Triaxial description of ^{188}Os and ^{188}Pt[†]

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A scheme based on variation after approximate angular momentum projection from a triaxially symmetric intrinsic wave function has been developed and applied to the study of ground-state band and γ band of ^{188}Os and ^{188}Pt nuclei. The intrinsic calculation has been performed in the framework of Hartree-BCS theory employing the pairing $+ Q\cdot Q$ interaction of Baranger and Kumar. The calculated level energies, $B(E2)$ values and electromagnetic moments compare quite well with the available experimental values. The position of the $K = 4^+$ band, many $B(E2)$ values and electromagnetic moments have been predicted. This study predicts relatively more rigid triaxial shapes for these nuclei than have been usually believed.

NUCLEAR STRUCTURE Variation after angular momentum projection with triaxial symmetry, ^{188}Os and ^{188}Pt; level energies, $B(E2)$ values, electromagnetic moments of ground and γ band, pairing + $Q\cdot Q$ interaction.

I. INTRODUCTION

The experimental and theoretical studies of the collective properties of nuclei in the Os and Pt region have been pursued with great excitement during the recent years. This being a shape transition region, it has provided the opportunity to test various models of nuclear structure. The extensive studies of Baranger and Kumar[1-4] employing the pairing + $Q \cdot Q$ force in the rare-earth region have been quite successful in accounting for the ground-state deformation. One beautiful feature of their theory is that simultaneously it can describe the ground band, β band, and γ band. They determine the collective parameters of the Bohr Hamiltonian and then solve the corresponding Schrödinger equation. These calculations have been very involved and contain very few adjustable parameters. However, their method has been undoubtedly the only method which provides a great wealth of prediction with very few parameters.

In this paper we aim at developing a different approach based on variation after angular momentum projection[5] theory which has been applied[6] in recent years to the study of high spin states with considerable success in this region. Over the years it has been rather well established that this theory, though apparently suitable for the description of nuclei having rotational spectra, can describe the spectra in the backbending region which are far from rotational in character. Hence, it appears that a scheme based on this type of theory should be able to describe transitional nuclei in the Os and Pt region.

The experimental evidence for the nonaxial collective motion in these nuclei has been quite convincing. So an intrinsic wave function characterized by both the symmetry parameter β and the asymmetry parameter γ needs to be considered and angular momentum projection from such a state must be done. Assuming the nuclear Hamiltonian to be separable into intrinsic and collective parts, we have developed an approximate angular momentum projection scheme from a triaxial intrinsic wave function which could simultaneously describe the ground-state band and γ band. The nuclear state in this scheme would have definite β and γ in contrast to that of Baranger and Kumar where the states are a superposition of all possible β and γ. It would be interesting to see how well such a picture would be valid. A further interest in this scheme arises from the recently developed interest[7,8] on the question of the existence of a static triaxial shape in this region. The static calculation[2] of Baranger and Kumar shows that only 3 nuclei out of 80 nuclei they considered are triaxial, but the deformation potential is very shallow and soft to γ deformation. For example, in case of ^{188}Os, they find that the triaxial solution is more bound than the axial solution by only 44 keV. Calculations[9] using the deformed oscillator model and Strutinsky shell correction yield similar results. In a paper[8] published recently, Giraud and Grammaticos have investigated the possible existence of static triaxial shapes in the framework of Hartree-Fock-Bogoliubov theory. In a few cases they find triaxial minima, but the potentials are very shallow. They feel that the zero-point collective motion might wash out these minima and thus have concluded that γ softness is a common feature. This would give rise to nonzero dynamical mean values of γ as is borne out in the dynamical calculations[4] of Baranger and Kumar. On the other hand, opposite views have been expressed by Lee *et al.*[7] who, from the experimental study of ^{192,194,196}Pt and model analysis of the results have concluded that these nuclei are not γ soft. Further in the study of odd A nuclei by Meyer-ter-Vehn[10] and Toki and Faessler,[11] the assumption of rigid triaxial shapes for the neighboring even-even nuclei yields satisfactory results in agreement with experiment. In view of the above, we hope our study would be quite useful. An outline of our theory is presented in Sec. II. The details of our calculation and the comparison of our results with experiment are discussed in Sec. III. The discussion and conclusions are given in Sec. IV.

