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Collective and noncollective excitations in odd-even $N = 83$ nuclei

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We have studied the level structure of the even-odd $N = 83$ nuclei $^{143}_{60}$Nd, $^{145}_{62}$Sm, $^{147}_{64}$Gd, and $^{149}_{66}$Dy. In order to explain the high level density above 2 MeV and low-lying high-spin states ($J > 13/2; E_x > 2.5$ MeV), a Hamiltonian is constructed which in addition to the collective vibrations of the core, describes also the two quasiparticle excitations of the protons in the $Z = 50-82$ shell. The results of the calculations are compared with the experimental data concerning the $N = 83$ nuclei.

[NUCLEAR STRUCTURE $N=83$ nuclei Nd, Sm, Gd, Dy. Collective proton two quasiparticle degrees of freedom. High-spin states. Calculated levels J, π; $B(E3)$.]

I. INTRODUCTION

The purpose of this paper is the study of the $N=83$ nuclei with an even number of protons. Numerical calculations are performed for $^{143}_{60}$Nd, $^{145}_{62}$Sm, $^{147}_{64}$Gd, and $^{149}_{66}$Dy about which many experimental data are available.[1-6] Until now, these nuclei have been studied within the framework of the unified model[7-10] only, which gives a good description of the energy spectrum below 2 MeV except for a number of states with two-particle–one-hole character. However, the energy spectrum above 2 MeV has two characterisitics which are not reproduced within a purely macroscopic particle-core coupling calculation, namely, (i) the strong increase of the level density above 2 MeV,[4] (ii) the observation of low-lying ($E_x \simeq 2.5$ MeV) high-spin states.[1-3] In order to remove these shortcomings, we have to consider also two-quasiparticle excitations of the protons in the $Z=50-82$ shell. We expect that as a result of coupling the neutron single-particle configurations with the various proton configurations, the level density will become larger and also that the same coupling with high-spin ($J^\pi = 8^+, 10^+, 7^-, 8^-, 9^-$) proton states can give rise to the high-spin states observed in the $N=83$ isotones. A detailed investigation of these assumptions is the purpose of this paper.

In Sec. II a Hamiltonian describing the collective as well as the proton two-quasiparticle excitation modes of the core nucleus ($N=82$) and their interaction with the extra ($N=83$) neutron is constructed. In Sec. III the proton-proton and proton-neutron interactions are discussed. Section IV deals with effects of configuration space truncation, and finally in Sec. V we discuss parameters and results.

II. CONSTRUCTION OF THE HAMILTONIAN

In order to construct a Hamiltonian describing the $N=83$ odd-mass nuclei with all characteristics as described earlier, we start from the Hamiltonian describing the corresponding $N=82$ core nucleus in a quasiparticle description. This Hamiltonian reads

$$H_{\rm qp} = \sum_\alpha \epsilon_a c_\alpha^+ c_\alpha + \frac{1}{4} \sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\, c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma. \quad (2.1)$$

The summations extend over all possible single-particle states in the $Z=50-82$ proton shell, and $V_{\alpha\beta\gamma\delta}$ stands for the matrix element of the residual two-body interaction between antisymmetric normalized two-particle states. This Hamiltonian can be handled further by means of the Bogoliubov-Valatin transformation for which we refer to Ref. 11. The Hamiltonian (2.1) can describe the full complexity of the $N=82$ core nucleus.

Alternatively, we can describe the collective excitations in the $N=82$ nucleus with a purely collective Hamiltonian

$$H_{\rm coll} = \sum_\lambda \hbar\omega_\lambda \sum_{\mu=-\lambda}^{\lambda} (b_{\lambda\mu}^\dagger b_{\lambda\mu} + \tfrac{1}{2}) , \quad (2.2)$$

however, neglecting a number of specific excitations of the protons in the 50–82 shell.

In our model we use the Hamiltonian $H_{\rm qp} + H_{\rm coll}$ for the core nucleus. However, a number of excitations are described twice by this Hamiltonian, firstly in a phonon representation and secondly in a quasiparticle (qp) representation.[12,13] Indeed, when the Hamiltonian $H_{\rm qp}$ is diagonalized in the 2qp space which consists of 2qp states coupled to a given spin and parity, one observes that one eigenstate separates from the rest. Moreover, this eigenstate is a linear superposition of all 2qp states with approximately equal amplitudes, and in this way it can be identified with a collective 1 phonon state. The problem of double counting of some specific excitations of the core nucleus can be solved approximately if one does not consider in the chosen configuration space of basic states, those linear combinations of 2qp states which give a microscopic description of the strong collective states. For instance, the lowest $J^{\pi}=2^{+}$ (3^{-}) states which can be described within a proton 2qp space correspond approximately to the macroscopic quadrupole (octupole) phonon states, and, hence, only the latter are admitted in the basis. The remaining overcompleteness of the basis (since the 1 phonon states do not coincide totally with their microscopic description in 2qp space) is expected to be small, and it is also rather unimportant for our purposes since we are mainly interested in high spin states (i.e., the sequences $J^{\pi}=6^{+}, 8^{+}, 10^{+}$ and $J^{\pi}=5^{-}, 6^{-}, 7^{-}, 8^{-}, 9^{-}$) which all have a pure 2qp character.

In the same way the 4qp states can be handled. In a first approximation we consider only those linear combinations of 4qp states which have a collective character, i.e., only two-phonon states are taken as basic states. In a second approximation we also consider one-phonon $\otimes$ 2qp states in the basis. All other 4qp states are always neglected in our calculations.

