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Exchange effects in nucleus-nucleus interaction

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The effects of antisymmetrization, represented by various nucleon-exchange terms in a resonating-group formulation, are studied in the case of nucleus-nucleus scattering. By examining the features of the effective local potentials which are constructed to yield the same Born scattering amplitudes as these exchange terms, it is found that the one-exchange and the core-exchange terms are the most important. In addition, this study shows that in scattering systems the one-exchange term has generally a substantial influence over a wide range of energies. On the other hand, the core-exchange term is important only when the nucleon-number difference of the interacting nuclei is rather small. Based on the results of this investigation, it can also be concluded that if a local-potential model is employed to phenomenologically analyze experimental scattering results, then the effective potential in this model may need to have an odd-even l-dependent character.

[NUCLEAR REACTIONS Effects of antisymmetrization; energy and spatial dependence of effective internuclear potentials.]

I. INTRODUCTION

One of the important findings from resonating-group investigations[1-3] is that, especially for scattering systems involving very light nuclei, the phase shift as a function of the relative orbital angular momentum shows a distinct zigzag behavior. This indicates that if one considers the nuclei as structureless and represents the interaction between them by an effective potential $\tilde{V}$, then because of the requirement to use a totally antisymmetric many-nucleon wave function in the microscopic formulation, this potential must contain an orbital-angular-momentum- or l-dependent component.[4] In fact, detailed examinations[2,5] of resonating-group results further show that, at relatively high energies well above the Coulomb barrier, this l-dependent part has a rather simple structure, i.e., essentially only odd-even or parity dependent. In other words, as a reasonable approximation at these energies, one can consider $\tilde{V}(\vec{R}')$ to have the simple form

$$\tilde{V}(\vec{R}') = V_D(R') + V_a(R') + V_b(R')P^{R'} , \qquad (1)$$

where $V_D(R')$ is an internuclear direct potential, and $V_a(R')$ and $V_b(R')P^{R'}$, with $P^{R'}$ as a Majorana space-exchange operator, are energy-dependent "exchange" potentials introduced to represent the main effects of antisymmetrization.

At relatively high energies, the presence of a Majorana term in the effective potential results in a cross-section rise at backward angles. In light-ion scattering, there have indeed been many experimental observations of such back-angle exchange rise in the cross section. For example, in ^{3}He + α scattering[6] at an incident energy of 35 MeV/nucleon and in α + ^{6}Li scattering[7] at an incident energy of 41.5 MeV/nucleon, it was experimentally found that the angular-distribution curves have well-formed V shapes, with a rapidly decreasing behavior in the forward angular region and a rapidly increasing behavior in the backward angular region. In heavy-ion scattering such as ^{16}O + ^{28}Si and so on,[8-13] back-angle cross-section data are also beginning to accumulate. These data were, however, taken at rather low incident energies of only a few MeV per nucleon. At these energies, resonance effects due to quasimolecular structures are quite important, and a clear discernment of the odd-even effect is somewhat more difficult.

In this investigation, we discuss some general features of exchange effects, arising from antisymmetrization, in the case where a nucleus A containing N_A nucleons is scattered by another nucleus B containing N_B ($N_B < N_A$) nucleons. What we shall do is to study the properties of the effective potentials resulting from various kernel terms in the resonating-group formulation. As will be seen, exchange effects may indeed be generally important and an omission of these effects may lead to substantial difficulties especially when the nucleon-number difference ($N_A - N_B$) is small.

In the next section, we present a brief description of the resonating-group formulation and discuss the features of the effective exchange potentials derived in the Born approximation. The spatial and energy dependence of the important one-exchange and core-exchange potentials are then examined in Sec. III. Finally, in Sec. IV, we summarize the results, and discuss the situation under which exchange effects are particularly significant and the introduction of a Majorana component becomes very important if a local-potential-model analysis of the experimental data is to be successfully made.

