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$N + d$ clustering of three-nucleon systems: D-state effects in (d,t) and $(d,{}^3\mathrm{He})$ reactions

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The S- and D-state radial components u_0 and u_2 of the relative motion between the clusters $d+n$ in ^{3}H necessary in a full finite range distorted-wave Born-approximation analysis of (d,t) reactions, are calculated using realistic triton and deuteron wave functions derived from the Reid soft-core potential. The parameter D_2 which provides a measure of the asymptotic D state to S state ratio is found to be almost entirely determined by the triton D state, the deuteron D-state contribution being about 10%. Using the Strayer and Sauer triton wave function, the value $D_2 = -0.17$ fm^2 is obtained after correcting the asymptotic behavior of the d-t overlap. This result suggests that the Reid soft-core potential overestimates D_2 by about 20% compared with values extracted from a local energy approximation analysis of (d,t) tensor analyzing power data. The difference between D_2 for (d,t) and $(d,{}^3\mathrm{He})$ is discussed.

[NUCLEAR STRUCTURE ^{3}H; calculated S and D states of the overlap integral with deuteron; deduced D_0 and D_2 for (d,t) and $(d,{}^3\mathrm{He})$ reactions.]

I. INTRODUCTION

It is well known that the deuteron D state has a very strong effect on the tensor analyzing powers of (d,p) and (d,n) reactions.[1] Similar effects have been shown to be present in (d,t) and $(d,{}^3\mathrm{He})$ reactions.[2-4] In these reactions it is the D-state component of the relative motion between the clusters $N+d$ in ^{3}H and ^{3}He which can have quite large effects on the tensor analyzing powers. When finite range effects are included in the distorted-wave Born-approximation (DWBA) theory using the local energy approximation[5] (LEA) the magnitude of the D-state effects is completely determined by a single parameter D_2 which can be adjusted in order to fit the data. The value of D_2 for (d,t) reactions calculated[2] with a very simple triton wave function is in reasonable agreement with the values of D_2 extracted from an LEA analysis of tensor analyzing power measurements.

In the present work we have calculated the S- and D-state components of the relative motion between the $d+n$ clusters in ^{3}H. From these relative motion wave functions we can calculate D_2 and discuss the information on the three-nucleon bound system contained in this parameter. The same wave functions are required in a full finite range DWBA calculation for (d,t) reactions which is required in order to include properly the D-state effects and therefore test the accuracy of the LEA.

In the DWBA the transition amplitude for a (d,t) reaction depends on the internal structure of the deuteron and triton through the matrix element

$$F(\vec{r})=\langle \chi_{1/2}^{\sigma_n}\varphi_d^{\sigma_d}(\vec{\rho})|V_{dn}|\varphi_t^{\sigma_t}(\vec{r},\vec{\rho})\rangle, \tag{1}$$

where V_{dn} is the interaction between the deuteron and the transferred neutron, $\varphi_d^{\sigma_d}$ and $\varphi_t^{\sigma_t}$ are the internal wave functions of the deuteron and triton, and $\chi_{1/2}^{\sigma_n}$ is the spin wave function of the transferred neutron. The vector $\vec{\rho}$ is the internal coordinate of the deuteron and $\vec{r}$ is the separation of the transferred neutron from the deuteron center of mass. Using the Schrödinger equation for the three-nucleon bound system we can write

$$F(\vec{r})=-(T+B)G(\vec{r}), \tag{2}$$

where

$$G(\vec{r})=\langle \chi_{1/2}^{\sigma_n}\varphi_d^{\sigma_d}(\vec{\rho})|\varphi_t^{\sigma_t}(\vec{r},\vec{\rho})\rangle. \tag{3}$$

T is the kinetic energy operator in the coordinate $\vec{r}$ and B is the difference between the binding energies of the triton and deuteron. Equation (3) shows that all information on the deuteron and triton internal structure which can be obtained from a DWBA analysis of (d,t) reactions is contained in the overlap integral $G(\vec{r})$.