The total Hamiltonian of the $N=83$ nuclei also describes the extra neutron outside the $N=82$ closed shell and its interaction with the underlying core. We have as a single-particle Hamiltonian

$$H_{\mathrm{sp}}=\sum_{\alpha} \epsilon_{a} c_{\alpha}^{\dagger} c_{\alpha}\,, \tag{2.3}$$

and as interaction with the collective degrees of freedom of the core[7]

$$H_{\mathrm{sp-coll}}=-\sum_{\lambda,\mu} \sum_{\alpha,\beta}\left(\frac{\pi}{2\lambda+1}\right)^{1/2} \xi_{\lambda} \hbar\omega_{\lambda}\left[b_{\lambda\mu}+(-1)^{\mu} b_{\lambda-\mu}^{\dagger}\right]\langle\alpha|Y_{\lambda\mu}|\beta\rangle c_{\alpha}^{\dagger} c_{\beta}\,, \tag{2.4}$$

where α and β denote neutron states in the $N=82-126$ shell.

However, $H_{\mathrm{sp-coll}}$ only describes the interaction of the extra neutron with the collective degrees of freedom. Thus, we still have to consider the remaining residual interaction between the extra neutron and the proton 2qp excitations within the $Z=50-82$ shell. This interaction is of the form

$$H_{\mathrm{sp-qp}}=\sum_{\alpha\beta\gamma\delta} V'_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma}\,. \tag{2.5}$$

Here, α and γ denote neutron states and β and δ proton states. $V'_{\alpha\beta\gamma\delta}$ is the matrix element of the residual proton-neutron interaction between two normalized proton-neutron states. After performing the Bogoliubov-Valatin transformation for the proton states (β,δ), we obtain

$$\begin{aligned} H_{\mathrm{sp-qp}}=&\sum_{\alpha\gamma\beta} v_{b}^{2} V'_{\alpha-\beta\gamma-\beta} c_{\alpha}^{\dagger} c_{\gamma}+\sum_{\alpha\beta\gamma\delta} V'_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\gamma}\left(s_{\delta} u_{b} v_{d} a_{\beta}^{\dagger} a_{-\delta}^{\dagger}+s_{\beta} v_{b} u_{d} a_{-\beta} a_{\delta}\right) \\ &\sum_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\gamma}\left(V'_{\alpha\beta\gamma\delta} u_{b} u_{d}-V'_{\alpha-\delta\gamma-\beta} s_{\beta} s_{\delta} v_{b} v_{d}\right) a_{\beta}^{\dagger} a_{\delta}\,. \end{aligned} \tag{2.6}$$

The total Hamiltonian of the model thus reads

$$H=H_{\mathrm{sp}}+H_{\mathrm{coll}}+H_{\mathrm{qp}}+H_{\mathrm{sp-coll}}+H_{\mathrm{sp-qp}}\,. \tag{2.7}$$

In this paper the Hamiltonian is used to describe odd-mass $N=83$ nuclei, but a number of other nuclei (for instance odd-mass $N=81$) could be described within the same model.

III. INTERACTIONS

For the proton-neutron interaction a schematic δ interaction is used. This type of force takes into account the short-range proton-neutron correlation rather well, and, moreover, simplifies the numerical calculations considerably. The diagonal matrix elements of the interaction $-V_{\mathrm{eff}}\left[(1-\alpha)+\alpha(\vec{\sigma}_{n}\cdot\vec{\sigma}_{p})\right]\delta(\vec{r}_{n}-\vec{r}_{p})$ are given in Ref. 14. Also, off-diagonal matrix elements have to be calculated in order to perform the diagonalization, and the following expression results:

$$\langle(\tfrac{1}{2}l_n j_n)(\tfrac{1}{2}l_p j_p)J| - V_{\mathrm{eff}}|(1-\alpha)+\alpha(\vec{\sigma}_n\cdot\vec{\sigma}_p)|\delta(\vec{r}_n-\vec{r}_p)|(\tfrac{1}{2}l_n' j_n')(\tfrac{1}{2}l_p' j_p')J'\rangle$$
$$=(-1)^{j_p+j_p'+l_n+l_n'}[(2j_n+1)(2j_n'+1)(2j_p+1)(2j_p'+1)]^{1/2}\langle j_p\tfrac{1}{2}j_n-\tfrac{1}{2}|J0\rangle\langle j_p'\tfrac{1}{2}j_n'-\tfrac{1}{2}|J'0\rangle\frac{V_{\mathrm{eff}}F^0}{2J+1}$$
$$\times\left\{\frac{1}{2}\left[1-(-1)^{j_n+j_n'+l_n+l_n'}\frac{[(2j_p+1)+(-1)^{j_p+j_n+J}(2j_n+1)][(2j_p'+1)+(-1)^{j_p'+j_n'+J'}(2j_n'+1)]}{4J(J+1)}\right]\right.$$
$$\left.-\alpha\tfrac{1}{2}[1+(-1)^{l_n+l_p+J}][1+(-1)^{l_n'+l_p'+J'}]\right\}\delta_{JJ'}\delta_{\pi\pi'}\,, \qquad (3.1)$$

where $\delta_{\pi\pi'}$ imposes conservation of parity and F^0 is the Slater integral for the radial wave functions of the harmonic oscillator.

Before using this δ force in the total Hamiltonian (2.7), we have tested this interaction in a simple model for doubly odd $N=81$ nuclei, in which we suppose that the protons of the 50–82 shell can be described by means of 1qp excitations only.[17] The best-fit values of α and V_{eff}, which we obtained from these calculations,[18] were used as starting values in the model for the $N=83$ nuclei.

