Electric dipole sum rules for the triton

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We apply Myers's technique to calculate sum rule values of the moments σ_2 and σ_3 for the triton photoeffect. We find that σ_2 agrees within 10% with the value found from Myers's cross sections. The disagreement found between values of σ_3 from sum rules and Myers's cross sections seems due to the sensitivity of the latter to inaccuracies in the high energy cross section.

[NUCLEAR REACTIONS Triton-photoeffect; hyperspherical harmonics; Sum rules.]

Myers *et al.*[1] used hyperspherical harmonics (hh) to calculate cross section moments σ_{-1}, σ_0, and σ_1 for the triton photoeffect. They compare values found from the cross section curve and from sum rules. In this note, we calculate σ_2 and σ_3 by both methods. We also calculate a more accurate value for σ_1 by including terms up to grand orbital $L=4$ in the hh expansion of the initial wave function and of the potential.

We use the same notation as Myers[1] and continue to limit ourselves to $E1$ transitions for a potential that commutes with the dipole operator. We first correct two misprints in Ref. 1: (i) Equation (2.22) should read $\sigma_{-1}=(2\pi^2/9)\alpha\langle i|r^2|i\rangle$. (ii) The "weight" in Table I should be specified as given in MeV.

The moment σ_1 is defined in Eq. (2.3) and evaluated in Eq. (4.1). We keep terms with $L=4$ in the expansion of the potential and the hh expansion of the wave function [Eqs. (3.9) and (3.8), respectively]. We find

$$\begin{aligned}\langle i|V(r,\Omega)|i\rangle &= 3\langle u_0(r)|V_0(r)|u_0(r)\rangle\\ &+ 6\sqrt{3}\,\langle u_0(r)|V_4(r)|u_4(r)\rangle\\ &+ 3\langle u_4(r)|V_0(r)|u_4(r)\rangle\,. \end{aligned} \qquad (1)$$

The matrix elements in Eq. (1) are evaluated numerically using hyperradial wave functions $u_0(r)$ and $u_4(r)$ and hypermultipoles $V_0(r)$ and $V_4(r)$ for the Volkov potential [see Ref. 1, Eq. (3.12)]. We obtain the sum rule value $\sigma_1=613$ MeV^2 mb. [Myers[1] used just the first term on the right-hand side of Eq. (1) and found $\sigma_1=580$ MeV^2 mb.]

We use the Heisenberg relation to find moments σ_2 and σ_3; σ_2 and σ_3 are proportional to the ground state expectation values of $\partial^2V/\partial z^2$ and $(\partial V/\partial z)^2$ respectively.[2] For the triton photoeffect in six dimensions the dipole operator is $e\eta_z/3^{1/2}$ [Ref. 1, Eq. (2.12)]. (The coordinate z in three dimensions is replaced by $\eta_z/3^{1/2}$.) We average $\partial^2V/\partial\eta_z^2$ and $(\partial V/\partial\eta_z)^2$ over five angles by Myers's technique [Eq. (2.21)][1] and truncate the hypermultipole expansion of $V(r,\Omega)$ at the first term. We find

$$\sigma_2=(\alpha/3)(2\pi\hbar^2/M)^2\langle i|[d^2V_0/dr^2+(5/r)dV_0/dr]|i\rangle \qquad (2)$$

and

$$\sigma_3=2\alpha(2\pi\hbar^2/M)^2\langle i|(dV_0/dr)^2|i\rangle\,. \qquad (3)$$

Numerical evaluation of the matrix elements in (2) and (3) for the Volkov potential give values of $\sigma_2=1.19\times10^4$ MeV^3 mb, and $\sigma_3=2.82\times10^5$ MeV^4 mb.

Values of photon energy E_γ, weight w, and cross section σ from Myers[1] are used with Eq. (3.7) to calculate the moments from the cross section. In Table I, we compare sum rule values of the moments and values found using cross sections. We also present values found using cross sections determined by Fang, Levinger, and Fabre[3] for coupled hh. (We double the Fang[3] cross sections given for the isospin $\frac{3}{2}$ final state.) We also reproduce Myers's values for σ_{-1}, σ_0, and σ_1.

Myers[1] and Fang[3] discussed the good agreement between values of moments σ_{-1} and σ_0 found from sum rule calculations and from the cross sections. Our more accurate calculation of the sum rule value of σ_1 reduces the disagreement with values from Myers's cross sections from 17% to 10%. The remaining 10% discrepancy is close to the 6% discrepancy found by Myers in comparing the Thomas-Reiche-Kuhn value $\sigma_0=39.8$ MeV mb with the value of σ_0 found from the cross sections. These results indicate that the severely truncated hh expansion of Myers gives reliable results for the photoeffect cross section for the Volkov potential of Wigner exchange character. The moments σ_2 and σ_3 give more weight to the cross section at higher energies. We again find only a 10% discrepancy between values of σ_2 found by our sum rule and from Myers's cross sections. However,

TABLE I. Comparison between moments for triton photoeffect.

Moment	Sum rule		Cross section	
	Myers [a]	Present	Myers [a]	Fang [b]
σ_{-1}	2.87 mb	...	2.86 mb [a]	2.80 mb [b]
σ_0	42.2, 39.8 MeV mb	...	42.2 MeV mb [a]	41.0 MeV mb [b]
σ_1	580 MeV^2 mb	613 MeV^2 mb	677 MeV^2 mb [a]	690 MeV^2 mb [b]
σ_2	...	1.19×10^4 MeV^3 mb	1.33×10^4 MeV^3 mb	1.43×10^4 MeV^{3}mb
σ_3		2.82×10^5 MeV^4 mb	4.99×10^5 MeV^4 mb	6.03×10^5 MeV^4 mb

[a] See Myers, Ref. 1

[b] See Fang, Ref. 3.

our values for the moment σ_3 have a large spread. We believe the sum rule value of σ_3 is accurate within 10%. (The percentage error due to the truncation should be similar to that of Myers's in calculation of the sum rule value of σ_1.) The 10-point quadrature formula [Eq. (3.7)][1] used in finding σ_3 from the cross sections gives a large weight to the highest energy point, at 201.7 MeV. This single point contributes almost 50% of σ_3, using either Myers's or Fang's cross sections. Calculations of high energy cross sections present a difficult numerical problem since their small values result from cancellations in the dipole overlap integral. The cross sections at 201.7 MeV are of order 10^{-5} of the peak cross sections, but they give major and poorly determined contributions to σ_3.

In summary, results shown in Table I are not inconsistent with good convergence of the hh expansion.

We could continue our sum rule calculation of σ_p to $p=4, 5, \ldots$ by further use of the Heisenberg relation. Equations analogous to (2) and (3) show that σ_p will continue to be finite for a smooth potential, such as the Volkov potential used in this paper. Then the cross section $\sigma(E_\gamma)$ should fall faster than $E_\gamma^{-(p+1)}$ asymptotically at high photon energy.

Our discussion above shows that $\sigma_4, \sigma_5, \ldots$ could not be accurately found from the cross sections, so a detailed comparison with sum rule calculations would not be worthwhile. However, in further work it might be possible to use the moments to reconstruct the cross section curve.

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[1]Karen J. Myers, K. K. Fang, and J. S. Levinger, Phys. Rev. C 15, 1215 (1977).

[2]J. S. Levinger, *Nuclear Photo-Disintegration* (Oxford Univ. Press, New York, 1961).

[3]K. K. Fang, J. S. Levinger, and M. Fabre de la Ripelle, Phys. Rev. C 17, 24 (1978).