Muon capture in ^{6}Li in the ditriton channel by the use of the elementary particle model

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The muon-capture rate for the reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ is calculated by the use of the elementary-particle model. The form factors describing the axial current matrix element are determined by pion-capture data for the reaction $\pi^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H}$ via the partially conserved axial vector current hypothesis and results based on the impulse approximation. The form factors describing the vector current matrix element are obtained from the reactions $\gamma + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{He}$ and ${}^3\text{H} + {}^3\text{He} \to \gamma + {}^6\text{Li}$ via the conserved vector current hypothesis. Two results are presented for which the assumptions vary, $\Gamma = 104.9$ sec^{-1} and $\Gamma = 160.5$ sec^{-1}.

[NUCLEAR REACTIONS Muon-capture ${}^6\text{Li}(\mu, \nu_\mu){}^3\text{H}{}^3\text{H}$ calculated Γ, $d\Gamma/d\nu$ using the elementary-particle model.]

I. INTRODUCTION

There has been a continuing interest in the muon-capture reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$, particularly because it might be a suitable candidate for determining the ν_μ mass.[1] This reaction has been studied theoretically[2] by means of an impulse-approximation calculation. In this paper we make use of the elementary-particle model[3] for calculating the capture rate.

The elementary-particle model has some advantages over the conventional impulse approximation treatment for problems of this type because it avoids the use of nuclear wave functions. The cross sections calculated by means of an impulse-approximation treatment sometimes depend sensitively on the wave functions which are, in general, not well known. Also in the elementary-particle approach the Pauli exclusion principle is incorporated directly into the matrix element.

In the elementary-particle model approach, the form factors describing the matrix element of the weak vector current are obtained from the electromagnetic form factors via the conserved-vector-current hypothesis (CVC). Information concerning the axial current form factors is usually obtained from β-decay or other data by making use of the partially conserved axial-current hypothesis (PCAC) and an impulse-approximation derived result.

In Sec. II of this paper we exhibit the general form of the weak current matrix element and obtain the form factors describing them. In Sec. III we obtain the muon-capture rate in the ditriton channel. In Sec. IV of the paper we discuss the results of the calculations presented.

II. STRUCTURE OF THE WEAK CURRENT MATRIX ELEMENT

The matrix element for the muon-capture process $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ may be written as

$$\langle {}^3\text{H}_1, {}^3\text{H}_2, \nu | H_w(0) | {}^6\text{Li}, \mu^- \rangle = \frac{G\cos\theta_C}{\sqrt{2}} \bar{u}_\nu \gamma^\lambda (1-\gamma_5) u_\mu \langle {}^3\text{H}_1, {}^3\text{H}_2 | J_\lambda^\dagger(0) | {}^6\text{Li} \rangle \quad (1)$$

to lowest order in G $(=1.02\times 10^{-5} m_p^{-2})$, the weak coupling constant, where θ_C is the Cabbibo angle ($\cos\theta_C = 0.98$), and

$$J^\mu(0) = V^\mu(0) - A^\mu(0) \quad (2)$$

is the weak hadronic current written in terms of the axial current $A^\mu(0)$ and vector current $V^\mu(0)$ parts, respectively. Thus the problem of obtaining the matrix element of the weak Hamiltonian for the process $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ reduces essentially to obtaining $\langle {}^3\text{H}{}^3\text{H} | A_\mu^\dagger(0) | {}^6\text{Li} \rangle$ and $\langle {}^3\text{H}{}^3\text{H} | V_\mu^\dagger(0) | {}^6\text{Li} \rangle$.

The reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ is structurally very similar to the reaction $\mu^- + d \to n + n + \nu_\mu$. Both the deuteron and ^{6}Li are 1^+ nuclei. Tritium is a spin $\frac{1}{2}$ nucleus and tritium and ^{3}He have the same isospin assignments as the pair n and p. Thus we expect the matrix elements $\langle nn | A_\mu^\dagger(0) | d \rangle$, $\langle nn | V_\mu^\dagger(0) | d \rangle$ to have precisely the same structure as $\langle {}^3\text{H}{}^3\text{H} | A_\mu^\dagger(0) | {}^6\text{Li} \rangle$ and $\langle {}^3\text{H}{}^3\text{H} | V_\mu^\dagger(0) | {}^6\text{Li} \rangle$, respectively.

