{W}}_{1,1'}]:\tilde{c}_1\tilde{c}_{1'}:, \tag{3.22}$$

$$H_c(t)=\frac{1}{2}\sum_{2,2'}[W^0_{2,2'}(1)+\bar{W}_{2,2'}]:c^\dagger_2c^\dagger_{2'}:. \tag{3.23}$$

The acceleration term is omitted and will be discussed in the next section. The operators $\hat{H}_b(t),\hat{H}_c(t)$ cancel against the two operator terms of (3.10) if one applies also Wick's theorem (in the backward direction) on the nonorthogonality part of $\hat{H}^{(4)}_{\rm int}(t)$ [see Eq. (2.24)]. Consequently only the generalized Hartree-Fock potential contributes to the successive transfer, which comes from the fact that real particles and not Bogoliubov quasiparticles are transferred. Thus unphysical processes where two particles are destroyed or created in the intermediate state cannot occur. If we define the square bracket of Eq. (3.21) as $T_{2,1}(t)$ we have

$$\hat{H}^{(2)}_{\rm act}(t)=\sum_{1,2}T_{2,1}(t)c^\dagger_2\tilde{c}_1. \tag{3.24}$$

One can neglect normal product dots, because the operators refer to different nuclei; so the contractions vanish. We let this operator act on the initial and final channel states (3.7) and (3.8) which yields:

$$\hat{H}^{(2)}_{\rm act}(t)\,|\Psi_\alpha\rangle=-\sum_{1,2}T_{2,1}(t)C_{n_1}C_{n_2}v_1u_2\,c^\dagger_{-1}|\,\omega_1,n_1-1,1)\times c^\dagger_2|\,\omega_2,n_2,2), \tag{3.25}$$

$$\langle\tilde{\Psi}_\delta|\hat{H}^{(2)}_{\rm act}(t)=\sum_{1,2}T_{2,1}(t)C^*_{n_2+1}C^*_{n_1-1}v_2u_1\times(\bar{\omega}_2,n_2,2|\tilde{c}_{-2}(\bar{\omega}_1,n_1-1,1|\tilde{c}_1, \tag{3.26}$$

where we used the following definitions:

$$|\,\omega_3,n_3,3)\equiv\oint dz\,z^{-n_3-1}\times\prod_{\substack{3'>0\\3'\neq|3|}}(u_{3'}+v_{3'}zc^\dagger_{3'}c^\dagger_{-3'})|0), \tag{3.27}$$

$$(\bar{\omega}_3,n_3,3|\equiv-\oint dz\,z^{-n_3-1}\times(0|\prod_{\substack{3'>0\\3'\neq|3|}}(u_{3'}+v_{3'}z\tilde{c}_{-3'}\tilde{c}_{3'}). \tag{3.28}$$

From (3.25) and (3.26) the structure of the intermediate states is clear: one pair level is blocked in either nucleus and one single unpaired nucleon is moving in this orbit. If one normalizes these states correctly, they form a biorthogonal, complete set in the space of allowed intermediate states,

$$|\Psi_\beta\rangle=\frac{1}{4\pi^2[{}_1R_1{}^1(1)_2R_0{}^1(2)]^{1/2}}c^\dagger_1|\,\omega_1,n_2-1,1)\times c^\dagger_2|\,\omega_2,n_2,2), \tag{3.29}$$

$$\langle\tilde{\Psi}_\beta|=\frac{1}{4\pi^2[{}_1R_1{}^1(1)_2R_0{}^1(2)]^{1/2}}(\bar{\omega}_2,n_2,2|\tilde{c}_2\times(\bar{\omega}_1,n_1-1,1|\tilde{c}_1$$

$$\langle\tilde{\Psi}_\beta|\Psi_{\beta'}\rangle=\delta_{\beta\beta'}$$

where β stands for a pair of indices 1, 2.

Inserting these results into Eq. (3.2) we get the amplitude for successive transfer,

$$f^{(2)}_\delta=-\frac{1}{\hbar^2}\int_{-\infty}^{+\infty}dt\int_{-\infty}^{t}dt'\sum_{1,2}T_{2,1}(t)T_{-2,-1}(t')v_1u_1v_2u_2\,e^{(i/\hbar)(E_\delta-E_\beta)t}e^{(i/\hbar)(E_\beta-E_\alpha)t'}16\pi^4C^*_{n_2+1}C^*_{n_1-1}C_{n_2}C_{n_1}\times{}_1R_1{}^1(1)_2R_0{}^1(2). \tag{3.30}$$

The question arises, whether in this formula the restriction to bound single-particle states in our basis and therefore the restriction in the set of allowed intermediate states raises difficulties. This is not so, as can be seen immediately. As we consider ground-state transfers, the second nucleon in a successive transfer must be put into the same level as the first one to reach again a completely paired state. If this level is very high, this state contains a highly excited pair and has therefore very little overlap with the ground state. Formally this is exhibited by the fact that the products uv in (3.30) are very small for single-particle states far from the Fermi energy. So continuum states will not contribute.

