Low lying spectra and electromagnetic transitions for ^{164}Dy, ^{166}Er, and ^{168}Yb nuclei

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Low-lying spectra of ^{164}Dy, ^{166}Er, and ^{168}Yb are studied in the framework of the pseudo SU(3) model allowing natural parity neutrons alone to be active. Various interband and intraband $B(E2)$ values are predicted and compared with experimental results and other model calculations. The agreement is quite reasonable. We predict a different level structure for the 2^+ quasirotational band in ^{164}Dy and 3^+ quasirotational band in ^{168}Yb. The present calculation shows that the enlargement of the natural parity neutron space will improve the predictions considerably.

[NUCLEAR STRUCTURE ^{164}Dy, ^{166}Er, ^{168}Yb, Pseudo SU(3) model, only natural parity neutrons, $\tilde{Q}$. $\tilde{Q}$ Interaction, calculated levels, $B(E2)$, ^{166}Er $-I_\gamma$, compared with experiment and other models.]

INTRODUCTION

Heavy deformed even-even nuclei are studied quite extensively in the framework of the unified model of Bohr and Mottelson.[1] The yrast cascades of even-even nuclei are studied microscopically in the projected Hartree-Fock scheme in great detail.[2] So far no attempt has been made to study these nuclei in a microscopic way where the characteristics of all bands can be understood in a unified way (microscopic approach means building wave functions from shell model states). This is because of the well-known complex mixing between shell-model states which gives rise to diagonalizing hopelessly large matrices. The pseudo SU(3) scheme suggested by Arima *et al.*[3] and Hecht and Adler[4] makes this problem tractable, at the same time giving meaningful results. Some of the preliminary calculations on odd-A nuclei[5,6] and odd-odd nuclei[7] support this viewpoint. However, even in this highly truncated scheme, the problem of heavy deformed even-even nuclei is quite formidable. To overcome this difficulty a simplified approach was recently suggested by the author.[8] (Hereafter we refer to this as SAK). Here (i) the proton configurations and (ii) the unnatural parity neutron configurations are assumed to be part of the core which carries zero angular momentum. Then a description for the structure of the low-lying positive parity states is sought out in terms of the leading and first few pseudo SU(3) irreducible representations (IR) corresponding to the natural parity neutrons. All the nuclei (ten in number) having eight natural parity neutrons in the $\tilde{\eta}=4$ shell are studied in this approach.[9] Assumptions (i) and (ii) are probably too drastic, but the fact that an extremely simple effective interaction [see Eq. (1)] accounts for the spectra of quite a number of similar nuclei with similar interaction strengths (see the results of Ref. 9) is very interesting and prompted us to do a fairly extensive calculation which will be presented in the next two sections. Although a good agreement with experiment is not expected, an extensive calculation is carried out to see (1) whether approximations (i) and (ii) have any validity, (2) the directions in which they are to be extended.

In the present paper the nuclei ^{164}Dy, ^{166}Er, and ^{168}Yb are studied using SAK. The method of calculation is given in Sec. I and the results are displayed in Sec. II. Some concluding remarks are made in Sec. III.

I. METHOD OF CALCULATION

The nuclei ^{164}Dy, ^{166}Er, and ^{168}Yb have 16 neutrons outside the magic core $N=82$. Now filling these 16 neutrons in order, in the Nilsson orbits that grow out of the spherical orbits $h_{9/2}$, $f_{7/2}$, $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $i_{13/2}$ it can be seen that, for deformation ≈ 0.3, six neutrons fill the Nilsson orbits that grow out of the $i_{13/2}$ orbit (these are called unnatural parity neutrons) and the remaining ten neutrons (these are called natural parity neutrons) fill the Nilsson orbits which can be assigned pseudo Nilsson quantum numbers (see Fig. 2 of Ref. 5) with pseudo major oscillator shell quantum number $\tilde{\eta}=4$. Thus the nuclei ^{164}Dy, ^{166}Er, and ^{168}Yb have ten natural parity neutrons in $\tilde{\eta}=4$ shell. A complete classification of various heavy deformed even-even nuclei, according to natural parity neutron number is given in Ref. 10. In the present

calculation, out of all possible pseudo SU(3) representations corresponding to ten natural parity neutrons, only the $(2\tilde{0},4)$ and $(2\tilde{2},0)$ SU(3) representations in $\tilde{S}=0$ partition are allowed. One can easily see that these representations can give rise to four bands. The ground state rotational band and the γ-vibration bands with $K=2^+$ and 4^+, grows out of the pseudo SU(3) representation $(2\tilde{0},4)$ and the β-vibration band grows out of the $(2\tilde{2},0)$ representation. In fact, these are the only prominent bands that are seen in the nuclei ^{164}Dy, ^{166}Er, and ^{168}Yb. Because of the very simple model space we have chosen, the largest matrix to be diagonalized is 4×4. The Hamiltonian is taken to be[9]