From general angular momentum and parity selection rules we conclude that $G(\vec{r})$ can contain only S- and D-state terms. Thus we can write

$$G(\vec{r})=\sum_{L=0,2}\sum_{\Lambda\sigma}u_L(r)Y_L^{\Lambda}(\hat{r})(L\Lambda s\sigma|\tfrac{1}{2}\sigma_t)\times(1\sigma_d\tfrac{1}{2}\sigma_n|s\sigma), \tag{4}$$

where s is the spin transfer; $s=\frac{3}{2}$ for $L=2$ and $s=\frac{1}{2}$ for $L=0$. The radial wave functions u_0 and u_2 describe the relative motion between the deuteron cluster and the remaining neutron in the triton. In low energy (d,t) reactions the dependence of the DWBA transition amplitude on u_0 and u_2 is to a large extent determined by the values of the zero-range normalization constant

$$D_0=-\sqrt{4\pi}\,B\int_0^\infty u_0(r)r^2dr \tag{5}$$

and the D-state parameter

$$D_2=\frac{1}{15}\frac{\int_0^\infty u_2(r)r^4dr}{\int_0^\infty u_0(r)r^2dr}\,. \tag{6}$$

Due to the factors r^2 and r^4, D_0 and D_2 are particularly sensitive to the tail of the radial wave functions u_0 and u_2. In the asymptotic region they go as

$$u_0^{\text{asymp}}(r)=C(2\alpha)^{1/2}\frac{e^{-\alpha r}}{r}, \tag{7}$$

$$u_2^{\text{asymp}}(r)=\epsilon C(2\alpha)^{1/2}\left(1+\frac{3}{\alpha r}+\frac{3}{(\alpha r)^2}\right)\frac{e^{-\alpha r}}{r}, \tag{8}$$

where $\alpha=(3B/4M\hbar^2)^{1/2}$, C and ϵ are dimensionless constants.

C is a well known parameter of the three-nucleon bound system which can be determined from various types of experiments[6] and in particular is related to D_0. An estimate of C can be obtained by substituting Eq. (7) into Eq. (5):

$$C\cong-\left(\frac{\alpha^3}{8\pi}\right)^{1/2}\frac{D_0}{B}. \tag{9}$$

On the other hand the only source of information on the asymptotic D-state to S-state ratio ϵ is provided by D_2. From Eqs. (6), (8), and (9) we obtain

$$\epsilon\cong\alpha^2D_2. \tag{10}$$

This approximation is known[7] to be very accurate in the deuteron with deviations of the order of 1%. In this case the deuteron quadrupole moment Q also gives as a rough estimate $\epsilon\cong\sqrt{2}\,Q\alpha^2$.

II. CALCULATIONS OF u_0 AND u_2 WITH REALISTIC WAVE FUNCTIONS

From the angular momentum coupling in the triton we can easily identify the contributions from different components of the deuteron and triton wave functions to the S- and D-state parts of the overlap function $G(\vec{r})$. The orbital angular momentum $\vec{L}$ in Eq. (4) is given by

$$\vec{L}=\vec{s}_t-(\vec{s}_1+\vec{s}_2+\vec{s}_3)-\vec{l}, \tag{11}$$

where $\vec{s}_t$ is the triton angular momentum, $\vec{s}_i$ are the three nucleon spins, and $\vec{l}$ is the deuteron orbital angular momentum. Thus

$$\vec{L}=\vec{l}'-\vec{l}, \tag{12}$$

where $\vec{l}'$ is the triton orbital angular momentum. Representing by $u_L^{ll'}$ the contribution to the radial wave function u_L from the overlap between the deuteron wave function component with orbital angular momentum l and the triton wave function component with orbital angular momentum l' and taking into account Eq. (12) we can write

$$u_0=u_0^{00}+u_0^{22}, \tag{13}$$

$$u_2=u_2^{02}+u_2^{20}+u_2^{22}. \tag{14}$$

Notice that the D-state part of the overlap can only be nonzero if either the deuteron or the triton wave function contains a D-state term. Furthermore, the most important contribution to u_2 comes from the triton D state and is therefore given by u_2^{02}. On the other hand u_0 is almost entirely determined by u_0^{00} except for a predictably small contribution from the overlap between the deuteron and triton D states.

