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Role of the small components of the nucleon-nucleus distorted wave in (p,π) reactions

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A formalism, based upon a covariant reduction of a Bethe-Salpeter equation for nucleon nucleus scattering, is presented. The aim is to focus attention upon the relativistic components of the nucleon nucleus scattering wave functions and the role they play in inelastic reactions such as the (p,π) reaction. One result is a renormalization of the pion-nucleon vertex function due to these components.

[NUCLEAR REACTIONS Relativistic effects in proton-pion reactions.]

I. INTRODUCTION

In a number of processes of the direct reaction type, for example (p,π) or (p,γ) reactions at intermediate energies, $P_{\text{Beam}} = 0.5$ GeV/c to 1.5 GeV/c, the impinging nucleon has a velocity comparable with the speed of light. Most analyses[1-5] of such processes include the use of distorted waves for the incoming proton. In this energy region the nucleon-nucleon cross sections have not leveled off to the 45 mb value characteristic of higher energy. In fact the pp total cross section doubles as P_{Beam} goes from 0.5 GeV/c to 1 GeV/c. Furthermore, the Fermi momentum of a struck target nucleon is approximately 280 MeV/c so that Fermi averaging of the nucleon-nucleon transition matrix over the combined energy of the projectile and target nucleon can be a delicate procedure.

The Glauber approximation[6] and the multiple scattering theories,[7,8] are fixed scatterer theories in that the target nucleons are considered frozen when struck. These approximations are generally capable of being adjusted so that good fits to the elastic nucleon-nucleus total and elastic differential cross sections are obtained. Inelastic reactions, such as the (p,π) reaction when treated in the distorted-wave Born approximation (DWBA), require off shell information not demanded in elastic scattering. Thus, in addition to the aspect of Fermi averaging in determining the appropriate distorted wave, the correct treatment of kinematics is also needed. It has been shown in the pion-nucleus problem that a covariant treatment of pion-nucleus scattering can lead to a more fundamental understanding of the problem.[9-11] Both the role of Fermi averaging and kinematic corrections are clarified and extension to higher energy[11] is shown to be compatible with fixed scatterer results. Whereas these effects may not be as important in nucleon-nucleus scattering because of the larger mass of the nucleon, the nucleon possesses another quality, namely spin, which requires a covariant treatment especially for (p,π) reactions.

The lower or relativistic components of nuclear wave functions have been dealt with in (p,π) reactions,[1] but only in the context of the bound state into which the nucleon is absorbed. The results of Miller and Weber are somewhat in doubt because the orthogonality constraints upon the bound state and continuum wave functions involved were not utilized.[12] It would appear that an appropriate technique by which to discuss the various aspects of high velocity nuclear motion in this problem is the covariant reduction scheme. Such methods in addition to their use in the aforementioned pion-nucleus problem have been employed in the nucleon-nucleon problem.[13-15] In addition the pionic disintegration of the deuteron[16] has been investigated including the role of the relativistic components of the deuteron wave function. However, the effects of the lower components of the nucleon-nucleus scattering wave have not been included.

The Dirac equation[17] has recently been applied to calculate the full distorted nucleon-nucleus wave using the vector and scalar meson transfer model[18] of the effective one body potential experienced by the nucleon. The fits to the elastic scattering data and polarization are excellent, however, the underlying optical model theory is not justified completely. In addition there is an apparent lack of nucleus recoil in the calculations, since the Dirac equation is solved in the presence of the effective one body potential whose parameters are then fitted to the data and the momentum transfer to the nucleus is ignored.

The aim of this paper is to construct a scheme for calculating the nucleon-nucleus scattering wave via a relativistic reduction of a covariant nucleon-nucleus equation. In the course of this construction in Sec. II the natural choice for the form of the reduced equation is the Dirac equation, however, the full effects of nuclear recoil are maintained. Furthermore, the lower components of the covariant nucleon-nucleus wave function are isolated and a scheme for constructing them suggested. A further reduction is made to a nonrelativistic notation. In Sec. III the wave function is investigated and the Moller scattering matrix is displayed.