$$H=a'\sum_i \tilde{l}_i^2+b'\sum_{i<j}\tilde{Q}(i)\cdot\tilde{Q}(j)$$
$$=a\sum_i \tilde{l}_i^2+b\tilde{L}(\tilde{L}+1)+c\tilde{C}_2, \qquad (1)$$

where $\tilde{C}_2$ is the bilinear Casimir invariant of the pseudo SU(3) group. In the present calculation the strengths a, b, and c are adjusted to get best fit with experiment. We make use of the fact that the interband separations depend largely on the $\sum_i \tilde{l}_i^2$ force, the intraband separations are governed by the $\tilde{L}(\tilde{L}+1)$ force and the separation between the ground state rotational band the the β-vibration band is governed by the $\tilde{C}_2$ force. For the $\sum_{i<j}\tilde{Q}(i)\cdot\tilde{Q}(j)$ force it is known that $c=-b/3$. In fact it can be seen by looking ahead that a fairly good spectrum is obtained for ^{164}Dy and ^{166}Er nuclei with $c=-b/3$. But for ^{168}Yb nuclei it is found that c should be varied independently. This amounts to saying that the force is not of pure quadrupole-quadrupole type for the ^{168}Yb nucleus, but it is an admixture of $\sum_{i<j}\tilde{Q}(i)\cdot\tilde{Q}(j)$ force and $\tilde{C}_2$ force. This process of varying the strength c is reminiscent of adjusting the coupling constant between $K=0^+$ band and the β-vibration band in the collective models.[11]

We use the Racah algebra available in Ref. 5 to decompose the Hamiltonian to SU(3) irreducible tensors and then the matrix elements are calculated using the coefficient of fractional parentage (CFP) expansion.

In the calculation of $B(E2)$ values, only the $T^{(\tilde{1}\tilde{1})\tilde{K}=1,\tilde{L}=2,\tilde{S}=0,J=2}$ tensor of the $\tilde{Q}_{op}$ [from here onwards superscripts ($\tilde{}$) will be omitted for brevity] is included. This piece is shown to give 95% of the predicted static quadrupole moments for odd-A nuclei.[6] We took the effective charge to be $e^n_{eff}=1$ and the R_0 value to be 1.4×10^{-13} cm as in Refs. 6 and 9).

II. RESULTS AND DISCUSSIONS

The energy spectra of ^{164}Dy, ^{166}Er, and ^{168}Yb nuclei are studied using the same IR (20, 4) and (22, 0). Although ^{170}HF falls into the same category, this nucleus is not studied as the available experimental information is very meager and the predictions do not change considerably from that of the above three nuclei, as we are using the same pseudo SU(3) representations for all the nuclei. The (22, 0) representation which is included in the present calculation, besides the leading pseudo SU(3) representation (20, 4) is found to play an important role, especially in predicting the interband transitions. First an attempt is made to study these nuclei by including only the (20, 4) representation, in the same spirit as in Ref. 5. It is found that we have to use an uncomfortably large strength for the $\sum l_i^2$ force (≈ 0.5 MeV). Even then, the agreement is reasonable only for the ground state band, but the 2^+ and 4^+ bands are found to be rather low in energy compared with the experiment (600–800 keV below). The intraband separations are also not good. We can see from Table I (col. 3) that the intraband transitions agree with the results obtained without the (22, 0) representation, but the interband transitions (Table II, col. 3) clearly show that matters will be much worse without the (22, 0) representation. This is precisely the point where the initial calculations in the pseudo SU(3) scheme failed.[5] Moreover, without (22, 0) representation we cannot account for the observed β-vibration band in ^{166}Er and ^{168}Yb nuclei. It is found that the (22, 0) representation mixes with the $K=0$ band of the (20, 4) representation considerably, but its mixing with the $K=2^+$ band is very small and zero for the $K=4^+$ band. For example, the off diagonal matrix element between (20, 4) $KL=4$ and (22, 0) $K=0$ $L=0$ is 3.914551, −0.054625, and 0.0 for $K=0$, 2, and 4, respectively. Due to this strong mixing between the two $K=0^+$ bands, the structure of the ground state band is changed considerably, but that of the others is almost unaltered. For example, see Table III for the wave functions with (22, 0) and without (22, 0) for $J=4^+$. Although the yrast band structure is changed by the (22, 0) representation, its character (the intraband separations and transitions) is not changed. At the same time the character of the other bands is improved considerably. In fact, the yrast band is shifted as a whole relative to the 2^+ γ-vibration band and the 2^+ γ-vibration band is shifted relative to the 4^+ γ-vibration band, giving right interband separations. Looking at Tables I, II, and IV we can infer that the (22, 0) representation improves considerably the interband transition probabilities. It is important to point out that the mixing between the (20, 4) and (22, 0) representations is understandable in terms of the fact that $\epsilon_H=2\lambda+\mu$ is the same for both of them, and the expectation value of the C_2 operator differ by $72c$ only.