II. EFFECTIVE INTERNUCLEAR POTENTIAL

A. Resonating-group formulation of nucleus-nucleus scattering

In the simplest, one-channel resonating-group formulation for $A+B$ scattering,[3] the trial wave function ψ is written as[14]

$$\psi=\mathcal{A}[\phi_A\,\phi_B F(\vec{R}_A-\vec{R}_B)Z(\vec{R}_{\mathrm{c.m.}})], \tag{2}$$

where $\mathcal{A}$ is an antisymmetrization operator, $\vec{R}_A$ and $\vec{R}_B$ are, respectively, center-of-mass coordinates of clusters A and B, and $Z(\vec{R}_{\mathrm{c.m.}})$ is any normalizable function describing the total c.m. motion. The functions ϕ_A and ϕ_B represent the internal structures of the clusters; they are chosen to be translationally invariant products of single-particle functions of the lowest configuration in harmonic-oscillator wells of width parameters α_A and α_B, respectively. The function $F(\vec{R})$ describes the relative motion between the clusters; it is obtained by solving the projection equation

$$\langle\delta\psi|H-E_T|\psi\rangle=0, \tag{3}$$

where E_T is the total energy of the system composed of cluster internal energies E_A and E_B, and the relative energy E in the c.m. system. The Hamiltonian H is a Galilean-invariant operator, given by

$$H=\sum_{i=1}^{N}T_i+\sum_{i<j=1}^{N}V_{ij}-T_{\mathrm{c.m.}}, \tag{4}$$

where $N=N_A+N_B$ is the total number of nucleons, $T_{\mathrm{c.m.}}$ is the kinetic-energy operator of the total center-of-mass, and V_{ij} is a nucleon-nucleon potential chosen to fit the two-nucleon scattering data especially in the low-energy region.

By using parameter representations for ψ and $\delta\psi$, i.e.,

$$\psi=\int\mathcal{A}[\phi_A\phi_B\,\delta(\vec{R}-\vec{R}'')\,Z(\vec{R}_{\mathrm{c.m.}})]\,F(\vec{R}'')\,d\vec{R}'', \tag{5}$$

$$\delta\psi=\int\mathcal{A}[\phi_A\phi_B\delta(\vec{R}-\vec{R}')\,Z(\vec{R}_{\mathrm{c.m.}})]\delta F(\vec{R}')\,d\vec{R}', \tag{6}$$

where $\vec{R}'$ and $\vec{R}''$ are parameter coordinates on which the antisymmetrization operator $\mathcal{A}$ does not act, we obtain from the projection equation (3) the following equation satisfied by the relative-motion function $F(\vec{R})$:

$$\int[\mathcal{H}(\vec{R}',\vec{R}'')-E_T\mathfrak{N}(\vec{R}',\vec{R}'')]F(\vec{R}'')\,d\vec{R}''=0, \tag{7}$$

where

$$\mathcal{H}(\vec{R}',\vec{R}'')=\langle\phi_A\phi_B\delta(\vec{R}-\vec{R}')Z\,|H|\,\mathcal{A}[\phi_A\phi_B\delta(\vec{R}-\vec{R}'')Z]\rangle, \tag{8}$$

$$\mathfrak{N}(\vec{R}',\vec{R}'')=\langle\phi_A\phi_B\delta(\vec{R}-\vec{R}')Z\,|\,\mathcal{A}[\phi_A\phi_B\delta(\vec{R}-\vec{R}'')Z]\rangle, \tag{9}$$

with the Dirac-bracket notation denoting an integration over all nucleon spatial coordinates and a summation over all spin and isospin coordinates. If we now write

$$\mathcal{A}=\mathcal{A}'\mathcal{A}_A\mathcal{A}_B, \tag{10}$$

where $\mathcal{A}_A$ and $\mathcal{A}_B$ are, respectively, antisymmetrization operators for the nucleons in clusters A and B, and $\mathcal{A}'$ is an antisymmetrization operator which interchanges nucleons in different clusters, then Eq. (9) can also be written as

$$\mathfrak{N}(\vec{R}',\vec{R}'')=\langle\phi_A\phi_B\delta(\vec{R}-\vec{R}')Z\,|\,\mathcal{A}'[\hat{\phi}_A\hat{\phi}_B\delta(\vec{R}-\vec{R}'')Z]\rangle, \tag{11}$$

with

$$\hat{\phi}_A=\mathcal{A}_A\phi_A,$$
$$\hat{\phi}_B=\mathcal{A}_B\phi_B. \tag{12}$$