The matrix elements $\langle nn | A_\mu^\dagger(0) | d \rangle$, and $\langle nn V_\mu^\dagger(0) | d \rangle$ have been obtained by the author in an earlier paper.[4] Thus we may write

$$\langle {}^3\text{H}{}^3\text{H}\,|\,V_\mu^\dagger(0)\,|\,{}^6\text{Li}\rangle = \eta\bar{u}(p_1)\left(\frac{F_1}{M_L{}^2}\epsilon_{\mu\nu\sigma\rho}\xi^\nu Q^\rho L^\sigma + \frac{F_2}{M_L}\gamma^\nu\epsilon_{\nu\rho\sigma\mu}\xi^\rho q^\sigma\right)\gamma_5 v(p_2), \tag{3a}$$

$$\langle {}^3\text{H}{}^3\text{H}\,|A_\mu^\dagger(0)\,|\,{}^6\text{Li}\rangle = \eta\bar{u}(p_1)\left(F_A\xi_\mu + F_P\frac{\xi\cdot Qq_\mu}{M_L{}^2}\right)\gamma_5 v(p_2), \tag{3b}$$

where $\eta = [m_T{}^2/(E_1E_2)]^{1/2}(2\pi)^{-1/2}(2L_0)^{-1/2}$, m_T and M_L are the triton and ^{6}Li masses, respectively, L_μ is the ^{6}Li four-momentum, E_1 and E_2 are the triton energies, ξ_μ is the ^{6}Li polarization vector, T_{1_μ} and T_{2_μ} are the triton four-momenta, and

$$\begin{aligned} Q_\mu &= T_{1_\mu} + T_{2_\mu}, \\ q_\mu &= T_{1_\mu} + T_{2_\mu} - L_\mu, \\ P_\mu &= T_{1_\mu} - T_{2_\mu}. \end{aligned} \tag{4}$$

The problem of calculating the muon-capture rate, therefore, reduces to the problem of determining the form factors F_A, F_P, F_1, and F_2. The form factor F_A is particularly important in this calculation because it dominates[4,5] the capture rate.

With respect to determining F_A there exists data on the branching ratio[6]

$$\Gamma(\pi^- + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^3\text{H})/\Gamma(\pi^- + {}^6\text{Li})_{\text{total}} = 3.4\times 10^{-4}. \tag{5}$$

Furthermore,[7] $\Gamma(\pi^- + {}^6\text{Li})_{\text{total}} \simeq \Gamma_{1s}$, the width of the $1s$ orbit of the pion. The width[8] $\Gamma_{1s} \simeq 0.15 \pm 0.5$ keV and thus

$$\Gamma(\pi^- + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^3\text{H}) \simeq 5.1\times 10^{-8}\ \text{MeV}. \tag{6}$$

The matrix element for this process $\langle TT\,|\pi^-\,{}^6\text{Li}\rangle$ can be related to the weak-axial current matrix element $\langle TT\,|A^{\dagger\mu}(0)\,|\,{}^6\text{Li}\rangle$ via the PCAC hypothesis[9]

$$\langle TT\,|\pi^-\,{}^6\text{Li}\rangle = \lim_{q^2\to m_\pi{}^2}\frac{(q^2 - m_\pi{}^2)}{m_\pi{}^2 f_\pi\sqrt{2}} \langle {}^3\text{H}{}^3\text{H}\,|\,\partial_\mu A^{\dagger\mu}(0)\,|\,{}^6\text{Li}\rangle, \tag{7}$$

where m_π is the pion mass and f_π ($f_\pi\cos\theta \simeq 96$ MeV) is the pion decay constant. Thus it is necessary to construct from Eq. (3b)

$$\langle {}^3\text{H}{}^3\text{H}\,|\,\partial_\mu A^{\dagger\mu}(0)\,|\,{}^6\text{Li}\rangle.$$