II. THEORY

A. The interaction

We take precisely the Hamiltonian of Baranger and Kumar[2] which consists of pairing and quadrupole interactions:

$$H=\sum_{\alpha\tau} \epsilon^{\tau}_{\alpha} C^{\dagger}_{\alpha} C_{\alpha} - \tfrac{1}{4}\sum_{\tau} G_{\tau} \sum_{\alpha\gamma} C^{\dagger}_{\alpha} C^{\dagger}_{\bar{\alpha}} C_{\bar{\gamma}} C_{\gamma} - \tfrac{1}{2}\chi' \sum_{\tau\tau'\mu} \alpha_{\tau}\alpha_{\tau'} \sum_{\alpha\beta\gamma\delta} \langle\alpha|Q^{2(\tau)}_{\mu}|\gamma\rangle\langle\delta|Q^{2(\tau')}_{\mu}|\beta\rangle C^{\dagger}_{\alpha} C^{\dagger}_{\beta} C_{\delta} C_{\gamma} \,, \tag{1}$$

where $\alpha, \beta, \ldots,$ stand for the complete list of spherical harmonic oscillator single particle quantum number $|nljm\rangle$, τ is the isospin projection, G_τ is the pairing force strength, χ' is the strength of $Q\cdot Q$ force,

$$Q^{2(\tau)}_{\mu} = \left(\frac{r^2}{b_\tau^{\,2}}\right) Y^{(\tau)}_{2\mu}(\theta,\varphi) \,,$$

and b_τ is the oscillator strength parameter. α_τ is a factor close to unity, the details of which are given in Ref. 2. Hartree-Bogoliubov calculation with the above interaction boils down to Nilsson-BCS calculation which we outline below briefly.

B. Solution of Nilsson Hamiltonian

For a triaxially symmetric system, the relevant Nilsson Hamiltonian is

$$h^{\tau}_{D} = h^{\tau}_{s} - D_0\alpha_\tau Y^{(\tau)}_{20}(\theta,\varphi) - D_2\alpha_\tau\left[Y^{(\tau)}_{22}(\theta,\varphi) + Y^{(\tau)}_{2-2}(\theta,\varphi)\right] \,, \tag{2}$$

where h^{τ}_{s} refers to the spherical single particle Hamiltonian, the deformation variables D_0 and D_2 have dimension of energy and are related to the usual parameters β and γ by

$$D_0 = \hbar\,\omega\beta\cos\gamma, \quad D_2 = \frac{1}{\sqrt{2}}\,\hbar\,\omega\beta\sin\gamma \,, \tag{3}$$

where $h\omega$ is the oscillator energy. Then one solves the eigenvalue equation

$$h^{\tau}_{D}|i\rangle = \eta_i|i\rangle \,, \tag{4}$$

where the eigenstates $|i\rangle$ have the structure

$$|i,\tau\rangle = \sum_{jm} C^{i\tau}_{jm}\,|jm\tau\rangle \tag{5}$$

and η^{τ}_{i}'s are the eigenvalues. The states $|i\rangle$ have $e^{i\pi J_z}$ symmetry and hence all m's differing by 2 will appear in the above summation.

C. BCS solution

A BCS transformation is performed from the Nilsson states to a deformed quasiparticle state. For proton and neutron, the energy gap Δ, chemical potential λ, and the corresponding wave functions U_i and V_i are generated separately. Thus the intrinsic wave function which is a function of β and γ is obtained as

$$\Phi(\beta,\gamma) = \prod_{i,\tau}\left(U^{\tau}_{i} + V^{\tau}_{i}\, a^{\dagger}_{i\tau} a^{\dagger}_{\bar{i}\tau}\right)|0\rangle \,, \tag{6}$$

where $|0\rangle$ is the closed shell vacuum state. The operator $a^{\dagger}_{i\tau}$ creates a particle in the Nilsson state $|i,\tau\rangle$ and $a^{\dagger}_{\bar{i}\tau}$ in the corresponding time reserved state.

D. Angular momentum projection

The nuclear stationary state $|\alpha IM\rangle$ can be expanded in terms of basis states having good band quantum number K as

$$|\alpha IM\rangle = \sum_K A_{\alpha IK}(\beta,\gamma)\Psi^I_{MK} \ , \tag{7}$$

where I, M are the angular momentum and the projection across Z axis in the laboratory frame and α is any other quantum number necessary to specify the states. We assume the nuclear Hamiltonian to be separable into intrinsic part H_i plus the rotation part H_r

$$H = H_i + H_r \ . \tag{8}$$

H_r has the usual form

$$H_r = A_x J_x^2 + A_y J_y^2 + A_z J_z^2 \tag{9}$$

and $A_x = 1/2\mathcal{I}_x$, $A_y = 1/2\mathcal{I}_y$, and $A_z = 1/2\mathcal{I}_z$ where $\mathcal{I}_x$, $\mathcal{I}_y$, and $\mathcal{I}_z$ are, respectively, the x, y, and z components of the moment of inertia. Equation (7) can be written in the form

$$|\alpha IM\rangle = \Phi(\beta,\gamma) \sum_{K\geq 0}{}' A_{\alpha IK}(\beta,\gamma)\left(\frac{2I+1}{16\pi^2(1+\delta_{K0})}\right)^{1/2} \times [D^{I*}_{MK} + (-1)^{I-K} D^{I*}_{M-K}] \ . \tag{10}$$

