PHYSICAL REVIEW C VOLUME 19, NUMBER 3 MARCH 1979

Electron-deuteron tensor polarization and the short range behavior of the deuteron wave function

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(Received 7 July 1978)

We investigate to what extent measurements of the polarization of recoil deuterons in electron-deuteron scattering would allow us to determine the short-range behavior of the deuteron wave function. We find that even if such measurements were performed out to $q = 10$ fm^{-1} with an error of $\pm 10\%$ and in some cases even within $\pm 1\%$, a considerable variation in the deuteron wave function inside 0.7 fm would still be compatible with such measurements. In particular it would be very difficult to employ such measurements to determine the presence of a hole in the deuteron wave function, unless complemented by firmly founded meson theoretical considerations. However, this does not in any way detract from their usefulness as a source of information on other deuteron properties, meson exchange currents and possibly three-nucleon forces.

[NUCLEAR REACTIONS Calculated electric deuteron form factor, polarization recoil deuterons electron-deuteron scattering, unitary transformations, short range behavior deuteron wave function.]

I. INTRODUCTION

There has recently been considerable interest in the possibility of using measurements of the tensor polarization P_e of recoil deuterons in electron-deuteron (ed) scattering to obtain information on the short-range behavior of the deuteron wave function.[1,2,3]

It has been shown by Hockert and Jackson[1] that a single measurement of the tensor polarization P_e of recoil deuterons in electron-deuteron (ed) scattering at a momentum transfer of $q^2 = 19.52$ fm^{-2} ($q \approx 4.5$ fm^{-1}) of $\pm 10\%$ accuracy could be combined with currently available data for the deuteron electric form factor to add considerably to our knowledge of the deuteron charge form factor. Specifically the uncertainty in the location of the zero of the deuteron charge form factor $G_0(q)$ could be greatly reduced by the inclusion of the suggested datum. The location of this zero is related to the degree of short-range repulsion in the *NN* interaction or in other words to the degree of "hardness" of "softness" of the core of the interaction.

Brady, Tomusiak, and Levinger[2] have shown that the node in $G_0(q)$ is sensitive to the "size" of the hole in the interior of the *S*-wave component of the deuteron wave function $u(r)$. The "size" of the hole is in turn dependent on whether the underlying potential has a hard, soft, or a super-soft core.

We therefore investigate here what latitude in the short-range ($r < 0.7$ fm) behavior of the deuteron wave function would remain if P_e were known at $q = 4.5$ fm^{-1}, the point at which the node in G_0 is expected to occur, with an accuracy of $\pm 10\%$. We find that a wide variety of behavior could be compatible with this datum if we do not assume *a priori* that there is a hole in the deuteron wave function and allow nodes or enhanced deuteron wave functions in the inner region.

Furthermore Moravcsik and Ghosh[3] have suggested that measurement of P_e in the region $q = 6$–10 fm^{-1} would be sensitive to the deuteron wave function within 0.5 fm and would yield a rather definitive determination of the wave function down to about 0.2 fm. Assuming that such measurements could be made to within an accuracy of about $\pm 10\%$ to $\pm 1\%$ we will show by means of examples that the ambiguity in the wave function in the inner region would remain much larger than expected.

Our lack of knowledge of the short-range behavior of the deuteron wave function is related to uncertainties in the off-shell behavior of the nucleon-nucleon (NN) interaction itself.

II. ELECTROMAGNETIC FORM FACTORS AND TENSOR POLARIZATION

We briefly summarize the equations for the relevant electromagnetic form factors of the deuteron in the impulse approximation.

The unpolarized cross section for elastic *ed* scattering is given by Gourdin[4] as

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\rm Mott} \{A(q) + B(q)[1 + 2(1+\nu)\tan^2\theta/2]\}, \tag{1}$$

where $(d\sigma/d\Omega)_{\text{Mott}}$ is the point scattering of the electron from the deuteron, $\nu = q^2/4M_d^2$ and θ is the electron scattering angle. The terms $A(q)$ and $B(q)$ are the deuteron electric and magnetic form factors, respectively. We can express $A(q)$ in terms of the charge and quadrupole form factors as follows:

$$A(q) = G_0^2(q) + G_2^2(q)\,, \tag{2}$$

where the charge form factor is given by

$$G_0(q) = 2G_{ES}\int_0^\infty (u^2 + w^2) j_0\left(\frac{qr}{2}\right) dr\,. \tag{3}$$