By defining further

$$\mathcal{A}'=1+\mathcal{A}'', \tag{13}$$

We can separate $\mathfrak{N}(\vec{R}',\vec{R}'')$ into two parts, i.e.,

$$\mathfrak{N}(\vec{R}',\vec{R}'')=\mathfrak{N}_D(\vec{R}',\vec{R}'')+\mathfrak{N}_E(\vec{R}',\vec{R}''), \tag{14}$$

where the direct part $\mathfrak{N}_D$ is

$$\mathfrak{N}_D(\vec{R}',\vec{R}'')=\langle\phi_A\phi_B\delta(\vec{R}-\vec{R}')Z\,|\,\hat{\phi}_A\hat{\phi}_B\delta(\vec{R}-\vec{R}'')Z\rangle, \tag{15}$$

and the exchange part $\mathfrak{N}_E$ is

$$\mathfrak{N}_E(\vec{R}',\vec{R}'')=\langle\phi_A\phi_B\delta(\vec{R}-\vec{R}')Z\,|\,\mathcal{A}''[\hat{\phi}_A\hat{\phi}_B\delta(\vec{R}-\vec{R}'')Z]\rangle. \tag{16}$$

Similarly, one can carry out an analogous procedure for $\mathcal{H}(\vec{R}',\vec{R}'')$ and obtain

$$\mathcal{H}(\vec{R}',\vec{R}'')=\mathcal{H}_D(\vec{R}',\vec{R}'')+\mathcal{H}_E(\vec{R}',\vec{R}''), \tag{17}$$

with

$$\mathcal{H}_D(\vec{R}',\vec{R}'')=\langle\phi_A\phi_B\delta(\vec{R}-\vec{R}')Z\,|H|\,\hat{\phi}_A\hat{\phi}_B\delta(\vec{R}-\vec{R}'')Z\rangle \tag{18}$$

and

$$\mathcal{H}_E(\vec{R}', \vec{R}'') = \langle \phi_A \phi_B \delta(\vec{R} - \vec{R}')Z \,|H|\, \mathcal{A}''[\hat{\phi}_A \hat{\phi}_B \delta(\vec{R} - \vec{R}'')Z]\rangle . \quad (19)$$

Next, we substitute Eqs. (14) and (17) into Eq. (7). After adopting appropriate normalization conditions for $\hat{\phi}_A$, $\hat{\phi}_B$, and $Z(\vec{R}_{c.m.})$, and carrying out some straightforward manipulation (for details, see Ref. 3), we obtain finally the following integrodifferential equation for $F(\vec{R}')$:

$$\left[-\frac{\hbar^2}{2\mu}\nabla_{\vec{R}'}{}^2 + V_D(\vec{R}') - E\right]F(\vec{R}') + \int K(\vec{R}', \vec{R}'')F(\vec{R}'')\,d\vec{R}'' = 0 , \quad (20)$$

where μ is the reduced mass, V_D is a direct potential, and $K(\vec{R}', \vec{R}'')$ is an energy-dependent kernel function given by

$$K(\vec{R}', \vec{R}'') = \mathcal{H}_E(\vec{R}', \vec{R}'') - E_T \mathfrak{N}_E(\vec{R}', \vec{R}'') . \quad (21)$$

From Eq. (20), one sees that, if the two clusters are considered as structureless, then the effective interaction between them must be both nonlocal and energy dependent.