Hence to make angular momentum projection we have to set up the Hamiltonian matrix in the basis and then the coefficients $A_{\alpha IK}$ [Eq. (7)] would be obtained by solving the eigenvalue equation

$$\sum_K \langle \Psi^I_{MK'} | H | \Psi^I_{MK}\rangle A_{\alpha IK}(\beta,\gamma) = E_I A_{\alpha IK'}(\beta,\gamma) \ . \tag{11}$$

The matrix element of the Hamiltonian H is

$$\langle \Psi^I_{MK'} | H | \psi^I_{MK}\rangle = \langle \Phi | H_i | \Phi\rangle \delta_{KK'} + \langle \Psi^1_{MK'} | A_x J_x^2 + A_y J_y^2 + A_z J_z^2 | \Psi^I_{MK}\rangle \ . \tag{12}$$

Putting $A_x = A - \delta$ and $A_y = A + \delta$, H_r reduces to

$$H_r = AJ^2 + (A_z - A) J_z^2 - \tfrac{1}{2}\delta(J_+^2 + J_-^2) \ . \tag{13}$$

We know that

$$\langle \Phi | H_i | \Phi\rangle = \langle \Phi | H | \Phi\rangle - A_x \langle \Phi | J_x^2 | \Phi\rangle - A_y \langle \Phi | J_y^2 | \Phi\rangle - A_z \langle \Phi | J_z^2 | \Phi\rangle \ . \tag{14}$$

Using Eqs. (13) and (14) in (12) we get

$$\langle \Psi^I_{MK'} | H | \Psi^I_{MK}\rangle = \left[\langle \Phi | H | \Phi\rangle - \frac{\langle \Phi | J_x^2 | \Phi\rangle}{2\mathcal{I}_x} - \frac{\langle \Phi | J_y^2 | \Phi\rangle}{2\mathcal{I}_y} - \frac{\langle \Phi | J_z^2 | \Phi\rangle}{2\mathcal{I}_z} + AJ(J+1) + (A_z - A)K^2 \right] \delta_{KK'} - \tfrac{1}{2}\delta\{[(J-K)(J+K+1)(J-K-1)(J+K+2)]^{1/2} \delta_{K',K+2} + [(J+K)(J-K+1)(J+K-1)(J-K+2)] \delta_{K',K+2}\} \ . \tag{15}$$

The correctness of the above formula can be tested by applying it to the following two cases:

1. Axially symmetric case

In this case $A_x = A_y$, $A_z = 0$ so $\mathcal{I}_x = \mathcal{I}_y = \mathcal{I}$ and $\delta = 0$. Taking the diagonal term which represents the energy of the state with angular momentum I belonging to the band K is

$$E_{I,K} = \langle \Psi^I_{MK} | H | \Psi^I_{MK}\rangle = \langle \Phi | H | \Phi\rangle - \frac{\langle \Phi | J^2 | \Phi\rangle}{2\mathcal{I}} + \frac{J(J+1)}{2\mathcal{I}} \ . \tag{16}$$

This agrees with the approximate projection formula derived by Das Gupta and Ginneken[12] and used by Nair and Ansari[13] in the study of backbending.

2. Triaxially symmetric case

Bouten, Elliot, and Pullen[14] have shown that the expression for energy $E_{I,K}$ of a state with angular momentum I and band quantum number K projected from a trixially symmetric intrinsic wave function Φ is

$$E_{I,K} = E_0 + \alpha K^2 + AI(I+1) \ , \tag{17}$$

where E_0, α, and A are characteristic of intrinsic structure. The diagonal term obtained from Eq.

(15) is

$$\langle\Psi^I_{MK}|H|\Psi^I_{MK}\rangle = E_{I,K}$$
$$= \langle\Phi|H|\Phi\rangle - \frac{\langle\Phi|J_x^2|\Phi\rangle}{2\mathcal{I}_x} - \frac{\langle\Phi|J_y^2|\Phi\rangle}{2\mathcal{I}_y} - \frac{\langle\Phi|J_z^2|\Phi\rangle}{2\mathcal{I}_z} + AJ(J+1) + (A_z - A)K^2 \quad (18)$$

which is similar to Eq. (17).

E. Scheme of calculation

(i) For a fixed value of β and γ, the Nilsson Hamiltonian [Eq. (2)] is solved, the eigenvalues and eigenstates [Eq. (4)] are obtained for protons and neutrons separately.

(ii) The BCS calculation is performed over the Nilsson states. The pairing gap Δ, the chemical potential λ, the wave functions U and V, and consequently the intrinsic wave function Φ [Eq. (6)] are obtained.

