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Effect of core vibration on the neutron-hole states of ^{207}Pb

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Three neutron-hole states—$1i_{13/2}$, $2f_{7/2}$, and $1h_{9/2}$—in ^{207}Pb are found to be affected strongly due to the coupling of these states with the 3^-, 5^-, 2^+, 4^+, and 6^+ vibration states of ^{208}Pb. The unperturbed shell model energies of these states are found at 2.00, 3.00, and 3.90 MeV, respectively. The fragmentations of $1i_{13/2}$, $1h_{9/2}$, and $1h_{11/2}$ states, as observed in a recent ^{208}Pb (^{3}He,α) reaction, can be quantitatively accounted for in terms of these altered level positions together with the hole-core coupling scheme.

[NUCLEAR STRUCTURE Deduced shell model energies of ^{207}Pb.]

It is universally believed[1] that ^{208}Pb is a very good closed shell nucleus. For this reason the nuclei in the neighborhood of ^{208}Pb offer a good testing ground for studying the validity of complex shell model calculations.[2-6] In all such calculations the observed single particle and hole states in ^{209}Pb, ^{209}Bi, ^{207}Pb, and ^{207}Tl are taken as the zeroth-order shell model states. But these states are actually the renormalized shell model states and they may differ from the zeroth-order shell model states due to the effect of core excitations. The importance of such core excitations was realized by Mottleson[7] and later Hamamoto[8] showed that two of the seven neutron states in ^{209}Pb – $2g_{9/2}$ and $1j_{15/2}$ should have the unperturbed energies at 0.30 and 2.00 MeV, respectively, relative to the observed $2g_{9/2}$ state in ^{209}Pb. These noticeable shifts in the energies are due to the coupling of the collective vibrations of ^{208}Pb with the neutron motion. Indeed, all four $1j_{15/2}$ states and four $2h_{11/2}$ states, observed in the ^{208}Pb(d,p) reaction work of Kovar *et al*,[9] can be understood in terms of this coupling scheme.[10]

In this work we shall see that the situation in ^{207}Pb is far more complicated as a result of neutron-hole core interactions. The $2f_{7/2}$, $1h_{9/2}$, and $1i_{13/2}$ states are appreciably affected by such interactions. The results of the present investigations have found convincing support from a recent[11] ^{208}Pb(^{3}He,α) reaction work with 206 MeV ^{3}He projectiles. We shall see that the fragmentations of the $1i_{13/2}$, $1h_{9/2}$, and $1h_{11/2}$ states, seen in the experimental work,[11] can be quantitatively understood in terms of the neutron-hole core interactions.

The details of the mathematical treatment are given in Ref. 10. The energies of the collective vibrations of ^{208}Pb, together with the zeroth-order shell model energies, ϵ_j, give the unperturbed energies of several $|\alpha j\rangle$ states, where

$$|\alpha j\rangle = |\lambda j';j\rangle . \tag{1}$$

These states interact with the zeroth-order shell model state $|\lambda=0,j\rangle$ through the interaction

$$H_{\text{int}} = k(r)(2\lambda+1)^{1/2}(\alpha_\lambda Y_\lambda)_0 . \tag{2}$$

The matrix element for this interaction is

$$\langle (j_2\lambda)j_1|H_{\text{int}}|j_1\rangle = \langle j_2|k(r)|j_1\rangle\langle j_2||Y_\lambda||j_1\rangle \times (2j_1+1)^{-1/2}\langle\alpha_\lambda\rangle , \tag{3}$$

where the amplitudes of the λ-pole vibration $\langle\alpha_\lambda\rangle$ are listed in Table I for the various collective states of ^{208}Pb. In Eq. (2),

$$k(r) = -r\frac{dV_0}{dr} , \tag{4}$$

V_0 being the spherical one particle potential.