II. REDUCTION SCHEME

In order to include the effects of recoil and to start from a relativistic basis, a Bethe-Salpeter equation is assumed to be valid for a nucleon of four-momentum W-P scattering from a spin zero nucleus of four-momentum P (see Fig. 1)

$$\langle W-P',P'|\mathfrak{M}|W-P,P\rangle=\langle W-P',P'|D|W-P,P\rangle + \frac{i}{2\pi}\int d^4P''\,\frac{\langle W-P',P'|D|W-P'',P''\rangle\langle W-P'',P''|\mathfrak{M}|W-P,P\rangle}{(\not{W}-\not{P}''-M)(P''^2-M_A{}^2)}\,. \tag{2.1}$$

Here $\mathfrak{M}$ is the invariant nucleon-nucleus scattering amplitude and D is the irreducible amplitude for nucleon-nucleus scattering, M is the mass of the nucleon and M_A the mass of the nucleus. The Dirac matrix $\mathfrak{M}$ appears in the S matrix for nucleon-nucleus scattering with all particles on their respective mass shells in the following manner:

$$\begin{aligned}\langle W'-P',S';P'|S|W-P,S;P\rangle=&\,\delta_{ss'}\delta^3(\vec{\mathrm{P}}-\vec{\mathrm{P}}')\delta^3(\vec{\mathrm{W}}-\vec{\mathrm{W}}')\\&-2\pi i\delta^4(W-W')(M^2/E(\vec{\mathrm{W}}-\vec{\mathrm{P}})E(\vec{\mathrm{W}}-\vec{\mathrm{P}}')4E_A(\vec{\mathrm{P}})E_A(\vec{\mathrm{P}}'))^{1/2}\\&\times\bar{U}_{s'}(\vec{\mathrm{W}}-\vec{\mathrm{P}}')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle U_s(\vec{\mathrm{P}})\,.\end{aligned} \tag{2.2}$$

If the conventions of Bjorken and Drell[19] are followed,

$$\mathfrak{M}=\mathfrak{M}_{\mathrm{BD}}/(2\pi)^3\,. \tag{2.3}$$

In equation (2.2) $E(\vec{\mathrm{W}}-\vec{\mathrm{P}})$ and $E_A(P)$ are the energies of the on-mass-shell nucleon and nucleus while $U_s(W-P)$ is a positive energy spinor for a nucleon of momentum W-P and spin s. In the evaluation of D one has to construct the amplitude for the scattering of a free nucleon from the ground state of the nucleus subject to the proviso that the nucleon-nucleus state never appears as an intermediate state. Some diagrams included in D are shown in Fig. 2. Pictured are single and double scattering contributions and pion production contributions. The aim of this work is not to calculate such diagrams involving fully off mass shell nucleon-nucleon scattering and pionic production amplitudes. Rather it is to isolate those ingredients of the lower components of scattering wave functions which can be parametrized by using the elastic scattering data from those requiring models.

To achieve this goal a reduction of Eq. (2.1)

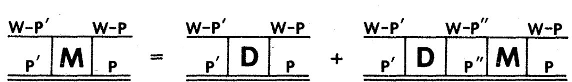

FIG. 1. A graphical representation of the Bethe-Salpeter equation, the open double lines represent the nucleus in its ground state and the single line the scattering nucleon.

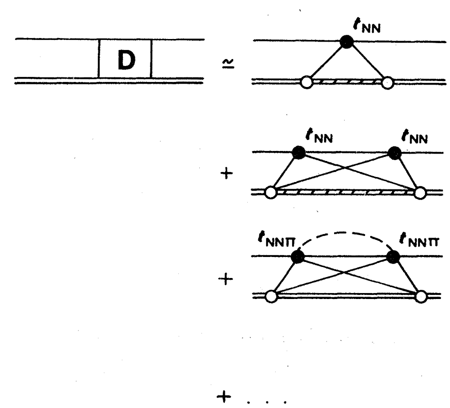

FIG. 2. Several terms in the decomposition of the irreducible amplitude D. Solid dots represent nucleon-nucleon invariant scattering amplitudes. Open circles represent vertex functions for the emission and absorption of a nucleon by the nucleus. Dashed lines represent propagating mesons.

to three dimensional form is made by placing the intermediate nucleus on its mass shell. This is most reasonable since in any virtual process in which the nucleus receives a momentum transfer, the large rest mass of the nucleus and the momentum content of nuclear wave functions will tend to prevent the momentum squared of the nucleus from deviating far from M_A^2.

The particular form of the reduction used here is to replace the Green's function G as follows:

$$\begin{aligned}\langle W-P', P'|G|W-P,P\rangle &\equiv \frac{i\delta^4(P-P')}{2\pi(\not W-\not P-M)(P^2-M_A{}^2)}\\ &\Rightarrow \frac{\delta(P^2-M_A{}^2)\delta^4(P-P')}{(\not W-\not P-M)}\theta(P^0)\theta(W-P)^2)\theta(W^0-P^0)\\ &= \frac{R^2(P)\delta(P^0-E_A(\vec{\mathrm{P}}))\delta^4(P-P')}{(\not W-\not P-M)}\theta(P^0)\theta((W-P)^2)\theta(W^0-P^0)\\ &\equiv\langle W-P',P'|G_R|W-P,P\rangle\,. \end{aligned}\tag{2.4}$$

