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Suppression of the isospin-1/2, three-body photodisintegration of ^{3}He

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The suppression of the isospin-1/2, three-body photodisintegration of ^{3}He is shown to result from two facts: (1) For the $S = 1/2$, $I = 1/2$ states of three nucleons, the spin-isospin symmetries lead to integral equations dominated by a single effective interaction V^+, which is the average of the NN s-wave spin-singlet and spin-triplet interactions and (2) This V^+ supports a bound state due to the nature of the NN effective-range parameters. In the approximation that the (1/2,1/2) states are generated from V^+ alone, it is clear that a fraction of the isospin-1/2 bremsstrahlung-weighted sum rule must come from two-body photodisintegration with a corresponding reduction of the three-body channel contribution, since the total isospin-1/2 sum rule is fixed. The substantial reduction of the actual physical isospin-1/2, three-body photodisintegration is directly related to the strength of V^+.

[NUCLEAR REACTIONS Photodisintegration of ^{3}He; exact three-body calculation; isospin sum rules.]

I. INTRODUCTION

Over ten years ago, Barton wrote a provocative paper asking for a physical explanation of certain simple features found in the photodisintegration of ^{3}He.[1] His query stemmed from the observation that the bremsstrahlung-weighted sum rules for the two possible breakup channels,

$$\gamma + {}^3\text{He} \to d + p$$
$$\to 2p + n \,,$$

are almost equal[2]:

$$J_{dp} = \int_0^{170\,\text{MeV}} dE_\gamma \frac{\sigma_{dp}(E_\gamma)}{E_\gamma} = 1.34 \pm 0.05 \text{ mb} \,, \tag{1}$$

$$J_{3N} = \int_0^{170\,\text{MeV}} dE_\gamma \frac{\sigma_{3N}(E_\gamma)}{E_\gamma} = 1.42 \pm 0.07 \text{ mb} \,, \tag{2}$$

i.e.,

$$J_{dp} \approx J_{3N} \,. \tag{3}$$

Here E_γ is the γ-ray energy and σ_i (E_γ) is the total cross section for breakup channel i. "Can the near equality of these sum rules be understood?" is one way of stating Barton's question. It can be rephrased slightly if three simplifying assumptions are made[3]: The ground state of ^{3}He is spatially symmetric, corrections to the long-wavelength limit are small, and ^{3}He photodisintegration is dominantly $E1$. The total bremsstrahlung-weighted $E1$ sum rule is then[4,1]

$$J_T = \int dE_\gamma \frac{\sigma_T(E_\gamma)}{E_\gamma} = \tfrac{4}{3}\pi^2 \alpha R^2 \,, \tag{4}$$

where $\sigma_T = \sigma_{dp} + \sigma_{3N}$, α is the fine-structure constant, and R is the ^{3}He matter radius. The total $E1$ sum rule can be decomposed according to breakup channel

$$J_T = J_{dp} + J_{3N} \,, \tag{5}$$

or the isospin of the final state

$$J_T = J_{1/2} + J_{3/2} \,. \tag{6}$$

Equation (6) follows from the completeness of the three-nucleon final states. From the assumption that ^{3}He is spatially symmetric,[4,1] we have

$$J_{1/2} = J_{3/2} \,. \tag{7}$$

Moreover, we know that the two-body breakup channel is isospin $\frac{1}{2}$, so

$$J_{dp} \leq J_{1/2} \,, \tag{8}$$

since three-body breakup leads to a final state which can have isospin $\frac{1}{2}$ or $\frac{3}{2}$. Equations (6)–(8) imply

$$J_{dp} \leq J_{1/2} = \tfrac{1}{2} J_T = \tfrac{1}{2}(J_{dp} + J_{3N}) \tag{9}$$

or

$$J_{dp} \leq J_{3N} \,. \tag{10}$$

Experiment, Eqs. (1)–(3), implies that the equality is close to the correct relation in Eq. (10), so that this is also the case for Eq. (8):

$$J_{dp} \approx J_{1/2} \,. \tag{8'}$$

Equation (8′) implies that the isospin-$\frac{1}{2}$ three-body photodisintegration of ^{3}He is severely suppressed compared to the isospin-$\frac{3}{2}$ component. To state Barton's question in another form, we quote him: "...we believe the observed exhaustion of the $I=\frac{1}{2}$ sum rule by the two-body channel to be so remarkable that a reasonably simple qualitative explanation is required." The *purpose* of this paper is to provide that explanation.

Since Barton's paper, two "exact" three-body calculations of the $E1$ photodisintegration of ^{3}He have been performed.[5,6] In both, the isospin-$\frac{1}{2}$ three-body breakup at low energies ($0 \leq E_\gamma \leq 40$ MeV) is considerably suppressed compared to the isospin-$\frac{3}{2}$. These calculations demonstrate that Eqs. (3) or (8′) are not exact, but approximate. Additionally, O'Connell and Prats[4] have shown by combining the bremsstrahlung-weighted sum rule with the unweighted sum rule that the suppression occurs mainly at low energies. Moreover, Barton pointed out that, on the basis of satisfying the sum rules with various complete sets for the three-body final states, any knowledge of the existence of the deuteron in the three-body channel must lead to some reduction in the isospin-$\frac{1}{2}$ three-body breakup contribution. Barbour and Phillips conjectured that the analytic property in the three-body amplitude which is most likely responsible for the reduction is the pole due to the triplet two-body channel.[5]

Our objective is to use the spin-isospin symmetries of the ^{2}S-isodoublet three-nucleon equations to simplify them and facilitate a comparison with the isoquartet equations. The method is to extend into the low-energy ^2S-isodoublet continuum the Wigner-supermultiplet approximation which leads to the spatial symmetry of the ground state. This is accomplished by neglecting the difference between the s-wave singlet and triplet two-nucleon interaction compared to their sum in the ^2S-isodoublet equations. It is then demonstrated that such an approximation works reasonably well.[7] With this foundation, we can compare the approximate isodoublet equations with the isoquartet equations in order to note their distinguishing features and shed light upon the suppression of the isospin-$\frac{1}{2}$ three-body photodisintegration of ^{3}He compared to that of the isospin-$\frac{3}{2}$.

We begin in Sec. II with a review of the three-nucleon $S=\frac{1}{2}$ isodoublet and isoquartet equations. Section II also contains an explanation of the approximation used in the isodoublet case. In Sec. III, the photodisintegration amplitudes and cross sections are derived (this work is related to that of Gibson and Lehman (GL)[6]) and the approximation made in the isodoublet equations is emphasized again. The approximation is justified with exact calculations in Sec. IV. Finally, Sec. V contains a discussion and Sec. VI, our conclusion.

II. EQUATIONS FOR THE $S=\frac{1}{2}$, ISODOUBLET AND ISOQUARTET STATES OF THREE NUCLEONS

When the $A=3$ nuclei are viewed as pure $^2S_{1/2}$ nuclei, their electric-dipole photodisintegration involves transitions from the ($S=\frac{1}{2}$, $I=\frac{1}{2}$) ground state to $(\frac{1}{2},\frac{1}{2})$ two-body and $(\frac{1}{2},\frac{1}{2})$ plus $(\frac{1}{2},\frac{3}{2})$ three-body final states. The objective of this section is to establish the form of these states and the equations they satisfy.