In the present calculation, we study only the low-

TABLE I. $B(E2)$ values for the yrast cascade of ^{164}Dy, ^{166}Er, and ^{168}Yb nuclei. WG results taken from the calculation of Warke and Guyne, Ref. 13. Experimental results are taken from Ref. 23 for ^{164}Dy and from Ref. 14 for ^{166}Er and ^{168}Yb. The numbers in the brackets are the ones generated without (22,0) SU(3) representation.

		$B(E2;\ J_i \to J_f)$ $(e^2 10^{-48}\ \text{cm}^4)$						
		^{164}Dy			^{166}Er		^{168}Yb	
J_i	J_f	Calc	WG	Expt.	Calc	Expt.	Calc	Expt.
2^+	0^+	1.310 (1.308)	1.038	1.081	1.334	1.1	1.366	1.1
4^+	2^+	1.851 (1.809)	1.405	1.572	1.872	1.7	1.925	...
6^+	4^+	1.997 (1.992)	1.475	1.426	2.031	1.6	2.069	...
8^+	6^+	2.026 (2.090)	1.506	1.422	2.057	1.85	2.082	...
10^+	8^+	2.007 (2.076)	1.602	1.975	2.033	2.1	2.037	...
12^+	10^+	1.896 (1.923)	1.856	2.25	1.907	2.1	1.864	...

lying positive parity states of different bands and no attempt is made to study the high spin states for the reasons stated in the beginning. The negative parity part of the spectra is also not studied.

Looking ahead it can be seen that the fit for the spectra is good up to 10^+ and is within 30 keV. The intraband transitions are well reproduced while the branching ratios and interband transitions do show deviations. A detailed study of each of the nuclei is presented below.

^{164}Dy nucleus

The predicted spectrum for the ^{164}Dy nucleus is compared with the experimental spectrum in Fig. 1. The best fit is obtained for $a = -0.2$, $b = 0.0125$, and $c = -0.004166$ MeV. The yrast cascade is well reproduced up to the 10^+ level and the spacings agree favorably with the experimental values[12] and with the results of Warke and Gunye.[13] The agreement is within 40 keV.

TABLE II. $B(E2)$ branching ratios for ^{164}Dy nucleus. $D1$ results taken from the work of Davydov and Filippov (Ref. 24). $D2$ results taken from the work of Alaga *et al.* (Ref. 25). $D3$ results taken from the work of Mottelson (Ref. 26). Experimental values are taken from Ref. 26. The numbers in the brackets are the ones generated without (22,0) SU(3) representation.

		$\frac{B(E2;\ J_iK_i=2 \to J_fK_f)}{B(E2;\ J_i'K_i'=2 \to J_f'K_f'=0)}$				
K_f	$J_i \to J_f / J_i' \to J_f'$	Calc	Expt	$D1$	$D2$	$D3$
	$2^+ \to 0^+ / 2^+ \to 2^+$	0.250 (0.048)	0.466	0.466	0.700	0.52
	$2^+ \to 4^+ / 2^+ \to 2^+$	0.173 (0.272)	0.060	0.100	0.050	0.087
0^+	$3^+ \to 4^+ / 3^+ \to 2^+$	2.524 (19.342)	0.600	0.590	0.400	0.750
	$4^+ \to 6^+ / 4^+ \to 4^+$	0.413 (0.393)	4	0.240	0.087	0.194
	$4^+ \to 2^+ / 4^+ \to 4^+$	0.003 (0.166)	0.115	0.120	0.340	0.160
2^+	$4^+ \to 2^+ / 4^+ \to 4^+$	7.837 (3.496)	9.435	5.500	0.572	1.550
0^+	$5^+ \to 6^+ / 5^+ \to 4^+$	13.93 (129.025)	4	...	...	...