Here

$$R(P)\equiv 1/[2E_A(P)]^{1/2}\,. \tag{2.5}$$

In the usual way if a new kernel K is defined via

$$K=D+D(G-G_R)K\,, \tag{2.6}$$

then $\mathfrak{M}$ may be expressed exactly in terms of K and G_R as

$$\mathfrak{M}=K+KG_R\mathfrak{M}\,. \tag{2.7}$$

If in (2.7) all states of the nucleus are taken on their mass shell then the following closed linear equation is obtained:

$$\begin{aligned}\langle W-P',P'|\hat{\mathfrak{M}}|W-P,P\rangle &=\langle W-P',P'|\hat{K}|W-P,P\rangle\\ &+\int d^3P''\langle W-P',P'|\hat{K}|W-P'',P''\rangle\hat{G}(W-P'')\\ &\times\langle W-P'',P''|\hat{\mathfrak{M}}|W-P,P\rangle\,.\end{aligned}\tag{2.8}$$

Here, for example

$$\langle W-P',P'|\hat{\mathfrak{M}}|W-P,P\rangle = R(\vec{\mathrm{P}})R(\vec{\mathrm{P}}')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle\Big|_{\substack{P^0=E_A(\vec{\mathrm{P}})\\ P'^0=E_A(\vec{\mathrm{P}}')}} \tag{2.9}$$

and

$$\hat{G}(W-P'')=1/(\not W-\not P''-M)\,. \tag{2.10}$$

Equation (2.8) is covariant if multiplied by $[R(P)R(P')]^{-1}$. The integration in (2.8) over P'' is such that

$$(W-P'')^2>0 \tag{2.11}$$

and

$$W^0-P^{0''}>0\,. \tag{2.12}$$

In the center of momentum the nucleon has momentum k'' and $W_\mu=(\sqrt{s},\vec{0})$ with s the square of the total four momentum of the nucleon-nucleus system. Constraint (2.11) becomes in this frame

$$(s-M_A{}^2)^2/4s>k''^2 \tag{2.13}$$

and is the tighter of the restrictions (2.11), (2.12) for all $s>0$, for example, at threshold $k''<M_N$. The restriction that the mass squared of the intermediate nucleon be greater than zero, besides shuttling all the tachyonlike dependence into Eq. (2.6), allows one to define off mass shell spinors[9,20] $U_s^*(W-P)$, $V_s^*(W-P)$, which obey the equations

$$[\not W-\not P-M^*(W-P)]U_s^*(W-P)=0\,, \tag{2.14}$$

$$[\not W-\not P+M^*(W-P)]V_s^*(W-P)=0\,, \tag{2.15}$$

where

$$M^*(W-P)\equiv[(W-P)^2]^{1/2}\,. \tag{2.16}$$

The U^* and V^* spinors are constructed as ordinary Dirac spinors except $E(W-P)$ is replaced by W^0-P^0 and M by $M^*(W-P)$. For a given value of $W-P$ subject to (2.11) and (2.12),

$$\bar{U}_s^*(W-P)U_s^*(W-P)=\bar{V}_s^*(W-P)V_s^*(W-P)=\delta_{ss'}\,, \tag{2.17}$$

$$\bar{U}_s^*(W-P)V_s^*(W-P)=0\,, \tag{2.18}$$

and they are complete in the sense that

$$\sum_s U_s^*(W-P)\bar{U}_s^*(W-P)-V_s^*(W-P)\bar{V}_s^*(W-P)=1 \tag{2.19}$$

or

$$\Lambda^{*(+)}(W-P)+\Lambda^{*(-)}(W-P)=1\,. \tag{2.20}$$

Equations (2.14) to (2.20) allow $\hat{G}$ to be written as

$$\hat{G}(W-P)=\frac{\not{W}-\not{P}+M}{M^*(W-P)^2-M^2}\times 1$$

$$=\sum_s \frac{U_s^*(W-P)\bar{U}_s(W-P)}{M^*(W-P)-M}$$

$$+\sum_s \frac{V_s^*(W-P)\bar{V}_s^*(W-P)}{M^*(W-P)+M}$$

or

$$\hat{G}(W-P)=\hat{G}^{(+)}(W-P)\Lambda^{*(+)}(W-P)$$
$$+\hat{G}^{(-)}\Lambda^{*(-)}(W-P)\,, \tag{2.21}$$