From the above discussion, one also sees that if the antisymmetrization between nucleons in different clusters is neglected or, in other words, if the antisymmetrization operator $\mathcal{A}'$ is set as unity, then the kernel functions $\mathcal{H}_E$ and $\mathfrak{N}_E$ will vanish. In this crude approximation, the effective intercluster potential will, therefore, just be the direct potential V_D which is a simple l-independent potential if a purely central nucleon-nucleon force, such as the one used in Ref. 5, which contains specifically no Majorana component, is employed.

The effects of antisymmetrization are, therefore, contained in the kernel functions $\mathcal{H}_E$ and $\mathfrak{N}_E$. In the following, we shall perform a Born-approximation study in order to learn some of the main consequences which are associated with these kernel functions.

B. Born-approximation study of the nucleus-nucleus interaction

The exchange-normalization kernel $\mathfrak{N}_E$ of Eq. (16) has the form

$$\mathfrak{N}_E(\vec{R}', \vec{R}'') = \sum_x \mathfrak{N}_E^x(\vec{R}', \vec{R}'') , \quad (22)$$

where

$$\mathfrak{N}_E^x(\vec{R}', \vec{R}'') = P_x \exp(-a_x \vec{R}'^2 - c_x \vec{R}' \cdot \vec{R}'' - a_x \vec{R}''^2) , \quad (23)$$

and x ($x \geq 1$) is the number of nucleons interchanged between the clusters and P_x is a polynomial in $\vec{R}'^2$, $\vec{R}' \cdot \vec{R}''$, and $\vec{R}''^2$. By using the complex-generator-coordinate technique developed recently,[2,3,15] one can derive general expressions for the coefficients a_x and c_x. These expressions are

$$a_x = \frac{\mu_0^2}{4x}\,\frac{x^2[(\alpha_A - \alpha_B)^2 + (2/\mu_0)(N_A + N_B)\alpha_A\alpha_B] + N_A N_B(1 - x/\mu_0)(\alpha_A + \alpha_B)^2}{N_A N_B(\alpha_A + \alpha_B) - x(N_A\alpha_A + N_B\alpha_B)} \quad (24)$$

and

$$c_x = -\frac{\mu_0^2}{2x}\,\frac{x^2(\alpha_A - \alpha_B)^2 + N_A N_B(1 - x/\mu_0)(\alpha_A + \alpha_B)^2}{N_A N_B(\alpha_A + \alpha_B) - x(N_A\alpha_A + N_B\alpha_B)} , \quad (25)$$

where μ_0 denotes the reduced nucleon number, given by

$$\mu_0 = \frac{N_A N_B}{N_A + N_B} . \quad (26)$$

It should be remarked that the kernel function $\mathcal{H}_E$ of Eq. (19) contains the same exponential factors in the limit case where the nucleon-nucleon potential[16] has a range approaching infinity [i.e., $\kappa \to 0$ in Eq. (16) of Ref. 5]. Since nucleon-exchange processes occur predominantly when the colliding nuclei are in close proximity, one may plausibly expect that the range of the nucleon-nucleon potential does not affect antisymmetrization effects to a large extent and the structures of $\mathcal{H}_E$ and $\mathfrak{N}_E$ may be rather similar. Therefore, we feel that a study of the properties of the kernel $\mathfrak{N}_E$ alone may yield useful information concerning the effects of antisymmetrization.[17] For a further understanding, one must of course examine in the future the general structure of $\mathcal{H}_E$. Because of the complicated nature of this latter kernel, this will be a difficult study but should certainly be worth carrying out.

Our next step is to derive effective local energy-dependent exchange potentials $\tilde{V}_x(\vec{R}')$ which yield, in the Born approximation, the same scattering amplitudes as the exchange-normalization kernel terms $\mathfrak{N}_E^x(\vec{R}', \vec{R}'')$. For this purpose, we first solve the equation

$$x_0^2(\alpha_A - \alpha_B)^2 + N_A N_B\left(1 - \frac{x_0}{\mu_0}\right)(\alpha_A + \alpha_B)^2 = 0\,, \tag{27}$$

to obtain a positive root for x_0 less than N_B. This can be easily done; in fact, the resultant value of x_0 turns out to be quite close to μ_0 (in the special case where $\alpha_A = \alpha_B$, x_0 is equal to μ_0), which is a consequence of the fact that the values of α_K and N_K ($K = A$ or B) are correlated. Then, by employing a procedure described previously,[18] we can find the following expressions for these effective potentials[19]:

