

Abrasion-ablation calculations of large fragment yields from relativistic heavy ion reactions

Luiz F. Oliveira, Raul Donangelo,* and John O. Rasmussen

Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

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Calculations of large mass fragment yields from high-energy heavy-ion reactions are performed based on the abrasion-ablation model. The geometrical picture of the clean-cut fireball model is used to calculate the number of participant nucleons in the abrasion stage and the excitation energy of the spectators (primary residues). A standard statistical evaporation code is used to calculate the ablation stage. Results from this model show an overall agreement with experimental data, although some systematic discrepancies are found and discussed. A frictional spectator interaction is introduced which increases the average excitation energy of primary fragments and improves the results considerably.

[NUCLEAR REACTIONS Relativistic Heavy-ions, abrasion-ablation model, peripheral reactions, calculated fragmentation cross sections.]

I. INTRODUCTION

A new field of nuclear research emerged from the possibility of accelerating heavy ions to energies of hundreds to thousands of MeV per nucleon in the Berkeley Bevalac. The great interest of this new research area was prompted by suggestions that under the extreme conditions of a relativistic heavy-ion collision many new exotic phenomena (Lee-Wick abnormal nuclear matter,¹ shock waves,² pion condensation³) would be produced. Although experimental evidence for the above processes have not been found yet, other interesting phenomena, such as factorization,⁴ scaling,⁵ and limiting fragmentation⁶ have been observed.

The subdivision of high-energy heavy-ion reactions into central and peripheral collisions is already well established among nuclear physicists. These two qualitatively different types of collisions are clearly distinguished by observation of streamer chamber⁷ and nuclear emulsion⁸ pictures. The central (or near central) collisions, which comprise about 10% of all cases, are characterized by an almost complete destruction of both the projectile and target nuclei. A large number of high-energy particles come out over a wide range of angles, i.e., these violent processes are high-multiplicity events. From the point of view of Swiatecki's participant-spectator model,⁹ practically all nucleons in both colliding partners are participants in a central collision. By contrast, in case of a peripheral event the momentum and energy transfers are relatively small. Only a few nucleons in the overlap zone effectively interact during the collision, i.e., the number of participant nucleons is small. As a result, a few particles are observed in the extreme forward cone

of laboratory angles with velocities approximately equal to that of the projectile. These particles originate from the fragmentation of the excited projectile. Simultaneously other particles are observed to have an almost isotropic distribution in the laboratory frame, and these particles are considered to be evaporation products from the excited target residue (target fragmentation). Many experiments have been performed to investigate one or the other of these processes. For a (recent) review of the experimental situation see Ref. 10.

In this paper we attempt to interpret results of target or projectile fragmentation experiments with high-energy heavy-ions.¹¹⁻¹³ We calculate cross sections for production of heavy fragments considered to be produced in peripheral collisions. We stress that the calculations described in this paper apply equally well for both target and projectile fragmentation processes. Which process one chooses to study is merely a choice of reference frame.

In Sec. II we present the overall scheme of the calculations, where following the abrasion-ablation model^{14,15} we have treated the reaction as a two-stage process. In Sec. III we give details of the abrasion stage calculations. The charge-to-mass dispersion in the abrasion products is considered in Sec. IV. In Sec. V we describe the ablation (or evaporation) stage of the reaction, and compare the results of a pure abrasion-ablation calculation with experimental data. These preliminary comparisons clearly show the need of an energy deposition higher than the extra surface energy term proposed by Swiatecki.⁹ As suggested in Ref. 15, we consider a frictional spectator interaction process (FSI) between the abraded nucleons and the spectator. In Sec. VI, result

of our calculations with FSI are compared with the same experimental data as before, and the agreement is found to be considerably better.

We remark that although we will be using the geometrical picture of the clean-cut fireball model¹⁶ for the fast (abrasion) stage, we will not refer to the other hypotheses of that model, since here we do not deal with the fireball itself (see Refs. 16–19). Instead, we concentrate our attention on the fate of the spectator (target or projectile) fragments.

II. COMPUTATIONAL PROCEDURE (Overall Scheme)

The abrasion-ablation model introduced by Bowman, Swiatecki, and Tsang,¹⁴ describes the reaction as a two-stage process. In the fast stage (abrasion) the nucleons within the overlap zone interact with each other and are sheared away from either the projectile or the target. The projectile fragment follows its trajectory with practically the same velocity as before, while the target fragment slowly recoils. Both fragments are excited as a result of the abrasion, and they later dissipate this excitation energy by undergoing particle evaporation (ablation).