As proton-proton interaction we used a Gaussian interaction of the form

$$V_{pp}=-V_0\exp(-\beta|\vec{r}_1-\vec{r}_2|^2)(P_s+tP_t)\,, \qquad (3.2)$$

with P_s and P_t the singlet and triplet projection operators, respectively. The choice of parameters V_0, β, and t will be discussed in Sec. V.

IV. MATRIX ELEMENTS AND CHOICE OF THE BASIS

We have to make a choice of our basic states in order to obtain matrices that can be treated numerically. Since the Hamiltonian of the core has a collective part as well as a quasiparticle part, the basis will include single-particle collective states ($|n\,\mathrm{ph}\otimes 1\mathrm{sp}\rangle$) as well as single-particle–two-quasiparticle states ($|2\mathrm{qp}\otimes 1\mathrm{sp}\rangle$). As single-particle states we consider all neutron states in the $N=82$-126 shell. For the collective core states we truncate up to $N_q=3$, $N_o=2$ and ($N_q=1$, $N_o=1$), where N_q (N_o) denotes the number of quadrupole (octupole) phonons. As proton 2qp states we consider a selected part of those 2qp states, which have no collective analog.

Concerning the more complex single-particle–collective–two-quasiparticle states $|(2\mathrm{qp}\otimes 1\mathrm{sp})\otimes 1\mathrm{ph}\rangle$, we made three different approximations:

(A1) We neglect the $|(2\mathrm{qp}\otimes 1\mathrm{sp})\otimes 1\mathrm{ph}\rangle$ states.

(A2) We calculate the effect of this truncation using second order perturbation theory, resulting in the following expression:

$$E_n'=E_n+\sum_m\frac{|V_{nm}|^2}{E_n^{(0)}-E_m^{(0)}}\,, \qquad (4.1)$$

where $E_n^{(0)}$ and $E_m^{(0)}$ are the zero order energies, E_n is an eigenvalue of the energy matrix, when it is diagonalized within the truncated space, and E_n' is the corresponding energy, corrected up to second order for the neglect of the states $|(2\mathrm{qp}\otimes 1\mathrm{sp})\otimes 1\mathrm{ph}\rangle$. The index m stands for all possible states of this type, and V_{nm} denotes the matrix elements of the perturbation Hamiltonian $(H_{\mathrm{sp\text{-}coll}}+H_{\mathrm{sp\text{-}qp}})$. Introducing the explicit expressions of the matrix elements V_{nm} into Eq. (4.1), the following results occur:

(i) energy correction for a $|2\mathrm{qp}\otimes 1\mathrm{sp}\rangle$ state:

$$E_{\mathrm{corr}}^{(2)}(|ab(J_{\mathrm{qp}})c;JM\rangle)=\sum_d\frac{\pi}{5(2j_c+1)}\frac{|\langle c||Y_2||d\rangle\xi_2\hbar\omega_2|^2}{\epsilon_c-\hbar\omega_2-\epsilon_d}+\sum_d\frac{\pi}{7(2j_c+1)}\frac{|\langle c||Y_3||d\rangle\xi_3\hbar\omega_3|^2}{\epsilon_c-\hbar\omega_3-\epsilon_d}\,, \qquad (4.2)$$

Where d runs over all neutron single-particle orbits in the 82–126 shell.

(ii) energy correction for a $|1\mathrm{ph}\otimes 1\mathrm{sp}\rangle$ state:

$$E_{\mathrm{corr}}^{(2)}(|((N_qR_q),(N_oR_o))Ra;JM\rangle)=\frac{1}{2j_a+1}\sum_{J_{\mathrm{qp}},b}\frac{2J_{\mathrm{qp}}+1}{\epsilon_a-\epsilon_b-E_{J_{\mathrm{qp}}}}$$
$$\times\left|\sum_{c,d}\chi_{J_{\mathrm{qp}}}^{(c,d)}[u_cv_dF'(abcd,J_{\mathrm{qp}})-(-1)^{j_c+j_d+J_{\mathrm{qp}}}u_dv_cF'(abcd,J_{\mathrm{qp}})]\right.$$
$$\left.\times\frac{1}{(1+\delta_{cd})^{1/2}}\right|^2. \qquad (4.3)$$

TABLE I. An illustration of the effect of the p-n interaction in the case of $^{143}_{60}$Nd. With K matrix the lowering of the unperturbed energy by diagonalizing the K matrix is meant.

J^{π}	$\frac{25}{2}^+$	$\frac{23}{2}^+$	$\frac{21}{2}^+$	$\frac{19}{2}^-$	$\frac{17}{2}^-$	$\frac{15}{2}^-$
Structure	$9^-\otimes f_{7/2}$	$8^-\otimes f_{7/2}$	$7^-\otimes f_{7/2}$	$6^+\otimes f_{7/2}$	$6^+\otimes f_{7/2}$	$4^+\otimes f_{7/2}$
K-matrix (MeV)	−0.103	−0.508	−0.423	0.065	−0.085	−0.262

Here the $\chi^{(c,d)}_{J_{qp}}$ represent the amplitudes for a particular configuration (c,d) with angular momentum J_{qp} within a proton 2qp description of doubly even $N=82$ nuclei, and b runs over all neutron single-particle orbits in the $n=82-126$ shell.

(A3) We calculate the energy matrix in the extended base, including the states $|(2\text{qp}\otimes 1\text{sp})\otimes 1\text{ph}\rangle$.

In all three approximations matrix elements of the Hamiltonian (2.7) between the various basic states have to be calculated. In the Appendix some particular important matrix elements are given explicitly.