(i) $x < x_0$: In this case, c_x has a negative value and the effective potentials are Wigner-type potentials which yield large Born scattering amplitudes only at forward angles. These potentials are

$$\tilde{V}_x(\vec{R}') = \tilde{P}_x \exp[-(k/k_x)^2]\exp[-(R'/R_x)^2]\,, \tag{28}$$

where k is the wave number given by $(2\mu E)^{1/2}/\hbar$ and

$$k_x = \left[\frac{\mu_0^2}{x}\left(\frac{N_A - x}{N_A}\,\alpha_A + \frac{N_B - x}{N_B}\,\alpha_B\right)\right]^{1/2}, \tag{29}$$

$$R_x = \left\{\frac{x^2(\alpha_A - \alpha_B)^2 + [N_A N_B - x(N_A + N_B)](\alpha_A + \alpha_B)^2}{x\alpha_A\alpha_B[N_A N_B(\alpha_A + \alpha_B) - x(N_A\alpha_B + N_B\alpha_A)]}\right\}^{1/2}. \tag{30}$$

(ii) $x > x_0$: In this case, c_x has a positive value and the effective potentials are Majorana-type potentials which yield large Born scattering amplitudes only at backward angles. These potentials are

$$\tilde{V}_x(\vec{R}') = \tilde{P}_x \exp[-(k/k_x)^2]\exp[-(R'/R_x)^2]P^{R'}\,, \tag{31}$$

where

$$k_x = \left[\frac{x\alpha_A\alpha_B}{(\alpha_A + \alpha_B) - x(\alpha_A/N_B + \alpha_B/N_A)}\right]^{1/2}, \tag{32}$$

$$R_x = \left\{\frac{[x(N_A + N_B) - N_A N_B](\alpha_A + \alpha_B)^2 - x^2(\alpha_A - \alpha_B)^2}{x\alpha_A\alpha_B[N_A N_B(\alpha_A + \alpha_B) - x(N_A\alpha_B + N_B\alpha_A)]}\right\}^{1/2}. \tag{33}$$

Also, in Eqs. (28) and (31), the functions $\tilde{P}_x$ are polynomials in k^2 and R'^2. Finally, for comparison, we write down here the expression for the direct nuclear potential which, in the limit of a zero-range nucleon-nucleon potential,[20] is

$$V_D(\vec{R}') = \tilde{P}_D \exp[-(R'/R_D)^2]\,, \tag{34}$$

where

$$R_D = \left[\frac{1}{\alpha_A}\left(1 - \frac{1}{N_A}\right) + \frac{1}{\alpha_B}\left(1 - \frac{1}{N_B}\right)\right]^{1/2}, \tag{35}$$

and $\tilde{P}_D$ is a polynomial in R'^2.

By examining the expressions for the characteristic wave number k_x and the characteristic range R_x, one can easily see that (i) for $x < x_0$, k_x and R_x decrease monotonically with x and have largest values when $x = 1$, and (ii) for $x > x_0$, k_x and R_x increase monotonically with x and have largest values when $x = N_B$.[21] This indicates, therefore, that among all exchange terms, the one-exchange term ($x = 1$) with characteristic values k_1 and R_1, and the core-exchange term ($x = N_B$) with characteristic values k_c and R_c are the most important. In fact, this finding has been amply verified by detailed resonating-group calculations. For example, in ^{3}He + α scattering[22] where $x_0 \approx \mu_0 = \frac{12}{7}$, the two-exchange term was found to be rather unimportant, and in ^{16}O + ^{16}O scattering[23] where $x_0 = \mu_0 = 8$, the one-, two-, and three-exchange effective potentials were found to have comparable depth but progressively shorter range.