To investigate more closely the matrix-element $T_{2,1}(t)$ we define

$$(\tilde{\psi}_2|\,\hat{V}_1\,|\,\psi_1)\equiv\sum_{1'}(\tilde{\psi}_2\tilde{\psi}_{1'}|\bar{v}|\psi_1\psi_{1'})v_{1'}{}^2, \tag{3.31}$$

and analogously $\hat{V}_2$. Then $T_{2,1}(t)$ may be written as

$$T_{2,1}(t)=\left(\tilde{\psi}_2\left|\frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m}+\hat{V}_1+\hat{V}_2\right|\psi_2\right). \tag{3.32}$$

Now we omit nonorthogonality corrections which imply the replacement (3.14) in $\hat{V}_1$, $\hat{V}_2$, and $T_{2,1}(t)$.

The single-particle wave functions satisfy a Hartree-Fock equation in their rest frames:

$$\left(\frac{\hat{p}^2}{2m}+\hat{\bar{V}}_1\right)\Big|\varphi_1\Big)=\epsilon_1|\varphi_1). \tag{3.33}$$

The quantities $|\varphi_1)$ and $\hat{V}_1$ are given by

$$\begin{aligned}|\varphi_1)&=\hat{G}_1^{\dagger}|\psi_1),\\ \hat{\bar{V}}_1&=\hat{G}_1^{\dagger}\hat{V}_1\hat{G}_1.\end{aligned} \tag{3.34}$$

Transforming into the moving frame we have

$$\left(\frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m}+\hat{V}_1\right)|\psi_1)=\epsilon_1|\psi_1). \tag{3.35}$$

In zeroth order in the overlap which corresponds to the neglect of nonorthogonality effects we obtain

$$\left(\psi_2\left|\frac{[\hat{p}-m\dot{R}_1(t)]^2}{2m}+\hat{V}_1\right|\psi_1\right)=\epsilon_1(\psi_2|\psi_1)=0, \tag{3.36}$$

and arrive at the simplified transfer matrix element

$$T_{2,1}(t)=(\psi_2|\hat{V}_2|\psi_1). \tag{3.37}$$

The change of this result due to nonorthogonality corrections up to first order in the overlap is discussed in Appendix A. Furthermore we use approximation (3.13) and neglect in the Galilean transformations $\hat{G}$, the shift operator in momentum space. So we work in the no-recoil approximation which is discussed in the next section. Taking $\hat{\bar{V}}_2$ as local and using the time-reversal properties of the BCS states we find the final formula for the amplitude of successive transfer:

$$f_{\delta}^{(2)}=-\sum_{j_1j_2}u_{j_1}(n_1-1)u_{j_2}(n_2)v_{j_1}(n_1)v_{j_2}(n_2+1)\frac{2j_1+1}{2}\frac{2j_2+1}{2}E_{j_2j_1}. \tag{3.38}$$

The $E_{j_2j_1}$ matrix plays the same role as $L_{j_2j_1}$ in the simultaneous transfer (3.17). It is defined by

$$\begin{aligned}E_{j_2j_1}=\frac{4}{(2j_1+1)(2j_2+1)}\sum_{\substack{m_1>0\\ m_2\gtrless 0}}\Bigg(&\int_{-\infty}^{+\infty}\frac{dt}{\hbar}e^{i\omega_{\delta\beta}t}e^{-(i/\hbar)\bar{\gamma}(t)}{}_{\rm CS}\langle j_2m_2|\hat{\bar{V}}_2e^{-(i/\hbar)r(t)\hat{p}}|\,j_1m_1\rangle_{\rm CS}^{*}\\ &\times\int_{-\infty}^{t}\frac{dt'}{\hbar}e^{i\omega_{\beta\alpha}t'}e^{-(i/\hbar)\bar{\gamma}(t')}{}_{\rm CS}\langle j_2m_2|\hat{\bar{V}}_2e^{-(i/\hbar)r(t')\hat{p}}|\,j_1m_1\rangle_{\rm CS}\\ &+\int_{-\infty}^{+\infty}\frac{dt}{\hbar}e^{i\omega_{\delta\beta}t}e^{-(i/\hbar)\bar{\gamma}(t)}{}_{\rm CS}\langle j_2m_2|\hat{\bar{V}}_2e^{-(i/\hbar)r(t)\hat{p}}|j_1m_1\rangle_{\rm CS}\\ &\times\int_{-\infty}^{t}\frac{dt'}{\hbar}e^{i\omega_{\beta\alpha}t'}e^{-(i/\hbar)\bar{\gamma}(t')}{}_{\rm CS}\langle j_2m_2|\hat{\bar{V}}_2e^{-(i/\hbar)r(t')\hat{p}}|j_1m_1\rangle_{\rm CS}^{*}\Bigg),\end{aligned} \tag{3.39}$$