To calculate the abrasion part of the reaction we could use the geometrical picture of either the clean-cut fireball model¹⁶ or the firestreak model.²⁰ In this paper we restrict ourselves to the former. From this part of the calculation we obtain the excitation energies and the cross sections for formation of the so-called primary residues. Then

we start statistical particle evaporation from each primary residue to obtain the partial yields of the final (observed) fragments. Finally we sum the partial yields of each final fragment over all primary residues, and obtain the final yields, which are to be compared with the experimental values. Figure 1 illustrates the entire procedure.

III. CLEAN-CUT (FIREBALL) ABRASION PROCESS

Two basic assumptions are included in the geometrical picture of the fireball model: Both nuclei are assumed to have sharp spherical surfaces and to follow straight-line trajectories. Therefore, the separation between spectator and target nucleons is given by the intersection of the two nuclear surfaces. The projectile scrapes the target, shearing away all nucleons located within the overlap zone of the two nuclei (participant nucleons). A geometrical view of this model is shown in Fig. 2. The number of nucleons a_T removed from the target is therefore calculated from geometrical consideration alone, i.e., for a given system a_T depends only on the impact parameter b [$a_T = a_T(b)$]. This number can be calculated by numerical integration of the overlap volume between a cylinder and a sphere. Instead, in this work we use the simpler approximate formulas derived by Swiatecki,⁹ which are shown by Morrissey *et al.*²¹ to be very good approximations for large impact parameters.

With $a_T(b)$ calculated as described above, the mass number for the primary residue is simply given by

$$A_{T'}(b) = A_T - a_T(b), \quad (1)$$

where A_T is the target mass number. From the inverse function ($b(A_{T'})$) the cross section for a primary residue of mass $A_{T'}$ is determined by

$$\sigma(A_{T'}) = \pi [b(A_{T'} + 0.5)^2 - b(A_{T'} - 0.5)^2]. \quad (2)$$

In this model the excitation energy of the abrasion products (primary residues) results from the rupture of the nucleon-nucleon bonds in the

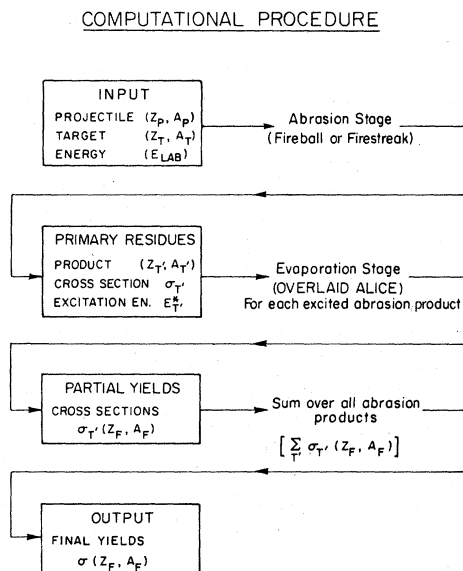


FIG. 1. Overall scheme of the calculations.

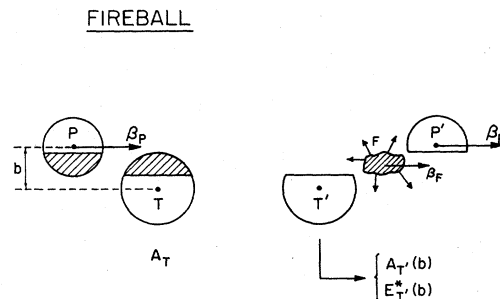


FIG. 2. Geometrical view of fireball abrasion.