Here u and w are the $^3S_1$ and $^3D_1$ partial wave components of the deuteron wave function, respectively, j_0 is a spherical Bessel function and the isoscalar form factor G_{ES} is given by

$$G_{ES} = G_{En} + G_{Ep}\,.$$

The proton and neutron electric form factors are taken to be those given by Janssens *et al.*[5]

$$G_{Ep} = 0.5\left(\frac{2.50}{1+q^2/15.7} - \frac{1.60}{1+q^2/26.7} + \frac{1.16}{1+q^2/8.19} - 0.06\right) \tag{4}$$

and

$$G_{En} = 0.5\left(\frac{2.50}{1+q^2/15.7} - \frac{1.60}{1+q^2/26.7} - \frac{1.16}{1+q^2/8.19} + 0.26\right). \tag{5}$$

Following Rand *et al.*[6] the tensor polarization for the recoil deuterons in *ed* scattering from aligned deuterons is given by

$$P_e = \frac{[2G_0G_2 + (1/\sqrt{2})G_2^2]}{(G_0^2 + G_2^2)}\,. \tag{6}$$

III. UNITARY TRANSFORMATIONS

The Schrödinger equation in the $^3S_1$-$^3D_1$ state can be written in the form

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{12} & H_{22} \end{pmatrix}\begin{pmatrix} |u\rangle \\ |w\rangle \end{pmatrix} = E\begin{pmatrix} |u\rangle \\ |w\rangle \end{pmatrix}. \tag{7}$$

We apply a unitary transformation,[7] of the form

$$W = \begin{pmatrix} U_1 & 0 \\ 0 & U_2 \end{pmatrix}, \tag{8}$$

to the wave function and make the following choice for U_1 and U_2:

$$\begin{aligned} &\text{(i) } U_1 = 1,\quad U_2 = U\,,\\ &\text{(ii) } U_1 = U,\quad U_2 = 1\,,\\ &\text{(iii) } U_1 = U_2 = U\,, \end{aligned} \tag{9}$$

where

$$U = 1 - 2|g\rangle\langle g|\,, \tag{10}$$

with $\langle g|g\rangle = 1$.

If $p(r)$ stands for $u(r)$ or $w(r)$ and $\bar{p}(r)$ for the appropriate transformed partial wave component, then it follows from Eq. (9) that for a transformation of finite range R we have

$$\int_0^R p^2(r)dr = \int_0^R \bar{p}^2(r)dr\,, \tag{11}$$

which in particular means that the percentage D state of the deuteron P_D is preserved.

We choose[8]

$$g(r) = \begin{cases} C(R-r)^\alpha(1-\beta r), & r \le R \\ 0, & r > R\,. \end{cases} \tag{12}$$

The constant C is determined by the normalization condition and we choose $\alpha > 2$ to ensure that the transformed wave functions besides being continuous have continuous first and second derivatives at R.

IV. THE SHORT-RANGE BEHAVIOR OF THE DEUTERON WAVE FUNCTION

We now intend to demonstrate that a measurement of P_e at $q = 4.5\ \text{fm}^{-1}$ to within $\pm 10\%$ accuracy would still allow a wide range of behavior of the deuteron wave function inside 0.7 fm. In particular this would not allow us to decide upon the presence of a hole in the deuteron wave function. To do this we carry out finite range unitary transformations on the supersoft-core (SSC) potential of de Tourreil and Sprung.[9]

We take as pseudodata for the deuteron electric form factor $A(q)$ that obtained nonrelativistically from the SSC interaction using Eqs. (4) and (5) for the nucleon electric form factors and assigned an error of $\pm 10\%$.

In Table I we list 12 interactions obtained using Eq. (12) with $R = 0.7$ fm, $\alpha = 2.1$ and for various β. All of these interactions have the same P_D and quadrupole moment in the impulse approximation (Q_D) as the SSC reference interaction while fitting the pseudodata for $A(q)$ to at least $q \ge 6.5\ \text{fm}^{-1}$, using Eqs. (4) and (5) for the nucleon electric form factors. Furthermore it is clear that the value of P_e at $q = 4.5\ \text{fm}^{-1}$ is always within the assumed error on the pseudodatum at that point. The deuteron partial wave components which suitably combined give the deuteron wave functions of Table I are plotted in Fig. 1 and show a wide variety of behavior inside 0.7 fm.

It is clear that, although we took P_e produced by the SSC potential at $q = 4.5\ \text{fm}^{-1}$ as our pseudodatum, a fit to this value of P_e does not require a

TABLE I. Interactions varying in the core region obtained using Eq. (12) with $R=0.7$, $\alpha=2.1$ and for various β. For the class of transformation refer to Eq. (9). All interactions have the same percentage D state (5.45%) and quadrupole moment (0.279 fm^2) as that of the SSC potential. For each interaction P_e at $q=4.5$ fm^{-1} is given.