TABLE III. Wave functions for $J=4^+$ states of ^{164}Dy nucleus. (For all the components in the wave function $S=0$).

SU(3) Representations	Band quantum numbers	Wave function
(20,4) and (22,0)	$K=0, J=4$	$0.842\,291 \mid (20,4)K=0L=4\rangle + 0.092\,708 \mid (20,4)K=2L=4\rangle$
		$0.001\,504 \mid (20,4)K=4L=4\rangle + 0.530\,988 \mid (20,0)K=0L=4\rangle$
	$K=2, J=4$	$-0.045\,070 \mid (20,4)K=0L=4\rangle + 0.993\,276 \mid (20,4)K=2L=4\rangle$
		$0.031\,062 \mid (20,4)K=4L=4\rangle - 0.102\,016 \mid (22,0)K=0L=4\rangle$
	$K=4, J=4$	$-0.008\,422 \mid (20,4)K=0L=4\rangle - 0.030\,015 \mid (20,4)K=2L=4\rangle$
		$0.999\,389 \mid (20,4)K=4L=4\rangle + 0.015\,770 \mid (22,0)K=0L=4\rangle$
	$K=0_2, J=4$	$-0.537\,069 \mid (20,4)K=0L=4\rangle + 0.062\,512 \mid (20,4)K=2L=4\rangle$
		$-0.015\,920 \mid (20,4)K=4L=4\rangle + 0.841\,068 \mid (22,0)K=0L=4\rangle$
(20,4)	$K=0, J=4$	$0.957\,346 \mid (20,4)K=0L=4\rangle + 0.288\,867 \mid (20,4)K=2L=4\rangle$
		$+ 0.006\,605 \mid (20,4)K=4L=4\rangle$
	$K=2, J=4$	$-0.288\,931 \mid (20,4)K=0L=4\rangle + 0.956\,853 \mid (20,4)K=2L=4\rangle$
		$+ 0.030\,832 \mid (20,4)K=4L=4\rangle$
	$K=4, J=4$	$0.002\,586 \mid (20,4)K=0L=4\rangle - 0.031\,425 \mid (20,4)K=2L=4\rangle$
		$+ 0.999\,503 \mid (20,4)K=4L=4\rangle$

The predicted 2^+ γ-vibration band is 90 keV above the experimental value. The $B(E2)$ value corresponding to the transition $2^+2^+ \rightarrow 0^+0^+$ (we use the notation $J_iK_i \rightarrow J_fK_f$) is 0.0204 against the experimental value 0.024. The levels observed at 1987, 2058, 2266, and 2418 keV are assigned to be 2^+, 3^+, 4^+, 5^+, and 6^+, respectively.[15] It is interpreted that they are members of the band built on the 2^+ quasiparticle state formed from the coupling of $[523]\,\frac{5}{2}^-$ and $[521]\,\frac{1}{2}^-$ neutrons. The present cal-

TABLE IV. Comparison of γ-ray relative intensities for ^{166}Er nucleus. E-1 results taken from Ref. 17.

		$BE2$ ($e^2 10^{-48}$ cm^4)		E_γ (keV)		T_γ (10^8 s^{-1})		Relative intensities		
$J_iK_iJ_fK_f$		Calc	$E1$	Calc	$E1$	Calc	$E1$	Calc	$E1$	Expt.
72	80	0.171	0.073	547	465	1007.17	194	8.48	0.14	0.28 ± 0.06[a]
72	62	0.653	0.536	179	154	14.79	5.7	0.121	0.004	...
72	60	0.002	0.029	908	830	121.59	1406	1.0	1.0	...
72	52	1.618	1.370	359	300	1179.75	410	9.70	0.29	0.31 ± 0.16[a]
62	80	0.055	0.020	368	311	45.68	7.1	0.18	0.01	...
62	60	0.89	0.72	729	675	2258.25	1230	8.90	2.25	1.3 ± 0.7[a]
62	40	0.002	0.006	1017	955	253.73	546	1.0	1.0	...
62	52	0.801	0.714	179	146	18.36	5.8	0.07	0.01	...
62	42	1.436	1.230	317	263	565.04	190	2.22	0.35	0.6 ± 0.4[a]
52	60	0.131	0.059	550	530	808.21	306	1.42	0.18	0.18 ± 0.03[a]
52	42	1.215	0.960	137	117	7.35	2.6	0.01	0.002	...
52	40	0.011	0.039	837	810	571.28	1670	1.0	1.0	...
52	32	0.993	1.219	245	215	108.04	53.2	0.19	0.032	0.08 ± 0.06[a]
42	60	0.035	0.013	412	410	51.03	19.6	0.03	0.01	...
42	32	1.631	1.380	118	96	4.51	1.4	0.002	0.001	...
42	40	0.089	0.072	700	690	1834.58	1397	1.0	1.0	...
42	22	0.760	0.062	204	170	32.67	10.6	0.02	0.01	...
42	20	0.001	0.012	891	880	346.10	732	0.19	0.52	0.56[b]
32	40	0.083	0.040	582	595	681.66	368	1.0	1.0	...
32	22	2.326	1.86	86	74	1.34	0.5	0.002	0.001	...
32	20	0.036	0.054	774	780	1221.37	1940	1.79	5.3	5.0[b]
22	40	0.013	0.004	496	522	44.33	17.5	0.07	0.01	...
22	20	0.036	0.058	688	707	677.92	1270	0.99	1.14	1.1[b]
22	00	0.021	0.030	772	787	682.96	1120	1.0	1.0	...

