Critical analysis of statistical methods used to detect resonances in nuclear reactions

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The effectiveness of several existing statistical tests in locating resonances in compound nuclear reactions above the Coulomb barrier is examined, and a new test based on the number of maxima at any single bombarding energy in a set of excitation functions is introduced. It was found that only the three tests based on the deviation functions, summed cross sections, and the distribution of maxima were suitable for the detection of resonances, and that the latter test was the most sensitive. The various tests were used to analyze $^{12}C(^{12}C,\alpha)^{20}Ne$, $^{12}C(^{14}N,\alpha)^{22}Na$, and $^{12}C(^{15}N,\alpha)^{23}Na$ excitation functions. Evidence for nonstatistical structure was found in all three reactions by examining the distribution of maxima.

[NUCLEAR REACTIONS $^{12}C(^{12}C,\alpha)^{20}Ne$, $E_{c.m.}=18.0-25.5$ MeV; $^{12}C(^{14}N,\alpha)^{22}Na$, $E_{c.m.}=10.2-18.1$ MeV; $^{12}C(^{15}N,\alpha)^{23}Na$, $E_{c.m.}=9.5-17.3$ MeV. Statistical model predictions. Statistical tests examined. Nonstatistical structure observed.]

I. INTRODUCTION

Recently there has been a great deal of experimental and theoretical study of resonance phenomena in heavy-ion compound nuclear reactions.[1-5] Two of the earliest features to be noted were that resonances were observed only in systems in which ^{12}C or ^{16}O was one of the reactants and in which the number of levels available in the compound nucleus was small.[5,6] In these reactions the resonant features are easily discernible from the normal statistical behavior due to correlations between different exit channels at the resonant energies. For many reactions, however, the resonant features, if they exist, are not very striking and at first glance do not appear radically different from statistical fluctuations. There is also a high probability of observing structure in excitation functions which appears to be intermediate structure, but is fully explainable in terms of the statistical model.[7] Standard statistical tests such as cross correlations and autocorrelations are not sensitive enough to detect the presence of only a few resonances in excitation functions.[1] Furthermore not all resonant features appear in the same manner. Thus, one needs a variety of statistical tests that are sensitive to the various types of resonant features.

It is our purpose in this paper to examine some of the different types of statistical tests previously used to find resonances and to determine their usefulness and sensitivity when applied to heavy-ion reactions. Further we introduce a test based upon the counting of maxima (this test was previously used[1] in a slightly different form). These tests are used to analyze excitation functions (differential cross sections at a single lab angle over a wide bombarding energy range) from the following reactions: $^{12}C(^{12}C,\alpha)^{20}Ne$, $^{12}C(^{14}N,\alpha)^{22}Na$, and $^{12}C(^{15}N,\alpha)^{23}Na$.

It is important to differentiate between resonant features and nonstatistical structures. Statistical tests can determine whether features of cross sections are inconsistent with the statistical model and thus locate nonstatistical structures. Resonances are not necessarily the only means of producing nonstatistical structure in excitation functions. In the present work, however, we can only search for nonstatistical structure. Assigning these phenomena as resonances requires additional information such as correlations in angle or observing them in other exit channels.

II. CRITIQUE OF STATISTICAL TESTS NOW IN USE

A. Deviation functions and sums of excitation functions

Tests based on sums of excitation functions and deviation functions have been the tests most frequently used to detect the presence of intermediate structure. The strength of these tests is that sums over a large number of states tend to average over the statistical fluctuations thus making correlated structure more apparent. One shortcoming of this method is that large fluctuations in a particular state with a large cross section might possibly be mistaken as nonstatistical structure. In order to avoid this problem deviation functions are sometimes used. There are two very similar deviation functions for N excitation functions

$$D_1(E)=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\sigma_i(E)}{\langle\sigma_i(E)\rangle}-1\right) \tag{1}$$

and

$$D_2(E)=\frac{1}{N}\sum_{i=1}^{N}\left|\frac{\sigma_i(E)}{\langle\sigma_i(E)\rangle}-1\right|, \tag{2}$$

where $\sigma_i(E)$ is the differential cross section at a given angle for the state i at bombarding energy E, and $\langle\,\rangle$ denotes an expectation value. At low energies this method works well but at higher energies there are three important characteristics that limit the effectiveness of deviation functions: (1) not all exit channels show the presence of a resonance in the same manner, (2) not all states in the same reaction channel show the presence of a resonance, and (3) the energy of a resonance may shift slightly for different exit channels.[8] Because of these effects the deviation functions and sums of excitation functions may not always show significant deviations from the average when resonances are present.