Because the capture process may be assumed to take place with both the pion and the lithium nucleus at rest the total laboratory space momentum for the tritons $\vec{Q}$ is zero. This fact combined with the fact that

$$\langle {}^3\text{H}{}^3\text{H}\,|\,\partial_\mu A^{\dagger\mu}(0)\,|\,{}^6\text{Li}\rangle = iq_\mu\langle {}^3\text{H}{}^3\text{H}\,|A^{\mu\dagger}(0)\,|\,{}^6\text{Li}\rangle$$

essentially eliminates those terms which normally dominate the matrix element of the axial current as may be seen from Eq. (3b). It is therefore necessary to modify Eq. (3b). To do this we add terms to Eq. (3b) to make it agree to the lowest order with those terms which survive in an impulse-approximation calculation of the same process, in which we assume that instead of dealing with the individual nucleons, we are dealing with three-body clusters within the nucleus.

The impulse approximation for $\langle {}^3\text{H}{}^3\text{H}\,|A_\mu^\dagger(0)\ {}^6\text{Li}\rangle$ is, under this assumption, given by

$$\langle {}^3\text{H}{}^3\text{H}\,|A_\mu^\dagger(0)\,|\,{}^6\text{Li}\rangle = \left\langle {}^3\text{H}{}^3\text{H}\,\left|\sum_{i=1}^{2}\,[\gamma_\mu F_A(q^2;{}^3\text{H}\longleftrightarrow{}^3\text{He}) + q_\mu F_P(q^2;{}^3\text{H}\longrightarrow{}^3\text{He})]\tau_{(i)}^- e^{iq\cdot r(i)}\gamma_5\right|\,{}^6\text{Li}\right\rangle. \tag{8}$$

This implies that the important terms in the pion capture process come from the 0th component of the current matrix element (we are using the Bjorken and Drell metric). The matrix element of the axial current Eq. (3b) when modified as

$$\langle {}^3\text{H}{}^3\text{H}\,|A_\mu^\dagger(0)\,|\,{}^6\text{Li}\rangle = \eta\bar{u}(p_1)\left[F_A\left(\xi_\mu + \frac{\xi\cdot\gamma}{2\sqrt{2}}\frac{L_\mu}{M_L}\right) + \left(\frac{\xi\cdot Q}{M_L} - \frac{\xi\cdot\gamma}{2\sqrt{2}}\right)\frac{q_\mu}{M_L}F_P\right]\gamma_5 v(p_2) \tag{9}$$

agrees in form with the impulse-approximation result under the same circumstances.[10] From Eq. (9)

$$\begin{aligned} \langle {}^3\text{H}{}^3\text{H}\,|\,\partial_\mu A^{\dagger\mu}(0)\,|\,{}^6\text{Li}\rangle &= i\bar{u}\left[\left(\xi\cdot Q + \frac{\xi\cdot\gamma}{2\sqrt{2}}\frac{q\cdot L}{M_L}\right)F_A + \left(\frac{\xi\cdot Q}{M_L} - \frac{\xi\cdot\gamma}{2\sqrt{2}}\right)\frac{q^2}{M_L}F_P\right]\gamma_5 v \\ &= i\bar{u}\left(\frac{\xi\cdot\gamma}{2\sqrt{2}}\frac{q\cdot L}{M_L}F_A - \frac{\xi\cdot\gamma}{2\sqrt{2}}\frac{q^2}{M_L}F_P\right)\gamma_5 v \end{aligned} \tag{10}$$

in the limit $Q\rightarrow 0$. It is next necessary to relate F_A and F_P. To do this we apply the formulation of PCAC due to Nambu[11] to Eq. (10) and obtain