A. $(\frac{1}{2},\frac{1}{2})$ States

The general form of the spin-doublet, isodoublet states in the notation established by Verde[8] is

$$\Psi_{1/2,1/2}(\widehat{123}) = N(\Psi^s_{1/2,1/2}\xi^a + \Psi''_{1/2,1/2}\xi' - \Psi'_{1/2,1/2}\xi'' - \Psi^a_{1/2,1/2}\xi^s)\,, \tag{11}$$

where the spatial components are

$$\begin{aligned} \Psi^s_{1/2,1/2}(\overline{123}) &= T^s g(1,\overline{23})\,, \\ \Psi''_{1/2,1/2}(1,\overline{23}) &= T'' h(1,\overline{23}) + T' H(1,\widehat{32})\,, \\ \Psi'_{1/2,1/2}(1,\widehat{23}) &= T' h(1,\overline{23}) + T'' H(1,\widehat{23})\,, \\ \Psi^a_{1/2,1/2}(\widehat{123}) &= T^s G(1,\widehat{32})\,, \end{aligned} \tag{12}$$

the spin-isospin functions are

$$\begin{aligned} \xi^a(\widehat{123}) &= \frac{1}{\sqrt{2}}(\chi'\eta'' - \chi''\eta')\,, \\ \xi'(\widehat{123}) &= \frac{1}{\sqrt{2}}(\chi'\eta'' + \chi''\eta')\,, \\ \xi''(\overline{123}) &= \frac{1}{\sqrt{2}}(\chi'\eta' - \chi''\eta'')\,, \\ \xi^s(\overline{123}) &= \frac{1}{\sqrt{2}}(\chi'\eta' + \chi''\eta'')\,, \end{aligned} \tag{13}$$

and N is a normalization constant depending on whether $\Psi_{1/2,1/2}$ represents a bound, plane-wave, or scattering state. The permutation operators are defined as

$$\begin{aligned} T^s &= (23) + (31) + (12)\,, \\ T' &= \tfrac{1}{2}\sqrt{3}\,[(31) - (12)]\,, \\ T'' &= -(23) + \tfrac{1}{2}[(31) + (12)]\,, \end{aligned} \tag{14}$$

and the spatial functions upon which they operate possess the following symmetry properties:

$$\begin{aligned} g(1,\overline{23}) &= g(1,\overline{32})\,, \\ h(1,\overline{23}) &= h(1,\overline{32})\,, \\ G(1,\widehat{23}) &= -G(1,\widehat{32})\,, \\ H(1,\widehat{23}) &= -H(1,\widehat{32})\,. \end{aligned} \tag{15}$$

The spin-$\frac{1}{2}$ functions, $\chi'(1,\widehat{23})$ and $\chi''(1,\overline{23})$, correspond to coupling the 23-pair to angular momentum 0 and 1, respectively, and then nucleon 1 to this combination for total angular momentum $\frac{1}{2}$. The isospin-$\frac{1}{2}$ functions, η' and η'', are similar. Note that the overall wave function is completely antisymmetric as are the spatially symmetric, antisymmetric, and mixed-symmetric pieces separately, i.e., $\Psi^s\xi^a$, $\Psi^a\xi^s$, and $(\Psi''\xi' - \Psi'\xi'')$, respectively.

To determine the equations which the basic spatial functions satisfy, we substitute Eq. (11) into Schrödinger's equation and project with the ξ^i. Following that, the equations for g, h, G, and H are determined by separating according to permutation symmetry. The system of equations is

$$\begin{bmatrix} E-H_0-V_{23}^{+}T^s & -V_{23}^{-}T'' & V_{23}^{-}T' & 0 \\ V_{23}^{-}T^s & E-H_0+V_{23}^{+}T'' & -V_{23}^{+}T' & 0 \\ 0 & -U_{23}^{+}T' & E-H_0-U_{23}^{+}T'' & U_{23}^{-}T^s \\ 0 & -U_{23}^{-}T' & -U_{23}^{-}T'' & E-H_0+U_{23}^{+}T^s \end{bmatrix} \begin{bmatrix} g(1,\overline{23}) \\ h(1,\overline{23}) \\ H(1,\widehat{23}) \\ G(1,\widehat{23}) \end{bmatrix} = 0 , \tag{16}$$

where E is the energy of the three-nucleon system, H_0 is the free, three-particle Hamiltonian, and the potentials are

$$V_{23}^{\pm} = \tfrac{1}{2}(V_{23}^{01} \pm V_{23}^{10}) , \qquad U_{23}^{\pm} = \tfrac{1}{2}(V_{23}^{00} \pm V_{23}^{11}) , \tag{17}$$

with V_{ij}^{st} representing the two-nucleon potential operator in the spin-s, isospin-t state of the pair i-j. We assume s-wave two-nucleon interactions so $V_{23}^{00} = V_{23}^{11} \equiv 0$, and both G and H satisfy

$$(E-H_0)\mathfrak{F} = 0 . \tag{18}$$

For the ground state and N-d scattering states, G and H are identically zero. Thus, g and h satisfy

$$\begin{bmatrix} E-H_0-V_{23}^{+}T^s & -V_{23}^{-}T'' \\ V_{23}^{-}T^s & E-H_0+V_{23}^{+}T'' \end{bmatrix} \begin{bmatrix} g(1,\overline{23}) \\ h(1,\overline{23}) \end{bmatrix} = 0 . \tag{19}$$

When $3N$ scattering states are of interest, G and H are not in general zero. Then, for s-wave interactions, Eqs. (16) apply with $U_{23}^{+} = U_{23}^{-} \equiv 0$. H and G are obtained from Eq. (18) and couple to the g and h equations through $V_{23}^{-}T'$ and $-V_{23}^{+}T'$, respectively. As we shall illustrate below, when H and G are nonzero they generate inhomogeneous terms in the equations for g and h. Therefore, in effect, Eqs. (19) apply for all cases apart from boundary conditions, i.e., inhomogeneous terms. Moreover, due to the relative weakness of V_{23}^{-} compared to V_{23}^{+}, Eqs. (19) are themselves almost uncoupled. This accounts for the dominance of the spatially symmetric component $\Psi^s = T^s g(1,\overline{23})$ in the three-nucleon ground state.[9]

Instead of the functions g, h, H and G which emphasize the symmetries, it is also convenient to use the functions which correspond to the pair interactions[10]:

$$v = (g+h)/2 , \quad u = (g-h)/2 , \quad q = (G+H)/2 , \quad r = (G-H)/2 . \tag{20}$$

The system of equations which they satisfy can be obtained directly from Eqs. (16):

$$\begin{bmatrix} [E-H_0-V_{23}^{10}(1+\frac{1}{4}P)] & -\frac{3}{4}V_{23}^{10}P & -\frac{1}{2}V_{23}^{10}T' & \frac{1}{2}V_{23}^{10}T' \\ -\frac{3}{4}V_{23}^{01}P & [E-H_0-V_{23}^{01}(1+\frac{1}{4}P)] & \frac{1}{2}V_{23}^{01}T' & -\frac{1}{2}V_{23}^{01}T' \\ -\frac{1}{2}V_{23}^{00}T' & \frac{1}{2}V_{23}^{00}T' & [E-H_0-V_{23}^{00}(1-\frac{1}{4}P)] & \frac{3}{4}V_{23}^{00}P \\ +\frac{1}{2}V_{23}^{11}T' & -\frac{1}{2}V_{23}^{11}T' & \frac{3}{4}V_{23}^{11}P & [E-H_0-V_{23}^{11}(1-\frac{1}{4}P)] \end{bmatrix} \begin{bmatrix} v(1,\overline{23}) \\ u(1,\overline{23}) \\ q(1,\widehat{23}) \\ r(1,\widehat{23}) \end{bmatrix} = 0 , \tag{21}$$

where $P=(31)+(12)$. Equations (21) reduce to the equivalent of Eqs. (18) and (19) by setting $V_{23}^{00}=V_{23}^{11}\equiv 0$:

$$\begin{bmatrix} E-H_0-V_{23}^{10}(1+\frac{1}{4}P) & -\frac{3}{4}V_{23}^{10}P \\ -\frac{3}{4}V_{23}^{01}P & E-H_0-V_{23}^{01}(1+\frac{1}{4}P)\end{bmatrix}\begin{bmatrix} v \\ u\end{bmatrix}=0, \quad (22)$$

where both q and r satisfy Eq. (18) and generate an inhomogeneous term in Eq. (22) for the $3N$ scattering case. Determination of the $(\frac{1}{2},\frac{1}{2})$ eigenstates by means of Eqs. (16) or (21) [(19) or (22)] is equivalent. Usually, Eqs. (19) ($h\equiv 0$) are used in the calculation of the ground state in order to take advantage of the weak coupling V_{23}^{-}.[9] When the coupling term is neglected a 100% spatially symmetric ground state is obtained, a good first approximation to the physical situation.