where

$$\hat{G}^{(\pm)}(W-P)=1/[\pm M^*(W-P)-M]\,. \tag{2.22}$$

The following definitions will be useful below, for example,

$$\langle W-P',P'|\hat{\mathfrak{M}}^{+-}|W-P,P\rangle$$
$$\equiv\Lambda^{*(+)}(W-P')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle$$
$$\times\Lambda^{*(-)}(W-P)\,,$$
$$\langle W-P',P'|\hat{\mathfrak{M}}^{++}|W-P,P\rangle$$
$$\equiv\Lambda^{*(+)}(W-P')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle$$
$$\times\Lambda^{*(+)}(W-P) \tag{2.23}$$

and

$$\langle W-P',s';P'|\hat{\mathfrak{M}}^{+-}|W-P,s;P\rangle$$
$$\equiv-\bar{U}_{s'}^*(W-P')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle$$
$$\times V_s^*(W-P)\,,$$
$$\langle W-P',s';P'|\hat{\mathfrak{M}}^{++}|W-P,s;P\rangle$$
$$\equiv\bar{U}_{s'}^*(W-P')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle$$
$$\times U_s^*(W-P)\,. \tag{2.24}$$

It is to be noted that quantities such as those appearing in Eq. (2.24) are invariant amplitudes with respect to the nucleon and, if the R factors are removed the entire function is invariant under Lorentz transformations.

Returning to Eq. (2.8) and utilizing (2.23), four equations can be developed for $\hat{\mathfrak{M}}^{++}\hat{\mathfrak{M}}^{-+}\hat{\mathfrak{M}}^{+-}$ and $\mathfrak{M}^{--}$. The two which relate directly to the lower components of the nucleon-nucleus scattering states are

$$\hat{\mathfrak{M}}^{++}=\hat{K}^{++}+\hat{K}^{++}\hat{G}^{+}\hat{\mathfrak{M}}^{++}+\hat{K}^{+-}\hat{G}^{-}\hat{\mathfrak{M}}^{-+}\,, \tag{2.25}$$

$$\hat{\mathfrak{M}}^{-+}=\hat{K}^{-+}+\hat{K}^{-+}\hat{G}^{+}\hat{\mathfrak{M}}^{++}+\hat{K}^{--}\hat{G}^{-}\hat{\mathfrak{M}}^{-+}\,. \tag{2.26}$$

By eliminating $\hat{\mathfrak{M}}^{-+}$ from (2.26) in (2.25) $\hat{\mathfrak{M}}^{++}$ satisfies the equation

$$\hat{\mathfrak{M}}^{++}=U^{++}+U^{++}\hat{G}^{+}\hat{\mathfrak{M}}^{++}\,, \tag{2.27}$$

where

$$U^{++}=\hat{K}^{++}+\hat{K}^{+-}\hat{G}^{-}(1-\hat{K}^{--}\hat{G}^{-})^{-1}\hat{K}^{-+}\,, \tag{2.28}$$

and $\hat{\mathfrak{M}}^{-+}$ is given as

$$\hat{\mathfrak{M}}^{-+}=(1-\hat{K}^{--}\hat{G}^{-})^{-1}\hat{K}^{-+}(1+\hat{G}^{+}\hat{\mathfrak{M}}^{++})\,. \tag{2.29}$$

The method of calculating the lower components of nuclear wave functions is now suggested. In (2.29) the factor $(1+\hat{G}^{+}\mathfrak{M}^{++})$ will be shown below to be associated with a nucleon-nucleus distorted wave. The factor $(1-\hat{K}^{--}\hat{G}^{-})^{-1}$ can be approximated by 1 since $\hat{K}^{--}$ is an optical potential for antinucleon-nucleus scattering. Although this potential may have the strength of the nucleon-nucleus optical potential, it is weighted by $\hat{G}^{-}\sim 1/(2M)$. Thus a reasonable approximation to $\hat{\mathfrak{M}}^{-+}$ is

$$\hat{\mathfrak{M}}^{-+}\simeq\hat{K}^{-+}(1+\hat{G}^{+}\hat{\mathfrak{M}}^{++})\,. \tag{2.30}$$

Relationship of $\hat{\mathfrak{M}}^{++}$ *to the Lippmann-Schwinger Equation.* Equation (2.27) may be cast into an equivalent Lippmann-Schwinger equation by first noting that in the c.m.

$$\hat{G}^{+}=\frac{1}{M^*(W-P'')-M}$$
$$=\frac{M^*(W-P'')+M}{W-E_A(\vec{k}'')+E_N(\vec{k}'')}\times\frac{W+W''}{W^2-W''^2}\,, \tag{2.31}$$

where

$$W\equiv(k_W{}^2+M_A{}^2)^{1/2}+(k_W{}^2+M^2)^{1/2}$$
$$=E_A(\vec{k}_W)+E_N(\vec{k}_W)\,, \tag{2.32}$$
$$W''\equiv(k''^2+M_A{}^2)^{1/2}+(k''^2+M^2)^{1/2}$$
$$=E_A(\vec{k}'')+E_N(\vec{k}'')\,. \tag{2.33}$$