$$F_P = \frac{q_0 M_L}{q^2 - m_\pi{}^2}F_A. \tag{11}$$

The matrix element $\langle {}^3\text{H}{}^3\text{H}\,|\,{}^6\text{Li}\pi^-\rangle$ becomes from Eqs. (7), (10), and (11)

$$\langle {}^3\text{H}{}^3\text{H}\,|\,{}^6\text{Li}\pi^-\rangle \simeq \bar{u}\left(\frac{\xi\cdot\gamma q_0}{4f_\pi}F_A\right)\gamma_5 v \tag{12}$$

and leads to the following matrix element squared for the process:

$$|M|^2=\frac{1}{3}\left(\frac{1}{4}\frac{q_0F_A}{f_\pi}\right)^2\left(\frac{(M_L+m_\pi)^2-4m_T^2}{m_T^2}\right). \quad (13)$$

Calculating the capture rate in the usual way and making use of the value of Γ given by Eq. (6), we obtain

$$|F_A|^2=1.76\times 10^{-5}. \quad (14)$$

However,[4] F_A is a function of several variables and can be written

$$F_A=F_A(Q\cdot L, q^2, P\cdot L). \quad (15)$$

For the case of pion capture, $F_A=F_A(m_\pi M_L, m_\pi^2, 0)$. It is necessary to have the general functional form for F_A given in Eq. (15). We found[4] in the case of deuterium that the form factors to the lowest order factorized as follows:

$$F_i(Q\cdot d, q^2, P\cdot d)\simeq F(Q\cdot d, P\cdot d)f_i(q^2),$$
$$i=1,2,A,P. \quad (16)$$

We make this assumption also for the case considered here and replace d_μ by L_μ. The function $f_A(q^2)$ is not known. However, it has been found by examining[12-14] the reaction $\nu+d\to p+p+\mu^-$ that at large q^2 values $f_A(q^2)$ for deuterium behaves like a nucleon form factor indicating that one nucleon participated in the reaction and the other was a spectator. If we take a cluster model seriously, at large q^2 we should expect a behavior characteristic of the interaction of the beam particles with one three-body cluster in the ^{6}Li while the other is essentially a "spectator." At low q^2, however, more "sharing" of energy among the nucleons would be expected. We therefore average these two behaviors. Assuming a dipole fit to $f_A(q^2)$ of the form

$$f_A(q^2)=\frac{f_A(0)}{(1-q^2/M_A^2)^2}, \quad (17)$$

we note that[15] $M_A^2=7.9m\pi^2$ for $^3\text{He}\longleftrightarrow{}^3\text{H}$ and $M_A^2=2.0m_\pi^2$ for $^6\text{Li}\longleftrightarrow{}^6\text{He}$ and that these denote the two extreme behaviors. We therefore average them[16] and use a value $M_A=4.95m_\pi^2$.

It is still necessary to obtain $F(Q\cdot L, P\cdot L)$ and, of course, $f_A(0)$. From experience with the deuteron case,[4] the $P\cdot d$ dependence is relatively unimportant and we assume

$$F(Q\cdot L, P\cdot L)\simeq F(Q\cdot L). \quad (18)$$

Since to the lowest order $F(Q\cdot L)$ is the same for all the form factors, it may be obtained from electromagnetic data.

There exist data concerning the reactions[17,18] $\gamma+{}^6\text{Li}\to{}^3\text{H}+{}^3\text{He}$ and $^3\text{H}+{}^3\text{He}\to\gamma+{}^6\text{Li}$. The electromagnetic form factors describing these processes may be related to the form factors describing the matrix element of the weak current via the CVC hypothesis[19]:

$$[I^-, J_\mu^{(3)}(0)]=[I^-, J_\mu^{\text{em}}(0)]$$
$$=V_\mu^\dagger(0), \quad (19)$$