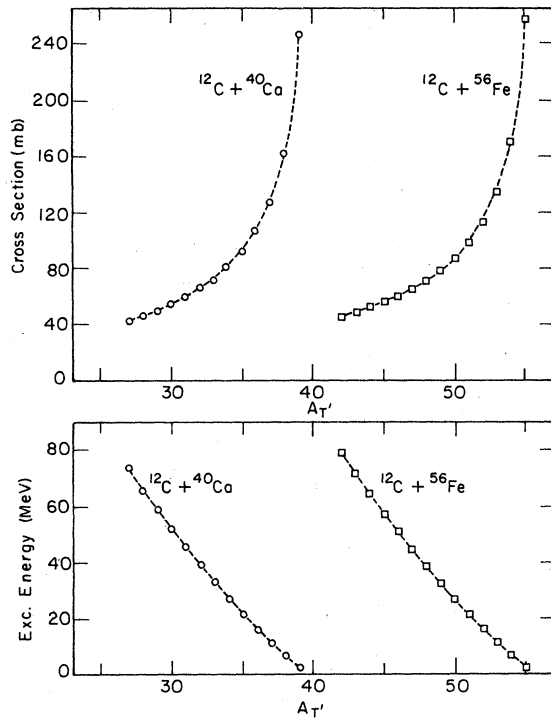


FIG. 3. Cross section and excitation energy as a function of the mass number of the primary residues. Calculated with the clean-cut fireball abrasion for two systems used later for comparisons.

intersection region. Since the number of bonds is roughly proportional to the intersection surface, we assume (as suggested in Ref. 9) the excitation energy to be proportional to the surface excess between the deformed abrasion product with a concave cylindrical surface gouged out of it and a spherical nucleus with the same volume. The proportionality constant is the nuclear surface tension, which is taken to be $0.95 \text{ MeV}/\text{fm}^2$.

In Fig. 3 we show the variation of both the cross section [as calculated from Eq. (2)] and the excitation energy with the mass of the primary residue, which is a function of the impact parameter. Figure 3 shows that while the cross sections for removing one or two nucleons are quite high, in both cases the excitation energies left are between 1 and 6 MeV, which are not enough to induce particle evaporation. This point will be important to bear in mind when comparing with experimental results.

IV. CHARGE-TO-MASS DISPERSION

The abrasion stage as described in Sec. III only calculates the number of struck nucleons $a_T(b)$

without specifying the proton-neutron ratio of the abraded nucleons. Before proceeding with the evaporation stage we must specify this ratio. This can be done in two very simple but rather different ways. The first, used in Ref. 16, assumes that the proton-neutron composition of the $a_T(b)$ abraded target (projectile) nucleons is exactly the same as that of the target (projectile). This implies that for each impact parameter b , $z(b)$ protons and $n(b)$ neutrons are ejected, which are determined by (in case of target fragmentation)

$$\begin{aligned} z(b) &= \frac{Z_T}{A_T} a_T(b), \\ n(b) &= \frac{N_T}{A_T} a_T(b), \end{aligned} \quad (3)$$

where Z_T and N_T are, respectively, the number of protons and neutrons of the target.

In our previous calculation²² we used a charge-to-mass dispersion (used also in Ref. 15), which assumes that each struck target nucleon has a Z_T/A_T probability of being a proton. Under this assumption, the cross section for production of a primary target residue ($Z_{T'}, A_{T'}$) is given by the hypergeometric distribution

$$\sigma(Z_{T'}, A_{T'}) = \frac{\binom{Z_T}{z} \binom{N_T}{n}}{\binom{A_T}{a}} \sigma(A_{T'}), \quad (4)$$

where $Z = z_T - Z_{T'}$, $n = N_T - N_{T'}$, and $a = n + z$ are, respectively, the number of protons, neutrons, and nucleons removed from the target, and $\sigma(A_{T'})$ is calculated by Eq. (2).

We understand that either calculation corresponds to a limiting situation: The former corresponds to a situation of complete correlation among the nucleons, such as would be the case if nuclear matter were a two-component crystal. The latter, on the other hand, considers no correlation at all between the proton and neutron distributions. The actual situation certainly lies between these two limiting cases.

In all calculations presented in this paper we use the no-correlation [Eq. (4)] charge-to-mass dispersion. The final results (after evaporation) are not very sensitive to the choice of charge dispersion relations for the case of element distributions (summed over A). On the other hand differences between the two limiting expressions are more pronounced in the case of isotopic distributions, with advantage to the uncorrelated expression [Eq. (4)], at least in the lighter elements.