Now since

$$W^2-W''^2=4(k_W{}^2-k''^2)/[1-(M_A{}^2-M^2)^2/(W^2W''^2)] \tag{2.34}$$

and

$$E_A(\vec{k}_W)+E_A(\vec{k}'')=(W+W'')[1+(M_A{}^2-M^2)/(WW'')]/2\,, \tag{2.35}$$

$$W-E_A(k'')+E_N(k'')=W[1-(M_A{}^2-M^2)/(WW'')]$$
$$=E_N^*(k'')+E_N(k'')\,, \tag{2.36}$$

one finds that

$$\hat{G}^{+}=\frac{M^*(W-P'')+M}{E_N^*(k'')+E_N(k_W)}\times\frac{2\mu(k'',k_W)}{(k_W{}^2-k''^2)}\,, \tag{2.37}$$

where

$$2\mu(k'',k_W)=[E_N^*(k_W)+E_N(k'')][E_A(k_W)+E_A(k'')]/(2W)\,. \tag{2.38}$$

Thus with

$$\bar{R}^2(W,k'') = [M^*(W-P'')+M] \times [2\mu(k'',k_W)]/[E_N^*(k'')+E_N(k_W)]\,, \quad (2.39)$$

and defining in the c.m.

$$\langle \vec{k}',s'|T(W)|\vec{k},s\rangle = \bar{R}(W,k')\langle W-P',s';P'|\mathfrak{M}^{++}|W-P,s,P\rangle \bar{R}(W,k) \quad (2.40)$$

and

$$\langle \vec{k}',s'|V(W)|\vec{k},s\rangle = \bar{R}(W,k')\langle W-P',s';P'|U^{++}|W-P',s';P\rangle \times \bar{R}(W,k)\,, \quad (2.41)$$

the ordinary Lippmann-Schwinger equation may be developed,

$$T(W) = V(W) + V(W)\frac{1}{k_W{}^2 - k_{\text{op}}{}^2}T(W)\,. \quad (2.42)$$

Summarizing this section, Eqs. (2.25) and (2.26) are a set of covariant integral equations for the scattering amplitude. The formal solution to (2.27) represents the covariant optical model prediction for the elastic scattering amplitude and (2.29) yields the remaining required information needed for various direct reaction inelastic processes. Equation (2.42) contains the same information as (2.27) reinterpreted in a Lippmann-Schwinger equation. One must note, however, that it is not quite the same since in intermediate integrations the restrictions (2.11) and (2.12) must be used. These restrictions should have only moderate consequences in determining the scattering wave functions since optical potentials can only support limited momentum transfers. It should be noted in addition that by using this reduction scheme a covariant transition amplitude having two nucleons off their mass shells is associated with a fully off energy shell T matrix.

To complete this section the relationship between the c.m. relative momenta and scattering angle are given in terms of the four-momenta involved in the elastic scattering. Let P, P' and $W-P$, $W-P'$ be the initial and final momenta of the nucleus and nucleon and define

$$k^\mu \equiv [(W\cdot P)(W-P)^\mu - W\cdot(W-P)P^\mu]/W^2\,, \quad (2.43)$$

$$k'^\mu \equiv [(W\cdot P')(W-P')^\mu - W\cdot(W-P')P'^\mu]/W^2\,, \quad (2.44)$$

then the magnitudes of the relative three-momenta in the c.m. are

$$|\vec{k}|^2 = -k_\mu k^\mu\,, \qquad |\vec{k}'|^2 = -k'_\mu k'^\mu\,, \quad (2.45)$$

and the c.m. scattering angle is given as

$$\cos\theta_{\text{c.m.}} = -(k_\mu k'^\mu)/(k_\mu k^\mu k'_\nu k'^\nu)^{1/2}\,. \quad (2.46)$$

III. WAVE FUNCTIONS

If the S-matrix for elastic scattering is developed in the Heisenberg picture one finds

$$\langle W'-P',s';P'|S|W-P,s;P\rangle = \delta^3(\vec{P}-\vec{P}')\delta^3(\vec{W}-\vec{W}')\delta_{ss'} - i(2\pi)^4\left(\frac{M}{E_N(\vec{W}-\vec{P}')}\right)^{1/2}\delta^4(W-W')/(2\pi)^{3/2}\lim_{W^0-P'^0\to E_N(\vec{W}-\vec{P}')}\bar{U}_{s'}(\vec{W}-\vec{P}')(\not{W}-\not{P}'-M) \times\langle P'|\psi(0)|W^{(+)}-P,s;P\rangle\,. \quad (3.1)$$