[a] Taken from Ref. 17.

[b] Taken from Ref. 27. The reason for the peculiar normalization of relative intensities is due to the particular nature of the experimental data of Ref. 17.

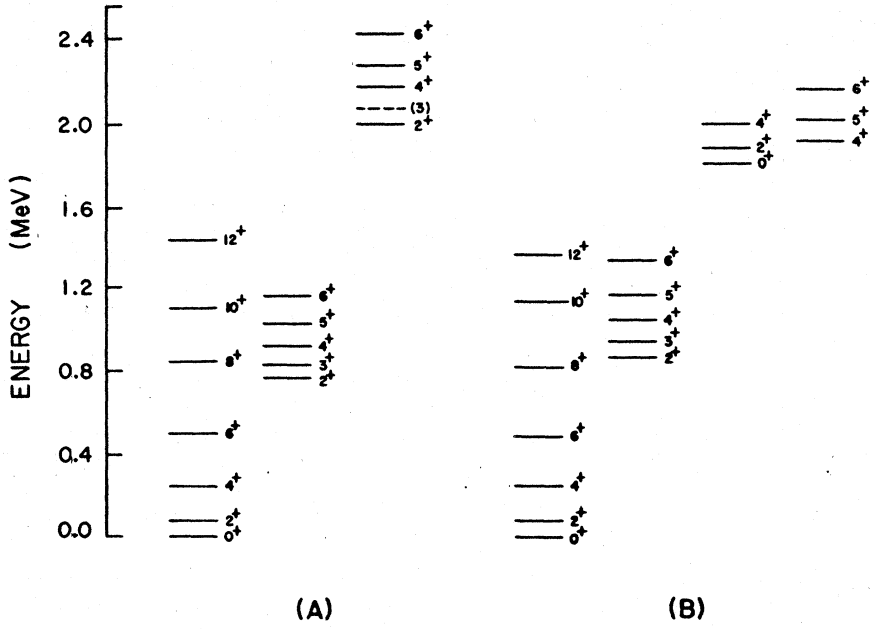

FIG. 1. Energy spectra for ^{164}Dy nucleus. (a) Experimental spectrum. The 10^+ and 12^+ levels are taken from Ref. 23. Other levels are taken from Ref. 12. (b) Calculated spectrum.

culation predicts a β-vibration band and a 4^+ ν-vibration band in this region. The branching ratio $B(E2;4^+4^+ \rightarrow 2^+0^+)/B(E2;4^+4^+ \rightarrow 4^+0^+)$ is found to be 0.308. For the ^{168}Yb nucleus which has the same number of natural parity neutrons as ^{164}Dy, we have a 4^+ level in the same region and the experimental value for the branching ratio is 0.3.[16] On the basis of this observation, we predict that the observed 4^+, 5^+, and 6^+ levels are members of a $K=4^+$ band. The spectra of ^{166}Er and ^{168}Yb have a 0^+ β-vibration band in the region where the 2^+ (1987 keV) level in ^{164}Dy spectra is observed and support the view that this may be a member of the β-vibration band. We expect the β-vibration band to start at about 1.8 MeV (within 200 keV error) and in fact a level is observed around 1.938 MeV (Ref. 15) whose level character is not established. In the region where the 3^+ level (2058 keV) is observed, we have a 4^+ level (from the β-vibration band) with the branching ratio $B(E2;4^+0_2^+ \rightarrow 2^+0_1^+)/B(E2;4^+0_3^+ \rightarrow 2^+0_1^+) \approx 14.5$. On the basis of the I(I+1) rule the 4^+ level should be at about 2100 keV. There are levels observed at about 2081 and 2126 keV,[15] and one can measure the $B(E2)$ branching ratios to see whether any one of them is our 4^+ level. The 3^+ level which is observed at 2058 keV can be predicted if we include the lower SU(3) representation (17, 7). It is worth mentioning that a 2^+ quasiparticle level whose intrinsic state grows out of $[523]\frac{5}{2}^-$ and $[521]\frac{1}{2}^-$ orbits is equivalent to the highest weight state of the (21, 2) SU(3) representation, which will have $S=1$ (for $S=0$, we cannot have an intrinsic state with $\epsilon_{max}=44$ and $\Lambda_{max}=1$; ϵ and Λ values can be predicted on the basis of the Nilsson diagrams as shown in Ref. 5). Usually the states with $S=1$ are expected to lie higher in energy than the place where the 2^+ level is observed. The experimentally observed negative parity band built on the 2^- band cannot be predicted in the present model as we are not including $i_{13/2}$ excitations.