An intermediate charge dispersion relation has been derived by Morrissey *et al.*²¹ They assume that fluctuations in the number of participant target protons are due to zero-point vibrations of the giant dipole resonance of the target nucleus. Charge dispersion curves obtained with their expression are somewhere in between the two limiting cases presented above, i.e., their distributions are always narrower than those obtained with Eq. (4). Another interesting analysis of the proton-neutron distribution in nuclei is given by Bondorf, Fai, and Nielsen.²³ They use a hydrodynamic model to express the ground-state correlations and investigate the possibility of observing such correlations after particle evaporation has occurred. Neither of these more complicated charge dispersion relations was used in this work, since here we are dealing with light systems where dispersion effects are not as important as for the heavier targets considered in Refs. 21 and 23.

V. THE ABLATION (EVAPORATION) STAGE AND FIRST COMPARISONS WITH EXPERIMENT

For this part of the calculations we use Blann's OVERLAID ALICE code,²⁴ which performs statistical evaporations of nucleons, deuterons, and α particles. We introduced some modifications and simplifications into ALICE to make it operational as a subroutine in our code. We use the option of the Myers-Swiatecki shell-corrected mass formula and a level density constant, $a = A/8 \text{ MeV}^{-1}$. In contrast to our previous calculations,²² we have now used the option of dropping the pairing term in the ground-state mass formula. This choice is because ALICE uses the same level density ($\rho(E) \propto E^{-2} \exp(2\sqrt{aE})$) for all nuclei, without the usual pairing shift of the excitation energy in the level density formula. Therefore, following Ericson,²⁵ we think that (for the range of products and excitation energies here) it is better approximation to drop the pairing term in the calculation of the binding energies, in order to compensate for its absence in the level density formula.

One of the difficulties of the clean-cut abrasion described in Sec. III is that it does not include transfer of angular momentum to the target fragment. Therefore, we assume zero angular momentum throughout the evaporation calculations. The most important effects of having a nonzero angular momentum distribution would be an enhancement of α -particle evaporation relative to that of nucleons. Shibata *et al.*¹¹ found that the maximum angular momentum transferred to the target fragments in the $^{12}\text{C} + ^{40}\text{Ca}$ at 400 MeV/N is about 5%. Such a small angular momentum value would have very little effect on the final

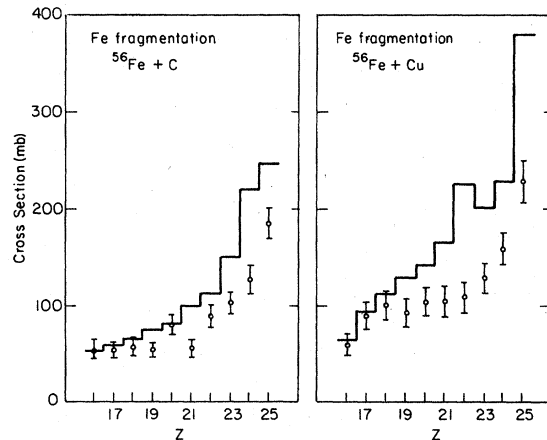


FIG. 4. Comparison between results of a pure abrasion-ablation calculation (solid line) and experimental data of Ref. 12.

results of our model.

In Fig. 4 we compare the results of our calculations (solid line) with the experimental data of Lindstrom *et al.*¹² They have studied the fragmentation of ^{56}Fe at 1.8 GeV/nucleon by several targets at Bevalac. Our calculated results for both cases presented (C and Cu targets) are higher than the experimental values, especially for the near projectile fragments ($Z=25$ and 24). We think we understand this feature: In the abrasion-ablation model as used here, the cross sections for the elements close to the projectile (or target, in case of a target fragmentation study) are really dominated by the abrasion stage. As stated at the end of Sec. III, the abrasion cross sections for the elements one or two nucleons removed from the projectile (target) are quite high (see Fig. 2), but the excitation energies calculated with the extra surface energy term alone are not enough to induce appreciable particle evaporation from these nuclei. An examination of Fig. 5 (which

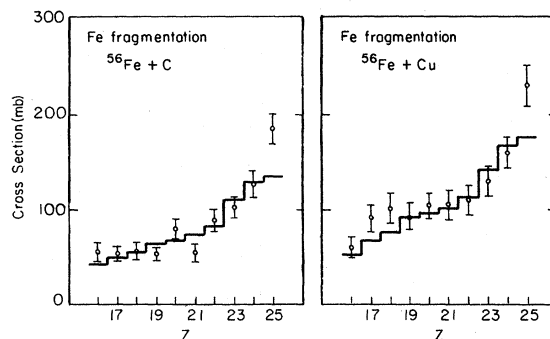


FIG. 5. Same as Fig. 4 but with FSI included in the theory.