III. ENERGY AND SPATIAL DEPENDENCE OF EXCHANGE POTENTIALS

In this section, we discuss the properties of the one-exchange effective potential $\tilde{V}_1(\vec{R}')$ and the core-exchange effective potential $\tilde{V}_c(\vec{R}')$. For clarity in discussion, we shall make the assumption

$$\alpha_A = \alpha_B = \alpha\,. \tag{36}$$

This does enable us to simplify our presentation but does not affect the conclusion in any significant way.

For the purpose of our discussion, we list below the expressions for the various relevant characteristic wave numbers and characteristic ranges[24]:

$$k_1 = [\mu_0(2\mu_0 - 1)\alpha]^{1/2}, \tag{37}$$

$$R_1 = \left[\frac{4(\mu_0 - 1)}{2\mu_0 - 1}\,\frac{1}{\alpha}\right]^{1/2}, \tag{38}$$

$$k_c = \left(\frac{N_A N_B}{N_A - N_B}\,\alpha\right)^{1/2}, \tag{39}$$

$$R_c = \left(\frac{4}{N_A - N_B}\,\frac{1}{\alpha}\right)^{1/2}, \tag{40}$$

$$R_D = \left(\frac{2\mu_0 - 1}{\mu_0}\,\frac{1}{\alpha}\right)^{1/2}. \tag{41}$$

In addition, it is important to note that in the polynomial factors $\tilde{P}_1$ and $\tilde{P}_D$, the highest powers of R'^2 are both given by $n_A + n_B$, where n_A and n_B are, respectively, the principal quantum numbers, in oscillator wells for clusters A and B, of the last shells to be filled. For example, in $\alpha + {}^{16}O$ scattering, the values of n_A and n_B are equal to 1 and 0, respectively. As for the polynomial factor $\tilde{P}_c$ occurring in $\tilde{V}_c$, its highest power of R'^2 is somewhat more difficult to determine; however, it has recently been derived in a paper by Baye, Deenan, and Salmon,[25] where it was shown that for those interesting cases in which N_A and N_B are nearly equal, this highest power is again approximately given by $n_A + n_B$.[26] Therefore, since the polynomial factors in $\tilde{V}_1$, $\tilde{V}_c$, and V_D have similar values for their highest powers in R'^2, it is appropriate to simply examine the exponential factors in order to decide the situations under which the effective potentials $\tilde{V}_1$ and $\tilde{V}_c$ make important contributions.

Let us first study the spatial dependence of $\tilde{V}_1$ and $\tilde{V}_c$. By comparing the values of R_1 and R_c with the value of R_D, we can make the following general remarks:

(i) The ratio R_1/R_D is given by

$$\frac{R_1}{R_D} = \left[1 - \frac{1}{(2\mu_0 - 1)^2}\right]^{1/2}, \tag{42}$$

which is smaller than but close to 1, indicating that the one-exchange term may be generally important. This is consistent with the results obtained in a number of previous investigations[27] where the purpose was to see if the phase-shift values calculated with the resonating-group method can be reasonably reproduced by a potential model in which one solves, instead of the integro-differential equation (20), a simpler equation

$$\left[-\frac{\hbar^2}{2\mu}\nabla_{\vec{R}'}{}^2 + \tilde{V}(\vec{R}') - E\right]F(\vec{R}') = 0, \tag{43}$$

with $\tilde{V}(\vec{R}')$ given by Eq. (1). Indeed, these investigations have invariably shown that the V_a term in Eq. (1) must have an appreciable magnitude in comparing with the V_D term. In addition, the fact that R_1 is less than R_D (Ref. 20) is also in agreement with an empirical finding,[28] obtained by potential-model analyses of p, ${}^3He$, and α scattering by ${}^{16}O$, that the range of V_a tends to be somewhat shorter than that of the direct nuclear potential V_D.

