PHYSICAL REVIEW C VOLUME 19, NUMBER 2 FEBRUARY 1979

Reactive content of the optical potential

J. M. Eisenberg*
Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel †
and Department of Physics, University of Virginia, Charlottesville, Virginia
(Received 7 April 1978)

A simple unitarity relation is derived for the elastic projectile-nucleus scattering amplitude. This result provides an alternative interpretation and justification for the *ordinary* distorted wave impulse approximation, and thus may serve to correct a misapprehension concerning this approximation which might erroneously be obtained from recent trends in the literature.

[NUCLEAR REACTIONS unitarity, optical potential, knock-out reactions, DWIA.]

Recent studies[1,2] of the reactive content of the optical potential in a single-scattering model, and applications[3] of this result to $(\pi, \pi N)$ reactions, have pointed out that the lowest-order optical potential has implicit within it a description of single-nucleon knockout with too little absorptive effect. It seems important to emphasize that this should not create an impression that—at least in knock-out reactions—the usual distorted wave impulse approximation (DWIA) may not be fully consistent, or that one should modify the DWIA for $(\pi, \pi N)$, say, so that the outgoing pion is *not* distorted and the outgoing nucleon is distorted *only* by the real potential which initially binds the nucleon. The ordinary DWIA would have the knock-out process governed by the matrix element

$$\langle \chi_{\pi'}^{(-)} \chi_{N'}^{(-)} \phi_{A-1} \Big| \sum_{i=1}^{A} t_{\pi N}^{(i)} \Big| \chi_{\pi}^{(+)} \phi_A \rangle , \tag{1}$$

where $t_{\pi N}^{(i)}$ is the pion-nucleon scattering amplitude for the ith nucleon and ϕ_A and ϕ_{A-1} are the initial and final nuclear wave functions for A and $(A-1)$ nucleons, respectively. Distorted waves with absorptive parts appear for the initial pion in $\chi_{\pi}^{(+)}$ *and for the final-state pion and outgoing nucleon* in $\chi_{\pi'}^{(-)}$ and $\chi_{N'}^{(-)}$. The conclusion which is inferred from the considerations of elastic unitarity for the lowest-order optical potential in Refs. 1–3 is that, in the final state, the pion there appears as a plane wave, $\chi_{\pi'}^{(-)} \to \phi_{\pi'}$, and the nucleon is distorted by a real binding potential. These authors correctly attribute this situation to the use of a single-scattering approximation. Nonetheless, the precise nature of the breakdown in this approximation has remained implicit and a dangerous impression may thus be left—not intended by those authors—that an approach which is in some sense "more consistent" than that used to derive the ordinary DWIA might suppress full final-wave distortions.

We shall therefore attempt a very simple, and essentially exact, derivation of the general unitarity relation for the elastic amplitude, in the course of which we hope to clarify both the conventional result and the unconventional one. Since the manipulations and considerations are very straightforward, and their presentation here justified only by the need to avoid the possibility of a serious misinterpretation, we strive for maximal concision by using the notation of Ref. 2, though with the suppression of the ubiquitous index e for "elastic" there. Thus the transition operator for scattering on the many-nucleon target at system energy E is given by the solution to the Lippmann-Schwinger equation

$$T(E) = V + VG(E)T(E) , \tag{2}$$

where $V = \sum_{i=1}^{A} v_i$ is the sum of the individual interactions and

$$G(E) = (E + i\epsilon - K - H_T)^{-1}, \quad \epsilon \to 0^+ , \tag{3}$$

with K the projectile kinetic energy operator and H_T the target Hamiltonian. The projection operator P is defined to project onto the target ground state, the remaining nuclear space being subsumed in $Q = 1 - P$. With standard and straightforward manipulations, we may then introduce an optical operator[4,5] $U(E)$ such that

$$T(E) = U(E) + U(E)PG(E)T(E) , \tag{4}$$

and the optical operator satisfies

$$U(E) = V + VQG(E)U(E) . \tag{5}$$

We now exploit the theorem on unitarity in Eqs. (44)–(46) of Ref. 2, namely, for operators such that

$$A(E)=B(E)+B(E)C(E)A(E)\,, \qquad (6)$$

the discontinuity

$$\Delta A(E)\equiv A(E+i\epsilon)^\dagger - A(E+i\epsilon)\equiv 2\pi i\,\mathrm{disc}A(E) \qquad (7)$$

is given by

$$\Delta A = A^\dagger \Delta C A + (A^\dagger C^\dagger + 1)\Delta B(1+CA)\,. \qquad (8)$$

Applying this to Eq. (4) yields immediately

$$\Delta T = T^\dagger \Delta(PG)T + [T^\dagger(PG)^\dagger + 1]\Delta U[1+(PG)T]\,, \quad (9)$$

while Eq. (5) gives the discontinuity for the optical operator equally directly as

$$\Delta U = U^\dagger \Delta(QG)U\,, \qquad (10)$$

since V is assumed energy independent. We note that the lowest-order optical potential does not satisfy this (exact) relationship, which may be viewed as a signal for the troubles noted in Refs. 1 and 2; on the other hand, Eq. (10) has the same form as the approximate result arising from nucleon correlations alone in the second-order optical potential. It is now convenient to introduce the strictly elastic phase space factor

$$\Lambda(E)\equiv \mathrm{disc}(PG)\,, \qquad (11)$$

and its strictly inelastic counterpart

$$M(E)\equiv \mathrm{disc}(QG)\,. \qquad (12)$$

Inserting Eq. (10) into Eq. (9) and using these definitions yields the result for the elastic unitarity

$$\mathrm{disc}T = T^\dagger \Lambda T + [T^\dagger(PG)^\dagger + 1]U^\dagger M U[1+(PG)T]\,. \qquad (13)$$