Interaction	Class	β (fm^{-1})	P_e
$I1$	(i)	−5.0	0.627
$I2$	(i)	2.2	0.626
$I3$	(i)	3.1	0.627
$I4$	(i)	3.6	0.627
$I5$	(ii)	−5.0	0.659
$I6$	(ii)	2.2	0.625
$I7$	(ii)	3.1	0.637
$I8$	(ii)	3.6	0.667
$I9$	(iii)	−5.0	0.660
$I10$	(iii)	2.2	0.625
$I11$	(iii)	3.1	0.638
$I12$	(iii)	3.6	0.668
SSC			0.626

suppressed deuteron wave function inside 0.7 fm. Our result does not disagree with the conclusion of Hockert and Jackson[1] who showed that a measurement of P_e at $q\approx 4.5$ fm^{-1} with 10% accuracy would place useful limits on the allowable variation in $G_0(q)$. These limits are not sufficiently narrow to enable us to distinguish clearly between different interactions inside 0.7 fm, unless one imposes further theoretical constraints on the "allowed" deuteron wave functions inside that distance, in particular the constraint of a deuteron wave function suppressed in the inner region. This presupposes a degree of confidence in the present state of meson theoretical calculations of the nucleon-nucleon potential at short distances, which appears to be hard to justify. Furthermore it is clear from Table I that an accuracy considerably better than ±10% is required in the measurement of P_e at $q\approx 4.5$ fm^{-1} to improve the situation significantly.

We now consider the suggestion of Moravcsik and Ghosh[3] that a determination of P_e in the region $q=6$–10 fm^{-1} would yield a rather definitive determination of the deuteron wave function in the interior region down to about 0.2 fm. Clearly this is a much more restrictive proposal than that of measurement of P_e at just a single point. The interactions $I2$, $I6$, and $I10$ fit $A(q)$ to within 4% and P_e to within 1% out to $q=10$ fm^{-1} while $I3$ likewise fits $A(q)$ and P_e out to $q=8.5$ fm^{-1}. In addition interactions ($I1, I4, I7, I11$) fit P_e out to $q=(9.5,\ 9.75,\ 8.75,\ 8.75)$ fm^{-1} except that $I7$ and $I11$ exceed the assumed error of ±10% at $q=5.5$ fm^{-1} (by 4%). Furthermore the pseudodata for $A(q)$ is fitted to $q=(8.5,\ 8.5,\ 7.75,\ 7.75)$ fm^{-1}, respectively. [The proton and neutron electric form factors given by Eqs. (4) and (5) have been used.] These wave functions are rather different from the suppressed SSC wave function inside 0.7 fm.

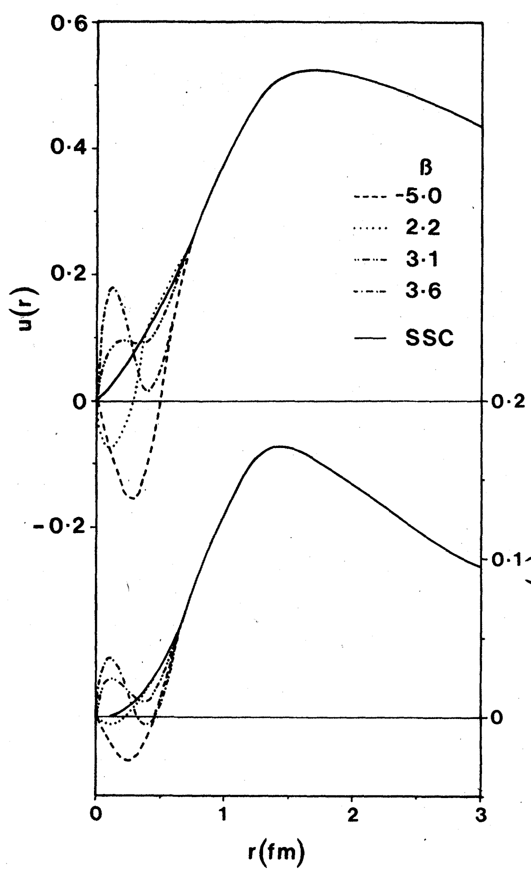

FIG. 1. The $^3S_1$ and $^3D_1$ deuteron partial wave components which suitably combined give the deuteron wave functions of Table I, plotted as a function of the radial distance in fm.

We conclude that a measurement of P_e at $q\simeq 4.5$ fm^{-1} with an accuracy of ±10% would not allow us to determine the short-range behavior of the deuteron wave function unambiguously. Furthermore we see that even a rather complete measurement of P_e in the range $q=6$–10 fm^{-1} with ±10% accuracy is unlikely to allow us to distinguish unambiguously between NN interactions inside 0.7 fm, considerably more accurate measurements being necessary. Even worse meson exchange currents could be expected to complicate the analysis considerably in the region $q=6$–10 fm^{-1} as well as (to a lesser degree) relativistic effects.