All the single hole states $3p_{1/2}$, $2f_{5/2}$, $3p_{3/2}$, $1i_{13/2}$, $2f_{7/2}$, $1h_{9/2}$, and $1h_{11/2}$ are included in the calculation. The energies ϵ_j for the first three of these states are taken from experiments as these states are found not to be fragmented. Energies for the remaining four states are adjusted till the positions of the observed $1i_{13/2}$, $2f_{7/2}$, and $1h_{9/2}$ states are correctly reproduced. The fragmentations of these three states together with those of the $1h_{11/2}$ state are listed in Table II and are compared with the ^{208}Pb(^{3}He,α) reaction data.[11]

It will be seen from Table II that apart from reproducing the main components of the $1i_{13/2}$ and $1h_{9/2}$ states, we get three groups of these states at 4.30, 5.00, and 6.00 MeV. The group at 4.30 MeV contains, in addition, two $1h_{11/2}$ fragments. In the experimental work[11] this group is identified

TABLE I. Collective states of ^{208}Pb (Ref. 10).

E (MeV)	λ	$\langle\alpha_\lambda\rangle$
2.61	3^-	0.046
3.20	5^-	0.017
4.08	2^+	0.025
4.32	4^+	0.024
4.42	6^+	0.015

TABLE II. The $1h_{9/2}$, $1h_{11/2}$, and $1i_{13/2}$ states in ^{207}Pb.

State nlj	E_j (Cal) MeV	a_{0j}^2	$\frac{\sum a_{0j}^2 E_j}{\sum a_{0j}^2}$	E_j (Expt) MeV	σ_{exp} ($\theta = 2°$) mb/sr
$1i_{13/2}$	1.63	0.92	1.63	1.634	17.0
$1h_{9/2}$	3.35	0.72	3.35	3.423	3.65
$1i_{13/2}$	4.12	0.01			
$1h_{11/2}$	4.35	0.04	4.30	4.20	1.3
$1h_{9/2}$	4.36	0.02			(0.9)
$1h_{11/2}$	4.45	0.02			
$1h_{9/2}$	4.75	0.04			
$1h_{9/2}$	4.94	0.02			
$1h_{9/2}$	5.03	0.05	5.00	5.10	3.4
$1h_{11/2}$	5.14	0.01			(1.1)
$1h_{9/2}$	5.23	0.01			
$1h_{9/2}$	5.40	0.09			
$1i_{13/2}$	5.75	0.03			
$1i_{13/2}$	6.34	0.01	6.00	6.10	3.0
$1i_{13/2}$	6.42	0.01			(1.0)
$1h_{11/2}$	6.89	0.12			
$1h_{11/2}$	7.24	0.02		6.6	
$1h_{11/2}$	7.96	0.02	...	9.9	9.6
$1h_{11/2}$	8.13	0.13			
$1h_{11/2}$	8.26	0.04			
$1h_{11/2}$	8.57	0.56			

as $1i_{13/2}$, but we find that the major contribution is from the $1h_{11/2}$ state through the $(3^-, 1i_{13/2})$ and $(5^-, 1i_{13/2})$ core coupled states. The identifications of the 5.00 and 6.00 MeV states are correctly done in Ref. 11 as $1h_{9/2}$ and $1i_{13/2}$, respectively. In the last column of Table II we have given estimates for the (^{3}He, α) cross section for the three bands assuming $\sigma_{\text{total}}(\theta = 2°)$ for $1i_{13/2}$, $1h_{9/2}$, and $1h_{11/2}$ to be 20.0, 5.0, and 10.0 mb/sr and multiplying these σ_{total} with the respective $a_{0j}{}^2$, the single hole strengths listed in the third column of Table II. We get order of magnitude agreement between the measured $\sigma(\theta = 2°)$ and the calculated $\sigma(\theta = 2°)$. The Q-value dependence of σ and the effect of the two step process (^{3}He, ^{3}He$'$, α) are neglected in these estimates. They will increase the calculated values of σ.

As will be evident from Table II, the $1h_{11/2}$ states are considerably fragmented and are spread over a region of 6.89 to 8.57 MeV as compared to the observed 6.6 to 9.9 MeV spread. This fragmentation is due to the proximity of the unperturbed $1h_{11/2}$ state with the $(2^+, 1h_{9/2})$, $(4^+, 1h_{9/2})$, and $(6^+, 1h_{9/2})$ core coupled states.