will be explained in the next section) also leads to the same conclusion: It is necessary to introduce a mechanism for energy deposition other than the extra surface energy. Hüfner *et al.*¹⁵ encountered a similar situation in their calculations and proposed a final-state interaction mechanism for energy deposition. We followed their idea and introduced a similar process in our calculations. We will call it frictional spectator interaction (FSI).

VI. FRICTIONAL SPECTATOR INTERACTION AND NEW COMPARISONS

Since the nucleon-nucleon elastic scattering cross section at high energies is largely forward peaked, it is a good approximation to assume (at such energies) that the incident projectile nucleon follows a straight-line trajectory without changing its initial direction, while a struck target nucleon moves in the plane perpendicular to the projectile direction. Based on the above approximation, we further assume that 50% of the struck target nucleons are directed toward the target spectator piece, depositing in it part of their energy.

In Ref. 26 the differential elastic scattering nucleon-nucleon cross section is presented as a function of t —the square of the four-momentum transfer to the target nucleon. From this angular distribution, we evaluated the average four-momentum transfer $\langle t \rangle$ by

$$\langle t \rangle = \frac{\int_0^\infty t(d\sigma/dt)dt}{\int_0^\infty (d\sigma/dt)dt}, \quad (5)$$

which is related to the average kinetic energy of the recoiling target nucleon by

$$\langle t \rangle = 2m_N \langle E_{\text{recoil}} \rangle. \quad (6)$$

From the energy dependence of $d\sigma/dt$ and Eqs. (5) and (6) we obtained the average recoil energy $\langle E_{\text{recoil}} \rangle$ as a function of the projectile laboratory energy/nucleon.

The real physical situation between colliding heavy ions is more complicated than this simple picture of individual nucleon-nucleon collisions. Therefore, we believe that the calculations presented above give only an upper limit to the energy imparted to a target nucleon during the collision, and are valid for the most grazing collisions, where the situation is closer to that of an individual N - N collision.

As the recoiling nucleon advances through the spectator piece, it loses energy in further N - N collisions. We calculate the deposited energy by assuming the recoiling nucleon energy to be given by²⁷

$$\frac{dE}{dx} = -\frac{\alpha}{\lambda} E, \quad (7)$$

where λ is the nucleon mean free path, and α is the fraction of energy lost in each collision. In the calculation presented here, we take $\alpha = 0.25$ (see Ref. 27) and

$$\lambda = 1/\rho\sigma_{NN}, \quad (8)$$

where $\rho = 0.17 \text{ fm}^{-3}$ is the nuclear matter density and σ_{NN} is the N - N cross section, assumed to be $\approx 300/E \text{ fm}^2$ (a fair approximation for $E < 150 \text{ MeV}$).

Finally, the average deposited energy $\langle E_{\text{FSI}} \rangle$ is calculated by averaging the deposited energy E_{dep} over all possible orientations θ on the plane of the spectator piece, i.e.,

$$\langle E_{\text{FSI}} \rangle = \frac{1}{\pi} \int_0^\pi E_{\text{dep}}(\theta) d\theta. \quad (9)$$

Table I presents the values we obtained for the three cases studied in this paper.

As stated before, we assume that each struck nucleon has a 50% chance of passing through the spectator, depositing $\langle E_{\text{FSI}} \rangle$ of excitation on the average. Therefore, each primary residue with mass number $A_T = A_T - a_T$ may have from 0 to a_T frictional spectator interactions according to a binomial distribution given by

$$\text{Prob}(m_{\text{FSI}}) = \frac{\binom{a_T}{m_{\text{FSI}}}}{2^{a_T}}. \quad (10)$$

The total excitation energy of a primary fragment (A_T), which has undergone m_{FSI} final-state interactions is given by

$$E^* = E_{\text{surf}}(a_T) + m_{\text{FSI}} \langle E_{\text{FSI}} \rangle, \quad (11)$$

where $E_{\text{surf}}(a_T)$ is the extra surface energy term, which is a function only of the number of nucleons removed.