The fact that interactions $I2$, $I6$, and $I10$ fit $A(q)$ to within 4% and P_e to within 1% out to $q=10$ fm^{-1} is rather discouraging. It seems to indicate that even the assumption of an unrealistically accurate measurement of P_e up to $q=10$ fm^{-1}, would not allow us to determine the behavior of the inner region of the *NN* interaction, unless it is complemented by a corresponding improvement in our theoretical knowledge of the *NN* interaction. If meson theory could rigorously exclude deuteron wave functions which have no hole inside 0.7 fm, like most of our examples, then of course a measurement of P_e within ±10% might perhaps be used to determine the size of the hole and the hardness or softness of the corresponding core. Our counter examples would not be relevant in that case.

V. CONCLUSION

Assuming the supersoft-core potential of de Tourreil and Sprung as our reference interaction to produce pseudodata for the *ed* tensor polarization P_e of the recoil deuterons and the deuteron electric form factor $A(q)$, we investigated to which extent a measurement of P_e at $q \simeq 4.5$ fm^{-1} within ±10% accuracy suggested by Hockert and Jackson[1] would reduce the latitude in the short-range behavior, i.e., less than 0.7 fm, of the deuteron wave function. It was found that such a measurement would not allow us to distinguish unambiguously between different types of short-range behavior of the deuteron, obtained by means of unitary transformations inside 0.7 fm, if we do not *a priori* require a suppressed deuteron wave function. Even a complete fit to the pseudodata for P_e within ±1% and to $A(q)$ within ±4% out to $q = 10$ fm^{-1}, would not allow us to determine the deuteron wave function inside 0.7 fm^{-1} uniquely, unless the presence of a hole in the deuteron wave function is assumed beforehand. However, it is still possible that experimental measurements of P_e within ±10% would allow us to distinguish between "hard" and "soft" interactions producing holes in the deuteron wave function and the size of these holes, as suggested by Hockert and Jackson[1] and Moravicsik and Ghosh.[3]

Our deuteron wave functions inside 0.7 fm may be regarded as unlikely and could probably only be produced by nucleon-nucleon interactions which have a considerable degree of nonlocality coupled to a relatively weak repulsive core. However, as the cases where only the $^3D_1$ eigenfunction is unitarily transformed indicate, the degree of nonlocality and the softness of the required interactions should not be overestimated. Some of these interactions would almost certainly, in view of the well known accuracy of the unitary pole approximation (UPA) in the three-nucleon system, produce results for the triton binding energy quite close to those obtained for our reference interaction, the SSC potential.

Requiring *a priori* that there must be a hole in the deuteron wave function also presupposes a perhaps unwarranted degree of confidence in the present meson-theoretical calculations of the nucleon-nucleon interaction at such short distances. In short our counter examples may represent unlikely interactions, but especially in view of the relative ease with which we produced them, indicating many more possibilities, they cannot be excluded rigorously in the present state of our knowledge. In any case our only aim was to demonstrate that even a rather complete measurement of P_e would by itself, not allow us to decide whether there is a hole in the deuteron wave function or not.

However, our previous comments do not, and were not intended to detract from the usefulness of such a measurement of the *ed* tensor polarization P_e. In the first place, as we already pointed out, it could be very valuable even for the short-range behavior of the deuteron, if supplemented by unassailable meson theoretical considerations. Furthermore it could also possibly provide information on other deuteron properties such as the percentage *D* state. In the second place we did not discuss the possible influence of meson exchange currents and did not touch the relation between the deuteron and the ^{3}He form factor. In this case the position of the first node and the height of the second maximum of the elastic electric form factor seem to be drastically changed by meson exchange effects. The remaining strong discrepancies between existing calculations and experiment are possibly an indication of strong three-nucleon forces or of a failure of present ideas on nucleon and nuclear structure.

It would therefore be very interesting to compare the elastic monopole form factor of ^{3}He and the deuteron directly and to see whether a similar strong influence of exchange currents manifests itself in both form factors. The possibility of studying the monopole and quadrupole form factors separately could, in conjunction with the three-nucleon data, possibly lead to a better understanding of meson exchange currents and three-nucleon forces. These arguments only indicate some of the possible uses to which an experimental measurement of the *ed* tensor polarization data could be put.

ACKNOWLEDGMENTS

The authors acknowledge financial support by the Council of Scientific and Industrial Research, Pretoria. One of us (HF) would like to thank McMaster University for a visiting professorship (1978–1979) and especially Professor D. W. L. Sprung for his warm hospitality.

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