On the other hand, the determination of the N-d and $3N$ continuum states is normally done with Eqs. (21). The reason is that Eqs. (21) lead immediately to integral equations according to the pair interactions of the Faddeev type[11]; for example, from Eqs. (21) for s-wave interactions

$$\begin{aligned} v&=v^{(0)}+(E-H_0-V_{23}^{10})^{-1}V_{23}^{10}(\tfrac{1}{4}Pv+\tfrac{3}{4}Pu+\tfrac{1}{2}T'q-\tfrac{1}{2}T'r),\\ u&=u^{(0)}+(E-H_0-V_{23}^{01})^{-1}V_{23}^{01}(\tfrac{3}{4}Pv+\tfrac{1}{4}Pu-\tfrac{1}{2}T'q+\tfrac{1}{2}T'r),\\ q&=q^{(0)},\\ r&=r^{(0)}.\end{aligned} \quad (23)$$

The inhomogeneous terms are constructed from solutions of

$$\begin{aligned}(E-H_0-V_{23}^{10})v^{(0)}&=0,\\ (E-H_0-V_{23}^{01})u^{(0)}&=0,\\ (E-H_0)q^{(0)}&=0,\\ (E-H_0)r^{(0)}&=0.\end{aligned} \quad (24)$$

Equations (23) were first used by Sitenko and Kharchenko to calculate the doublet N-d scattering length.[10] For that case,

$$\begin{aligned} v^{(0)}&=\phi_{\vec{p}}^{(0)}(1)\phi_B(\overline{23}),\\ u^{(0)}&=q^{(0)}=r^{(0)}=0\end{aligned} \quad (25)$$

defines the two-body N-d channel, where $\phi_{\vec{p}}^{(0)}(1)$ represents the N moving freely with c.m. momentum $\vec{p}$ and $\phi_B(\overline{23})$ is the deuteron wave function. For $3N$ states, there are four possibilities defined by the following choices of inhomogeneous terms:

$$v^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{10}(\overline{23}),\quad u^{(0)}=q^{(0)}=r^{(0)}=0, \quad (26)$$

$$u^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{01}(\overline{23}),\quad v^{(0)}=q^{(0)}=r^{(0)}=0, \quad (27)$$

$$q^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23}),\quad v^{(0)}=u^{(0)}=r^{(0)}=0, \quad (28)$$

$$r^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23}),\quad v^{(0)}=u^{(0)}=q^{(0)}=0, \quad (29)$$

where $\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{st}(\overline{23})$ is a scattering-wave-function solution of the operator $E-H_0-V_{23}^{st}$ and $\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23})$ is a solution to Eq. (18). The relative momentum of particles 2 and 3 is $\vec{k}$.

In closing this subsection on the $(\frac{1}{2},\frac{1}{2})$ states, it should be emphasized that the basic structure of the Faddeev-type equations for the bound, N-d, and $3N$ states is the same; it is the inhomogeneous terms which distinguish the different states. In terms of the v, u, q, and r functions, there are two coupled equations:

$$\begin{aligned} V&=V^{(0)}+(E-H_0-V_{23}^{10})^{-1}V_{23}^{10}(\tfrac{1}{4}PV+\tfrac{3}{4}PU),\\ U&=U^{(0)}+(E-H_0-V_{23}^{01})^{-1}V_{23}^{01}(\tfrac{3}{4}PV+\tfrac{1}{4}PU).\end{aligned} \quad (30)$$

The bound, N-d, and $3N$ states are obtained by solving Eqs. (30) under the following conditions:

(a) bound state

$$\begin{aligned} &v=V,\quad q=r\equiv 0,\\ &u=U,\quad V^{(0)}=U^{(0)}\equiv 0;\end{aligned} \quad (31)$$

(b) N-d state

$$\begin{aligned} &v=V,\quad q=r\equiv 0,\\ &u=U,\quad V^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_B(\overline{23}),\quad U^{(0)}\equiv 0;\end{aligned} \quad (32)$$

(c) 3N states [according to (st)]

(i) (10)

$$\begin{aligned} &v=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\overline{23})+V,\quad q=r\equiv 0\\ &u=U,\\ &V^{(0)}=\tfrac{1}{2}(T_{pk}^{s}-T_{pk}'')\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{10}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})],\\ &U^{(0)}=\tfrac{1}{2}(T_{pk}^{s}+T_{pk}'')\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{01}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})];\end{aligned} \quad (33)$$

(ii) (01)

$$\begin{aligned} &v=V,\quad q=r\equiv 0,\\ &u=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\overline{23})+U,\\ &V^{(0)}=\tfrac{1}{2}(T_{pk}^{s}+T_{pk}'')\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{10}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})],\\ &U^{(0)}=\tfrac{1}{2}(T_{pk}^{s}-T_{pk}'')\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{01}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})];\end{aligned} \quad (34)$$

(iii) (00)

$$\begin{aligned} &v=V,\quad q=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23}),\\ &u=U,\quad r\equiv 0,\\ &V^{(0)}=\tfrac{1}{2}T_{pk}'\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{10}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})],\\ &U^{(0)}=-\tfrac{1}{2}T_{pk}'\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{01}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})];\end{aligned} \quad (35)$$

(iv) (11)

$$\begin{aligned} &v=V,\quad q\equiv 0\\ &u=U,\quad r=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23}),\\ &V^{(0)}=-\tfrac{1}{2}T_{pk}'\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{10}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})],\\ &U^{(0)}=\tfrac{1}{2}T_{pk}'\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{01}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})].\end{aligned} \quad (36)$$

The various $3N$ inhomogeneous terms are obtained from Eqs. (23) by substituting the appropriate inhomogeneous term from the set Eqs. (26)–(29). For example, Eqs. (23) and (29) yield Eqs. (30) and (36) through the following types of manipulations:

$$\begin{aligned}
-(E-H_0-V_{23}^{10})^{-1}V_{23}^{10}\tfrac{1}{2}T'r &= G_{23}^{10}(E)\,V_{23}^{10}\tfrac{1}{2}T'\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23})\\
&= \tfrac{1}{2}T'_{pk}G_{23}^{10}(E)\,V_{23}^{10}\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\overline{23})\\
&= -\tfrac{1}{2}T'_{pk}[i\epsilon\, G_{23}^{10}(E)\,V_{23}^{10}G_0(E)\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\overline{23})]\\
&= +\tfrac{1}{2}T'_{pk}\{i\epsilon\,[G_{23}^{10}(E)-G_0(E)]\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\overline{23})\}\\
&= -\tfrac{1}{2}T'_{pk}\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{10}(23)-\phi_{\vec{k}}^{(0)}(\overline{23})]\ ,
\end{aligned}$$

where $E\equiv E+i\epsilon=k^2/M+3p^2/4M+i\epsilon$ is invariant under particle permutations and T'_{pk} permutes the external-momentum quantum numbers in contrast to T' which permutes the internal coordinates.