where J_μ^{em} is the electromagnetic current density, $J_\mu^{(3)}$ is the isovector part of J_μ^{em}, and I^- is the isospin lowering operator. Since $|{}^3\text{H}{}^3\text{H}\rangle$ is in an isospin state $I=1$, $I_3=-1$ and ^{6}Li is in an $I=0$ state we obtain

$$\langle{}^3\text{H}{}^3\text{H}|V_\mu^\dagger(0)|{}^6\text{Li}\rangle=2\langle{}^3\text{H}{}^3\text{He}|J_\mu^{\text{em}}(0)|{}^6\text{Li}\rangle, \quad (20)$$

where $|{}^3\text{H}{}^3\text{He}\rangle$ is in an $I=1$, $I=0$ state. The matrix element $\langle{}^3\text{H}{}^3\text{He}|J_\mu^{\text{em}}|{}^6\text{Li}\rangle$ may be written[4]

$$\langle{}^3\text{H}{}^3\text{He}|J_\mu^{\text{em}}(0)|{}^6\text{Li}\rangle=\eta\bar{u}\left[\frac{F_a}{M_L}\epsilon_{\mu\nu\sigma\rho}\xi^\nu Q^\rho L^\sigma + \frac{F_b}{M_L}\gamma^\nu\epsilon_{\nu\rho\sigma\mu}\xi^\rho q^\sigma\right]\gamma_5 v \quad (21)$$

so that

$$\sqrt{2}F_a=F_1 \text{ and } \sqrt{2}F_b=F_2. \quad (22)$$

Again making use of Eq. (16) and noting that for the two electromagnetic processes mentioned $q^2=0$, we see that $|F_a-F_b|=F(Q\cdot L)|f_a(0)-f_b(0)|$. We normalize $F(Q\cdot L)$ (Ref. 20) so that is is unity at its maximum. Thus we are ready to obtain its functional form.

The one difficulty remaining is that we must have the ^{3}He-^{3}H in an $I=1$, $I_3=0$ state. However, the data isolate the $E1$ contribution which comes[17] from the $^3P_3$ partial wave so that the ^{3}He-^{3}H system is in the needed isospin state. The matrix element squared for either of the two electromagnetic processes may be written as

$$|M|^2=\alpha e^2\frac{k^2(F_a-F_b)^2}{m_T^2}, \quad (23)$$

where $\alpha=\frac{1}{6}$ for $\gamma+{}^6\text{Li}\to{}^3\text{H}+{}^3\text{He}$, $\alpha=\frac{1}{4}$ for $^3\text{H}+{}^3\text{He}\to\gamma+{}^6\text{Li}$, and k is the γ momentum. Calculating the cross sections in the usual way and normalizing $F(Q\cdot L)$ as mentioned above, we find that the data are well fitted by

$$|F(Q\cdot L)|^2=\{1-0.33\exp[-9.59\times10^{-2}(q_0-20)^2]\}\{20.84+2.01\exp[-1.589\times10^{-3}(q_0-95)^2]$$
$$-0.357q_0\exp[-6.01\times10^{-5}(q_0-16.5)^2]\}[(q_0-16.5)^2+12.04]^{-1}, \quad (24)$$

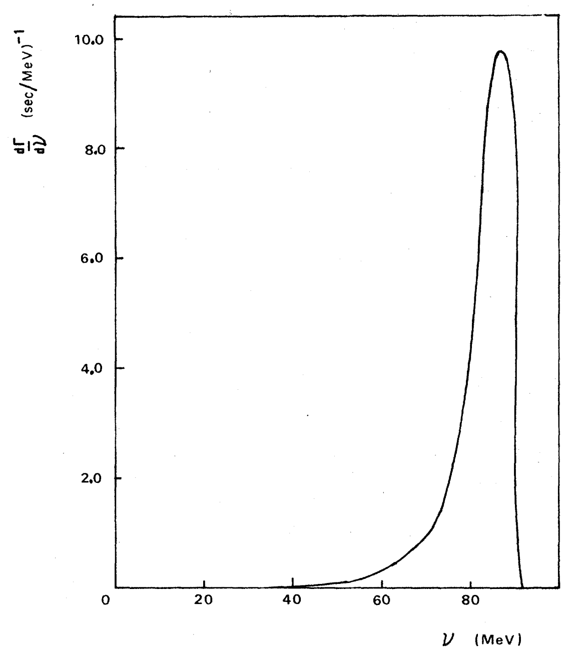

FIG. 1. Plot of the differential capture rate $d\Gamma/d\nu$ as a function of ν, the neutrino energy. The curve is plotted for the value $f_A(0) = 0.296$.