B. Autocorrelation functions

The autocorrelation function for each state i is defined as

$$R_i(\epsilon)=\left\langle\frac{\sigma_i(E+\epsilon)\sigma_i(E)}{\langle\sigma_i(E+\epsilon)\rangle\langle\sigma_i(E)\rangle}-1\right\rangle_{\text{av}}, \tag{3}$$

where $\langle\,\rangle$ denotes the expectation value, but must be determined experimentally by calculating the average cross sections for an energy interval I and by then averaging $R_i(\epsilon)$ over the entire excitation function. The usefulness of the autocorrelation function to detect the presence of intermediate structure was demonstrated by Singh *et al.*[9] in studies of the ^{27}Al$(p,\alpha)^{28}$Si reaction. This technique, first suggested by Papallardo,[10] involves studying changes in the variance of an excitation function $R_i(\epsilon=0)$ as the length of the averaging interval is increased.

When broad structures are present the variance will increase steadily with increasing averaging interval until the averaging interval is comparable to the width of the structure. At this point the variance remains constant until the averaging interval becomes larger than the structure width.[10] This plateau is very distinct when the structure has a much larger width than the fluctuation width and it is one of the dominant features of the excitation function. Unfortunately, this method does not allow comparison with statistical model predictions so random fluctuations may be mistaken as nonstatistical structures.

C. Cross-correlation function

Cross-correlation coefficients have been used to look for correlated structure in excitation functions.[9,11] The cross-correlation coefficient between two excitation functions i and j is given by

$$C_{ij}=\left\langle\left(\frac{\sigma_i(E)}{\langle\sigma_i(E)\rangle}-1\right)\left(\frac{\sigma_j(E)}{\langle\sigma_j(E)\rangle}-1\right)\right\rangle[R_i(0)R_j(0)]^{-1/2}. \tag{4}$$

Since the statistical model predicts that the cross correlation should be zero with some variation due to the finite range of the data (FRD),[12] the results of this test can be directly compared to the statistical model. Using a running average to calculate average cross sections removes the effect of gradual trends in the data and thus reduces the average correlation coefficients. Relating nonstatistical results of a cross-correlation analysis to a particular resonant feature is difficult because the cross-correlation coefficient is not dependent on energy and thus cannot single out a localized feature of the excitation function.

D. Energy dependent cross-correlation function

One cross-correlation coefficient[11] that can determine energies of resonances is

$$C(E)=\frac{2}{N(N-1)}\sum_{i>j=1}^{N}\left(\frac{\sigma_i(E)}{\langle\sigma_i(E)\rangle}-1\right)\times\left(\frac{\sigma_j(E)}{\langle\sigma_j(E)\rangle}-1\right)[R_i(0)R_j(0)]^{-1/2}, \tag{5}$$

where the subscripts i and j are indices used to label the excitation functions. A running average can be used for average cross sections. This function gives indications of correlated bumps and dips in the cross sections. The major difficulty with using this function is that it is not a true correlation coefficient. $C(E)$ is not bounded by 1 and -1 and can not easily be altered to a function that is bounded without destroying its usefulness. Because it is not a true correlation coefficient, standard methods cannot be used to determine the statistical model prediction for the range of $C(E)$. One can obtain an estimate of the probability distribution of $C(E)$ values using a generalization of Chebyshev's inequality which states that[13]

$$P(C(E)\geq t(M_n)^{1/n})\leq 1/t^n. \tag{6}$$

Here P is the probability of observing a value of $C(E)$ greater than $t(M_n)^{1/n}$ where

$$M_n=\langle|C(E)|^n\rangle \tag{7}$$

for n greater than 1. For $n=2$ this is the standard Chebyshev's inequality

$$P(|C(E)-\langle C(E)\rangle|/\text{var}[C(E)]^{1/2}\geq t)\leq t^{-2}, \tag{8}$$

where var$(C(E))$ is the variance of $C(E)$.