B. $(\frac{1}{2},\frac{3}{2})$ States

The spin doublet, isoquartet states have the general form[10]

$$\Psi_{1/2,3/2}(\widehat{123})=N(\psi''_{1/2,3/2}\chi'-\psi'_{1/2,3/2}\chi'')\eta^s\ , \tag{37}$$

where $\eta^s(\overline{123})$ is an isospin-$\frac{3}{2}$ function for three nucleons. The spatial functions are defined as

$$\begin{aligned}
\psi'_{1/2,3/2}(1,\widehat{23}) &= T'w(1,\overline{23})+T''s(1,\widehat{23})\ ,\\
\psi''_{1/2,3/2}(1,\overline{23}) &= T''w(1,\overline{23})+T's(1,\widehat{32})\ ,
\end{aligned} \tag{38}$$

with

$$\begin{aligned}
w(1,\overline{23}) &= w(1,\overline{32})\ ,\\
s(1,\widehat{23}) &= -s(1,\widehat{32})\ .
\end{aligned} \tag{39}$$

The basic functions w and s satisfy the equations

$$\begin{bmatrix} E-H_0+V_{23}^{01}T'' & -V_{23}^{01}T' \\ V_{23}^{11}T' & E-H_0+V_{23}^{11}T'' \end{bmatrix}\begin{bmatrix} w \\ s \end{bmatrix}=0\ , \tag{40}$$

which can be transformed into the Faddeev-type equations

$$\begin{aligned}
w &= w^{(0)}-(E-H_0-V_{23}^{01})^{-1}V^{01}(\tfrac{1}{2}Pw-T's)\ ,\\
s &= s^{(0)}
\end{aligned} \tag{41}$$

for s-wave interactions, where

$$\begin{aligned}
(E-H_0-V_{23}^{01})w^{(0)} &= 0\ ,\\
(E-H_0)s^{(0)} &= 0\ .
\end{aligned} \tag{42}$$

The only states of interest are of the $3N$ type and there are two possibilities:

$$w^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{01}(\overline{23}),\quad s^{(0)}=0 \tag{43}$$

or

$$w^{(0)}=0,\quad s^{(0)}=\phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23})\ . \tag{44}$$

When Eqs. (43) and (44) are respectively substituted into Eq. (41), it becomes apparent that only one basic Faddeev-type equation is required for the $(\frac{1}{2},\frac{3}{2})$ case with the different $3N$ states distinguished by the inhomogeneous term involved. That equation is

$$W=W^{(0)}-\tfrac{1}{2}(E-H_0-V_{23}^{01})^{-1}V_{23}^{01}PW\ . \tag{45}$$

The two possible $3N$ states are generated as follows:

(i) (01)

$$\begin{aligned}
w &= \phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\overline{23})+W\ ,\\
s &\equiv 0\ ,\\
W^{(0)} &= -T''_{pk}\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{01}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})]\ .
\end{aligned} \tag{46}$$

(ii) (11)

$$\begin{aligned}
w &= W\ ,\\
s &= \phi_{\vec{p}}^{(0)}(1)\phi_{\vec{k}}^{(0)}(\widehat{23})\ ,\\
W^{(0)} &= T'_{pk}\phi_{\vec{p}}^{(0)}(1)[\phi_{\vec{k}}^{01}(\overline{23})-\phi_{\vec{k}}^{(0)}(\overline{23})]\ .
\end{aligned} \tag{47}$$

The equations for the isoquartet states are given here to complete the review of the equations that determine the eigenstates relevant to the trinucleon photodisintegration amplitudes, for an important later comparison with the isodoublet equations. Moreover, we emphasize that the isodoublet and isoquartet states belong to orthogonal subspaces of the Hilbert space of three-nucleon eigenstates. This is the reason for different equations determining their spatial components, Eqs. (30) and (45), respectively. Thus, for s-wave two-nucleon interactions, the isodoublet equations depend on both V^{10} and V^{01}, whereas the isoquartet equation depends *only* on V^{01}.

C. Approximation for $(\frac{1}{2},\frac{1}{2})$ States

It was pointed out in part A of this section that in dealing with isodoublet states either Eqs. (16) or (21) can be used. Also, it was emphasized that while Eqs. (16) are usually used for the ground state, Eqs. (21) were most convenient for the continuum states since the inhomogeneous terms identify the interaction channel. In contrast, consider the integral-equation equivalent of Eqs. (16) for s-wave interactions:

$$\begin{aligned} g &= g^{(0)} + (E-H_0-V_{23}^+)^{-1}(V_{23}^+Pg + V_{23}^-T''h - V_{23}^-T'H) \,, \\ h &= h^{(0)} + (E-H_0-V_{23}^+)^{-1} \\ &\quad\times(-V_{23}^-T^sg - \tfrac{1}{2}V_{23}^+Ph + V_{23}^+T'H) \,, \\ H &= H^{(0)} \,, \\ G &= G^{(0)} \,, \end{aligned} \tag{48}$$

where

$$\begin{aligned} (E-H_0-V_{23}^+)g^{(0)} &= 0 \,, \\ (E-H_0-V_{23}^+)h^{(0)} &= 0 \,, \\ (E-H_0)H^{(0)} &= 0 \,, \\ (E-H_0)G^{(0)} &= 0 \,. \end{aligned} \tag{49}$$

Whether it be bound, N-d, or $3N$ states under consideration, there is only a pair of Faddeev-type equations to be solved:

$$\begin{aligned} \mathcal{G} &= \mathcal{G}^{(0)} + (E-H_0-V_{23}^+)^{-1}(V_{23}^+P\mathcal{G} + V_{23}^-T''\mathcal{H}) \,, \\ \mathcal{H} &= \mathcal{H}^{(0)} - (E-H_0-V_{23}^+)^{-1}(V_{23}^-T^s\mathcal{G} + \tfrac{1}{2}V_{23}^+P\mathcal{H}) \,, \end{aligned} \tag{50}$$

where $\mathcal{G}^{(0)}$ and $\mathcal{H}^{(0)}$ determine the state under consideration. To illustrate, consider N-d states. Then there are two cases:

$$\mathcal{G}^{(0)} = \phi_{\mathrm{p}}^{(0)}(1)\phi_B^+(\overline{23}), \quad \mathcal{H}^{(0)} \equiv 0 \tag{51}$$

and

$$\mathcal{G}^{(0)} = 0, \quad \mathcal{H}^{(0)} = \phi_{\mathrm{p}}^{(0)}(1)\phi_B^+(\overline{23}) \tag{52}$$

with $g=\mathcal{G}$, $h=\mathcal{H}$, and $H=G\equiv 0$. $\phi_B^+(\overline{23})$ can be called the "Majorana deuteron" since it is obtained from V_{23}^+ which is the average of the central potentials in the triplet-even and singlet-even states of the two-body system.