(iii) With the wave function Φ, the moments of inertia $\mathcal{I}_x$, $\mathcal{I}_y$, and $\mathcal{I}_z$ are calculated using the cranking formula. Then the Hamiltonian matrix [Eq. (15)] for each angular momentum state is set up and diagonalized to obtain the stationary state energies and the corresponding eigenfunctions. Then the values of β and γ are changed and the steps from (i) to (iii) are repeated until the minima in the (β,γ) space are obtained for each state of the ground band. Then the levels of the γ band and $K=4$ bands are the higher states coming from the diagonalization of the energy matrices at the above minima. This guarantees the mutual orthogonality of all the states.

F. Electromagnetic moments

Here we derive the expressions which are used in our calculation of spectroscopic quadrupole moments, $E2$ transition probabilities and magnetic dipole moments.

1. $E2$ transition probability

In the lab system the $E2$ operator is given by

$$\mathfrak{M}(E2,m) = \sum_p e_p r_p^2 y_{2m}(\Omega p) , \quad (19)$$

where $m=0$, ±1, ±2 and r_p, Ω_p and e_p represent, respectively, the radial coordinate, angular coordinate, and the effective charge of the particles outside the core. The transition probability is given by

$$B(E2,\alpha_i I_i \to \alpha_f I_f) = \frac{2I_f+1}{2I_i+1} |\langle\alpha_f I_f \| \mathfrak{M}(E2) \| \alpha_i I_i\rangle|^2 . \quad (20)$$

The $E2$ operator [Eq. (19)] is related to the corresponding operator $\mathfrak{M}'(E2,K)$ in the intrinsic frame as

$$\mathfrak{M}(E2,m) = \sum_K D^{2*}_{mK}\mathfrak{M}'(E2,K) . \quad (21)$$

Using (10), (21), and (20) we obtain

$$B(E2,\alpha_i I_i \to \alpha_f I_f) = \left[\sum_{\substack{K_i K_f \geq 0\\ K}} \frac{1}{[(1+\delta_{K_i0})(1+\delta_{K_f0})]^{1/2}} \{C(I_i 2 I_f; K_i K K_f) + (-1)^{I_i} C(I_i 2 I_f; -K_i K K_f)\} \times \langle\Phi_{I_i}|\mathfrak{M}'(E2,K)|\Phi_{I_f}\rangle A_{\alpha_i I_i K_i} A_{\alpha_f I_f K_f}\right]^2 , \quad (22)$$

where C refers to usual Clebsch-Gordan coefficients.

2. The spectroscopic quadrupole moment

The spectroscopic quadrupole moment of a state $|\alpha I\rangle$ is given by

$$Q(\alpha I) = \tfrac{16}{5}\pi\langle\alpha I, M=I|\mathfrak{M}(E2,0)|\alpha I, M=I\rangle . \quad (23)$$

Using (10) and (21) in the above expression, $Q(\alpha I)$ is obtained as

$$Q(\alpha I) = \left(\frac{16\pi}{5}\right)^{1/2} C(12I;\, I0I) \sum_{\substack{K_i K_f \geq 0\\ K}} \frac{1}{[(1+\delta_{K_i0})(1+\delta_{K_f0})]^{1/2}} \{C(I2I;K_i K K_f) + (-1)^I C(I2I;-K_i K K_f)\} \times \langle\Phi|\mathfrak{M}'(E2,K)|\Phi\rangle A_{\alpha I K_i} A_{\alpha I K_f} . \quad (24)$$

3. *Magnetic dipole moment*

Following Baranger and Kumar[1,3] we use the cranking model expression for the intrinsic magnetic moment. The K component of the intrinsic gyromagnetic ratio in this model is

$$g_K=\frac{1}{(\mathcal{g}_K+\mathcal{g}_K^C)}\left[g_K^C\mathcal{g}_K^C+\sum_{ij}{}' \frac{(U_iV_j-U_jV_i)^2|\langle i|J_K|j\rangle\langle j|M_K|i\rangle|}{E_i+E_j}\right], \quad (25)$$

where E_i and E_j are the energies of the quasiparticle states i and j, respectively, and M is the usual single particle magnetic moment operator

$$\vec{M}=g_l\vec{l}+g_s\vec{s}$$

with different gyromagnetic ratios g_l and g_s for neutrons and protons. g_K^c and $\mathcal{g}_K^c$ are the core gyromagnetic ratio and core moment of inertia whose values are the same as in Ref. 2. The K-component μ_K of the intrinsic magnetic moment is given by

$$\mu_K=g_KI_K\,. \quad (26)$$

Following the same procedure, the magnetic moment operator in the laboratory can be expressed as

$$\mathfrak{M}(M1,m)=\sum_K\mathfrak{M}'(M1,K)D^{1*}_{mK}\,. \quad (27)$$

The expectation values of the operator $\mathfrak{M}'$ in the intrinsic states are written as

$$\langle\Phi|\mathfrak{M}'(M1,0)|\Phi\rangle=(3/4\pi)^{1/2}\mu_3\,,$$
$$\langle\Phi|\mathfrak{M}'(M1,\pm1)|\Phi\rangle=\mp(3/8\pi)^{1/2}(\mu_1\pm i\mu_2)\,. \quad (28)$$