(ii) The characteristic range R_c decreases with increasing value of the nucleon-number difference,

$$\delta = N_A - N_B, \tag{44}$$

between the clusters A and B. This means that one expects the core-exchange effect to become less important as δ increases. Indeed, we have reached a similar conclusion based on the results of many resonating-group calculations.[2,27] There it was found that the degree of odd-even l-dependence, exhibited by the calculated phase shift, turns out to be quite strong in scattering systems involving two s-shell nuclei where δ is small, and weak in systems such as $\alpha + {}^{16}O$ and $n + {}^{40}Ca$ where δ takes on much larger values. In addition, of course, the finding that core-exchange effects are important in ${}^{12}C + {}^{13}C$ and ${}^{12}C + {}^{16}O$ scattering but not in $\alpha + {}^{40}Ca$ scattering[31,32] supports the assertion reached by our present analysis.

Next, we examine the energy dependence of exchange effects by studying the expressions for k_1 and k_c given by Eqs. (37) and (39). For this we make a reasonable, though arbitrary, assumption that the effective potentials $\tilde{V}_1$ and $\tilde{V}_c$ will become rather unimportant when their energy-dependent exponential factors [see Eqs. (28) and (31)] acquire a value less than, say, e^{-4}. Adopting this criterion, one can then easily find that the one-exchange term has a significant influence when $E/\mu_0 < \tilde{E}_1$,[33] where

$$\tilde{E}_1 = \frac{\hbar^2}{2M}\,\frac{4(2\mu_0 - 1)}{\mu_0}\,\alpha, \tag{45}$$

and M is the nucleon mass, and the core-exchange term has a significant influence when $E/\mu_0 < \tilde{E}_c$, where

$$\tilde{E}_c = \frac{\hbar^2}{2M}\,\frac{4(2-\xi)^2}{1-\xi}\,\frac{1}{\delta}\,\alpha, \tag{46}$$

with $\xi = \delta/N_A$ $(0 < \xi < 1)$. In Table I, the values of $\tilde{E}_1$ and $\tilde{E}_c$ calculated for some representative systems are listed. Here it is seen that the one-exchange term is important over a wide energy range for all these systems. On the other hand, because of the factor $1/\delta$ occurring in Eq. (46), the core-exchange term has a slow energy de-

TABLE I. Values of $\tilde{E}_1$ and $\tilde{E}_c$ in various systems.

System	δ	α (fm^{-2})	$\tilde{E}_1$ (MeV/nucleon)	$\tilde{E}_c$ (MeV/nucleon)
$n+\alpha$	3	0.52		90
${}^3H+\alpha$	1	0.45	53	152
$\alpha+{}^{16}O$	12	0.36	50	16
${}^{16}O+{}^{17}O$	1	0.32	50	106
${}^{16}O+{}^{20}Ne$	4	0.30	47	25
${}^{16}O+{}^{40}Ca$	24	0.27	43	5

pendence only when δ is relatively small.

Besides the δ dependence discussed above, $\tilde{E}_c$ has also a specific mass dependence. From Eq. (46), one notes that $\tilde{E}_c$ depends on N_A not only implicitly through the width parameter α,[24] but also explicitly through the quantity ξ. It should be noted, however, that comparing with its δ dependence, the mass dependence of $\tilde{E}_c$ is relatively weak. For example, in $^3H+\alpha$ and $^{16}O+^{17}O$ scattering where the values of δ are the same, it is seen from Table I that the corresponding values of $\tilde{E}_c$ do differ, but only by a rather moderate amount.

It should be remarked that even in the case where the core-exchange potential has a small magnitude, one may still find noticeable effects on the differential scattering cross section in situations where partial-wave scattering amplitudes strongly cancel one another. Generally, these situations occur at large backward angles when the scattering energies are relatively high. For example, in $p+\alpha$ scattering at a rather high energy of 125 MeV, there is the experimental observation of a rise, though small, in the scattering cross section at angles larger than about 140°.[34,35]

Finally, we must emphasize that the considerations given here are made in the Born approximation and based mainly on the features of the exchange-normalization kernel. Therefore, the results obtained have only semiquantitative significance at relatively high energies. However, we do believe that especially the dependence of the core-exchange effect on the factor δ is very likely a realistic prediction and this particular finding should be very useful when one attempts to construct potential models for the analyses of experimental scattering results.