It has already been emphasized that it is a good approximation to neglect V_{23}^- when solving Eqs. (50) for three-nucleon bound states. We *propose* to test this approximation for the low-energy three-nucleon continuum states in trinucleon photodisintegration. If it is a good approximation, then we shall be able to make clear-cut comparisons between the isodoublet and isoquartet three-body photodisintegration of ^{3}He. The source of the strong suppression in the isodoublet channel then will be evident. The key to this comparison is Eqs. (50) with $V_{23}^-=0$, (45), (37), and (11). Equations (50) become uncoupled:

$$\begin{aligned} \mathcal{G} &= \mathcal{G}^{(0)} + (E-H_0-V_{23}^+)^{-1}V_{23}^+P\mathcal{G} \,, \\ \mathcal{H} &= \mathcal{H}^{(0)} - \tfrac{1}{2}(E-H_0-V_{23}^+)^{-1}V_{23}^+P\mathcal{H} \,, \end{aligned} \tag{50$'$}$$

that is, the symmetry classes have decoupled. The spatially symmetric component of $\Psi_{1/2,1/2}$ is generated from a single equation for $\mathcal{G}$ and the mixed-symmetric component is generated from a single equation for $\mathcal{H}$. Note that the equation for $\mathcal{H}$ is now formally identical with the isoquartet equation for W, except V_{23}^+ replaces V_{23}^{01}. Their identity, apart from the potential, occurs because both $\mathcal{H}$ and W generate mixed-symmetric functions. As we shall see, this comparison of the $\mathcal{H}$ equation in Eq. (50′) and the W equation, Eq. (45), will be crucial to our discussion of the isodoublet three-body photobreakup suppression.

With the above background, we are now ready to look at the photodisintegration amplitudes.

III. PHOTODISINTEGRATION AMPLITUDES

In this section, we derive the ^{3}He photodisintegration amplitudes using the form of the states as given in Sec. II. Our emphasis will be placed on the form of the amplitudes, their relationship to the work of GL,[6] and the manner in which the approximation of Sec. II C is implemented.

As is clear from the Introduction, we are interested only in the $E1$ photodisintegration of ^{3}He. Our interest lies with the final states, so the ground state is taken to be the dominant spatially symmetric part. The $E1$ operator,

$$H' = \frac{e}{2}\sum_{i=1}^{3}(\hat{\epsilon}\cdot\vec{r}_i)\tau_3^{(i)} \,, \tag{53}$$

then operates on

$$|\psi_B\rangle = \psi_0^s\frac{1}{\sqrt{2}}(\chi'\eta'' - \chi''\eta') \tag{54}$$

to yield

$$H'|\psi_B\rangle = \frac{-e}{2\sqrt{3}}\,\hat{\epsilon}\cdot\left[\frac{2}{\sqrt{3}}\vec{\rho}\xi' - \vec{r}\xi'' - \left(\frac{2}{\sqrt{3}}\vec{\rho}\chi' - \vec{r}\chi''\right)\eta^s\right]\psi_0^s \,, \tag{55}$$

where the notation is that of Ref. 6. The $(\frac{1}{2},\frac{1}{2})$ and $(\frac{1}{2},\frac{3}{2})$ amplitudes are easily obtained from Eq. (55) by use of Eqs. (11) and (37), respectively. Specifically for $(\frac{1}{2},\frac{1}{2})$,

$$\langle\Psi_{1/2,1/2}|H'|\psi_B\rangle = -\frac{e}{\sqrt{6}}\langle h(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle + \frac{e}{2\sqrt{2}}\langle H(1,\widehat{23})|\hat{\epsilon}\cdot\vec{r}|\psi_0^s\rangle \tag{56}$$

$$= -\frac{e}{\sqrt{6}}\langle v(1,\overline{23}) - u(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle + \frac{e}{2\sqrt{2}}\langle q(1,\widehat{23}) - r(1,\widehat{23})|\hat{\epsilon}\cdot\vec{r}|\psi_0^s\rangle , \tag{57}$$

where $N = -(\frac{2}{3})^{1/2}$ for continuum states. Similarly, the $(\frac{1}{2},\frac{3}{2})$ amplitude is

$$\langle\Psi_{1/2,3/2}|H'|\psi_B\rangle = \frac{e}{\sqrt{3}}\langle w(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle - \frac{e}{2}\langle s(1,\widehat{23})|\hat{\epsilon}\cdot\vec{r}|\psi_0^s\rangle , \tag{58}$$

where $N = -(\frac{1}{3})^{1/2}$.

A. Two-body breakup

Two-body photodisintegration of ^{3}He leads to only an $I=\frac{1}{2}$ final state. Therefore, Eq. (57) applies in conjunction with Eq. (32) or Eq. (56) with Eqs. (51) and (52):

$$\langle\Psi^{Nd}_{1/2,1/2}|H'|\psi_B\rangle = -\frac{e}{\sqrt{6}}\langle h(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle \tag{59}$$

$$= -\frac{e}{\sqrt{6}}\langle v(1,\overline{23}) - u(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle . \tag{60}$$

In Born approximation, i.e., $v = V^{(0)}$ and $u = U^{(0)}$, we recognize Eq. (60) as

$$B_t(z,\vec{\mathrm{p}}) = -\frac{e}{\sqrt{6}}\langle\phi^{(0)}_{\vec{\mathrm{p}}}(1)\phi_B(\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle , \quad z = \frac{3p^2}{4} - \gamma^2, \tag{61}$$

of GL1, and[12]

$$M_2^t(z,\vec{\mathrm{p}}) = \langle\Psi^{Nd}_{1/2,1/2}|H'|\psi_B\rangle . \tag{62}$$

The two-body differential cross section is obtained in the standard way:

$$d\sigma = \frac{2\pi^2}{\hbar c}E_\gamma\left|\mathfrak{M}_2^t\left(\frac{3p^2}{4} - \gamma^2, \vec{\mathrm{p}}\right)\right|^2 \sin^2\theta\rho_f , \tag{63}$$

where $M_2^t(z,\vec{\mathrm{p}}) = \hat{\epsilon}\cdot\hat{p}\mathfrak{M}_2^t(z,p)$, E_γ is the photon energy, θ is the center-of-mass angle of the outgoing nucleon with respect to the incident photon direction, and ρ_f is the density of final states.

B. Three-body breakup

Three-body photodisintegration of ^{3}He has two possible isospin final states: $\frac{1}{2}$ or $\frac{3}{2}$.

There are four possible isospin-$\frac{1}{2}$ amplitudes generated from Eqs. (33)–(36), where the classification is according to the spin-isospin state of particles 2 and 3. Each amplitude has the general form of Eq. (57). As an example, consider the $st = 10$ amplitude. We obtain

$$\langle\Psi^{3N(10)}_{1/2,1/2}|H'|\psi_B\rangle = -\frac{e}{\sqrt{6}}\langle\phi^{(0)}_{\vec{\mathrm{p}}}(1)\phi^{(0)}_{\vec{\mathrm{k}}}(\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle - \frac{e}{\sqrt{6}}\langle V(1,\overline{23}) - U(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle , \tag{64}$$

where the first term is $C_{10}^{1/2}$ and the second term is $F_{10}^{1/2} + S_{10}^{1/2}$ in the notation of GL2.[6] $F_{10}^{1/2}$ is explicitly obtained by replacing V and U by $V^{(0)}$ and $U^{(0)}$, respectively.[13] In the same manner, all the $C^{1/2}_{S_{23}I_{23}}$, $F^{1/2}_{S_{23}I_{23}}$, $S^{1/2}_{S_{23}I_{23}}$ amplitudes of GL2 can be generated. In fact, the GL M-amplitudes are

$$M^{1/2}_{S_{23}I_{23}} = \langle\Psi^{3N(st)}_{1/2,1/2}|H'|\psi_B\rangle , \tag{65}$$

where $S_{23} = s$ and $I_{23} = t$.