By comparing (3.1) with (2.2) the following identity holds:

$$\lim_{W^0-P'^0\to E_N(W-P')}\bar{U}_{s'}(\vec{W}-\vec{P}')(\not{W}-\not{P}'-M)\langle P'|\psi(0)|W-P,s;P\rangle = (2\pi)^{-3/2}(M/E_N(\vec{W}-\vec{P}))^{1/2}\left(\frac{1}{2E_A(\vec{P})}\right)^{1/2}\left(\frac{1}{2E_A(\vec{P}')}\right)^{1/2} \times \bar{U}_{s'}(\vec{W}-\vec{P}')\langle W-P',P'|\mathfrak{M}|W-P,P\rangle U_s(\vec{W}-\vec{P})\,. \quad (3.2)$$

Here all particles are on their respective mass shells. In order to construct a wave function which makes contact with the transition amplitudes defined in the previous section, it is necessary to make an off shell prescription. The most natural one is the identification

$$(\not{W}-\not{P}'-M)\langle P'|\psi(0)|^{(+)}W-P,s;P\rangle = (2\pi)^{-3/2}\left(\frac{M}{E_N(W-P)}\right)^{1/2}\left(\frac{1}{2E_A(P)}\right)^{1/2}\left(\frac{1}{2E_A(P')}\right)^{1/2} \times\langle W-P',P'|\mathfrak{M}|W-P,P\rangle U_s(W-P)\,. \quad (3.3)$$

Equation (3.3) is to hold for the final nucleon off shell. The definition of a wave function

$$\langle W-P',P'|\Psi_{W-P,s;P}\rangle \equiv (2\pi)^{3/2}\langle P'|\psi(0)|^{(+)}W-P,s;P\rangle\left(\frac{E_N(W-P)}{M}\right)^{1/2} \tag{3.4}$$

and Equation (2.9) allow (3.3) to be rewritten as

$$(\not{W}-\not{P}'-M)\langle W-P',P'|\Psi_{W-P,s;P}\rangle=\langle W-P',P'|\hat{\mathfrak{M}}|W-P,P\rangle U_s(\vec{W}-\vec{P}). \tag{3.5}$$

Expanding the wave function in the following manner

$$\langle W-P',P'|\Psi_{W-P,s;P}\rangle=\sum_{s'}\langle W-P',s',P'|\Psi^+_{W-P,s;P}\rangle U^*_{s'}(W-P')$$
$$-\sum_{s'}\langle W-P',s',P'|\Psi^-_{W-P,s;P}\rangle V^*_{s'}(W-P'), \tag{3.6}$$

and inserting (3.6) into (3.5) yields the following half off energy shell (one nucleon off mass shell) equations

$$(M^*(W-P')-M)\langle W-P',s';P'|\Psi^+_{W-P,s;P}\rangle=\langle W-P',s';P'|\hat{\mathfrak{M}}^{++}|W-P,s;P\rangle, \tag{3.7a}$$

$$-(M^*(W-P')+M)\langle W-P',s';P'|\Psi^-_{W-P,s;P}\rangle=\langle W-P',s';P'|\hat{\mathfrak{M}}^{-+}|W-P,s;P\rangle. \tag{3.7b}$$

In order to invert these equations the inhomogeneous free wave Ψ_0 is defined as

$$\langle W-P',s';P'|\Psi^+_{0\,W-P,s;P}\rangle=\delta^3(\vec{P}-\vec{P}')\delta_{s's}, \tag{3.8a}$$

$$\langle W-P',s;P'|\Psi^-_{0\,W-P,s;P}\rangle=0. \tag{3.8b}$$

The inversion of equations (3.7) in abbreviated form is

$$\begin{pmatrix}|\Psi^+\rangle\\|\Psi^-\rangle\end{pmatrix}=\begin{pmatrix}|\Psi_0^+\rangle\\|\Psi_0^-\rangle\end{pmatrix}+\begin{pmatrix}\hat{G}^+\hat{\mathfrak{M}}^{++} & 0\\0 & \hat{G}^-\hat{\mathfrak{M}}^{--}\end{pmatrix}\begin{pmatrix}|\Psi_0^+\rangle\\|\Psi_0^-\rangle\end{pmatrix}, \tag{3.9}$$

resulting in the solved form for the wave function. By substituting (2.25) and (2.26) and using (3.7) the integral equation obeyed by the wave function is

$$\begin{pmatrix}|\Psi^+\rangle\\|\Psi^-\rangle\end{pmatrix}=\begin{pmatrix}|\Psi_0^+\rangle\\|\psi_0^-\rangle\end{pmatrix}+\begin{pmatrix}\hat{G}^+ & 0\\0 & \hat{G}^-\end{pmatrix}\begin{pmatrix}\hat{K}^{++} & \hat{K}^{+-}\\\hat{K}^{-+} & \hat{K}^{--}\end{pmatrix}\begin{pmatrix}|\Psi^+\rangle\\|\Psi^-\rangle\end{pmatrix}. \tag{3.10}$$