where we have introduced the quantities

$$\omega_{\delta\beta}=\frac{E_\delta-E_\beta}{\hbar}\qquad\omega_{\beta\alpha}=\frac{E_\beta-E_\alpha}{\hbar}$$

$$r(t)=R_1(t)-R_2(t)$$

$$\bar{\gamma}(t)=\gamma_1(t)-\gamma_2(t)$$

[see Eq. (3.15) and also (2.3) of DH].

If all E matrix elements have similar phases we have the same enhancement as in the simultaneous transfer. The explicit expressions for the $E_{j_2j_1}$ and $L_{j_2j_1}$ matrix elements are given in Ref. 14.

Several approximations have been made to calculate the amplitudes (3.19) and (3.38): In both cases we neglect in the Galilean transformation (3.15) the shift operators in momentum space. This means neglecting recoil effects in the two-nucleon transfer matrix elements (form factors). Furthermore we omit pair breaking in the successive transfer as we do not include $H_{\rm int}^{(4)}(t)$ in the second-order matrix element. This operator would produce broken pairs in one nucleus as intermediate state in the two step transfer. We discuss these approximations in the next section.

It should be noted that there exists an alternate formalism for the description of two nucleon transfer by Götz *et al.*[7] The reader who is interested in connections and differences between the two formulations may consult Appendix B.

IV. NUMERICAL CALCULATIONS AND RESULTS

A. Potentials and wave functions

As an example, the formalism is applied to the ^{120}Sn(^{120}Sn, ^{118}Sn)^{122}Sn reaction. The tin isotopes are single-closed-shell nuclei because the protons form a closed-shell configuration. This implies spherical symmetry which leads to a high degeneracy of the neutron levels at the Fermi energy. Therefore tin nuclei are very good superconductors. In the general case, where neutron and proton shells are not closed, one finds de-

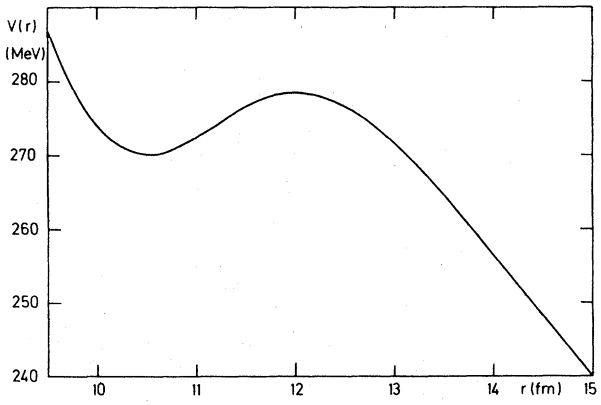

FIG. 1. The nucleus-nucleus potential as a function of the relative distance r between the two ^{120}Sn nuclei. At $r=12$ fm lies the barrier of 278.4 MeV. The potential was calculated in the EDF formalism (see Refs. 15 and 16) and contains the Coulomb repulsion assuming two uniformly charged spheres of radius $R=1.20\times(A^{1/3}+A^{1/3})$ fm.

formed ground states. Then one has a smaller density of single-particle states at the Fermi energy and superconductivity is less marked for these systems.

Considering collisions below the interaction barrier we use a classical orbit determined by the internuclear Coulomb and strong interactions. Because very little mass, energy, and no charge is transferred we neglect recoil effects in the orbital motion. The nucleus-nucleus potential was determined from the energy-density formalism (EDF) (Ref. 15) in the parametrization of Ref. 16. It is very similar to the proximity potential and is given by the expression

$$V_N(r)=-\frac{A_1^{1/3}A_2^{1/3}}{A_1^{1/3}+A_2^{1/3}}V_0\exp(-0.27\cdot s^2),\quad s\geq 0$$
$$s=r-1.0(A_1^{1/3}+A_2^{1/3}),\quad V_0=30\text{ MeV} \tag{4.1}$$

where A_1, A_2 are the mass numbers of the colliding nuclei.