Likewise, we can write down the two isospin-$\frac{3}{2}$ amplitudes from Eq. (58) with Eqs. (46) and (47):

$$M^{3/2}_{S_{23}I_{23}} = \langle\Psi^{3N(st)}_{1/2,3/2}|H'|\psi_B\rangle . \tag{66}$$

As an example, consider $st = 01$:

$$M^{3/2}_{01} = \frac{e}{\sqrt{3}}\langle\phi^{(0)}_{\vec{\mathrm{p}}}(1)\phi^{(0)}_{\vec{\mathrm{k}}}(\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle + \frac{e}{\sqrt{3}}\langle W(1,\overline{23})|\hat{\epsilon}\cdot\vec{p}|\psi_0^s\rangle \tag{67}$$

$$= C^{3/2}_{01} + (F^{3/2}_{01} + S^{3/2}_{01}) . \tag{68}$$

The three-body differential cross section is obtained from

$$d\sigma = \frac{4\pi^2}{\hbar c}E_\gamma\sum_{s_{23}=0}^{1}\left|\mathfrak{M}_3^{s_{23}}\left(\frac{3p^2}{4} + k^2, \vec{\mathrm{p}}, \vec{\mathrm{k}}\right)\right|^2_{\text{pol av}}\rho_f . \tag{69}$$

where E_γ is the photon energy and ρ_f is the density of final states. The form of $\mathfrak{M}_3^{s_{23}}$ depends on which nucleon is designated particle 1. For example, if the neutron is particle 1, then

$$\mathfrak{M}_3^{s_{23}} = -(\tfrac{2}{3})^{1/2}M^{1/2}_{s_{23}1} + (\tfrac{1}{3})^{1/2}M^{3/2}_{s_{23}1} , \tag{70}$$

or if the neutron is particle 2

$$\mathfrak{M}_3^{s_{23}} = \frac{1}{\sqrt{2}}\left[-M^{1/2}_{s_{23}0} + (\tfrac{1}{3})^{1/2}M^{1/2}_{s_{23}1} + (\tfrac{2}{3})^{1/2}M^{3/2}_{s_{23}1}\right] . \tag{71}$$

C. The model for $(\frac{1}{2},\frac{1}{2})$ amplitudes

In Sec. II C, an important approximation for the $(\frac{1}{2},\frac{1}{2})$ states is discussed. In this section, we point out how this approximation is implemented and tested. Hereafter, we shall refer to this approximation as the *model*. We emphasize that this approximation applies to the $(\frac{1}{2},\frac{1}{2})$ states *only*, as can be seen from the dynamical equations Secs. II A and II B.

The approximation is to neglect V_{23}^- compared to V_{23}^+. As is well known,[3] this is a good approximation for the three-nucleon ground states and it explains why they are predominantly spatially symmetric. Our purpose is to determine whether it remains a fairly good approximation for the low-energy three-nucleon continuum states. If it does, we can then compare the *model* $(\frac{1}{2},\frac{1}{2})$ amplitudes and equations with their $(\frac{1}{2},\frac{3}{2})$ counterparts to reach an understanding of the isospin-$\frac{1}{2}$, three-body, ^{3}He photodisintegration suppression.

In order to check the validity of the model, we calculate two- and three-body photodisintegration of ^{3}He by the methods of GL.[6] The s-wave separable potentials chosen are those of Tabakin,[14] since the singlet and triple interactions differ only in the strength parameter and still give a fair representation of the respective scattering length and effective ranges. The V_{23}^+ is obtained by a simple average of the singlet and triplet strength parameters as shown in Table I. V_{23}^+ supports a two-body bound state with binding energy 0.43 MeV. This number is essentially model independent and simply reflects the average of the reciprocal singlet and triplet s-wave n-p scattering lengths,

$$\frac{1}{2}\left(\frac{1}{a_t}+\frac{1}{a_s}\right)=\frac{1}{2}\left(\frac{1}{5.397}+\frac{1}{(-23.715)}\right)=\frac{1}{13.97}\ \text{fm}^{-1},$$

which in zero-range approximation implies a two-body bound state of 0.21 MeV. Thus, V_{23}^+ will have a bound state no matter how sophisticated the two-nucleon potentials which generate it, provided they give a reasonable fit to the scattering lengths and effective ranges. This is a basic property of the two-nucleon system.

How are the cross sections calculated for the model? Neglecting V_{23}^- or setting $V_{23}^- \approx 0$ is equivalent to forcing

$$V_{23}^{10} \approx V_{23}^{01} \tag{72}$$

or

$$V_{23}^{10} \approx V_{23}^{+}\,, \quad V_{23}^{01} \approx V_{23}^{+}\,, \tag{73}$$

where V_{23}^+ is the *average* of the singlet and triplet s-wave potentials. Since V_{23}^+ always supports a bound state, there are now two two-body breakup channels, one corresponding to the triplet as usual, but also one due to the singlet since $V_{23}^{01} \approx V_{23}^+$. Except for their spin-isospin properties, they are indistinguishable since they are both derived from V_{23}^+. Therefore, the two-body cross section for the model is obtained from Eq. (63) by adding incoherently the singlet amplitude $\mathfrak{M}_2^s$. Clearly $\mathfrak{M}_2^s = \mathfrak{M}_2^t$ for the model, so Eq. (63) can be used as it stands multiplied by 2 if $\mathfrak{M}_2^t$ is calculated in the usual manner, but with V^{10} and V^{01} replaced by V^+ throughout. The $(\frac{1}{2},\frac{1}{2})$ three-body cross sections are calculated as usual, except V^{10} and V^{01} are replaced by V^+.

How do the model $(\frac{1}{2},\frac{1}{2})$ equations for three-body breakup compare with the $(\frac{1}{2},\frac{3}{2})$ equations? Firstly, the equations which determine the key functions, $\mathcal{K}$ for $(\frac{1}{2},\frac{1}{2})$ and W for $(\frac{1}{2},\frac{3}{2})$, are identical as can be seen by comparing Eqs. (50′) and (45), except V_{23}^+ is present in the $\mathcal{K}$ equation and V^{01} in the W equation. Secondly, the $(\frac{1}{2},\frac{1}{2})$ amplitudes are a factor of $\sqrt{2}$ smaller than the $(\frac{1}{2},\frac{3}{2})$ [compare Eq. (56) with Eq. (58)], but according to Eq. (70) this factor is removed when they are combined. It is known that the isospin amplitudes combine incoherently in the total cross section, so the isospin-$\frac{1}{2}$ and isospin-$\frac{3}{2}$ three-body-breakup total cross sections can be calculated separately and compared. Therefore, if the model is a fairly good approximation to the calculation with $V_{23}^- \neq 0$, we can begin to understand differences between isospin-$\frac{1}{2}$ and isospin-$\frac{3}{2}$ results. Differences can be attributed to the difference between V_{23}^{01} and V_{23}^+, the latter being the effective potential in the isospin-$\frac{1}{2}$ case. Our next step then is to check the validity of the model.

IV. TEST OF THE MODEL

The model is checked in two ways. Firstly, the photodisintegration cross sections are calculated as a function of E_γ with the Tabakin interactions when V_{23}^- is *not* neglected (hereafter referred to as Tabakin) and for the model. The Tabakin and model results are then compared. Secondly, the bremsstrahlung-weighted sum rules are evaluated,

TABLE I. Potential parameters.