V. PARAMETERS AND RESULTS

Numerical calculations were performed for the nuclei $^{143}_{60}$Nd, $^{145}_{62}$Sm, $^{147}_{64}$Gd, and $^{149}_{66}$Dy, with the following general results.

(i) All unperturbed high-spin states [i.e., when one considers only the single-particle contributions of the Hamiltonian (2.7)] occur too high in energy. Various calculations performed within the three different approximations mentioned in the previous section showed that lowering of the high-spin states is mainly the result of two effects:

(1) the K-matrix elements (see Appendix) of the proton-neutron interaction. Explicit calculations showed that K-matrix elements can lower the energy considerably. For instance, in the case of ^{143}Nd, the effect becomes particularly important for the $J^{\pi}=\frac{23}{2}^+$ and $J^{\pi}=\frac{21}{2}^+$ states (see Table I).

(2) the extension of the total basis to the more complex $|(2\text{qp}\otimes 1\text{sp})\otimes 1\text{ph}\rangle$ states. The effect of this extension has been calculated up to second order perturbation theory using formulas (4.2) and (4.3). From Table II one can see that high-spin states which are built from the configuration $|2\text{qp}\otimes 2f_{7/2}\rangle$ are lowered by $\Delta E_x=0.5$ MeV on the average.

(ii) The results of the unified model calculations concerning the spectrum below 2 MeV are not altered by introducing 2qp excitations. This is not obvious since the excitation energy and the spectroscopic factors of the $|n\text{ph}\otimes 1\text{sp}\rangle$ states

TABLE II. Energy corrections in MeV due to the basis truncation. In the left column the single-particle configuration is mentioned, together with the nature of the state to which it is coupled. The abbreviation coll stands for any collective state and 2qp stands for any two-quasiparticle state.

	$^{143}_{60}$Nd$_{83}$	$^{145}_{62}$Sm$_{83}$	$^{147}_{64}$Gd$_{83}$	$^{149}_{66}$Dy$_{83}$
$\lvert p_{1/2}\otimes \text{coll}\rangle$	−0.039	−0.037	−0.035	−0.047
$\lvert p_{3/2}\otimes \text{coll}\rangle$	−0.071	−0.059	−0.049	−0.048
$\lvert f_{5/2}\otimes \text{coll}\rangle$	−0.041	−0.216	−0.067	−0.068
$\lvert f_{7/2}\otimes \text{coll}\rangle$	−0.061	−0.048	−0.041	−0.033
$\lvert h_{9/2}\otimes \text{coll}\rangle$	−0.092	−0.059	−0.063	−0.048
$\lvert i_{13/2}\otimes \text{coll}\rangle$	−0.122	−0.056	−0.101	−0.055
$\lvert f_{7/2}\otimes \text{2qp}\rangle$	−0.544	−0.502	−0.434	−0.508

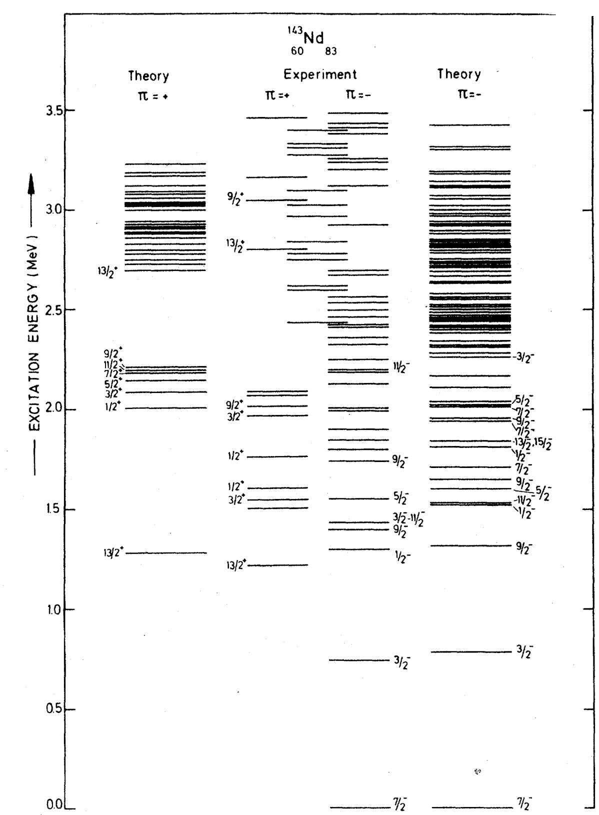

FIG. 1. Density of states for the nucleus $^{143}_{60}$Nd. The experimental spectrum is taken from Refs. 4,5 and references cited therein. The parity assignments ($\pi=+$, $\pi=-$) are shown in different columns. Experimental levels with unknown parity are shown in the middle.

TABLE III. Single–particle energies in MeV.

	Proton single–particle energies					Neutron single–particle energies					
	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$g_{7/2}$	$h_{11/2}$	$p_{1/2}$	$p_{3/2}$	$f_{5/2}$	$f_{7/2}$	$h_{9/2}$	$i_{13/2}$
$^{143}_{60}Nd_{83}$	3.28	3.55	1.25	0.00	3.50	1.80	1.30	2.32	0.00	1.26	1.60
$^{145}_{62}Sm_{83}$	3.43	3.85	0.85	0.00	3.15	2.00	1.21	2.30	0.00	1.35	1.50
$^{147}_{64}Gd_{83}$	3.35	3.85	0.60	0.00	2.7	2.05	1.45	2.30	0.00	1.35	1.40
$^{149}_{66}Dy_{83}$	3.35	3.85	0.40	0.00	2.3	2.10	1.25	2.30	0.00	1.35	1.30

could possibly be changed because of two reasons:

(1) the 2qp-core coupling, i.e., the influence of $|2qp \otimes 1sp\rangle$ states on $|nph \otimes 1sp\rangle$ states via the interaction matrix elements of the type $\langle 2qp \otimes 1sp|H|1sp\rangle$. These matrix elements turn out to be very small and so will be the changes in excitation energy and spectroscopic factors of the $|nph \otimes 1sp\rangle$ states.