In Table I the intraband $B(E2)$ transition probabilities for the yrast cascade up to 12^+ are compared with the experimental results and the variation after projection (VAP) calculations of Warke and Gunye. The agreement is within 20%. In Table II the $B(E2)$ branching ratios are compared with experimental values and rotor model calculations. The agreement for the branching is reasonable except for the $4^+2^+ \rightarrow 2^+0^+/4^+2^+ \rightarrow 4^+0^+$ branching ratio which is too small by a factor of 40. This may be due to the fact that the mixing with the other lower SU(3) representations [i.e., (17, 7), (18, 5), (14, 10), etc.] is important for 4^+ levels and this particular transition may be sensitive to admixtures. The situation is similar in the case of ^{166}Er and ^{168}Yb nuclei also. A large

number of 4^+ levels observed in these nuclei do support this viewpoint. Besides this particular case, the agreement for others is within a factor of 4. In view of the extreme truncation scheme we are using this is all that can be expected.

^{166}Er nucleus

The predicted spectrum for the ^{166}Er nucleus is compared with the experimental spectrum in Fig. 2. The best fit is obtained for $a = -0.168$, $b = 0.014$, and $c = -0.004666$ MeV. The yrast band is reproduced rather satisfactorily and compares favorably with the asymmetric rotor model predictions of Cline and Reich.[17] The 2^+ γ-vibration band is predicted at about 20 keV above the experimental value. The $B(E2)$ transition $2^+2^+ \rightarrow 0^+0^+$ is predicted to be 0.021 against the experimental value of 0.027.[14] The intraband separations are reasonable except for the 7^+ level which is coming 70 keV above the experimental value. The β-vibration band is at about the same energy (within 10 keV) as the experimental value. We predict a 4^+ γ-vibration band at about 1.7 MeV. From the spectra of ^{168}Yb which has the same number of natural parity neutrons as ^{166}Er, we expect that within 1.8 ± 0.2 MeV one should find this band. The unnatural parity states $5^-, 6^-$, etc. observed[18] at about 1.7 MeV can be accounted for, if we include $i_{13/2}$ excitations.

The intraband $B(E2)$ values for the yrast cascade are compared with experimental values in Table I. It can be seen that the agreement is good. For ^{166}Er, an extensive study of the branching ratios is made by Cline and Reich using the asymmetric rotor model of Davydov and Filippov. They used two more parameters in addition to the four usual parameters (rotation-vibration coupling constant and the three moment of inertia parameters) and the $B(E2)$ values for almost all transitions from the γ-vibration band to the ground state band are studied. From Table IV we can see that the predictions of SAK are surprisingly in good agreement with their calculations. The energy spacings ($E_\gamma = E_{J_i} - E_{J_f}$) and the $B(E2)$ values are well reproduced. Notable deviations occured for the transitions $7^+2^+ \rightarrow 6^+0^+$ and $4^+2^+ \rightarrow 2^+0^+$. The reason may lie in the fact that our 7^+ is a pure state, as it can be generated by the (20, 4) representation alone, and mixing with the lower representation (17, 7) may be important here. We have calculated the intensities for the transitions $J_iK_i \rightarrow J_fK_f$ using the relation[19]

$$T_\gamma = 1.23 \times 10^{-2} E_\nu^5(\text{keV}) B(E2)\ \text{s}^{-1}. \qquad (2)$$

The T_γ values and the relative intensities of γ-transitions are compared with the results of Cline and Reich, and the experimental values. The agreement is within a factor of 4. Notable deviations