where $q_0 = (Q - L)\cdot L/M_L$ and units are compatible with q_0 expressed in MeV. Combining Eqs. (14), (16), (17), and (24) we obtain

$$F_A(m_\pi M_L, m_\pi^2, 0) = f_A^2(0)\times 2.01\times 10^{-4} = 1.75\times 10^{-5} \quad (25a)$$

or

$$f_A(0) = 0.296. \quad (25b)$$

We also note that in Ref. 4 a relation was obtained for determining $f_A(0)$ in the deuteron case

$$f_A(0) = F_A(0, n \longleftrightarrow p)\sqrt{2}/[F_V(0, n\longleftrightarrow p) + F_M(0, n \longleftrightarrow p)].$$

If we assume a cluster model and replace the quantities[21] $F_i(0, n \longleftrightarrow p)$ by $F_i(0, {}^3\mathrm{H} \longleftrightarrow {}^3\mathrm{He})$ we obtain

$$f_A(0) = 0.38, \quad (26)$$

which is in reasonable agreement with Eq. (25b).

The form factors F_1 and F_2 describing the matrix element of the vector current appear in the muon-capture rate in the form $|F_1 - F_2|^2$. From the data[22] in Refs. 16 and 17 and Eq. (22)

$$|F_1(0) - F_2(0)| \simeq 4.49. \quad (27)$$

We assume the same q^2 dependence given by Eq. (17). The $Q\cdot L$ dependence is given by Eq. (24). The muon-capture rate for this process does not depend sensitively on the behavior of $|F_1 - F_2|$. Thus the form factors in Eqs. (3a) and (3b) are obtained and the muon-capture rate can be determined.

III. CAPTURE RATE

The matrix element squared for the process $\mu^- + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \nu_\mu$ is given by

$$|M|^2 = \frac{2}{6m_\mu m_T^2}\left\{F_A^2(m_\mu + M_L)\frac{(m_\mu + M_L - \nu)}{2}[3m_\mu\nu + 2m_\mu^2\nu^2/(m_\mu^2 - 2\nu m_\mu - m_\pi^2) + \nu^3 m_\mu^3/(m_\mu^2 - 2\nu m_\mu - m_\pi^2)^2] + (F_1 - F_2)^2 m_\mu \nu^3\right\}. \quad (28)$$

This leads to a capture[23] rate of

$$\Gamma = 104.9\ \mathrm{sec}^{-1} \quad (29)$$

for the value of $f_A(0)$ given by Eq. (25b) and

$$\Gamma = 160.5\ \mathrm{sec}^{-1} \quad (30)$$

for the value given by Eq. (26).

In Fig. 1 we plot $d\Gamma/d\nu$, the capture rate as a function of the neutrino energy. We note that in calculating the muon-capture rate as well as the pion-capture rate we have made use of the correction factor[3] $C_{\mathrm{Li}} = 0.928$ which takes into account the spread of the charge in ^{6}Li.

IV. CONCLUSION

A calculation of the muon-capture rate for the process $\mu^- + {}^6\mathrm{Li} \to T + T + \nu_\mu$ by the use of the impulse approximation has already been mentioned.[2] The result obtained was

$$\Gamma = 160\ \mathrm{sec}^{-1} \quad (31)$$

in close agreement with the second value given by Eq. (30) obtained here. Because of the number of assumptions that were needed to obtain Eqs. (29) and (30), it is best to treat these results as theoretical estimates rather than precise calculations. However, the reasonable agreement of the results is encouraging. Obviously, the accuracy of the calculation presented here would be much improved if data were available to better determine F_A. Data for the photoproduction reaction $\pi + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \gamma$ or $\gamma + {}^6\mathrm{Li} \to {}^3\mathrm{H} + {}^3\mathrm{H} + \pi$ would be particularly useful.