Figures 1(a)–1(c) show $C(E)$ for the ^{12}C(^{12}C, α)^{20}Ne, ^{12}C(^{14}N, α)^{22}Na, and ^{12}C(^{15}N, α)^{23}Na excitation functions (these data will be discussed in Sec. III). There are several energies where

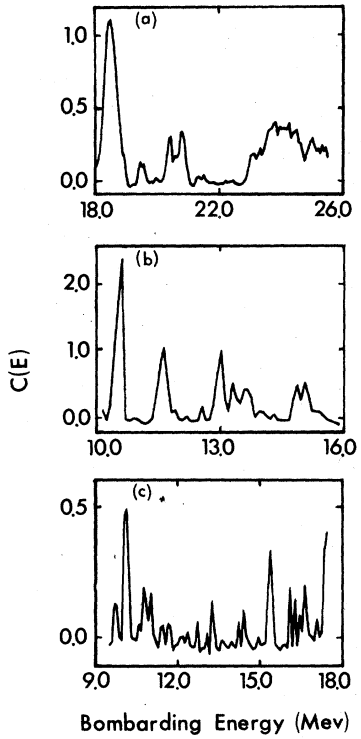

FIG. 1. The energy dependent correlation coefficients for (a) $^{12}C(^{12}C, \alpha)^{20}Ne$, (b) $^{12}C(^{14}N, \alpha)^{22}Na$, and (c) ^{12}C $(^{15}N, \alpha)^{23}Na$ reactions calculated using Eq. (5). The bombarding energies are in the c.m. system.

$C(E)$ exceeds one, but we found that the magnitude of $C(E)$ sometimes depends on the length of the energy interval used to calculate the average cross section. In Sec. III we will briefly examine the reliability of using $C(E)$ to predict nonstatistical structures. We can see from Eq. (7) that very large deviations are needed before the probabilities become small. A better method of calculating the probabilities would make the $C(E)$ function more useful.

E. Runs statistic

The runs distribution and length of runs distribution were first determined by Mood.[14] These distributions were utilized by James,[15] Moore,[16] and Baudinet-Robinet and Mahaux[17] to detect intermediate structure in excitation functions. In using this test a single "run above" is defined as an unbroken string of observed quantities above a predetermined reference value r; a "run below" is similarly defined. We let u be the total number of runs both above and below a reference value in a sample. If p the probability for a value above r and q the probability for a value below r are unknown then when one observes N_1 data values above r and N_2 data values below r the expected numbers of runs $E(u)$ and variance in the number of runs $\mathrm{var}(u)$ are given by[15]

$$E(u) = \frac{2N_1N_2}{N_1+N_2} + 1 \tag{9a}$$

and

$$\mathrm{var}(u) = \frac{2N_1N_2(2N_1N_2 - N_1 - N_2)}{(N_1+N_2)^2(N_1+N_2-1)}. \tag{9b}$$

Whenever both N_1 and N_2 are greater than 10 the variable

$$X = \frac{|u - E(u)| - \frac{1}{2}}{\mathrm{var}(u)^{1/2}} \tag{10}$$

is approximately normally distributed with a mean of zero and a variance of one.[17]

The runs test can tell us if there are correlations within a single excitation function. Statistical model excitation function cross sections are not random sequences of numbers but are correlated when the energy step size is comparable to the coherence width. Thus any test which checks for randomness within a single excitation function is sure to find nonrandom behavior in such excitation functions and cannot be used to support the presence of other types of nonstatistical structure.

F. Distribution of maxima

Another test that can be used to check for nonstatistical structure is based on counting the number of maxima that occur at each energy in a group of excitation functions. Resonances may result in an anomalously high number of maxima in a given energy region; thus by comparing statistical model predictions for the number of maxima to what is observed, one may be able to locate nonstatistical components in excitation functions. The predicted number of maxima in each excitation function depends only slightly on the spin and energy of the final levels,[18,19] the number of effective channels, and any direct component[20,21]; these effects are small and can be neglected. For example, variations in Γ for the three reactions studied here were very near the variations found in Γ for the synthetic excitations studied by Van der Woude.[20] Statistical effects on the number of maxima are small,[21] less than 3% in the excitation functions considered here. Variations in the number of maxima with energy within a single excitation function are smaller than the expected variation taking sample size into account.[19,22] Since the effects discussed above are small, one can consider the number of maxima in each excitation function as different statistical estimates of the same number. Also since the number of maxima is linearly proportional to the energy interval,[23] we conclude that the