As can be seen from Fig. 1 the full potential between two tin nuclei shows an interaction barrier at a distance of 12.0 fm between the centers of the nuclei and at a potential energy of 278.4 MeV. It has a shallow pocket because we are just in the region of Z_1, Z_2 values where the pocket vanishes. Because in experiment the barrier is not very well known, the relevant quantity is not the absolute energy of relative motion in the collision $E_{\text{c.m.}}$ but its difference to the barrier top $E_{\text{barrier}}-E_{\text{c.m.}}$. The trajectories have been calculated numerically in this potential.

The simultaneous transfer of Eqs. (3.17) and (3.19) is due to the nucleon-nucleon force. We take an effective interaction of Gaussian form as used in shell-model calculations, especially the Ferrel-Visscher force[17]:

$$v(r_{ij})=\exp\left[-\left(\frac{r_{ij}}{\mu}\right)^2\right](V_e\hat{P}_{es}+V_o\hat{P}_{ot})$$
$$V_e=-33\text{ MeV},\quad V_o=19\text{ MeV},\quad \mu=1.732\text{ fm}. \tag{4.2}$$

Here the $\hat{P}$ operators project on the even-singlet and odd-triplet states. Arvieu[18] showed that by calculations in the tin region that forces with $|V_e\mu^2|=110$ MeV fm^2 yield optimum fits. The Ferrel-Visscher force underestimates this by 11 MeV fm^2. The singlet part seems to be rather well established, whereas the contributions from the triplet part seem to be very uncertain (see listings in Ref. 19). The transfer calculation showed very little contributions from the triplet force, so this uncertainty is not important.

In the simultaneous amplitude, harmonic-oscillator (HO) wave functions are used for the bound particles for simplicity of the calculation. According to Moszkowski's prescription[20] the oscillator parameter was $b=2.24$ fm. The influence of the wrong asymptotic behavior of these functions will be discussed in Sec. IV B. In the successive amplitude, however, wave functions of a finite Woods-Saxon well, expanded in HO functions, were chosen. Using ten expansion coefficients the asymptotic behavior was correct up to nearly twice the nuclear radius. In the parametrization of Ref. 21 the Woods-Saxon potential reads

$$V(r)=-V_0[\mathfrak{F}(r)-\lambda g(r)\vec{\sigma}\vec{l}],$$
$$\mathfrak{F}(r)=\left[1+\exp\left(\frac{r-r_0A^{1/3}}{\alpha}\right)\right]^{-1},$$
$$g(r)=\left(\frac{\hbar}{2Mc}\right)^2\frac{1}{r}\frac{d\mathfrak{F}(r)}{dr} \tag{4.3}$$
$$r_0=1.25\text{ fm},\quad \alpha=0.65\text{ fm},\quad \lambda=25$$

The depth V_0 was adjusted to reproduce the neutron binding energy (well-depth method). Since Sn nuclei are single-closed-shell nuclei only neutrons contribute to superconductivity, and we have to take into account the single-particle states being filled in this region, i.e., the $3s_{1/2}, 2d_{3/2}, 2d_{5/2}, 1g_{7/2}, 1h_{11/2}$ levels. In the intermediate state of the successive transfer we have one extra nucleon outside the superconducting core. The binding energy of these neutrons is just the binding energy of the core plus the excitation energy of the one-quasiparticle state. These excitation energies were taken from Ref. 21, where they were calculated according to the prescription:

$$E_{l_j}=\frac{\sum_i S_i(l_j)E_i(l_j)}{\sum_i S_i(l_j)}. \tag{4.4}$$

The sum runs over the experimental states of the

^{119}Sn and ^{121}Sn nuclei over which the strength of the quasiparticle state distributes. $S_i(l_j)$ are spectroscopic factors in a (d,p) reaction. In the even-even isotopes the simple one neutron separation energy was used to determine the wave functions. In the energy exponentials of (3.39) the energies of Eq. (4.4) were also taken into account.

The transferring potential, which is in principle the Hartree-Fock potential, was approximated by a Woods-Saxon well with parameter set (4.2) except for $\lambda = 0$ and a depth of $V_0 = 50$ MeV. Similar parameters were used in Ref. 21 as real part in single nucleon optical potentials. The u, v coefficients in Eqs. (3.19) and (3.38) have been taken from Kisslinger and Sorensen.[22] The values of Arvieu[18] would not change the results.

Finally we neglect the acceleration term in Eq. (2.23), because the inertial force per nucleon, which is exerted by the motion on the trajectory, is much smaller than the nuclear interaction, which is felt by the nucleon in the contact region where the transfer takes place.

B. Transfer results

In both transfer amplitudes, recoil effects were neglected which implies local transfer matrix elements (form factors). The inclusion of recoil would lead to a scaling constant in the relative position vector as well as to a change of angular momentum selection rules.[7,23] The scaling constant deviates from 1 by the ratio of the transferred mass to the mass of the nucleus; so this is a small quantity in our reaction. Secondly one may estimate easily the angular momentum mismatch which would destroy the coherence in ground state to ground state transfer. It starts to be important for scattering angles below 150° for our reaction conditions. After all for scattering close to the barrier and around 180° recoil effects are very small (for an extensive investigation see Ref. 23).