If Ψ^- is eliminated above or equations (3.7a), (3.8a), and (2.27) are used, the part of the wave function involved in the elastic scattering amplitude Ψ^+ obeys the equation

$$|\Psi^+\rangle=|\Psi_0^+\rangle+\hat{G}^+U^{++}|\Psi^+\rangle, \tag{3.11}$$

where U^{++} is the effective operator defined in (2.28). In turn the nonrelativistic version of (3.11) is obtained by defining in the c.m.

$$\langle W-P',s;P'|\psi^+_{\mathrm{NR}\,W-P,s;P}\rangle$$
$$=[\bar{R}(W,k')]^{-1}\langle W-P',s';P'|\psi^+_{W-P,s;P}\rangle$$
$$\times\bar{R}(W,k_W) \tag{3.12}$$

and using Eq. (2.41) so that

$$|\Psi^+_{NR}\rangle=|\psi_0^+\rangle+\frac{1}{k_W{}^2-k_{\mathrm{op}}{}^2}V_{(W)}|\Psi^+_{NR}\rangle. \tag{3.13}$$

Finally, the covariant (except for factors of $2E_A$) wave function in (3.4) may be written as

$$\langle W-P',P'|\Psi_{W-P,s;P}\rangle=\delta^3(\vec{P}-\vec{P}')U_s(P)+\sum_{s'}\{[M^*(W-P')-M]^{-1}\langle W-P',s';P'|\hat{\mathfrak{M}}^{++}|W-P,s;P\rangle U^*_{s'}(W-P')$$
$$+[M^*(W-P')+M]^{-1}\langle W-P',s';P'|\hat{\mathfrak{M}}^{-+}|W-P,s;P\rangle V^*_{s'}(W-P')\}$$
$$\equiv\langle W-P',P'|\hat{\Omega}^+|W-P,P\rangle U_s(P). \tag{3.14}$$

It is the $\hat{\Omega}^+$ operator defined in (3.14) which may be inserted into reduced Feynman graphs such as the one shown in Fig. 3 and is equivalent to the use of a distorted wave. The operator $\hat{\Omega}^+$ may be decomposed

$$\hat{\Omega}^+=\hat{\Omega}^{++}+\hat{\Omega}^{-+}, \tag{3.15}$$

where

$$\hat{\Omega}^{\pm+}=\Lambda^{*\pm}(W-P')\hat{\Omega}^+\Lambda^+(W-P). \tag{3.16}$$

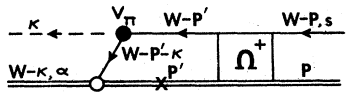

FIG. 3. The DWBA in graphical form for proton induced pion production in the single nucleon model.

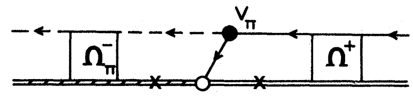

FIG. 4. The same as 3 without the final state distortion and with momentum labels appropriate to Eq. (3.17).

It is $\hat{\Omega}^{++}$ which is the Moller operator corresponding to the nonrelativistic wave function in (3.12). Consider Fig. 4 which is part of DWBA for the (p,π) reaction in the model where pion emission is from one nucleon. The distortion of the outgoing pion wave is omitted here. It is certainly of great importance but has been discussed in other works and the methods of handling this distortion have been developed.[9,10] If the S matrix for this graph is evaluated by placing the intermediate nucleus on its mass shell it yields

$$S=-2\pi i\delta^4(W-W')\,\frac{1}{[2\omega(\vec{k})]^{1/2}}\left(\frac{M}{E_N(W-P)}\right)^{1/2}$$
$$\times\int d^3P'\langle W-\kappa,\alpha|\bar{\Psi}(0)|P'\rangle\hat{\phi}^+_\pi\cdot\vec{V}_\pi(W-P'-\kappa|\kappa|W-P')\langle W-P',P'|\hat{\Omega}^+|W-P,P\rangle U_s(\vec{W}-\vec{P})\,. \qquad (3.17)$$