We use the definition

$$I_\pm=I_1\pm iI_2$$
and $\quad (29)$
$$g_\pm=\tfrac{1}{2}(g_1\pm g_2)\,.$$

Using (26)–(29) and Eq. (10), we obtain the expression for magnetic dipole moment as

$$\begin{aligned}\mu(\alpha I)&=\left(\frac{4\pi}{3}\right)^{1/2}\langle\alpha II|\mathfrak{M}(M1,0)|\alpha II\rangle\\
&=C(I1I,I0I)\sum_{K_iK_f\geq0}\frac{A_{\alpha IK_i}A_{\alpha IK_f}}{[(1+\delta_{K_i0})(1+\delta_{K_f0})]^{1/2}}\\
&\quad\times[g_+\{R_{I,-K_i}C(I1I;-K_i+1,-1,-K_f)+(-1)^IR_{I,-K_i}C(I1I;-K_i+1,-1,K_f)\\
&\qquad+R_{I,K_i}C(I1I;K_i+1,-1,K_f)+(-1)^IR_{I,K_i}C(I1I;K_i+1,-1,-K_f)\}\\
&\quad+g_-\{R_{I,K_i}C(I1I;-K_i-1,-1,-K_f)+R_{I,-K_i}C(I1I;K_i-1,-1,K_f)\\
&\qquad+(-1)^IR_{I,K_i}C(I1I;-K_i-1,-1,K_f)+(-1)^IR_{I,-K_i}C(I1I;K_i-1,-1,K_f)\}\\
&\quad+g_3K_i\{C(I1I;K_i0K_f)+(-1)^IC(I1I;K_i,0,-K_f)\}]\,, \end{aligned} \quad (30)$$

where

$$R_{I,K}=[\tfrac{1}{2}(I-K)(I+K+1)]^{1/2}$$

III. RESULTS

A. Parameters and minima

Following Baranger and Kumar[2] we have chosen a core consisting of 40 protons and 70 neutrons. For the protons $N=4$ and 5 oscillator shells and for the neutrons $N=5$ and 6 oscillator shells have been taken as active shells. The values of the parameters G, single particle energies, etc., are exactly the same as has been used by Baranger and Kumar in Ref. 2 and then our calculation has no adjustable free parameters.

In Table I we have presented a comparison of the various minima obtained in our calculation. The axial and triaxial intrinsic minima are the ones obtained by minimizing in the space of β and (β,γ), respectively, which correspond also to the potential minima $V(\beta,\gamma)$. The triaxial intrinsic minima are more bound by only 44.0 keV, and 50.6 keV, respectively, for ^{188}Os and ^{188}Pt. For

TABLE I. Description of axial and triaxial minima. The energies E's and the γ's are in MeV and degrees, respectively.

Nucleus		Intrinsic calculation Axial	Intrinsic calculation Triaxial	Angular momentum projection Axial	Angular momentum projection Triaxial
$^{188}_{76}$Os	β	0.178	0.182	0.180	0.178
	γ	0.0	22.0	0.0	26.0
	E	-308.782	-308.826	-310.254	-310.968
$^{188}_{78}$Pt	β	0.170	0.174	0.170	0.170
	γ	0.0	20.0	0.0	31.0
	E	-305.076	-305.126	-305.563	-306.288

^{188}Os, Baranger and Kumar[2] have also reported the same result in Ref. 2. However, when variation after angular momentum projection is performed for the ground state following our above scheme, the triaxial minima get more bound than the corresponding axial minima by 0.714 MeV and 0.724 MeV for ^{188}Os and ^{188}Pt, respectively. Thus, our calculation predicts more rigid triaxial shapes compared to that of Baranger and Kumar and also of Götz *et al.*[9] This result seems to support the view of Lee *et al.*[7] who strongly feel that Pt nuclei are quite γ stable in sharp contrast to the generally held notion of very γ soft character of these nuclei. From a comparison of the equilibrium values of β and γ between axial and triaxial cases, we find that the values of γ are larger for the latter than for the former. It is satisfying to note that our value of γ for ^{188}Os is 26° which agrees remarkably with the mean value (26.5°) obtained by Baranger and Kumar[4] in their dynamical theory.

B. Level energies

In Tables II(a) and III(a) we have presented the calculated and experimental energies, the equilibrium values of β and γ and the wave function $A_{\alpha IK}$ which are the weight factors of various K

TABLE II. (a) ^{188}Os: Energies (MeV) of the levels of ground state band, γ band and $K = 4$ band and the corresponding equilibrium values of β and γ and the components $A_{\alpha IK}$'s of wave functions [see Eq. (10)]. (b) ^{188}Os: The moments of inertia ($\mathscr{I}$), the quadrupole moments (QM) and magnetic moments (MM) are given in MeV^{-1}, e b and μ_N, respectively, of the various states. Quantities in parentheses refer to experimental values.