The zeroth-order shell model energies of the neutron-hole states in ^{207}Pb are listed in Table III. The ϵ_j for the $2f_{7/2}$ and $1h_{9/2}$ states can be modified by about 100 keV towards the higher excitation region if the couplings of the $3s_{1/2}$ and $2d_{3/2}$ states (at about 7.5 MeV excitation) with the 3^- and 5^- collective states of ^{208}Pb are considered. Since the energies of these two positive parity states are not known, their effect on the $2f_{7/2}$ and $1h_{9/2}$ cannot be quantitatively estimated. But from the ^{208}Pb(p, d) reaction[12,13] data we can say that the primary fragments of these two positive parity states are not seen up to an excitation of 7.5 MeV. So the quoted corrections of 100 keV for the tabulated ϵ_i of $2f_{7/2}$ and $1h_{9/2}$ states are the upper limit of correction. It is probably less than 100

TABLE III. The ϵ_j for ^{207}Pb and the ground state wave function of ^{206}Pb.

State	$3p_{1/2}$	$2f_{5/2}$	$3p_{3/2}$	$1i_{13/2}$	$2f_{7/2}$	$1h_{9/2}$	$1h_{11/2}$
ϵ_j (MeV)	0	0.570	0.897	2.0	3.00	3.90	8.00
C_j^2 (Cal)	0.59	0.24	0.07	0.06	0.02	0.01	...
C_j^2 (Expt) (Ref. 12)	0.60	0.17	0.14	0.05	0.03	0.01	...

keV.

In Table III we have also listed the component C_j^2 of the ground state wave function of ^{206}Pb calculated in terms of the pairing interaction using the listed ϵ_j values. These are compared with the experimental estimates of Lanford.[12] From the agreement observed between these two sets of C_j^2 we can conclude that our adopted ϵ_j's for ^{207}Pb are consistent with the observed ground state structure of ^{206}Pb.

To summarize, the present research advocates the use of the revised zeroth-order energies for the neutron-hole states, as listed in Table III. It is necessary to examine to what extent these energies will modify the shell model calculations[2-6] on ^{206}Pb, ^{205}Pb, and ^{204}Pb. It is also likely that some of the proton particle and hole states of ^{208}Pb should be corrected for such particle-core interactions. For quantitative estimates of such corrections we need good reaction data for the weaker fragments of the states in ^{209}Bi and ^{207}Tl.

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[1] N. Stein, in *Proceedings of the International Conference on Properties of Nuclear States*, *Montreal, 1969*, edited by M. Harvey (University of Montreal Press, Montreal, 1969).

[2] C. W. Ma and W. W. True, Phys. Rev. C 8, 2313 (1973).

[3] G. H. Herling and T. T. S. Kuo, Nucl. Phys. A181, 113 (1972).

[4] J. Vary and J. N. Ginocchio, Nucl. Phys. A166, 479 (1971).

[5] T. F. Harvey and D. M. Clement, Nucl. Phys. A176, 592 (1971).

[6] J. B. McGrory and T. T. S. Kuo, Nucl. Phys. A247, 283 (1975).

[7] B. R. Mottelson, Proceedings of the International Conference on Nuclear Structure, Tokyo, edited by J. Sawada [J. Phys. Soc. Japan, Suppl. 24, 87 (1968)].

[8] I. Hamamoto, Nucl. Phys. A155, 362 (1970).

[9] D. G. Kovar, N. Stein, and C. K. Bockelman, Nucl. Phys. A231, 266 (1974).

[10] P. Mukherjee, I. Mukherjee, and R. Majumdar, Nucl. Phys. A294, 73 (1978).

[11] J. Van de Wiele, E. Gerlic, H. Langevin-Jolit, and G. Duhamel, Nucl. Phys. A297, 61 (1978).

[12] W. A. Lanford, Phys. Rev. C 11, 815 (1975).

[13] S. M. Smith, P. G. Roos, C. Moazed, and A. M. Bernstein, Nucl. Phys. A173, 32 (1971).