Potential	Strength λ (fm^{-3})	Inverse range β (fm^{-1})	Scattering length a (fm)	Effective range r_0 (fm)
V^{10}	0.220	1.15	5.68	2.09
V^{01}	0.148	1.15	−21.25	2.74
V^{+}	0.184	1.15	11.20	2.34

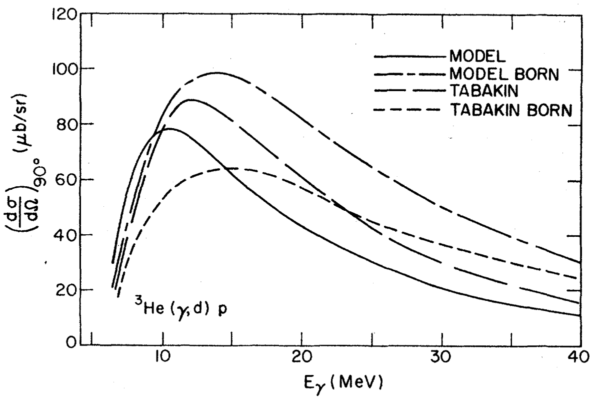

FIG. 1. Comparison of the model and Tabakin results for the two-body breakup 90° differential cross section.

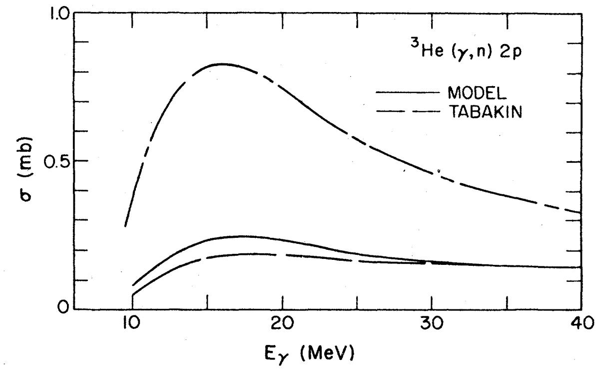

FIG. 3. Isospin contributions to the total cross section for three-body photodisintegration of ^{3}He. The unlabeled, - - -, curve is the isospin-$\frac{3}{2}$ cross section. The two labeled curves are the Tabakin and model isospin-$\frac{1}{2}$ cross section predictions.

checked for consistency, and the Tabakin and model results are compared.

The ground-state wave function used in all of our calculations is spatially symmetric and derived from the average potential V_{23}^{+} generated with Tabakin's parameters (see Table I). The predicted three-nucleon binding energy is 9.33 MeV and the matter radius is calculated to be 1.55 fm. Since the same ground-state wave function is used throughout, all differences discussed below are attributable to the final states.

Figures 1 and 2 display the two-body breakup curves. Figure 1 contrasts Tabakin and model curves for both the Born amplitudes and the complete calculations. There is a large difference between the two Born curves, but this is not surprising. The approximation leading to the model is not made in Born approximation, but is made in the full dynamical equations. That is why the Tabakin and model curves for the complete calculation are in much better agreement. The model curve rises more rapidly, peaks at a lower value, and then remains below the Tabakin curve. There is approximatly a 20% difference between the two curves for a given E_γ. One might expect this after comparing the difference to the sum of the Tabakin triplet and singlet interaction strengths (see Table I):

$$\frac{\lambda_{10}-\lambda_{01}}{\lambda_{10}+\lambda_{01}} \approx 0.2 \ .$$

Figure 2 compares the two curves with the experimental data. The 90° differential cross sections are used to ensure no $E2$ contribution.

The three-body-breakup curves are displayed in Figs. 3 and 4. In Fig. 3, the unlabeled curve corresponds to the isospin-$\frac{3}{2}$ three-body breakup.

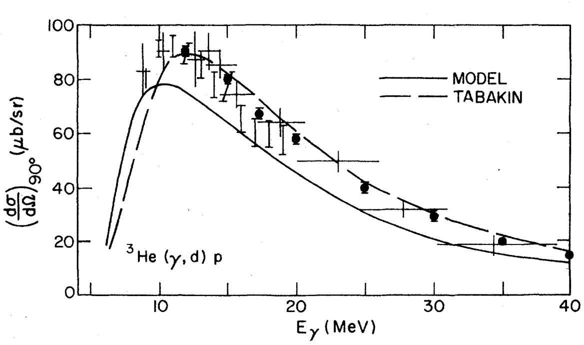

FIG. 2. Comparison of the model and Tabakin predictions with experiment for the two-body breakup 90° differential cross section. The data points are as follows: (●) Ticcioni *et al.*, Ref. 15, (*I*) Berman *et al.*, Ref. 16, and (+) Stewart *et al.*, Ref. 17.

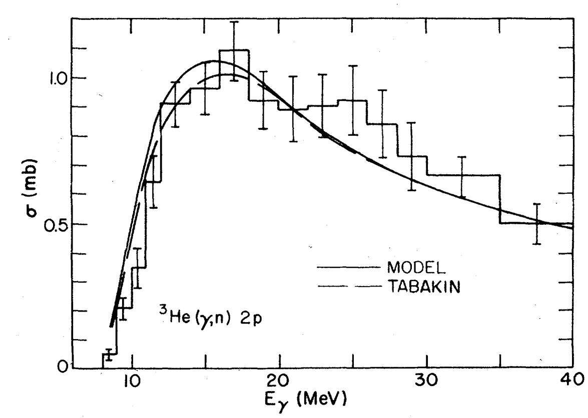

FIG. 4. Comparison of the model and Tabakin three-body photodisintegration cross sections with experiment. The two curves are obtained by adding the isospin-$\frac{3}{2}$ curve to the two isospin-$\frac{1}{2}$ curves of Fig. 3. The data are from A. N. Gorbunov, Ref. 1.

There is no model curve for isospin-$\frac{3}{2}$, because the model applies only to the $(\frac{1}{2},\frac{1}{2})$ states. The two labeled curves are the Tabakin and model isospin-$\frac{1}{2}$ curves. Their differences are similar to the two-body breakup. Note that the isospin-$\frac{1}{2}$ three-body breakup, Tabakin or model, is considerably suppressed compared to the isospin-$\frac{3}{2}$. Moreover, the fact that the model isospin-$\frac{1}{2}$ cross section is somewhat larger than the Tabakin curve is consistent with the fact that the model two-body cross section is somewhat lower than the Tabakin. This is expected on the basis of the sum rules [see Eqs. (4), (6), and (7)]:

$$J_{(1/2)(\text{total})} = J_{(1/2)(Nd)} + J_{(1/2)(3N)} = \tfrac{2}{3}\pi^2\alpha R^2 .$$

Since $J_{(1/2)(\text{total})}$ is fixed by the matter radius of the ground state, an increase of $J_{(1/2)(3N)}$ implies that $J_{(1/2)(Nd)}$ must decrease and vice versa. Finally, in Fig. 4, we compare the calculated three-body cross sections with experimental data. The two curves are obtained by adding the isospin-$\frac{1}{2}$ curves in Fig. 3.

The contributions to the bremsstrahlung-weighted sum rule

$$J_i = \int dE_\gamma \frac{\sigma_i(E_\gamma)}{E_\gamma}$$

for the various final channels are evaluated for the cross sections displayed in the figures. The results are summarized in Table II. When the three-body ground state is spatially symmetric the isodoublet and isoquartet contributions to the sum rule should be equal, Eq. (7); i.e., the values in the third and fourth columns should be equal. This is the case within the errors of the calculations.[18] Furthermore, the Tabakin and model isodoublet contribution should be equal, since they arise from two different bases of the same subspace of the three-nucleon Hilbert space. They are equal within errors. Moreover, the total sum rule should be equal to $\frac{4}{3}\pi^2\alpha R^2$ with $R = 1.55$ fm (the Tabakin matter radius) which yields 2.31 mb, again in good agreement with the figures in the last column of Table II. We note that in the Tabakin calculation 75% of the isodoublet contribution goes into the two-body channel, compared to 70% for the model calculation. The difference is due to the supermultiplet approximation ($V_{23}^- = 0$) used in the model.