(2) the extension of the basis to the $|(2qp \otimes 1sp) \otimes 1ph\rangle$ states. One observes from Table II that the energy levels below 2 MeV (all of nature $|nph \otimes 1sp\rangle$) are not much affected (shifted by about $\Delta E_x = -0.05$ MeV) by this extension.

(iii) Calculations pointed out that the δ force which we used as proton-neutron interaction can reproduce in most cases the correct ordering of high-spin states.

(iv) The level density above 2 MeV becomes very large, as was qualitatively expected (see Fig. 1), and no further detailed comparison between theory and experiment is possible for low-spin states in this energy region. A possible way to handle this problem should be based on statistical arguments.

In Tables III and IV all parameters are collected which we have used for diagonalizing the energy matrices within an extended basis. The proton single-particle energies and the parameters of the Gaussian interaction are determined by the study of the corresponding $N=82$ nuclei,[16] whereas the neutron single-particle energies and the collective parameters are taken from Ref. 15. We have estimated the parameters for $^{149}_{66}$Dy by linear extrapolation, since in the previous works no fits were done for $^{149}_{66}$Dy, and, moreover, no experimental data exist on $^{148}_{66}Dy_{82}$. The value of $\hbar\omega_3$ was taken as $\hbar\omega_3=1.80$ MeV because this parameter probably goes through a minimum for $Z=64$. In this way the variation of the $E3$ transition probability $B(E3; \frac{13}{2}^+ \to \frac{7}{2}^-)_{exp}$ with the number of protons can be explained, as is shown in Fig. 2.

In Figs. 3–5 we show experimental level schemes together with the theoretical predictions according to the various approximations. We indicate for each level the structure of the main component in its wave function. In the case of strong admixture we give two components. The abbreviations Q and O stand for quadrupole phonon and octupole phonon, respectively.

In Ref. 1, positive parity was tentatively assigned to the levels of $^{143}_{60}$Nd between 2.0 and 2.5 MeV ($J^\pi = \frac{15}{2}^{(+)}, \frac{11}{2}^{(+)}, \frac{17}{2}^{(+)}, \frac{19}{2}^{(+)}$). On the basis of our theoretical results a negative parity assignment is very tempting, and in this way the particular levels could be explained as the coupling of a $J^\pi=4^+$ or 6^+ state with the $2f_{7/2}$ neutron state. Also, the $\Delta J=1$ γ cascade starting from $\frac{25}{2}^{(+)}$ which is experimentally observed in $^{143}_{60}$Nd can be associated with the $J^\pi=9^- - 8^- - 7^- - 6^+ - 4^+$ sequence in $^{142}_{60}$Nd.

In Fig. 4 a comparison is made between the two approximations (A2) and (A3) in the case of $^{145}_{62}$Sm. It is noticed that explicit diagonalization

TABLE IV. Unified model parameters and interaction parameters.

	Unified model parameters				Gauss interaction			p-n interaction	
	$\hbar\omega_2$ (MeV)	$\hbar\omega_3$ (MeV)	ξ_2	ξ_3	t	β	V_0 (MeV)	α	V_{eff} (MeV fm^3)
$^{143}_{60}Nd_{83}$	1.58	2.08	1.4	1.4	0.2	3.4	−36.8	0.21	400
$^{145}_{62}Sm_{83}$	1.65	1.81	1.2	1.6	0.2	3.4	−37.8	0.15	300
$^{147}_{64}Gd_{83}$	1.58	1.60	1.1	1.7	0.2	3.4	−40.0	0.15	342
$^{149}_{66}Dy_{83}$	1.60	1.80	1.0	1.8	1.0	3.4	−41.0	0.11	342

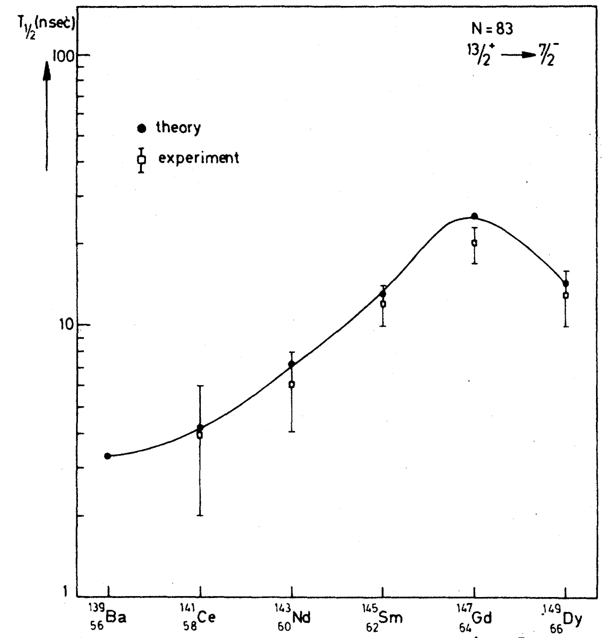

FIG. 2. The transition probability $B(E3;\frac{13}{2}^+ \rightarrow \frac{7}{2}^-)$ as a function of the mass number A in the $N=83$ isotones.