As stated in Sec. IV A the simultaneous transfer amplitude was calculated using HO wave functions. These exhibit wrong behavior in the asymptotic region outside the nucleus. Several methods have been developed to remedy this.[24] For numerical simplicity we proceed as follows: We utilize the fact that we know the successive amplitude calculated with Woods-Saxon wave functions (WS) which have the correct asymptotic behavior. For fixed relative kinetic energy $E_{c.m.}$ and fixed scattering angle $\theta_{c.m.}$ we calculate the simultaneous and successive amplitudes with pure HO wave functions for various oscillator parameters b. The amplitudes then change strongly due to the varying range of the HO wave functions. The ratio between the amplitudes, however, remains constant within narrow limits. This reflects the fact that the matrix elements for the two processes depend on the asymptotic behavior of the wave functions in the same way. Calculating this ratio for every $\theta_{c.m.}$ and $E_{c.m.}$ and assuming that this ratio also holds if one would calculate both simultaneous and successive amplitudes with WS wave functions, one arrives at a corrected simultaneous amplitude from the knowledge of the successive amplitude for WS functions. In every case the simultaneous amplitude is much smaller than the successive one.

In Fig. 2 the absolute values of the corrected amplitude of simultaneous transfer ($f_c^{(1)}$) together with the successive amplitude ($f^{(2)}$) is given as a function of the kinetic energy of relative motion in

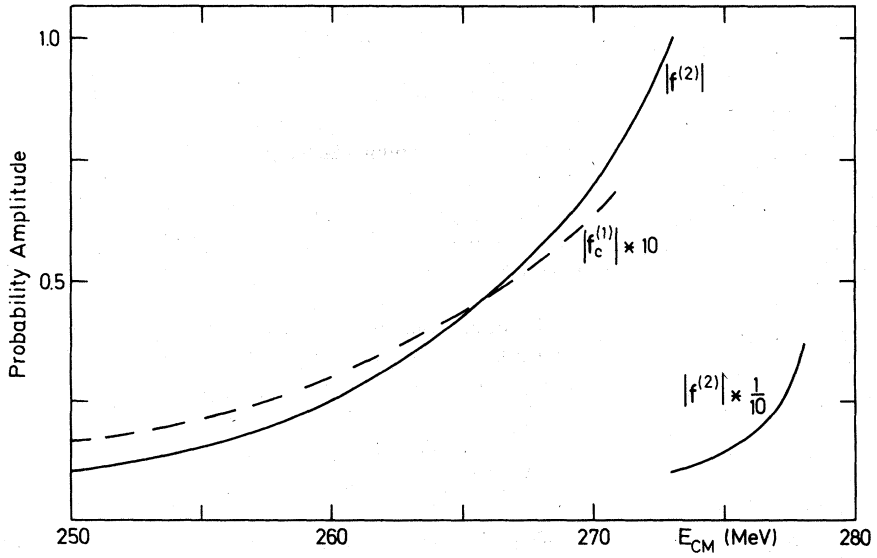

FIG. 2. Absolute values of transfer amplitudes for simultaneous ($f_c^{(1)}$) and successive ($f^{(2)}$) pair transfer as a function of the kinetic energy of relative motion of the two ^{120}Sn nuclei at infinity. The c.m.-scattering angle is fixed at 180°.

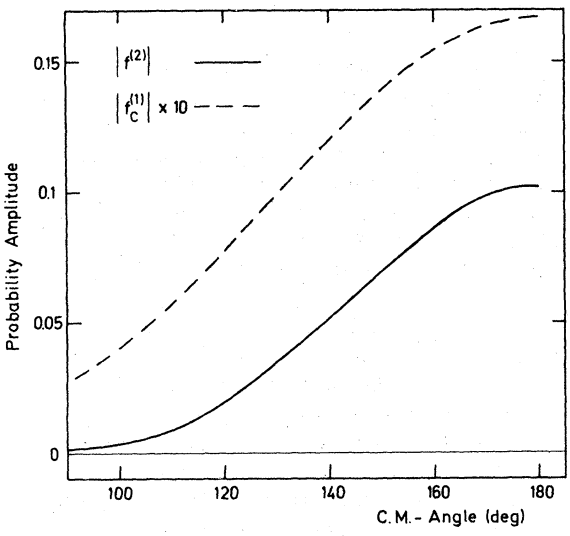

FIG. 3. Like Fig. 2 as a function of the c.m.-scattering angle at fixed relative energy $E_{c.m.} = 250$ MeV.