(a)

Level I	E_{ex}	E_{th}	β	γ	Wave function $K=0$	$K=2$	$K=4$	$K=6$	$K=8$	$K=10$
0^+	0.0	0.0	0.178	26.0	1.0					
2^+	0.155	0.163	0.186	18.0	0.998	0.050				
4^+	0.478	0.478	0.198	14.0	0.996	0.086	0.001			
6^+	0.948	0.884	0.218	27.0	0.751	0.644	0.142	0.008		
8^+	1.588	1.361	0.234	27.0	0.695	0.682	0.225	0.033	0.001	
10^+	2.244	1.915	0.242	28.0	0.641	0.696	0.312	0.078	0.009	0.000
$2'^+$	0.633	0.656	0.186	18.0	-0.050	0.999				
$3'^+$	0.790	0.808	0.186	18.0	0.0	1.0				
$4'^+$	0.966	1.216	0.198	14.0	-0.086	0.996	0.011			
$4''^+$		3.394	0.198	14.0	0.000	-0.011	0.999			

(b)

Level I	$\mathscr{I}_x$	$\mathscr{I}_y$	$\mathscr{I}_z$	QM	MM
0^+	13.641	5.689	3.075	0.0	0.0
2^+	14.372	8.108	1.789	-1.420 (-1.47)	0.586 (0.62)
4^+	15.242	9.890	1.300	-1.875	1.206
6^+	18.032	7.829	4.873	-0.412	1.750
8^+	19.591	8.750	5.552	-0.385	2.343
10^+	20.280	8.847	6.246	-0.251	2.918
$2'^+$	14.372	8.108	1.789	1.420	0.656
$3'^+$	14.372	8.108	1.789	0.0	0.931
$4'^+$	15.242	9.890	1.300	-0.903	1.246
$4''^+$	15.242	9.890	1.300	2.777	1.360

TABLE III. (a) ^{188}Pt: Same as Table II (a). (b) ^{188}Pt: The moment of inertia ($\mathcal{I}$), the quadropole moments (QM) and magnetic moments (MM) are given in MeV^{-1}, e b, and μ_N, respectively, of the various states.

(a)

Level					Wave function					
I	E_{ex}	E_{th}	β	γ	$K=0$	$K=2$	$K=4$	$K=6$	$K=8$	$K=10$
0^+	0.0	0.0	0.170	31.0	1.0					
2^+	0.266	0.245	0.186	20.0	0.996	0.083				
4^+	0.671	0.671	0.198	17.0	0.984	0.176	0.003			
6^+	1.185	1.219	0.214	22.0	0.853	0.518	0.059	0.002		
8^+	1.782	1.862	0.226	22.0	0.790	0.602	0.111	0.008	0.000	
10^+	2.436	2.603	0.238	23.0	0.728	0.660	0.183	0.023	0.001	0.000
$2'^+$	0.606	0.713	0.186	20.0	-0.083	0.996				
$3'^+$	0.936	0.920	0.186	20.0	0.0	1.0				
$4'^+$		1.287	0.198	17.0	-0.176	0.984	0.024			
$4''^+$		3.006	0.198	17.0	0.001	-0.024	0.999			

(b)

Level I	$\mathcal{I}_x$	$\mathcal{I}_y$	$\mathcal{I}_z$	QM (e b)	MM (μ_N)
0^+	11.923	3.719	3.748	0.0	0.0
2^+	14.018	7.212	2.223	-1.259	0.427
4^+	15.239	8.826	1.913	-1.549	0.836
6^+	16.993	8.679	3.201	-0.886	1.324
8^+	18.192	9.524	3.463	-0.841	1.741
10^+	19.287	10.268	3.980	-0.733	2.162
$2'^+$	14.018	7.217	2.223	1.259	0.696
$3'^+$	14.018	7.217	2.223	0.0	0.842
$4'^+$	15.239	8.826	1.913	-1.020	0.965
$4''^+$	15.239	8.826	1.913	2.594	1.538

components in the wave function [Eq. (10)] for ^{188}Os and ^{188}Pt, respectively. Since we have not taken into account the core, following the standard practice,[6] we have incorporated the effect of the core approximately by multiplying a renormalization factor X to all the calculated level energies. The value of X is determined from the relation

$$XE_{cal}(I=4)=E_{ex}(I=4),$$

where E_{cal} and E_{ex} are, respectively, the calculated and experimental level energies of the lowest (ground band) $I=4$ state. These renormalized values are presented in Tables II(a) and III(a).