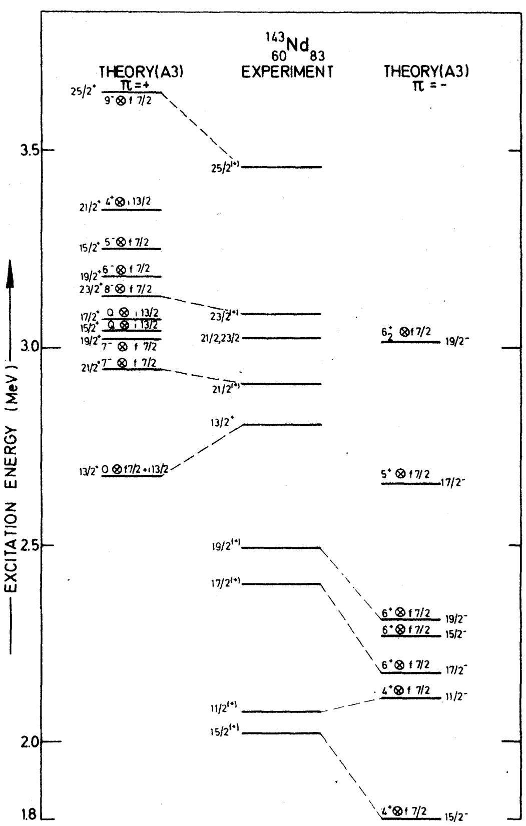

FIG. 3. High-spin states of the nucleus ${}^{143}_{60}$Nd. Only the lowest levels with $J > \frac{13}{2}$ and $E_x > 2$ MeV are shown in the theoretical spectrum. The experimental spectrum is taken from Ref. 1.

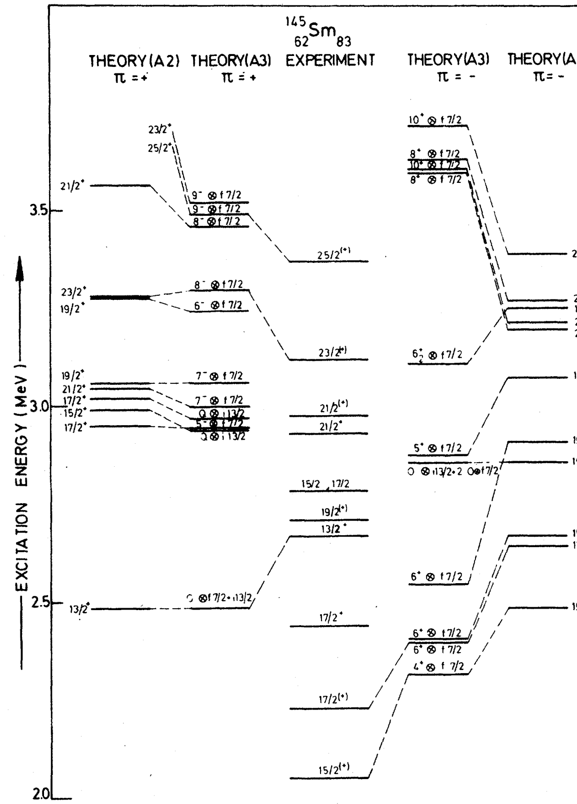

FIG. 4. High-spin states of the nucleus ${}^{145}_{62}$Sm. Only the lowest levels with $J > \frac{13}{2}$ and $E_x > 2$ MeV are shown in the theoretical spectrum. The results of two different approximations (A2) and (A3) are shown. The experimental spectrum is taken from Ref. 1.

within the extended basis yields better results than a calculation up to second order perturbation theory, regarding as examples the $J^\pi = \frac{25}{2}^+$ level with structure $|9^- \otimes 2f_{7/2}\rangle$ and also the $J^\pi = \frac{15}{2}^-, \frac{17}{2}^-$ levels with structure $|4^+ \otimes 2f_{7/2}\rangle$, $|6^+ \otimes 2f_{7/2}\rangle$. Nevertheless, the same problem arises for the parity assignment of the experimental levels $J^\pi = \frac{15}{2}^{(+)}$ and $J^\pi = \frac{17}{2}^{(+)}$ at an excitation energy of 2.050 MeV and 2.230 MeV, respectively. Moreover, we cannot associate the theoretical $J^\pi = \frac{21}{2}^+$ level at 3.00 MeV with an experimental level uniquely since two $J = \frac{21}{2}$ levels occur in that energy region.

In Fig. 5, a comparison is made between the two approximations (A1) and (A2) in the case of ${}^{149}_{66}$Dy. The energy correction (second order perturbation theory) shifts the high-spin states towards the correct energy region, but spacing and ordering of the levels differs from the experiment. These shortcomings are probably due to the particular choice of a zero-range proton-neutron interaction. In this case also we can asso-