the c.m. system $E_{c.m.}$. The scattering angle is fixed to $\theta_{c.m.} = 180°$. At about $E_{c.m.} = 273$ MeV (about 5.4 MeV below the barrier) perturbation theory starts to diverge, i.e., the transfer probability becomes larger than one for the successive amplitude. This breakdown of perturbation theory means that the probability for the two pair transfer is not simply given by the square of the one pair transfer probability. Higher order transfers must be very important but can only be calculated in a coupled-channel calculation. The solutions of these calculations would have the properties as deduced from simple models[5] if pair breaking processes are sufficiently weak. Under the same condition it can be said according to our calculation that multiple pair transfer (at least two pair transfer) is possible at $E_{c.m.} = 273$ MeV due to successive transfer mechanism. The simultaneous transfer is much smaller and would not allow multiple pair transfer.

The phase of $f_c^{(1)}$ is purely positive imaginary while the phase of $f^{(2)}$ varies little between 108° and 115° (depending on $\theta_{c.m.}$ and $E_{c.m.}$) which yields a somewhat constructive interference between the amplitudes. Because in Eq. (3.19) all $L_{j_2 j_1}$ have the same sign and in (3.38) all $E_{j_2 j_1}$ have very similar phases, both processes gain the same enhancement by superconductivity. The same result was found by Hashimoto and Kawai for (p,t) reactions.[25] In our calculation the enhancement factor ranges from 100 to 1000 in the cross section.

In Fig. 3 the absolute values of the amplitudes are given as a function of the c.m. scattering angle at the fixed energy $E_{c.m.} = 250$ MeV. Below 150° the amplitudes should decrease even more rapidly due to angular momentum mismatch.

C. Dependence on the parameters

Checking the reliability of the results, the parameters have been changed to see the effect on the amplitudes. A change of the range parameter of the Ferrel-Visscher force from $\mu = 1.732$ to 2.0 fm increases the simultaneous amplitude by about 25%. The change in the transferring potential to parameters $r_0 = 1.27$ fm or $\alpha = 0.75$ fm [see (4.3)] yields an increase of the successive amplitude by about 17%. These parameters do not influence the results strongly. The biggest uncertainty, however, comes from the single-particle wave functions themselves. If one increases the magnitude of the exponential tail by some factor, the amplitudes are increased by the same factor to the fourth power as can be seen from (3.12) and (3.39). Calculating the wave functions with respect to a Woods-Saxon well with parameters $r_0 = 1.3$ fm and $\alpha = 0.65$ fm the amplitude for successive transfer increases by about a factor 1.7 to 2. The ratio of simultaneous to successive transfer should not be changed (see discussion in Sec. IV B).

D. Nonorthogonality effects

To estimate nonorthogonality numerically two calculations have been done: the norm (A5) and the nonorthogonality contribution (A7) to successive transfer have been evaluated. All quantities were calculated for numerical simplification at the distance of closest approach for head-on collisions at $E_{c.m.} = 250$ and 273 MeV. Because the correction term should be integrated over the whole trajectory and nonorthogonality corrections are largest at the distance of closest approach our results are necessarily upper limits for the true corrections.

The norm (A5) gives results between 0.01 and 0.06 for $E_{c.m.} = 250$ MeV and between 0.03 and 0.1 for $E_{c.m.} = 273$ MeV. So condition (A5) is fairly well satisfied. To obtain a numerical estimate of nonorthgonality effects in the successive transfer we consider the quantity

$$\epsilon = \sum_{1,2} T_{2,1}(t_0) T_{-2,-1}(t_0) v_1 u_1 v_2 u_2 \qquad (4.4)$$

at the distance of closest approach (time $= t_0$). Remembering (3.13) this is a direct measure of $f^{(2)}$. Comparing ϵ with matrix element (3.37) and with matrix element (A8) one finds a reduction of ϵ by 9% ($E_{c.m.} = 250$ MeV) or by 23% ($E_{c.m.} = 273$ MeV) due to nonorthogonality. Because of the double averaging over the trajectory in the time integra-

tion (3.30) the true corrections should be much smaller. So the amplitude for successive transfer would still diverge at slightly higher energies (a reduction of 23% would be compensated at $E_{c.m.} = 274$ MeV). The simultaneous transfer would be influenced in the same way by nonorthogonality. Because the amplitude is already small we did not consider nonorthogonality corrections in this case.