An analysis of the weightage factors $A_{\alpha IK}$ presented in Tables II(a) and III(a) reveals that $I=0,2,4,6,8,10$ predominantly belong to the $K=0$ band. For $I=0,2,4$ the mixing of other bands is very small. However, with progressive increase of I, the mixing increases very rapidly. For $I=10$, in case of ^{188}Pt, the probability of $K=0$ is only 53% and the remaining strength is spread mainly in $K=2$, $K=4$, $K=6$ components. In case of ^{188}Os, $I=10$ contains only 41% of $K=0$, and 49% of $K=2$. Though $I=6,8,10$ are very much mixed states, we will refer to them as belonging to ground state band in our discussion. The $I=2'$, $3'$, and $4'$ states have the probability of the $K=2$ component more than 97% for both nuclei. Hence these states would beong to γ band. The $I=4''$ states have the probabilities of $K=4$ component close to 99% for both the nuclei and hence these states belong to the $K=4$ band.

An analysis of the level energies shows that the calculated energies agree quite well with the experiment for all the levels. As for the ground-state band the maximum discrepancy occurs for 10^+ levels. Even in this case the level energy is reproduced within 7% and 14% of the experimental values for ^{188}Pt and ^{188}Os, respectively. The positions of the γ-vibrational band head are higher by only 107 and 23 keV, respectively, for ^{188}Pt and ^{188}Os. The γ-band spreads are given correctly in either case. Experimentally the positions of the $K=4^+$ bandhead have not yet been measured. Our calculation predicts them to be 3.394 and 3.006 MeV, respectively, for ^{188}Os and ^{188}Pt.

C. Equilibrium values of β and γ

The equilibrium values of β and γ [see Tables II(a), III(a)] for various levels do show the expected behavior. With the increase of angular momentum, the β value increases which is a manifestation of the centrifugal stretching. Up to $I=6$, the value of β increases by 22% and 23% of their values at ground state for ^{188}Os and ^{188}Pt nuclei, respectively. However, these numbers increase by 33% and 41% for $J=10$ levels. For ^{188}Os, Kumar and Baranger have calculated the root mean square values of γ and β. Their rms values of β are in general larger than the equilibrium values of β calculated by us.

For all the states except the ground states of ^{188}Pt, the equilibrium values of γ do not exceed 30. However, in the calculation of Baranger and Kumar,[4,2] for a few levels in ^{188}Os the root mean square value of γ slightly exceeds 30. The root mean square values of γ up to $I=4$ calculated[2] by Baranger and Kumar for ^{188}Os show that γ decreases. We find a similar trend up to $I=4$. However, with increase of I further, i.e., for $I=6, 8, 10$, the value of γ increases with the increase of I. Baranger and Kumar have not calculated these states. This rising trend of γ with increase of angular momentum is indeed observed in cranked Hartree-Bogoliubov calculation.[15] It has also been anticipated by Bohr and Mottelson[16] that a nucleus would favor triaxial intrinsic shape energetically at higher angular momentum states. It is worth mentioning here that in our scheme the $2', 3'$ states (γ band) and 2^+ state (ground band) originate from the same intrinsic structure. Hence the values of β and γ for these three states are identical. Due to the same reason 4^+ (ground band), $4^{+\prime}$ (γ band) and $4^{+\prime\prime}$ ($K=4^+$ band) have the same values of β and γ.

D. Electromagnetic moments and moments of inertia

In Tables II(b) and III(b) the three components of moments of inertia, spectroscopic quadrupole moments and magnetic dipole moments are presented for ^{188}Os and ^{188}Pt, respectively. In the ground-state band, the x and y components of the moment of inertia increase with the increase of angular momentum, whereas the Z component, which is the smallest of the three, first shows some decreasing trend for 2^+ and 4^+, and then increases consistently with the increase of angular momentum. This overall increasing trend is very much expected in view of the centrifugal stretching effect.

For the calculation of quadrupole moments and transition probabilities, we have used the effective charge of proton as $e_p = 1 + 1.7\,(Z/A)$ and of neutron as $e_n = 1.7\,(Z/A)$. These values of effective charges have also been taken by Baranger and Kumar in Ref. 4. For only 2^+ state of ground-state band in ^{188}Os, the experimental value of the quadrupole moment has been given by Hoehn *et al.*[17] which is presented in parentheses in the table. The value given by Russo *et al.*[18] and Lane *et al.*[19] are -1.26 and -1.32, respectively: Our result for this state compares well with that of Hoehn. The signs of the quadrupole moment of $2'$ state (γ band) and 2^+ (g band) are positive and negative, respectively. This feature of the results of Baranger and Kumar is borne out in our calculation. Our values of the quadrupole moments are somewhat larger than those of Baranger and Kumar for the few states of ^{188}Os they have given. Regarding magnetic moment, our results compare well with those of Baranger and Kumar. For the

TABLE IV. $B(E2, i \rightarrow f)$ values in $e^2 \times 10^{-48}$ cm^2 (quantities in parentheses refer to experimental values).