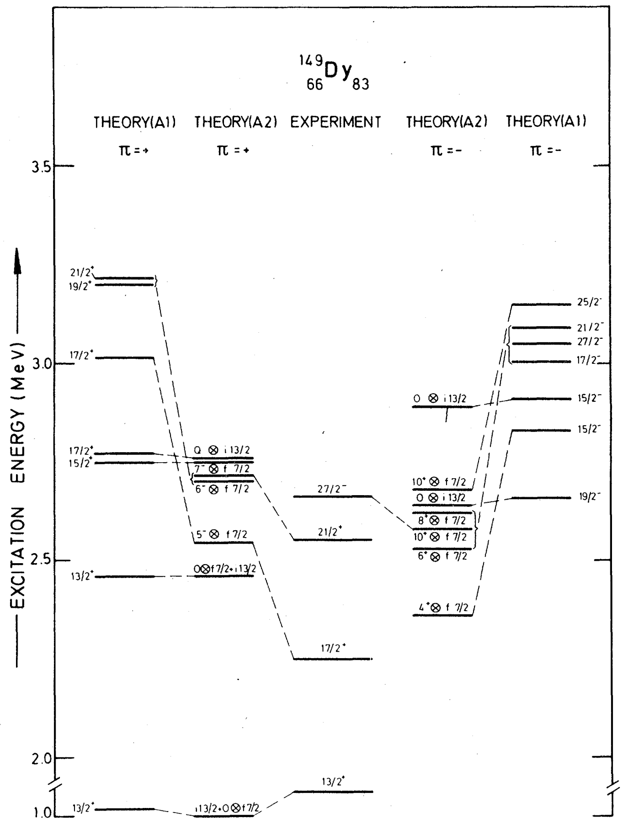

FIG. 5. High-spin states of the nucleus ${}^{149}_{66}$Dy. The experimental spectrum is taken from Ref. 2. Only a limited number of the calculated levels is shown. The left column represents the levels without correcting for the truncation of the basis, and the column in the middle shows the spectrum with this correction taken into account.

ciate the γ cascade $J^\pi=\frac{27}{2}^- - \frac{21}{2}^+ - \frac{17}{2}^+ - \frac{13}{2}^+$ with the γ sequence $J^\pi=10^+ - 7^- - 5^- - 3^-$ in the corresponding $N=82$ nucleus. We also calculated the half-life of the 0.1104 MeV $E3$ transition $J^\pi=\frac{27}{2}^- \to \frac{21}{2}^+$, assuming a three particle configuration for both the initial and final state. This assumption is quite realistic for the $J^\pi=\frac{27}{2}^-$ level, since it almost coincides with the $(\pi h_{11/2} \otimes \pi h_{11/2})_{10^+} \otimes \nu f_{7/2}$ three-particle configuration. In the case of the $J^\pi=\frac{21}{2}^+$ level with structure $7^- \otimes \nu f_{7/2}$, the $J^\pi=7^-$ state does not coincide with a pure two-particle configuration, and, moreover, its structure depends critically on the single-particle energies used. Assuming a $\pi h_{11/2} \otimes \pi d_{5/2}$ configuration for the $J^\pi=7^-$ state, we calculated a half-life of 0.040 s, and in the case of a $\pi h_{11/2} \otimes \pi g_{7/2}$ configuration we obtain 0.173 s. In calculating these half-lives an effective charge $e_p=1.5e$ has been used together with the experimental conversion coefficient $\alpha_{tot}=22$. We conclude that the $(\pi h_{11/2} \otimes \pi g_{7/2})_{7^-} \otimes \nu f_{7/2}$ configuration is the most probable for the $J^\pi=\frac{21}{2}^+$ state.

VI. CONCLUSIONS

The results of our study of $N=83$ isotones on the basis of a macroscopic particle-core coupling with the inclusion of anharmonic effects of proton two-quasiparticle excitations can be summarized as follows.

(i) The correct prediction of the unified model concerning the energy and the spectroscopic factors of levels below 2 MeV are not altered.

(ii) The level density of the theoretical spectra increases above 2 MeV, but no detailed comparison with the experimental levels is possible.

(iii) Many high-spin states occur as a result of the coupling of the neutron single-particle $2f_{7/2}$ configuration with the two quasiparticle states of two sequences in the corresponding $N=82$ nuclei (the $J^\pi=4^+ - 6^+ - 8^+ - 10^+$ and the $J^\pi=5^- - 6^- - 7^- - 8^- - 9^-$ sequence). Moreover, the $\Delta J=1$ γ cascade observed in ${}^{143}_{60}$Nd can be associated with an analog γ cascade in the $N=82$ nucleus, whereas in the Dy isotopes correspondence should exist between the $J^\pi=\frac{27}{2}^- - \frac{21}{2}^+ - \frac{17}{2}^+ - \frac{13}{2}^+$ γ cascade and the $J^\pi=10^+ - 7^- - 5^- - 3^-$ γ cascade (not observed experimentally as yet). No such observations are possible in the case of ${}^{145}_{62}$Sm, reflecting the more complex decay properties of this nucleus.

(iv) Energy levels are shifted towards the correct energy region as a result of the proton-neutron interaction and the extension of the basis. The latter effect can be estimated fairly well by second order perturbation theory, but explicit diagonalization of the Hamiltonian matrix in an extended basis yields better results.

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APPENDIX

All quantities G', F', T', R', K', defined with a prime, refer to proton-neutron matrix elements. We give here the explicit expression of two matrix elements in which $\bar{G}^+(abJc, \mathcal{J}\mathfrak{M})$ denotes the normalized antisymmetric state of two quasiparticles coupled to J, which is coupled with j_c to $\mathcal{J}$, i.e.,

$$\bar{G}^+(abJc,\mathcal{J}\mathcal{M}) = \frac{1}{(1+\delta_{ab})^{1/2}} \sum_{Mm_am_bm_c} \langle j_am_aj_bm_b|JM\rangle\langle JMj_cm_c|\mathcal{J}\mathcal{M}\rangle a^\dagger_\alpha a^\dagger_\beta c^\dagger_\gamma : \tag{A1}$$