In the present calculations of the overlap function $G(\vec{r})$ we use the deuteron wave function of Reid[8] (soft core)

$$\varphi_d^{\sigma_d}(\vec{\rho})=\sum_{l=0,2}(lm1\sigma|1\sigma_d)\omega_l(\rho)Y_l^m(\hat{\rho})\chi_1^{\sigma}\eta_2^{\tau}, \tag{15}$$

where χ_1^{σ} and η_2^{τ} are the spin triplet and isospin singlet wave functions.

A. Approximate triton D state of Jackson and Riska

In order to estimate D_2 for (d,t) reactions we have first used the model triton D state of Jackson and Riska,[9] which is obtained in first order perturbation theory. In momentum space it is given by

$$\langle\vec{q},\vec{k}\,|D\rangle=\frac{1}{B_t-q^2/m-3k^2/4m}\times\langle\vec{q},\,\vec{k}|V_T^{12}+V_T^{23}+V_T^{13}|S\rangle, \tag{16}$$

where $\vec{q}$ and $\vec{k}$ are the momenta conjugate to $\vec{\rho}$ and $\vec{r}$, respectively, B_t is the triton binding energy, V_T^{ij} is the tensor force between nucleons i and j, and $|S\rangle$ is the normalized triton S-state wave function. This simple approximate D-state wave function gives a very accurate description of the asymptotic region which makes it particularly convenient for our purpose.

The three tensor force terms in Eq. (16) give quite different contributions to u_2. In order to analyze these contributions we represent by 1 and 2 the nucleons in the deuteron and by 3 the transferred neutron. Apart from the energy factor, present in the three terms, the tensor interaction between nucleons 1 and 2 yields

$$\langle\vec{q},\vec{k}|V_T^{12}|S\rangle=\int e^{i(\vec{q}\cdot\vec{\rho}+\vec{k}\cdot\vec{r})}V_T(\rho)S_{12}\psi_S\,d\vec{\rho}\,d\vec{r}, \tag{17}$$

where

$$\psi_S=v(\vec{r},\vec{\rho})\xi_0 \tag{18}$$

is the triton S-state wave function and

$S_{12}=(\vec{\sigma}_1\cdot\hat{\vec{\rho}})(\vec{\sigma}_2\cdot\hat{\vec{\rho}})-\frac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2$

is the tensor force operator. In Eq. (18)

$$\xi_0=\frac{1}{\sqrt{2}}(\chi_1\eta_2-\chi_2\eta_1) \tag{19}$$

is the full antisymmetric spin and isospin part of the S state, $\chi_1(\eta_1)$ and $\chi_2(\eta_2)$ represent 3-nucleon spin (isospin) wave functions,[10] and $v(\vec{r},\vec{\rho})$ its normalized radial part. Performing the angular integration in Eq. (17) we obtain

$$\langle\vec{q},\vec{k}|V_T^{12}|S\rangle=-4\left(\frac{6}{5\pi}\right)^{1/2}\sum_m Y_2^{m*}(\hat{\vec{q}})[\sigma_1\times\sigma_2]_m{}^2\xi_0\int j_0(kr)\,j_2(q\rho)V_T(\rho)v(\vec{r},\vec{\rho})r^2\rho^2dr\,d\rho, \tag{20}$$

where $[\sigma_1\times\sigma_2]_m{}^2$ represents the second rank tensor product of the spin operators and j_L the spherical Bessel functions.

For the tensor interaction between the particles 2 and 3 an analogous derivation leads to the following expression:

$$\begin{aligned}\langle\vec{q},\vec{k}|V_T^{23}|S\rangle=\frac{8\sqrt{6}}{5}\sum_m(-1)^m[\sigma_3\times\sigma_2]_{-m}^2\xi_0\sum_{LL'l'\alpha\beta}[Y_\alpha(\hat{\vec{q}})\times Y_\beta(\hat{\vec{k}})]_m{}^2\\ \times i^{L'-L}(2L+1)(2L'+1)(2l'+1)(L'0l'0|\beta 0)(L0l'0|\alpha 0)(L0L'0|20)\\ \times\begin{Bmatrix}L & L' & 2\\ \beta & \alpha & l'\end{Bmatrix}\int V_T(\rho)v(\vec{r},\vec{\rho})j_L(\tfrac{1}{2}q\rho)j_{L'}(\tfrac{3}{4}k\rho)j_{l'}(\tfrac{1}{2}kr)j_{l'}(qr)\rho^2r^2d\rho\,dr.\end{aligned} \tag{21}$$