E. Coulomb excitation

Winnik[5] has shown in a numerical calculation using a schematic model that Coulomb excitation strongly suppresses the pair transfer amplitude. He found an effective decrease of the pair transfer coupling constant by 25% which leads, in the region where perturbation theory is correct, to a reduction of the cross section by a factor of 2. Checking Winnik's results with the analytic formula of Baltz *et al.*[26] one finds that Winnik underestimates Coulomb excitation. Therefore we calculated the probability for Coulomb excitation using the deBoer-Winther code. We found that the probability for both nuclei to be in the ground state after the collision was 30% for energies at the barrier, which is far below Winnik's value of about 50%. Therefore we expect a decrease of the pair transfer amplitude by about 50% instead of Winnik's 25%. This shows that Coulomb excitation strongly suppresses the pair transfer process. But even the reduction of the amplitude by a factor of 2 allows multiple pair transfer at 2.5 MeV below the barrier. In all these calculations only the Coulomb excitation of the first 2^+ level in ^{120}Sn was considered. We want to stress, however, that the inclusion of higher excited states does not change the results, because it is expected that higher BE2 values are much smaller than predicted by a pure phonon model.[27] This leads to a bottleneck effect because electromagnetic transitions to higher states are strongly suppressed. In every case a better estimate of the pair transfer amplitude would require a coupled-channel calculation including Coulomb excitation.

V. DISCUSSION OF THE RESULTS

In the ^{120}Sn(^{120}Sn, ^{118}Sn)^{122}Sn reaction under backward angles we find the amplitude of the successive pair transfer mechanism to be about a factor of 6 larger than the simultaneous one. This implies the case of weak correlations of Ref. 7. The nucleons are preferentially transferred by the Hartree-Fock potential of the reaction partner in two steps rather than in one step as a correlated pair. The same relation between simultaneous and successive transfer amplitudes has been found by Broglia *et al.*[8] fairly below the barrier. It is important to note that both mechanisms gain the same enhancement due to superconductivity.[14,25]

Because the successive transfer is so dominant we consider the consequences of this process in the following. As already mentioned in the Introduction, we define the nuclear Josephson effect as a multiple transfer of nucleon pairs between the ground states of superconducting nuclei in a heavy-ion collision. Asking for the existence of the nuclear Josephson effect we must distinguish three different regions for the relative kinetic energy of the colliding nuclei.

(i) Fairly below the barrier (more than 10 MeV) perturbation theory is valid. So the probability for the transfer of two pairs is approximately the square of the probability for one pair transfer, which is clearly smaller than 1. Therefore multiple pair transfer is strongly suppressed.

(ii) Immediately below the barrier (about 5 MeV) perturbation theory diverges, i.e., the transfer probability is larger than 1. So multiple pair transfer is possible and could only be calculated by coupled-channel calculations, which would yield an oscillating excitation function in the pair transfer channels typical for the existence of Josephson currents.[5] However, two types of pair breaking mechanisms will reduce this effect. Including Coulomb excitation, we expect that perturbation theory does not diverge before 3 MeV below the barrier. Pair breaking channels due to nuclear forces should be of direct character below the barrier like one nucleon transfers and inelastic excitations. They produce absorption in the pair transfer channels. Because these processes do not gain the enhancement due to superconductivity, we expect them to be weak for energies where Josephson currents just establish.

(iii) Above the barrier the nuclei interpenetrate deeply (Fig. 1) and many different reaction channels open which lead to strong absorption and probably suppress the Josephson currents. Such a result was found in a recent paper by Broglia *et al.*[28] by using a WKB technique with complex turning points.

As a consequence one may expect the Josephson effect just below the barrier where the collision is very slow and very soft. Little is known about absorption for these nuclei. If the pair breaking effects are larger than expected in (ii) the Josephson effect could be destroyed: however, the pair transfer channel must be very strong. It should be mentioned that the semiclassical approximation is still valid at 3 MeV below the barrier, i.e., contributions from classically forbidden regions are very small.

Finally we want to mention a possible experi-

ment to detect the nuclear Josephson effect. As the pair transfer cross sections are of the order of the Rutherford cross section between $\theta_{c.m.} = 180°$ and $\theta_{c.m.} = 150°$ we expect rather big cross sections of several tens of μb's for the pair transfer. Instead of measuring directly the fragments after the collision, one may detect them by activity measurements.[29] In a $^{112}Sn - ^{112}Sn$ collision for example, where all relevant properties are the same as in the $^{120}Sn - ^{120}Sn$ case (as we have checked), one may detect the unstable ^{110}Sn isotope by its radioactive decay products. Because in a central collision the ^{110}Sn nucleus is stopped in the target and all fragments from contaminant reactions are not stopped due to the high kinetic energy of the incoming ^{112}Sn nucleus, it is a rather clean experiment.