probability of observing a maximum at any energy in any excitation function is the same. Thus the distribution of the number of maxima at each energy in any group of excitation functions should follow a binomial distribution. The probability parameter p for the binomial distribution can be determined by counting the number of maxima in all the excitation functions and dividing by the number of points. The statistical error on the binomial parameter p is given by

$$\mathrm{var}(p) = \frac{p(1-p)}{n} \leq \frac{1}{4n}, \tag{11}$$

where n is the total number of points checked for maxima. The probability for observing N or more maxima $P(N)$ in N_m excitation functions is

$$P(N) = \sum_{k=N}^{N_m} \frac{N_m!}{k!(N_m-k)!} p^k (1-p)^{N_m-k}. \tag{12}$$

We can further strengthen the test by adding other conditions to the maxima, such as that $\sigma(E)/\langle\sigma(E)\rangle$ be greater than a given reference value r and that it be significantly higher than its nearest neighbors. In all cases the binomial parameter must be redetermined using the new conditions.

The tests based on the distribution of maxima can be used to study structures whose maxima are spread over several energies. This situation can be handled in a very straightforward way by making p the probability for a maximum in the energy interval $E \pm \Delta E$. Of the three reactions studied here all showed possible nonstatistical structures based on the number of maxima occurring at two or three adjacent energies. Exactly how large an energy inverval can be used effectively in this test depends on the relative sizes of the energy spacing of the data points, the coherence width of the compound nucleus and the widths of any nonstatistical structures present. The likelihood of finding nonstatistical structure does decrease as the energy width is increased due to the smaller nonstatistical structure to fluctuating background ratios. In general the shifts in energy from one reaction channel to the next are expected to be less than the decay width of the structure. Resonances that were previously reported in light ion reaction channels had widths ranging from 100 to 400 keV.[6] Thus we expect energy shifts of roughly this size. For each of the three reactions we examined, our largest energy interval was about 250 keV. This width should be large enough to detect most nonstatistical structure present.

To illustrate how these ideas fit the data, Figs. 2(a)–2(c) show typical fits to several observed distributions. In each case the parameter p was the probability for a maxima under the given conditions determined by counting the number of such

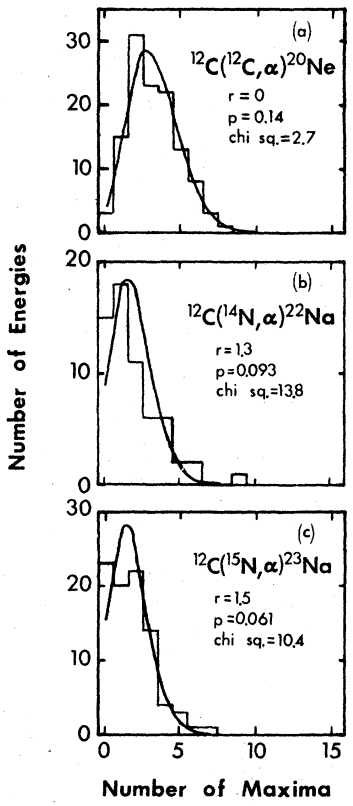

FIG. 2. The histograms show the observed distributions of the number of bombarding energies with a given number of maxima at a single bombarding energy for the excitation functions of the $^{12}C(^{12}C, \alpha)^{20}Ne$, $^{12}C(^{14}N, \alpha)^{22}Na$, and $^{12}C(^{15}N, \alpha)^{23}Na$ reactions under the conditions given. The solid curves show the statistical model predictions calculated from the binomial distribution using the deduced probability p for a maximum under the same conditions.

maxima in a group of excitation functions and dividing by the number of points checked. As can be seen from Fig. 2 the fits are quite good, but there are some energies where the number of maxima is larger than might be expected by the statistical model. We wish to determine at which energies these large numbers of maxima occur.