Finally, we note that from Fig. 1, the differential muon-capture rate $d\Gamma/d\nu$ is strongly peaked in the upper range of the neutrino energy. This, of course, implies that the differential capture rate is peaked in the lower range of the triton energy, which is in agreement with the results of Ref. 2.

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[10]Because the divergence of the axial current enters into the pion capture, and because $\vec{Q}=0$ while $q_\mu = q_0 - \vec{Q}$, it is clear that we need look only at the matrix element of $\langle f|A_0^\dagger|i\rangle$. Furthermore, because the form of the impulse approximation and the elementary-particle model are so different, a comparison must be made on a case by case basis between Eq. (8) and Eq. (9). For example, taking the axial form factor term of Eq. (8)

$$\langle {}^3\mathrm{H}{}^3\mathrm{H}|\sum^2 \gamma_0^{(i)} F_A(q^2)\tau_{(i)} e^{iq\cdot r^{(i)}}|{}^6\mathrm{Li}\rangle$$

and noting that the isopinor for ^{6}Li can be written

$$\chi^I_{\mathrm{Li}} = \left[\begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right]$$

and

$$\chi^I_{{}^3\mathrm{H}{}^3\mathrm{H}} = \begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2,$$

we obtain

$$\langle {}^3\mathrm{H}{}^3\mathrm{H}\sum_{i=1}^{2} \gamma_0^{(i)} F_A(q^2)\tau_{(i)} e^{iq\cdot r^{(i)}}|{}^6\mathrm{Li}\rangle = \frac{2}{\sqrt{2}}\langle {}^3\mathrm{H}{}^3\mathrm{H}|[\gamma_0^{(1)}\gamma_5^{(1)}e^{iq\cdot r(1)} - \gamma_0^{(2)}\gamma_5^{(2)}e^{iq\cdot r(2)}]F_A(q^2)|{}^6\mathrm{Li}\rangle$$

$$\simeq \frac{2R}{\sqrt{2}}\chi'\left[\left(1, \frac{-\sigma_1\cdot p_1}{E_1+m}\right)\begin{pmatrix}1&0\\0&-1\end{pmatrix}_1\begin{pmatrix}0&1\\1&0\end{pmatrix}_1\begin{pmatrix}1\\ \frac{\sigma_1\cdot p_1}{E_1+m}\end{pmatrix}\right.$$

$$\left. -\left(1, \frac{\sigma_2'\cdot p_2'}{E_2'+m}\right)\begin{pmatrix}1&0\\0&-1\end{pmatrix}_2\begin{pmatrix}0&1\\1&0\end{pmatrix}_2\begin{pmatrix}1\\ \frac{\sigma_2\cdot p_2}{E_2+m}\end{pmatrix}\right]\chi,$$

where primes denote final state quantities and χ is the appropriate spinor. If, for example, we take $\chi' = \binom{1}{0}_1\binom{1}{0}_2$ and $\chi = \binom{1}{0}_1\binom{1}{0}_2$, we obtain

$$\langle {}^3\mathrm{H}{}^3\mathrm{H}|\sum \gamma_0^{(i)} F_A(q^2)\tau_i e^{i\vec{q}\cdot\vec{r}(i)}|{}^6\mathrm{Li}\rangle = R\frac{\sqrt{2}}{2}P_Z F_A(q^2),$$

where we have included the wave function integration in R. If we look at the corresponding case in the elementary particle made from Eq. (9)