In (3.17) $\langle W-\kappa,\alpha|\bar{\Psi}(0)|P\rangle$ is the wave function of the nucleon absorbed by the nucleus whose internal quantum state is labeled α, $\vec{V}_\pi$ is the pion nucleon vertex function. By using (3.15) and the approximation (2.30) equation (3.17) may be rewritten

$$S=-2\pi i\delta^4(W-W')\,\frac{1}{(2W)^{1/2}(\vec{k})}\left(\frac{M}{E_N(\vec{W}-\vec{P})}\right)^{1/2}$$
$$\times\int d^3P'd^3P''\langle W-\kappa,\alpha|\bar{\Psi}(0)|\vec{P}'\rangle\hat{\phi}^{*+}_\pi\cdot V^{\text{eff}}_\pi(W-P'-\kappa|\kappa|W-P'')\langle W-P'',P''|\hat{\Omega}^{++}|W-P,P\rangle U_s(W-P)\,, \qquad (3.18)$$

where

$$\vec{V}^{\text{eff}}_\pi(W-P'-\kappa|\kappa|W-P'')\cong\vec{V}_\pi(W-P'-\kappa|\kappa|W-P')\delta^3(\vec{P}'-\vec{P}'')$$
$$+\int d\vec{P}'''V_\pi(W-P'-\kappa|\kappa|W-P''')\hat{G}^-(W-P''')\langle W-P''',P'''|\hat{K}^{-+}|W-P'',P''\rangle\,. \qquad (3.19)$$

Thus a spin renormalization to the pion nucleon vertex results if it is desired to use nonrelativistic nucleon nucleus distorted waves as in (3.18). Because of the off diagonal spinorial nature of the pion nucleon vertex, if γ_5 theory is used, the effects of the second term in (3.19) may be important. The size of these effects depends crucially upon the model dependent evaluation of the operator $\hat{K}^{-+}$. This question is under current investigation.

IV. FURTHER OBSERVATIONS

The facts that the amplitudes $K^{++}, K^{-+}, \mathfrak{M}^{++}$, etc. are covariant and that the scattering of a spin $\frac{1}{2}$ partical from a spin zero partical is being described allows one to write, for example,

$$\langle W-P',s';P'|\hat{K}^{++}|W-P,s;P\rangle=R(\vec{P}')R(\vec{P})\bar{U}^*_s(W-P')\Big[k^{++}(W^2,(P-P')^2,M^{*2}(W-P'),M^{*2}(W-P))$$
$$+h^{++}(W^2,(P-P')^2,M^{*2}(W-P'),$$
$$\times M^{*2}(W-P))\frac{\not{P}'+\not{P}}{2M_A}\Big]U^*_s(W-P)\,. \qquad (4.1)$$

Here k^{++} and h^{++} are invariant functions of their arguments. The advantage of having invariant quantities to calculate is that kinematic ambiguities are dispensed with and this has proven of great utility in the pion nucleus scattering problem.[9,10]

On shell in the c.m. the elastic scattering amplitude,

$$\langle W-P',s';P'|\mathfrak{M}^{++}|W-P,s;P\rangle=\frac{W}{2ME_A}\langle\vec{k}'_W,s'|T(W)|\vec{k}_W,s\rangle\,. \qquad (4.2)$$

If T is expanded in partial waves,

$$\langle \vec{k}'_W, s'|T(W)|\vec{k}_W, s\rangle = \sum_{\substack{J\,m \\ \omega=\pm 1}} X_s^\dagger \cdot Y^{J\,m}_{l=J+\omega/2}(\hat{k}'_W) Y^{J\,m\dagger}_{l=J+\omega/2}(\hat{k}_W) \cdot X_s \langle k_W|T^{J,\omega}_{(W)}|k_W\rangle \tag{4.3}$$

then

$$\langle k_W|T^{J,\omega}_{(W)}|k_W\rangle = -\frac{2}{\pi k_W} \frac{[\eta^{J,\omega}(W)e^{2i\delta^{J,\omega}_{(W)}} - 1]}{2i}. \tag{4.4}$$

If a separable model is used to fit the phase shifts and inelasticity parameters δ and η similar to that constructed by Londergan, McVoy, and Moniz[21] for the pion nucleon problem, then the half off shell T matrix may be generated and with the half off shell version of (2.40) $\mathfrak{M}^{++}$ may be extended to the region where one nucleon is off its mass shell. Alternatively $\mathfrak{M}^{++}$ may be developed using Glauber theory for the higher beam momenta. In estimating the corrections implied by the use of (3.19) in (3.18) the remaining uncertainty is in the calculation of $\hat{K}^{-+}$.

V. SUMMARY

A covariant reduction scheme appropriate for the nucleon-nucleus problem at intermediate energies has been outlined. The central result of this work is that for the (p,π) reaction when calculated in the single nucleon emission model, the pion nucleon vertex is to be corrected by a factor dependent upon the lower component of the nucleon-nucleus scattering wave function. In addition ambiguities associated with using a Dirac equation ignoring nuclear recoil are clarified.

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