Figure 3 shows the detailed behavior of $P(N)$ as a function of r for three energies measured in the $^{12}C(^{14}N, \alpha)^{22}Na$ reaction excitation functions. The dashed line in Fig. 3 also shows the behavior of p as a function of r. The strong oscillations in $P(N)$ for all three energies are the result of a change in the number of maxima observed at the particular energy being studied. These oscillations are not due to sample size effects on p. This is clear because of the lack of correlation in the large oscillations at the different energies and the relative smoothness of p as a function of r for r less

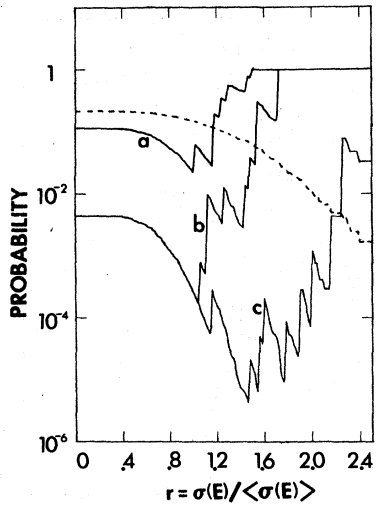

FIG. 3. The probability $P(N)$ of [Eq. (12)] for three energies from the $^{12}C(^{14}N, \alpha)^{22}Na$ reaction is shown as a function of r. The c.m. bombarding energies are 10.89, 13.29, and 10.62 MeV for curves a, b, and c, respectively. The dashed curve shows the behavior of p, the probability of a maximum, as a function of r.

than 2. Sample size effects on p and thus $P(N)$ can be seen in the small oscillations in p for r greater than 1.2 and in similar oscillations in $P(N)$ which are most easily seen in the downward sloping segments of the solid curves in Fig. 3.

When interpreting the results of the distribution of maxima tests for the various choices of r it is important to realize that nonstatistical structure will not necessarily have low probabilities of occurrence for all values of r. This is easily seen for large values of r because one can always choose an r so large that none of the excitation functions contain a maximum for which $\sigma(E)/\langle\sigma(E)\rangle$ is greater than r. For small values of r the expected number of maxima is often so high that the probability for a maximum in all the excitation functions in one energy region is not extremely small. Since we are looking for a distribution of maxima in a small energy interval that is substantially different from the predicted distribution, we need only consider the range of r values where the predicted and observed distributions can be significantly different. Thus we restricted our study to r values between 1.0 and 1.5 (the precise limits for the range is somewhat arbitrary). Any structure which shows a small probability for any value of r can be considered nonstatistical (though most nonstatistical structures will have small probabilities for a range of r values). Confidence in calling any structure nonstatistical increases for smaller probabilities. This is illustrated by the three solid curves in Fig. 3. The curves b and c show $P(N)$ values for structures that have low probabilities of occurrence over a wide range of r values and could be called nonstatistical. The curve in c shows much lower values than in b over a larger range in r, and one has more confidence in calling this structure nonstatistical. The structure whose $P(N)$ values are shown by curve a does not exhibit probability values that would normally be considered nonstatistical. By sampling a few values of r one can find most nonstatistical structure. Any arbitrary dependence on the choices of r resulting in missed structure can be removed by examining more values of r.

III. EVIDENCE FOR THE EXISTENCE OF NONSTATISTICAL PHENOMENA

Of the tests described in Sec. II only those based on the number of maxima and deviation functions were used to locate nonstatistical structure. Because these tests will yield nonstatistical results when there are correlations between different exit channels we can use nonstatistical results as evidence of resonant behavior.[24] Of course the greater the deviation from statistical predictions the stronger the evidence, but for smaller deviations the separation between nonstatistical and fluctuation behavior is nontrivial. Making the separation based solely on the results of large deviations such as seen in the exit channels of the ^{12}C-^{12}C reactions may mean missing similar but weaker effects in other reactions. In making our decisions we followed Dayras *et al.*[25] and called any event with a probability of 0.01 or less nonstatistical. At this probability level we would expect, from simple statistical considerations, about one such event over the entire range of the excitation functions.

A. The reaction $^{12}C(^{12}C,\alpha)^{20}Ne$

Twenty-three excitation functions ($\theta_{lab} = 5°$) for ^{20}Ne excitation energies from 0.0 to 17.44 MeV with bombarding energies taken in 62.5 keV steps for center of mass energies from 18.0 to 25.5 MeV (Ref. 1) were analyzed with the tests described in Sec. II. Fortune *et al.*[1] postulated the existence of four resonances at 18.4, 18.6, 19.0, and 19.4 MeV (c.m.) based on the number of peaks they observed at these energies.

Table I shows the $P(N)$ values for selected bombarding energies under several conditions on the maxima. The number of digits to the right of the decimal for the energies listed indicates whether the $P(N)$ values are for $P(N)$ values determined from single energies (two digits) or from a range of energies (one digit). The energies listed in Table I are the only ones that showed consistently

TABLE I. Statistical model probabilities for observed structures.