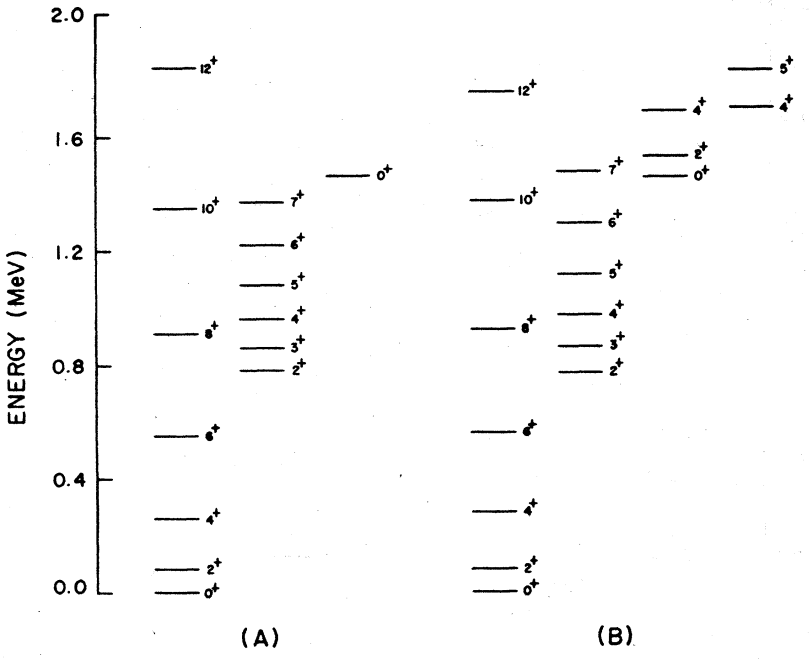

FIG. 2. Energy spectra for ^{166}Er nucleus. (a) Experimental spectrum (taken from Ref. 12). (b) Calculated spectrum.

occurred for the relative intensities of $7^+2^+ \rightarrow J_f K_f$ transitions. The reason lies in the fact that the intensity of the $72 \rightarrow 60$ transition is used in predicting these relative intensities, and only this particular transition is not well predicted by SAK for the reasons mentioned above. Besides this case, the intensity corresponding to the transition $52 \rightarrow 60$ is coming to be eight times larger than the experimental values. This may also be due to the fact that the 5^+ state is coming from the (20, 4) representation alone. On the whole a very large number of $B(E2)$ values are predicted well by SAK.

^{168}Yb nucleus

The predicted spectrum for ^{168}Yb is compared with the experimental spectrum in Fig. 3. The best fit is obtained for $a = -0.15$, $b = 0.0145$, and $c = -0.008$ MeV. The larger strength used for the C_2 force clearly shows that the coupling between the β-vibration band and the ground state rotational band is large compared to the one for the ^{164}Dy and ^{166}Er nuclei. The ground state band is well reproduced as usual. The 2^+ γ-vibration band is coming at about 100 keV below the experimental value. The observed β-vibration band is starting 40 keV above the experimental value and the 4^+ band is 150 keV above the experimental value. The intraband separations are reasonably good. The level observed at 1.479 MeV is assigned to be a 3^+ level.[20] The 4^+ level observed at 1.551 MeV is interpreted as a member of the 3^+ band.[20] The calculated $B(E2; 4^+4^+ \rightarrow 2^+0^+)/B(E2; 4^+4^+ \rightarrow 4^+0^+) = 0.167$ agrees well with the experimental value (0.31 ± 0.6).[16] This result forces us to think that the 4^+ level observed at 1.551 MeV is the band head for $K = 4^+$ band. The 3^+ level may be coming from the pseudo SU(3) representations (17, 7) or (18, 5). The unnatural parity band 5^-, 6^-, 7^- observed at 2 MeV (Ref. 21) cannot be predicted as we are neglecting unnatural parity neutron configurations. Experimentally many 4^+ levels are observed around 2.2 MeV (Ref. 16) (as many as five levels). This may be due to the fact that the lower SU(3) representations (18, 5) and (14, 10) have the same expectation value for the C_2 operator and the 4^+ levels generated out of these representations are lying close together.