In Fig. 5 we show a comparison between the results of our calculations with FSI and the same experimental data of Fig. 4. The agreement is generally better than before, except for $Z = 25$ which now presents the opposite situation than before, i.e., the calculated value is less than the experimental result. We interpret this discrepancy at $Z = 25$ as an indication of the effects of other

TABLE I. Average energy deposited by a nucleon undergoing FSI.

Spectator	Laboratory energy (MeV/ N)	$\langle E_{\text{FSI}} \rangle$ (MeV)
⁴⁰ Ca	400	35.3
⁵⁶ Fe	1800	38.8
Cu	2000	41.5

peripheral processes not included in our calculations, such as Coulomb dissociation via the giant $E1$ resonance of the projectile^{28,29} or a dissociation of the projectile in the nuclear field of the target.²² The former has indeed been observed by Heckman and Lindstrom³⁰ for the case of ^{12}C and ^{16}O (2.1 GeV/A) fragmentation. They observed that such process is negligible for light targets, but becomes very important for heavy ones. In the present case (Fe fragmentation) Coulomb dissociation is certainly negligible for ^{12}C targets, but it may very well account for some of the missing cross section at $Z=25$ for the case of Cu targets. For heavier targets the cross section for $Z=25$ elements increases much more than those for any other Z ,¹² and the discrepancy between the experimental data and the results of our model becomes more pronounced, reflecting the above discussion.

A comparison between the results of our calculation and the experimental results of the TOSABE Group¹¹ is shown in Fig. 6. They utilized in-beam γ ray techniques to study the fragmentation of ^{40}Ca following interaction with high-energy alpha and carbon projectiles. Most nearly measuring nuclide yields are the intensities of the $2^+ \rightarrow 0^+$ transitions in doubly even products. Also a few radioactive isotopes (Shown on the left side of Fig. 6) were identified by measuring the off-beam spectra between the one-second beam spills. The histograms represent the experimental data, and the full and dashed lines are, respectively, the results of our calculations with and without FSI. Again the inclusion of FSI substantially improves the results, especially for those isotopes with masses close to the target mass.

Shibata *et al.*¹¹ compared their Ca fragmentation data (both with alpha and carbon projectiles) with

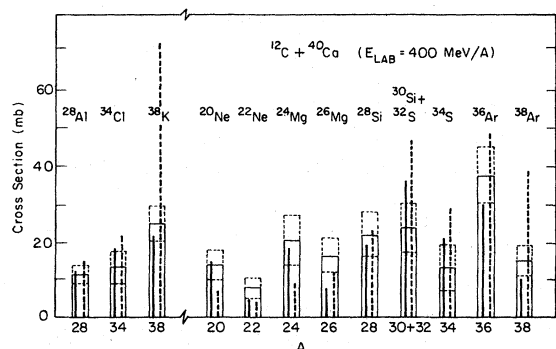


FIG. 6. Comparison between theoretical results and experimental data of Ref. 11. Histograms correspond to experimental data, and full and traced bars are results of our calculations with and without FSI, respectively. Dotted lines on histograms indicate experimental error bars.

data resulting from Ca bombardment by π^- and proton projectiles. Figure 7 was replotted from their work with the inclusion of our theoretical results for ^{12}C projectile. A feature clearly observed in this figure is the development of a plateau for yields corresponding to the heavier projectiles in contrast to the monotonic falloff of yields resulting from the lighter ones. Such plateau is well reproduced by the present theoretical results. It was suggested¹¹ that plateau formation for the carbon data reflects the great range of intermediate excited products formed at various impact parameters during the fast initial collision process, while the monotonic falloff of the pion yields mostly represents statistical nucleon and alpha evaporation from excited ^{40}Ca or its nearest neighbors. Within the context of our theory, their suggestion is equivalent to saying that production of a wide range of primary products in the abrasion-type process gives the distinctive plateau associated with the heavier projectiles.