Reaction	$E_{c.m.}$	Probabilities calculated using Eq. (12) $r = 1.0$	$r = 1.1$	$r = 1.2$	$r = 1.3$	$r = 1.4$	$r = 1.5$
$^{12}C(^{12}C,\alpha)^{20}Ne$							
	18.4	0.015	4.6×10^{-3}	0.020	8.9×10^{-3}	5.4×10^{-3}	0.020
	18.6	0.016	7.3×10^{-3}	0.017	0.037	0.021	0.042
	20.75	0.053	0.027	0.071	9.3×10^{-3}	0.021	9.5×10^{-3}
	21.9	7.4×10^{-3}	3.2×10^{-3}	1.7×10^{-3}	0.018	5.4×10^{-3}	0.020
	22.44	7.4×10^{-3}	3.2×10^{-3}	3.5×10^{-3}	1.5×10^{-3}	1.4×10^{-3}	5.7×10^{-3}
	24.75	4.3×10^{-3}	7.3×10^{-3}	0.017	9.3×10^{-3}	0.021	0.15
$^{12}C(^{14}N,\alpha)^{22}Na$							
	10.6	1.3×10^{-5}	1.7×10^{-6}	1.9×10^{-6}	9.4×10^{-8}	6.2×10^{-9}	6.8×10^{-8}
	13.29	2.6×10^{-4}	5.1×10^{-4}	4.8×10^{-3}	7.8×10^{-3}	3.1×10^{-3}	0.034
	13.7	6.9×10^{-4}	3.3×10^{-3}	2.9×10^{-3}	7.1×10^{-3}	1.7×10^{-3}	1.5×10^{-3}
$^{12}C(^{15}N,\alpha)^{23}Na$							
	11.47	0.038	0.015	0.018	5.3×10^{-3}	0.020	0.025
	12.4	1.7×10^{-3}	0.043	0.050	0.057	0.020	6.0×10^{-3}
	15.4	5.7×10^{-3}	5.0×10^{-3}	7.5×10^{-4}	2.0×10^{-3}	3.9×10^{-4}	2.1×10^{-3}
	16.2	0.041	0.014	0.021	0.017	5.0×10^{-3}	7.0×10^{-3}

low values of $P(N)$ as r varied. The majority of energies have been excluded from the table as they do not show low probabilities in any of the tests. As can be seen from Table I the criterion outlined above is satisfied by the structures at 18.4, 18.6, 20.75, 21.9, 22.44, and 24.75 MeV. Only two of these energies, 18.4 and 18.6 MeV, agree with those given in Ref. 1. These structures are the most striking of those seen in this reaction. When the results from the distribution of maxima test are compared to the energy dependent correlation function shown in Fig. 1(a) one observes that they are only slightly correlated with the energies for the nonstatistical structures. Secondly, the strongest peak in Fig. 1(a) does not correspond to the energy with the most maxima but instead exhibits a peak between 18.4 and 18.6 MeV; thus it indicates the presence of one rather broad structure in this energy region. This broad structure is also seen in the deviation function. Because of this we decided that what was previously called two resonances is probably only one. If resonances do occur at 19.0 and 19.4 MeV their strength is spread out enough so as not to be detected by this test. The maxima at the latter two energies may be due to the correlated minima reported by Greenwood, *et al.*[11] at $E_{c.m.} = 19.2$ MeV.

B. The reaction $^{12}C(^{14}N,\alpha)^{22}Na$

Twenty excitation functions for ^{22}Na levels less than 5.44 MeV excitation energy were analyzed by the use of the distribution of maxima tests. These twenty excitation functions were for α particle cross sections measured from bombarding energies of $E_{c.m.} = 10.15$ to 18.09 MeV at $\theta_{lab} = 7°$.[26] From the probabilities given in Table I we see that there is evidence for nonstatistical structure at three energies: $E_{c.m.} = 10.6$, 13.3 and 13.7 MeV. The structure at 10.6 MeV is very striking considering just the maxima at 10.6 MeV, but there are also a large number of maxima at the next two lower energies 10.5 and 10.4 MeV. This indicates the presence of a broad structure whose strength is spread over these three energies. This structure is also seen in the $C(E)$ distribution and in the deviation function where the peak at 10.6 MeV has a statistical probability for occurrence of about 0.002. The structure in the total cross section has a rather wide width with $\Gamma_{c.m.}$ being about 400 keV. Because of this we conclude that there is only one structure near 10.6 MeV and not three closely spaced structures.