(This equation ignores the problem of connectedness discussed in detail in Ref. 2 [see below their Eq. (47)] and is therefore expected to be valid only where rearrangement is small.) To identify a given inelastic amplitude, this must be compared with the general unitarity relation

$$\mathrm{disc}T = T^\dagger \Lambda T + \sum_{n\neq e} T_n^\dagger \Lambda_n T_n = T^\dagger \Lambda T + T_{\mathrm{inel}}^\dagger M T_{\mathrm{inel}}\,, \qquad (14)$$

where n refers to inelastic channels whose amplitudes are obtained from T_{inel}. Thus a particular inelastic amplitude for a transition from the elastic channel e to channel r, whose nucleon state is projected from Q by R, is given by Eq. (13) as

$$T_r = RU[1+(PG)T]P\,. \qquad (15)$$

Now the optical operator can also be viewed as satisfying the relationship

$$U\cong \sum_{i=1}^{A}\tau^i + \sum_{i=1}^{A}\tau^i(QG)U = \sum_{i=1}^{A}\tau^i + U(QG)\sum_{i=1}^{A}\tau^i\,, \qquad (16)$$

where for lucidity an innocuous approximation is made which is valid to order A^{-1} and is easily eliminated by noting that our projection operators refer to antisymmetrized nuclear states and using the techniques of Ref. 5 [compare Eqs. (7)–(10) of Ref. 2]. The single-scattering operator satisfies

$$\tau^i(E) = v_i + v_i QG(E)\tau^i(E)\,. \qquad (17)$$

Thus Eq. (13) can be rewritten, in a form exact to order A^{-1}, as

$$\mathrm{disc}T = T^\dagger \Lambda T + [T^\dagger(PG)^\dagger + 1][U^\dagger(QG)^\dagger + 1] \times \sum_i \tau^{i\dagger} M \sum_j \tau^j[1+(QG)U][1+(PG)T]\,. \qquad (18)$$

This is to be contrasted with Eq. (56) of Ref. 2 where the optical potential has been approximated already in Eq. (9) as

$$PUP \approx P\sum_{i=1}^{A}\tau^i P\,, \qquad (19)$$

and the factors $[1+(QG)U]$ and its Hermitian conjugate are lost. For the inelastic amplitude of Eq. (15) we can write, using the second form in Eq. (16),

$$T_r = R[1+U(QG)]\sum_{i=1}^{A}\tau^i[1+(PG)T]P\,, \qquad (20)$$

again exact to order A^{-1}. If we now invoke this same hierarchy of diagonal terms here, we will select out of Q only the state R, obtaining the lowest-order DWIA result for the pertinent transition operator

$$T_r \cong R[1+U(RG)]\left[R\sum_{i=1}^{A}\tau^i P\right][1+(PG)T]P\,; \quad (21)$$

higher-order corrections to the DWIA have been treated in Ref. 6. In Eq. (21), the diagonal factor $[1+(PG)T]P$ distorts the initial pion in the full optical potential, while the diagonal factor

$$R[1+U(RG)] \cong R\left[1+\sum_{i=1}^{A}\tau^i(RG)\right]$$

distorts the outgoing pion in a lower-order optical potential, which is defined with respect to the final nuclear state R and not the initial one P. The inclusion of the factor $R[1+U(RG)]$ is the analog here of calculating elastic scattering with the amplitude which is the solution for the optical potential, and thus iterates successive diagonal scatterings on the ground state, rather than using lowest-order impulse approximation for the elastic amplitude. The key nondiagonal step is in the "hard" collision $R\Sigma_i \tau^i P$. Lastly, the final nuclear system in the state R is to be an eigenstate of the full nuclear Hamiltonian and so, for knock-out processes, will contain optical distortion for the outgoing nucleon when projection is made for a particular state of the residual nucleus. Thus Eq. (21) shows that the ordinary DWIA indeed emerges from a treatment of elastic unitarity which is *not* restricted to lowest order.

It is a pleasure to thank the Center for Advanced Studies of the University of Virginia for its support while part of this work was carried out, and to acknowledge a very useful exchange with Dr. E. F. Redish on the subject of this work.

---

*Work supported in part by the U.S.-Israel Binational Science Foundation, the Israel Academy of Science, and by the Israel Center for Immigrant Absorption in Science.

†Permanent address.

[1]P. C. Tandy, E. F. Redish, and D. Bollé, Phys. Rev. Lett. 35, 921 (1975).

[2]P. C. Tandy, E. F. Redish, and D. Bollé, Phys. Rev. C 16, 1924 (1977).

[3]A. W. Thomas and R. H. Landau, Phys. Lett. 77B, 155 (1978).

[4]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).

[5]A. K. Kerman, H. McManus, and R. M. Thaler, Ann. Phys. (N.Y.) 8, 551 (1959).

[6]A. Gal and J. M. Eisenberg, Phys. Rev. C 14, 1273 (1976).