A last comparison is made between our results and the experimental data of Cumming *et al.*¹³ They performed radiochemical studies of the spallation products from copper irradiated with 80 GeV ^{40}Ar ions, measuring cross sections for the production of 35 radioactive nuclides. We restrict the comparison to the heaviest isotopes, which are the most appropriate to our model due

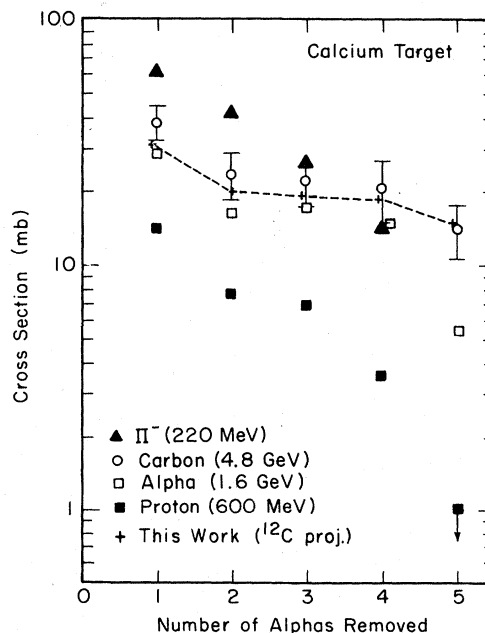


FIG. 7. ^{40}Ca fragmentation yields (Ref. 11) for so-called α nuclei as a function of number of α particles removed from target. The line connecting theoretical results is merely to guide the eyes.

TABLE II. Comparison between theoretical results (with and without FSI) and experimental Cu spallation yields (from Ref. 13).

Isotope	Experiment	Calculation without FSI	Calculation with FSI
^{64}Cu	64.0 ± 15.0	50.8	25.4
^{61}Cu	32.0 ± 6.0	43.0	15.9
^{57}Ni	1.87 ± 0.28	3.51	1.08
^{56}Ni	0.1 ± 0.8	1.5	0.1
^{60}Co	31.0 ± 13.0	38.0	15.2
^{58}Co	37.0 ± 7.0	12.7	42.5
^{57}Co	51.0 ± 5.0	22.6	25.9
^{56}Co	17.6 ± 1.4	16.6	8.0
^{55}Co	3.15 ± 0.22	7.7	1.48
^{59}Fe	5.9 ± 0.9	29.4	3.4
^{52}Fe	0.35 ± 0.07	1.95	0.30
^{56}Mn	9.1 ± 1.6	23.0	5.8
^{54}Mn	47.2 ± 1.8	28.5	29.9
^{52}Mn	17.3 ± 0.2	12.2	14.5
^{51}Cr	51.9 ± 1.7	24.4	26.9
^{48}Cr	0.92 ± 0.05	3.48	0.76
^{48}V	26.8 ± 1.1	19.1	20.3
^{48}Sc	1.73 ± 0.17	8.3	0.77
^{47}Sc	7.56 ± 0.37	11.2	3.4
^{46}Se	21.0 ± 1.4	14.4	9.1
^{44}Sc	31.8 ± 2.3	18.9	19.8
^{43}So	8.5 ± 3.5	11.2	9.0

to their sure origin in peripheral collisions. The general trend of the data is fairly well reproduced by our calculations, but our results with FSI (see Table II) are a factor of 2 lower than the data, with a few exceptions.³¹

VII. COMMENTS AND CONCLUSIONS

The abrasion-ablation model with the clean-cut fireball abrasion and the OVERLAID ALICE evaporation code has proven to be very useful in studying peripheral reactions between high-energy heavy ions. However, the yields obtained with the pure abrasion-ablation model for the elements with masses very close to the target mass are too high as compared to experimental data. The reason for this lack of agreement is believed to be the low values of the excitation energy after removal of one or two nucleons from the target, as calculated

assuming only an extra surface energy contribution. This led us to include a frictional spectator interaction (FSI) mechanism in the calculations. The agreement between the theoretical and experimental results improved substantially in most cases. It is important to point out that there are no free parameters in the calculations presented in this work.

Several modifications to the present model are possible, the most important of them is the use of a firestreak rather than a fireball geometry for the abrasion stage with the option of a diffuse nuclear surface. This introduces several new features in the calculations and is the subject of another work to be published elsewhere.³² Other charge dispersion relations incorporating ground-state correlations may also be used to investigate the effects of such correlation on the final product distribution.

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- *Present address: Instituto de Fisica, Universidade Federal do Rio de Janeiro, Cidade Universitária, Rio de Janeiro, RJ, Brazil.
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