(i) $$\langle\tilde{0}|\bar{G}(abJc,\mathcal{J}\mathcal{M})H\bar{G}^+(deJ'f,\mathcal{J}'\mathcal{M}')|\tilde{0}\rangle = \{[(\epsilon_c+E_a+E_b)D(ab\,de,J)+Q(ab\,de,J)]\delta_{cf}\delta_{JJ'} + K'(abJc,deJ'f;\mathcal{J})\}\frac{\delta_{\mathcal{J}\mathcal{J}'}\delta_{\mathcal{M}\mathcal{M}'}}{[(1+\delta_{ab})(1+\delta_{de})]^{1/2}}. \tag{A2}$$

where we have used the abbreviations D and Q of Ref. 11 and

$$\begin{aligned} K'(ab\,Jc,de\,J'f;\mathcal{J}) = &-\sum_{J_1}(-1)^{j_a+j_d+J+J'}(2J_1+1)[(2J+1)(2J'+1)]^{1/2}\begin{Bmatrix} j_a & j_c & J_1 \\ \mathcal{J} & j_b & J\end{Bmatrix}\begin{Bmatrix} j_d & j_f & J_1 \\ \mathcal{J} & j_e & J'\end{Bmatrix}T'(cafd,J_1)\delta_{eb} \\ &-\sum_{J_1}(-1)^{j_a+j_d+J}(2J_1+1)[(2J+1)(2J'+1)]^{1/2}\begin{Bmatrix} j_a & j_c & J_1 \\ \mathcal{J} & j_b & J\end{Bmatrix}\begin{Bmatrix} j_e & j_f & J_1 \\ \mathcal{J} & j_d & J'\end{Bmatrix}T'(cafe,J_1)\delta_{bd} \\ &-\sum_{J_1}(-1)^{j_a+j_d+J'}(2J_1+1)[(2J+1)(2J'+1)]^{1/2}\begin{Bmatrix} j_b & j_c & J_1 \\ \mathcal{J} & j_a & J\end{Bmatrix}\begin{Bmatrix} j_d & j_f & J_1 \\ \mathcal{J} & j_e & J'\end{Bmatrix}T'(cbfd,J_1)\delta_{ea} \\ &+\sum_{J_1}(2J_1+1)[(2J+1)(2J'+1)]^{1/2}\begin{Bmatrix} j_b & j_c & J_1 \\ \mathcal{J} & j_a & J\end{Bmatrix}\begin{Bmatrix} j_e & j_f & J_1 \\ \mathcal{J} & j_d & J'\end{Bmatrix}T'(cbfe,J_1)\delta_{ad}, \end{aligned} \tag{A3}$$

with

$$T'(abcd,J) = u_bu_dG'(abcd,J) + v_bv_dR'(abcd,J), \tag{A4}$$

$$R'(abcd,J') = -\sum_J (2J+1)\begin{Bmatrix} j_a & j_b & J \\ j_c & j_d & J'\end{Bmatrix}G'(abcd,J), \tag{A5}$$

$$G'(abcd,J) = \langle j_al_aj_bl_bJ|\hat{V}_{pn}|j_cl_cj_dl_dJ\rangle ; \tag{A6}$$

(ii) $$\langle\tilde{0}|c_\alpha\hat{H}\bar{G}^+(bcJd,\mathcal{J}\mathcal{M})|\tilde{0}\rangle = (-1)^{j_d+J-\mathcal{J}}\left(\frac{2J+1}{2j_a+1}\right)^{1/2}[u_bv_cF'(adbc,J) - (-1)^{j_b+j_c-J}u_cv_bF'(abcd,J)]\frac{\delta_{\mathcal{J}j_a}\delta_{\mathcal{M}m_a}}{(1+\delta_{bc})^{1/2}}, \tag{A7}$$

with

$$F'(acdb,J') = \sum_J (2J+1)(-1)^{J+j_c+j_d}\begin{Bmatrix} j_a & j_b & J \\ j_d & j_c & J'\end{Bmatrix}G'(abcd,J). \tag{A8}$$

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[1]Z. Haratym, J. Kownacki, J. Ludziejewski, Z. Sujkowski, L. E. De Geer, A. Kerek, and H. Ryde, Nucl. Phys. A276, 299 (1977).

[2]A. M. Stefanini, P. Kleinheinz, and M. R. Mayer, Phys. Lett. 62B, 405 (1976).

[3]R. Broda, M. Ogawa, S. Lunardi, M. R. Maier, P. J. Daly, and P. Kleinheinz, Z. Phys. A285, 423 (1978).

[4]J. C. Veefkind, D. Spaargaren, J. Blok, and K. Heyde, Z. Phys. A275, 55 (1975).

[5]W. Booth, S. Wilson, and S. S. Ipson, Nucl. Phys. A238, 301 (1975).

[6]W. Booth and S. Wilson, Nucl. Phys. A247, 126 (1975).

[7]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1975), Vol. II.

[8]K. Heyde and P. J. Brussaard, Nucl. Phys. A104, 81 (1967).

[9]K. Heyde and G. Vanden Berghe, Nucl. Phys. A126, 381 (1969).

[10]G. Vanden Berghe, K. Heyde, and M. Waroquier, Nucl. Phys. A165, 662 (1971).

[11]T. T. S. Kuo, E. U. Baranger, and M. Baranger, Nucl. Phys. 79, 513 (1966).

[12]L. S. Kisslinger and R. A. Sorensen, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. 32, No. 9 (1960).

[13]M. Baranger, Phys. Rev. 120, 957 (1960).

[14]K. Sasaki, Nucl. Phys. 71, 95 (1965).

[15]K. Heyde, M. Waroquier, and H. Vincx, Phys. Lett. 57B, 429 (1975).

[16]M. Waroquier and K. Heyde, Z. Phys. 268, 11 (1974).

[17]L. Funke, W. D. Fromm, H. J. Keller, R. Arlt, and P. M. Gopytsch, Nucl. Phys. A274, 61 (1976).

[18]P. Van Isacker (unpublished).