$$\eta\bar{u}(p_1')\frac{(\xi\cdot\gamma L_0)}{2\sqrt{2}M_L}\gamma_5 v(p_2')F_A \simeq \left(1, 0, \frac{-p_{1Z}'}{E_1'+m}, \frac{-p_{1+}'}{E_1'+m}\right)[\gamma_1 + i\gamma_2]\gamma_5\begin{bmatrix}\frac{P_2^-}{E_2'+m}\\ \frac{P_{2Z}'}{E_2'+m}\\ 0\\ 1\end{bmatrix}\frac{1}{2\sqrt{2}}F_A$$

$$\simeq \frac{\sqrt{2}}{2}P_z F_A(q^2, Q\cdot L, P\cdot L).$$

The other cases and terms are similarly done.

[11]There is insufficient data to use the Gell-Mann-Levy form of PCAC for the process described. However, the differences in the two formulations do not lead to large differences in the matrix element of the axial current. (See Ref. 14.) For the Nambu formulation of PCAC see Ref. 9.

[12]S. L. Mintz, Phys. Rev. D 10, 3017 (1974).

[13]S. L. Mintz, Phys. Rev. D 13, 639 (1976).

[14]S. L. Mintz (unpublished).

[15]See, for example, C. W. Kim and S. L. Mintz, Nucl. Phys. B27, 621 (1971).

[16]If we take the two extreme values, we find that $M_A^2 = 2.0m^2_\pi$ would lead to $\Gamma = 10.2$ sec^{-1} and $M_A^2 = 7.9_\pi{}^2$ would lead to $\Gamma = 181.3$ sec^{-1}.

[17]Y. M. Shin, D. M. Skopik, and J. J. Murphy, Phys. Lett. 55B, 297 (1975).

[18]E. Ventura, C. C. Chang, and W. E. Meyerhof, Nucl. Phys. A173, 1 (1971).

[19]See Ref. 4.

[20]This is just done for convenience. From Eqs. (13) and (16) any factor normalizing $F(Q\cdot L)$ simply forces a change in $f_A(0)$ by 1 over that factor. Since the product of $F(Q\cdot L)$ and $f_A(0)$ occurs in the total form factor $F_A(q^2, Q\cdot L, P\cdot L)$ any normalization factor will cancel out.

[21]The numbers $F_A(q_2 = 0.96m_\mu{}^2;\ {}^3\mathrm{He} \longleftrightarrow {}^3\mathrm{H}) = -1.046$, $F_V(q^2 = 0.96m_\mu{}^2;\ {}^3\mathrm{He} \longleftrightarrow {}^3\mathrm{H}) = 0.811$, and $F_M(q^2 = 0.96m_\mu{}^2;\ {}^3\mathrm{He} \longleftrightarrow {}^3\mathrm{H}) = -4.69$ are given in J. Frazier and C. W. Kim, Phys. Rev. 177, 2560 (1969). Assuming approximately the same q^2 dependence for all the form factors, we obtain the value given by Eq. (26).

[22]Equation (23) determines the quantity $|F_a - F_b|$ but from Eq. (16) $|F_a - F_b| \simeq F(Q\cdot L) f_a(q^2) - F_b(q^2)|$. We use the value of $F(Q\cdot L)$ given by Eq. (24) and assume for the form of $f_a(q^2) - f_b(q^2) = [f_a(0) - f_b(0)]/(1 - q^2/M_v{}^2)^2$. Since we have no way of determining M_v, guided by the deuteron case we use the value for M_A. When these substitutions are made in Eq. (23), we obtain a value for $f_a(0) - f_b(0)$. Then using Eq. (22) we obtain the value for $|F_1(0) - F_2(0)|$ given here.

[23]We assume an angular dependence given by averaging the angular dependences given for $q_0 = 22.3$ MeV and $q_0 = 26.6$ MeV in Ref. 17. This leads to $f(\theta) = 1 + (0.075)P_1 - (0.99)P_2$, where P_1 and P_2 are Legendre polynomials and θ is the angle between q and one of the outgoing tritons (see the argument given in Ref. 4).