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APPENDIX A

Introducing the overlap matrix $M_{3,4} = (\psi_3 | \psi_4)$ and separating the diagonal part, we obtain the matrix K [see Eq. (2.13) of DH],

$$M = 1 + K . \tag{A1}$$

If $K = 0$, the basis is orthogonal; so the matrix elements of K are a direct measure of the nonorthogonality. K has the properties

$$K_{1,2} = (\psi_1 | \psi_2) , \quad K_{1,1'} = K_{2,2'} = 0 . \tag{A2}$$

In first order of the nonorthogonality we get [see (2.12)]

$$M^{-1} = 1 + \tilde{K} = (1 + K)^{-1} \approx 1 - K . \tag{A3}$$

Equation (2.13) reads now in first order in K,

$$(\tilde{\psi}_1 | = (\psi_1 | - \sum_2 (\psi_1 | \psi_2)(\psi_2 | = (\psi_1 | + (\psi_1^{NO} | . \tag{A4}$$

A similar expression holds for $(\tilde{\psi}_2 |$. A necessary condition that nonorthogonality contributions are small is that the norm of $(\psi_1^{NO} |$ is small:

$$[(\psi_1^{NO} | \psi_1^{NO})]^{1/2} = \Big[\sum_2 |(\psi_1 | \psi_2)|^2 \Big]^{1/2} \ll 1 , \tag{A5}$$

Because the successive transfer is the dominant process, we also investigate the nonorthogonality contributions to this process. Carrying out the calculation following (3.32), including nonorthogonality up to first order as defined in (A4) and neglecting all interaction matrix elements which transfer more than one particle, we arrive at the correction term,

$$T_{2,1}^{NO}(t) = - \sum_{1',2'} (\psi_2 | \psi_{1'})(\psi_{1'} \psi_{2'} | v | \psi_1 \psi_{2'}) v_{2'}^2 + \sum_{1',2'} (\psi_{2'} | \psi_{1'})(\psi_2 \psi_{1'} | v | \psi_{2'} \psi_1) v_{2'}^2 . \tag{A6}$$

Because the first term is dominant and we use a local Woods-Saxon potential for the generalized HF potential (neglecting exchange effects), our first order nonorthogonality correction for the transfer matrix-element is:

$$T_{21}^{NO}(t) = - \sum_{1'} (\psi_2 | \psi_{1'})(\psi_{1'} | \hat{V}_2 | \psi_1) \tag{A7}$$

where $\hat{V}_2$ is now the HF potential of nucleus 2. The corrected successive transfer matrix-element reads:

$$T_{2,1}^{c}(t) = (\psi_2 | \hat{V}_2 | \psi_1) + T_{2,1}^{NO}(t) . \tag{A8}$$

APPENDIX B

Because the formulation of Götz *et al.*[7] for two-nucleon transfer will be more common to most readers, we try to compare the two formalisms. First we state the main features of Ref. 7. The total Hamiltonian is decomposed according to the different channels, which leads to different channel-Hamiltonians implying different systems of basis functions for different channels. By their very definition basis functions referring to different channels are not orthogonal on each other. Treating the T matrix up to second order one is led, according to the different Hamiltonians, to different representations such as prior and post representations for the first order T matrix and prior-prior, prior-post, post-prior, and post-post representations for the second order T matrix. If one neglects correlations between the nucleons, the transferring potential is only the HF potential, which is a one-body operator: Then simultaneous transfer is not possible. In the formulation of Ref. 7 it turns out in this case that simultaneous transfer vanishes only due to a cancellation of the first order T-matrix element and the nonorthogonality term of the second order T matrix. This means there is no clear cut distinction between simultaneous and successive transfer,

because these processes are not simply given by first and second order T-matrix elements.

In the DH formalism, however, only one Hamiltonian is used in second quantization. The basis is determined by a BCS (or HF) calculation, which means that one puts the effects of two-body forces acting within one nucleus as far as possible into one-body operators such as the HF potential in (2.19). But there are still two-body matrix elements which scatter between the nuclei and which are not taken into account by the basis. These matrix-elements are then responsible for the simultaneous transfer (3.9). If one neglects two-body forces, the simultaneous transfer amplitude automatically vanishes. The HF potential exerted by the nucleons of one nucleus on the nucleons of the other is responsible for successive transfer (3.37), which means that there is a clear and unique distinction between simultaneous and successive transfer. Furthermore nonorthogonality enters differently: each process may be corrected for these effects separately and there is no interweaving of the perturbation expansion and nonorthogonality corrections as in Ref. 7. However, the DH formalism has two disadvantages: (i) It may be applied only to heavy systems, because one has only one Hamiltonian. So properties such as the HF potential must not change strongly in the transfer process. (ii) It is not possible to transfer clusters with more than two particles simultaneously because there are at most two body operators.

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