Clearly, the above results demonstrate that the model is a good approximation for the purposes of making the comparison of interest and understanding the suppression of the isospin-$\frac{1}{2}$ three-body photodisintegration of ^{3}He. Beyond that, the results demonstrate that the approximate symmetry which leads to the ground states of the trinucleons being ~90% spatially symmetric carries over into the low-energy continuum, for states with the same spin-isospin quantum numbers as the ground state, to within 20%. Thus, the approximation of neglecting V_{23}^- compared to V_{23}^+, i.e., the Wigner SU_4 (supermultiplet) approximation, carries into the low-energy $(\frac{1}{2},\frac{1}{2})$ continuum states. It appears to be another example to add to those of Dyson[19] where an approximate symmetry works much better than expected.

V. $I=\frac{1}{2}$, THREE-BODY SUPPRESSION

The demonstration that the model is a reasonable approximation to the $(\frac{1}{2},\frac{1}{2})$ continuum states permits us to use it in explaining the suppression of the isospin-$\frac{1}{2}$ three-body photodisintegration compared to the isospin-$\frac{3}{2}$. The key is to compare the model equations with the isospin-$\frac{3}{2}$ equations. To emphasize it again (see Sec. III C), we first note that the isospin amplitudes enter incoherently into the calculation of the total cross section, but the isospin-$\frac{1}{2}$ amplitude is multiplied by $-(\frac{2}{3})^{1/2}$ and the isospin-$\frac{3}{2}$ by $(\frac{1}{3})^{1/2}$ [see Eq. (70)].[20] These factors are compensated by the fact that the isospin-$\frac{1}{2}$ amplitude is smaller by a factor of $\sqrt{2}$ compared to the isospin-$\frac{3}{2}$ [compare Eqs. (56) and (58)]. Finally, the dynamical equations which generate the key functions, $\mathcal{H}(1,\overline{23})$ for isospin-$\frac{1}{2}$ and $W(1,\overline{23})$ for isospin-$\frac{3}{2}$, are identical *except* for the interactions [compare Eqs. (45) and (50′)]. Therefore, the only difference is that the isospin-$\frac{3}{2}$ results are generated from V^{01}, whereas the isospin-$\frac{1}{2}$ results are generated from V^+ in the model. The suppression of the isospin-$\frac{1}{2}$ three-body photodisintegration of ^{3}He compared to the isospin-$\frac{3}{2}$ is therefore due to a difference in the properties of V^+ compared to V^{01}.

The difference between V^{01} and V^+ has already

TABLE II. Isospin and total bremsstrahlung–weighted sum rules (mb).

Case	two–body	isospin-$\frac{1}{2}$ three–body	total	isospin-$\frac{3}{2}$ three–body	total
Tabakin	0.89 ± 0.02	0.29 ± 0.01	1.18 ± 0.03	1.08 ± 0.02	2.26 ± 0.05
Model	0.79 ± 0.02	0.34 ± 0.01	1.13 ± 0.03		2.21 ± 0.05

been pointed out in Sec. III C. V^{01} is the s-wave, spin-singlet two-nucleon interaction which does not possess a bound state. On the other hand, V^+ is the average of the s-wave spin-singlet and triplet two-nucleon interactions, which, due to the nature of the effective-range parameters of the two-nucleon system, does possess a bound state, i.e., it is a stronger interaction than V^{01}. If the strength of V^{01} were arbitrarily increased in our calculations to values which lead to a two-body bound state, the isospin-$\frac{3}{2}$ three-body breakup cross section would decrease as the contribution to an isospin-$\frac{3}{2}$ two-body cross section increased. The isospin-$\frac{1}{2}$ three-body breakup is suppressed because the effective, or dominant, interaction in this channel is V^+ which possesses a bound state. *If* V^{01} were weaker such that V^+ did not support a bound state, the two-body photodisintegration of ^{3}He would be much suppressed and three-body photodisintgeration greatly enhanced compared to the actual physical situation. The bremsstrahlung-weighted sum rule requires that any enhancement of the two-body-breakup channel contribution necessarily leads to a decrease in the three-body breakup channel contribution since their sum remains constant:

$$J_{(1/2)\,(\text{total})} = J_{(1/2)\,(Nd)} + J_{(1/2)\,(3N)} = \tfrac{2}{3}\pi^2 \alpha R^2 \ .$$

In exact three-body calculations, the three-body channel "knows" about the two-body channel through the dynamics, i.e., the kernel of the model equation has a pole corresponding to the existence of the bound state of V^+, whereas the isospin-$\frac{3}{2}$ equation has no such pole. The amount of suppression is related to the strength of V^+.

The above comparison was made possible by invoking the inherent symmetries possessed by the $(\frac{1}{2}, \frac{1}{2})$ states of the three-nucleon system. What had previously been established as an important symmetry of the three-nucleon ground states was extended into the low-energy continuum of the $(\frac{1}{2}, \frac{1}{2})$ states. Then, by comparing the isospin-$\frac{1}{2}$ model equations with the isospin-$\frac{3}{2}$ equations, and noting that only the spatial functions which generate mixed-symmetry components of the $(\frac{1}{2}, \frac{1}{2})$ states enter the isospin-$\frac{1}{2}$ amplitudes, it is apparent that the only difference is V^+ in place of V^{01}. Therefore, it is not the deuteron pole or triplet interaction alone which is responsible for the suppression. It is instead the symmetries of the $(\frac{1}{2}, \frac{1}{2})$ states which make V^+ the dominant interaction and the fact that the two-nucleon s-wave effective-range parameters are such that V^+ supports a bound state which combine to suppress the isospin-$\frac{1}{2}$ three-body photodisintegration.

VI. CONCLUSION

In summary, the suppression of the isospin-$\frac{1}{2}$, three-body photodisintegration of ^{3}He stems from two points: (1) The symmetries of the $(\frac{1}{2}, \frac{1}{2})$ states lead to $V^+ = \frac{1}{2}(V^{01} + V^{10})$ being the effective (dominant) interaction for isospin-$\frac{1}{2}$. (2) V^+ supports a bound state due to the nature of the low-energy two-nucleon parameters. The first point is a statement of the approximate validity of the Wigner supermultiplet theory for the continuum. The second point states the physics of the s-wave *NN* interaction: V^{10} is more "bound" than V^{01} is "unbound" such that the average V^+ is itself "bound." Therefore, in the approximation where the $(\frac{1}{2}, \frac{1}{2})$ states are generated from V^+ alone, a portion of the isospin-$\frac{1}{2}$ bremsstrahlung-weighted sum rule must come from two-body breakup with a corresponding reduction in the three-body breakup, since the total isospin-$\frac{1}{2}$ sum rule is a constant. The fact that the reduction of the three-body-breakup contribution is substantial is related to the strength of V^+.

If V^+ supported no bound state, the model would show explicitly that most of the isospin-$\frac{1}{2}$ photodisintegration cross section would lie in the three-body channel. The fact that V^+ does possess a bound state (because V^{10} is more "bound" than V^{01} is "unbound") is the underlying cause of the transfer of cross section from the isospin-$\frac{1}{2}$ three-body channel to the two-body channel through the off-shell singlet amplitude as was discussed in GL1.

In closing, it appears that whether a given reaction is dominated by the particular channel having the lowest threshold,[21] as is the case here where the isospin-$\frac{1}{2}$ two-body channel robs the isospin-$\frac{1}{2}$ three-body channel, depends upon whether the dominant effective interactions support the bound system corresponding to that lowest threshold.

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