The intraband transition probabilities for the ground state band are given in Table I. Only the $2^+ \rightarrow 0^+$ $B(E2)$ value is observed experimentally.[14] The calculated results for other transitions may be compared with experimental values whenever they are measured. In Table V, $B(E2)$ branching ratios are compared with the experimental results

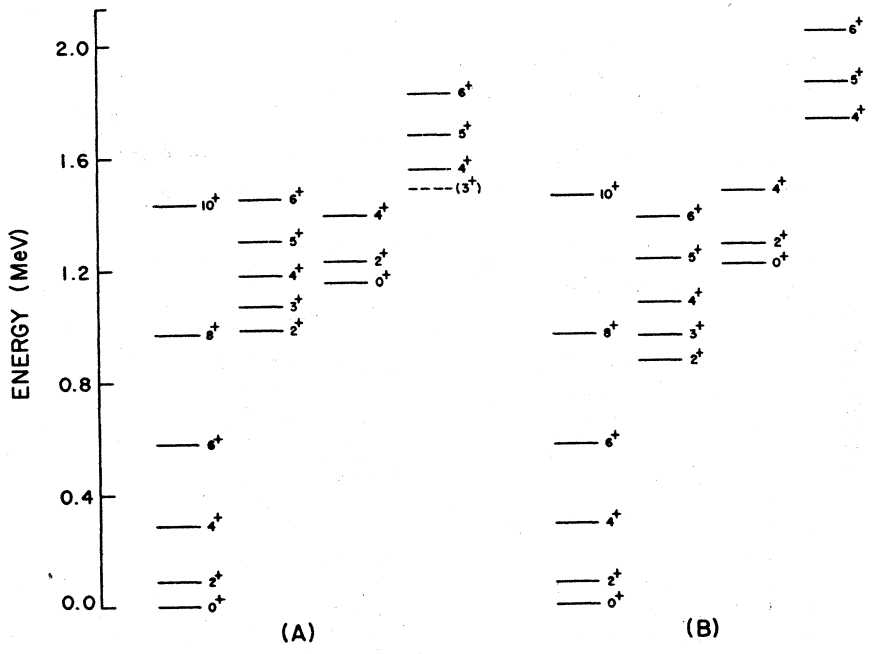

FIG. 3. Energy spectra for ^{168}Yb nucleus. (a) Experimental spectrum. The 3^+, 4^+, 5^+, and 6^+ levels at about 1.5 MeV are taken from Ref. 19. Other levels are taken from Ref. 12. (b) Calculated spectrum.

TABLE V. $B(E2)$ branching ratios for ^{168}Yb nucleus. Y1: Experimental results taken from Ref. 16. Y2: Experimental results taken from Ref. 28.

	$\frac{B(E2;\ J_i K_i = 2 \to J_f K_f = 0)}{B(E2;\ J_i' K_i = 2 \to J_f' K_f = 0)}$		
$J_i \to J_f / J_i' \to J_f'$	Calc	Y1	Y2
$2^+ \to 0^+ / 2^+ \to 2^+$	0.33	0.57	0.44
$3^+ \to 4^+ / 3^+ \to 2^+$	1.67	0.64	1.00
$4^+ \to 2^+ / 4^+ \to 4^+$	0.03	0.17	0.11
$5^+ \to 6^+ / 5^+ \to 4^+$	0.437	1.10	1.90
$6^+ \to 4^+ / 6^+ \to 6^+$	0.012	0.058	...

of Wilson and that of Charvet *et al.* The agreement is within a factor of 4.

III. CONCLUDING REMARKS

An extensive calculation of $B(E2)$ values and branching ratios for the nuclei ^{164}Dy, ^{166}Er, and ^{168}Yb are presented in the present paper. The primary aim here is to see the applicability of SAK and to look for the directions in which it is to be extended to get better agreement with the experiment. With the results of the present calculation, we see that the expansion of natural parity neutron space will improve matters considerably as shown by the inclusion of the (22, 0) representation. The mixing effects are reflected in the right direction, in the predictions of interband separations and transition probabilities. Besides including $S=0$ states, $S=1$ states may also be included (for example to account for the observed 3^+ levels) to get better agreement. These seem to be feasible with the development of fast computer codes to calculate reduced matrix elements.[22] However, these codes are to be modified to suit the $\eta=4$ shell calculations. In the near future, we hope to succeed in this attempt.

No attempt is made in the present paper to study high spin states. Here one has to include $i_{13/2}$ neutron configurations and proton configurations.[2] If some work is done on 9-SU(3) algebra,[29] the available Racah algebra makes product space calculations feasible.

On the whole the predictions of SAK are quite reasonable, keeping in mind its extreme simplicity. However, we clearly note that to get more meaningful numbers one should relax the assumption in SAK in a "naive fashion" before making extensive calculations.

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