IV. DISCUSSION AND CONCLUSION

In this investigation, the effect of the Pauli principle on the scattering of composite nuclei is examined by considering the structure of the exchange-normalization kernel occurring in the resonating-group formulation. Essentially, the procedure we use for this examination is to construct effective local exchange potentials which yield, in the Born approximation, the same scattering amplitudes as the various nucleon-exchange terms in this kernel function. Then, by studying the features of these effective potentials, we obtain the interesting finding that, at least at relatively high energies, the one-exchange and the core-exchange terms are the most important ones among all exchange terms.

In addition, we find that in all scattering systems the one-exchange term has an important influence over a wide range of energies. The core-exchange term, on the other hand, is generally important only when the nucleon-number difference of the interacting nuclei is rather small. This means that in a scattering calculation where the nuclei involved have a large difference in mass and where there is no strong clusterization in the heavier nucleus, such as $\alpha+^{208}Pb$ scattering, it might be a reasonable approximation to consider only the one-exchange term, thus substantially reducing the computational difficulty associated with a many-nucleon resonating-group investigation.

In the Born approximation, the effective local potentials corresponding to one-exchange and core-exchange terms are Wigner-type and Majorana-type potentials, respectively. This indicates that, if a local-potential model is constructed to analyze experimental scattering results, then the effective potential in this model (i.e., the real central part of the optical potential) must, in general, contain a Majorana exchange component [see Eq. (1)], or in other words, the effective potential must have an odd-even l-dependent (i.e., parity dependent) character. In this respect, it is interesting to note that such odd-even potential models have already been successfully used to fit experimental data of $p+^3He$, $p+\alpha$, $^3He+\alpha$, and $\alpha+^6Li$ scattering.[6,36-38] Also, in heavier systems such as $^{12}C+^{13}C$ and $^{12}C+^{16}O$,[13,29,39] the application of such models has similarly yielded satisfactory agreement with measured results.

From this investigation, it also becomes clear why the conventional optical model, in which the effective potential contains no Majorana component, works quite well for light-ion scattering by medium and heavy-weight nuclei. The main reason for this is evidently that, in these cases, the nucleon-number difference of the interacting nuclei is sufficiently large such that, except for fitting scattering data at large backward angles, the core-exchange term has little influence and, therefore, can be reasonably omitted from the calculation.

For a better understanding of the core-exchange effect, it will be useful to conduct both theoretical and experimental investigations at energies of about 20 to 50 MeV/nucleon for scattering systems in which the nuclei involved have similar mass. In addition, it is important that the experimental measurement should cover as large an angular region as feasible. Currently, there are experimental data of this type in light-ion scattering, such as $^3He+\alpha$ or $\alpha+^6Li$ scattering[6,7] where the angular-distribution curve has a distinct V shape,[40] indicating a clear separation of the core-exchange effect from other scattering mechanisms. In heavy-ion scattering, the most interesting problem would obviously be a systematic study of the scattering of ^{16}O ion by ^{17}O, ^{18}O, and other heavier targets. At present, we know of no experimental data satisfying the above-mentioned criteria for these systems. We hope, however, that with the availability of new, higher-energy accelerator facilities, these data may yet be forthcoming.

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[20]With a finite-range nucleon-nucleon potential, the direct nuclear potential V_D will have a range somewhat longer than that given by Eq. (35).

[21]The largest possible value of x will depend, of course, on the nucleon configurations of clusters A and B as described by the internal functions $\hat{\phi}_A$ and $\hat{\phi}_B$. In most cases, however, this value will turn out to be equal to N_B. For the special case where N_A is equal to N_B, such as $^{40}Ar+^{40}Ca$ scattering, this largest value will be somewhat smaller than N_B. Also, if the two clusters happen to be identical (e.g., $^{16}O+^{16}O$ scattering), then the largest value of x will effectively be given by either $N_B/2$ or $(N_B-1)/2$ depending upon whether N_B is even or odd.

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