C. The reaction $^{12}C(^{15}N,\alpha)^{23}Na$

Twenty-eight excitation functions for states in ^{23}Na less than 8.94 MeV excitation energy, which were measured in 89 keV steps for center of mass bombarding energies from 9.51 MeV to 17.33 MeV at $\theta_{lab} = 7°$,[27] were analyzed by the statistical tests described in Sec. II. The only test that showed evidence of nonstatistical fluctuations was the test based on the distribution of maxima. Referring to Table I we see evidence for the presence of four nonstatistical structures at $E_{c.m.} = 11.5$, 12.4, 15.4, and 16.2 MeV. The most prominent structure located at 15.4 MeV, can be based on the maxima at 15.29, 15.38, and 15.47 MeV. Also the nonstatistical nature of the structure at 16.2 MeV is based on the maxima from two adjacent energies.

In Ref. 2 a much larger number of correlated

structures were advocated. The fact that the present test does not detect these structures may result from the abnormally large value of p, the binomial probability for a maximum, that such a large number of correlated maxima could produce. However, the observed value of p for the $^{12}C(^{15}N, \alpha)^{23}Na$ excitation functions is consistent with that predicted using $\Gamma = 14 \exp[-4.69(A/E_x)^{1/2}]$ MeV and the methods described in Ref. 22 for excitation energy E_x and nucleus of mass A.

IV. DISCUSSION

Statistical tests do not prove the existence of resonances; they merely indicate where they are most likely to be found. As long as the probability for some structure is not zero, it is possible that it can be explained by the statistical model. Therefore, further evidence is needed, such as the measurement of different angular momentum dependences on and off such structures, or correlations in angle or exit channel, before one can claim a resonance has been found.

The major difference between the structures observed here and the quasimolecular structures observed at lower energies is that these structures are not nearly as pronounced as the quasimolecular resonances. Further these structures do not appear in all of the excitation functions, the strongest at 10.6 MeV in $^{12}C(^{14}N, \alpha)^{22}Na$ appears in 18 of 20 levels. This leads one to believe that there is either a selection rule in operation or the structures are not due to entrance channel resonances.

The energies of the $^{12}C(^{12}C, \alpha)^{20}Ne$ structures fit in well with the quasimolecular model. Whether the structures in the other reactions can be understood with this model would require the location of more structures at other energies, in other channels and the measurement of angular momentum values at each of the structures. The reactions in which these structures occur are basically statistical in nature and the structures, with the possible exception of the one at 10.6 MeV in the $^{12}C(^{14}N, \alpha)^{22}Na$ reaction, produce only minor deviations from statistical model predictions for average cross sections or coherence widths.[26] Gomez del Campo[2] has suggested that the structures in the $^{12}C(^{15}N, \alpha)^{23}Na$ reaction are due to the excitation of isolated high-spin states in the compound nucleus. Such high-spin states would have an angular momentum above the Yrast line but below the critical angular momentum cutoff.

Another possibility arises in light of calculations with synthetic S matrices by Moldauer.[28] These calculations showed that when large transmission coefficients are involved in the calculation of compound nuclear cross sections a broad resonance tends to show up strongly in all channels. Moldauer's calculation was done using a much smaller number of compound nuclear states than are present in these reactions so that an extrapolation from his calculations to the cases studied here is not obvious. Generally the resonances Moldauer is considering are much broader than those observed here, nevertheless the correlated peaks we observe could be a manifestation of this effect in this higher density region.

V. CONCLUSION

The analysis of the three sets of excitation functions has produced evidence for nonstatistical structures in all three sets of excitation functions. This evidence is based on the number of correlated maxima observed at each energy. Other statistical tests, such as those that look for nonstatistical deviations in the summed cross sections, do not give evidence for these nonstatistical effects with the exception of the structure at 10.6 MeV in $^{12}C(^{14}N, \alpha)^{22}Na$.

The tests do not give an indication of the cause of such structures; thus more work needs to be done to determine whether these are some type of resonance phenomena or whether they represent a breakdown of the statistical model due to the presence of large transmission coefficients or small density of high-spin states.

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