J_i^π	J_f^π	$^{188}_{76}$Os	$^{188}_{76}$Pt
0^+	2^+	2.716	2.215
		(2.75)	(2.60)
0^+	$2'^+$	0.339	0.378
		(0.250)	
2^+	4^+	1.588	1.376
		(1.41)	
$2'^+$	$4'^+$	0.621	0.497
		(1.05)	
$2'^+$	4^+	0.005	0.011
$2'^+$	$4''^+$	0.128	0.164
2^+	$2'^+$	0.144	0.214
		(0.146)	
2^+	$4'^+$	0.030	0.018
		(0.020)	
2^+	$4''^+$	0.0002	0.001
$3'^+$	4^+	0.080	0.172
$3'^+$	$4'^+$	0.977	0.795
$3'^+$	$4''^+$	0.059	0.102
2^+	$3'^+$	0.110	0.121
$2'^+$	$3'^+$	1.372	1.218
4^+	$4'^+$	0.097	0.160
		(0.159)	
4^+	$4''^+$	0.00005	0.00037
$4'^+$	$4''^+$	0.016	0.028
4^+	6^+	1.273	1.230
		(1.68)	
$4'^+$	6^+	0.498	0.153
$4''^+$	6^+	0.007	0.001
6^+	8^+	2.021	1.654
8^+	10^+	2.162	1.798

TABLE V. The branching ratio $B(E2, i \to f)/B(E2, i \to f')$.

			$^{188}_{77}$Os			$^{188}_{78}$Pt	
I_i	I_f/I'_f	expt.	BK	Present theory	expt.	BK	Present theory
$2'^+$	$2^+/0^+$	2.92 ± 0.15	10.95	2.250	2.7 ± 0.7		2.892
$4'^+$	$4^+/2^+$	14.3 ± 3.6		6.236			16.0
2^+	$4^+/0^+$	2.565 ± 0.25	2.765	2.920			3.115

2^+ state (ground state) the experimental value is available for ^{188}Os only and this is presented in parentheses. Our calculation predicts it to be 0.586 μ_N which is in good agreement with the experimental value 0.62 μ_N.[20]

In our calculation of $B(E2)$ values, we have used the expression (22) given previously. The various $B(E2)$ values for ^{188}Os and ^{188}Pt are presented in Table IV. The available experimental quantities[21,22] are presented in parentheses. For ^{188}Os altogether eight $B(E)$ values for intraband and interband transitions (between ground band and γ band) are experimentally known. In most cases our results agree quite well with experimental data. The transition probabilities provide a test of the nuclear wave function and consequently the model used to calculate them. Hence, in view of our good agreement for both the intraband and interband $B(E2)$ values, our model appears to be quite successful. Baranger and Kumar[4,2] have not calculated many of the transitions like $4 \to 6$, $6 \to 8$, $4 \to 4$, etc. However, the ones they have calculated agree in trend with our results. In Table V, we have presented a comparison of the available experimental branching ratios with our calculated values. We have also included the predictions of Baranger and Kumar for the cases wherever available. It can be seen from Table V that our results compare well with experiment.

IV. DISCUSSION AND CONCLUSION

We have developed a scheme to study simultaneously the ground-state band and γ band of nuclei with static triaxial intrinsic shape. The scheme is based on the variation after an approximate angular momentum projection from an intrinsic wave function having triaxial symmetry. We have applied this theory to the study of ^{188}Os and ^{188}Pt using pairing $+ Q\cdot Q$ interaction the parameters of which have been kept fixed. The level energies up to $I=10$ in the ground-state band and up to $I=4$ in the γ band have been calculated and are in good agreement with experiment. The position of the $K=4$ band has been predicted. The quadrupole moments, magnetic moment, and the $E2$ transition probabilities calculated with our wave function agree closely with experiment. For ^{188}Os, the results of the dynamic calculation of Baranger and Kumar are available up to $I=4$ states in the ground-state band and γ band and these compare quite well with ours. It is thus quite satisfying to note that our wave function characterized by fixed β and γ is adequate to describe the two bands and the results are comparable to those of Baranger and Kumar who use a wave function smeared over a large region of the β-γ plane. This may be due to our arriving at lower minima in (β, γ) space as a result of variation after angular momentum projection. However, our theory in the present form is inadequate to describe the β band simultaneously and needs to be improved.

Our calculations predict deeper triaxial minima compared to the corresponding axial ones for both the nuclei. Thus this study somewhat supports the view[7] of more rigid triaxial shapes for both the nuclei than have been usually believed. Similar conclusion has been arrived at by Faessler *et al.*[23] in their recent study. However, we feel that our minima are not deep enough to justify the existence of a strong γ deformation and more work needs to be done in this respect. The present result is quite encouraging and the method presented here with improvement may be quite useful in the study of the rare-earth region.

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