The tensor interaction between the particles 1 and 3 gives an identical expression except that σ_3 and σ_2 are substituted by σ_1 and σ_3 respectively.

For the triton S state we use the wave function of Jackson, Lande, and Sauer[11] and we take only the main component[12] that has no angular dependence on $\vec{r}$ or $\vec{\rho}$. The triton D state is generated according to Eq. (12) from the same S-state wave function. With this model for the triton u_2^{20} is zero because v is a scalar in $\vec{\rho}$. Due to the same reason the only contributions to u_2^{02} arise from the tensor force between the nucleon pairs 13 and 23. The term u_2^{22} in Eq. (14) is probably small and was not estimated with these wave functions.

Since the approximate triton D-state wave function is given in momentum space it is particularly easy to calculate D_2 directly. If we represent the Fourier transform of the overlap $G(\vec{r})$ by

$$f_L(k)Y_L^\Lambda(\hat{\vec{k}})=\int d\vec{r}\,e^{i\vec{k}\cdot\vec{r}}u_L(r)Y_L^\Lambda(\hat{\vec{r}}), \tag{22}$$

the radial function $f_2(k)$ has the following expansion for small momenta:

$$f_2(k)=a_2k^2+a_4k^4+\cdots. \tag{23}$$

It can be easily proved[13] from Eqs. (6) and (22) that D_2 is proportional to the coefficient of the k^2 term and is given by

$$D_2=-a_2\bigg/\left(4\pi\int_0^\infty u_0(r)r^2dr\right). \tag{24}$$

In first order perturbation theory with the wave function described we obtain a triton D state with a probability of 4% and $D_2=-0.14$ fm^2. The calculation can be improved if we introduce a normalization constant on the right-hand side of Eq. (16) and require that the sum of the triton S- and D-state probabilities to be equal to 1. Our final result for D_2 is then

$$D_2=-0.648[P_D/(1-P_D)]^{1/2}\ \text{fm}^2, \tag{25}$$

where P_D is the triton D-state probability. For a triton D-state probability of 8%, Eq. (25) gives $D_2=-0.20$ fm^2.

B. Triton wave function of Strayer and Sauer

In view of the rather crude approximations involved in the previous calculation we have chosen a more realistic triton wave function to obtain the radial overlap functions u_0 and u_2. We use the triton wave function of Strayer and Sauer[14] which is generated through a variational calculation from the soft-core Reid nucleon-nucleon potential and gives a reasonable agreement with the three-nucleon ground state properties. It is expressed in a translationally invariant basis of harmonic oscillator functions which is convenient to perform the numerical calculation of the overlap with the deuteron.

Following essentially the same notation of Ref. 14 we write the projection of the triton wave function into an isospin zero state for particles 1 and 2, as

$$\varphi_t^{\sigma_t}(\vec{r},\vec{\rho}) = \sum_{\substack{l'=0,2\\ \lambda'\sigma'}} \zeta_{l'}\, b_{l'}\,(l'\lambda' s'\sigma'|\tfrac{1}{2}\sigma_t) \sum_{\substack{N_1L_1N_2L_2\,(L_1+L_2=\text{even})\\ n_1l_1n_2l_2\ (l_1\,\text{even})}} C(N_1L_1N_2L_2)\, i^{l_1+2n_1}\langle n_1l_1n_2l_2,\, l'\{|N_1L_1N_2L_2,\, l'\rangle \times \sum_{m_1m_2}(l_1m_1l_2m_2|l'\lambda')\,|n_1l_1m_1\rangle |n_2l_2m_2\rangle\,, \qquad (26)$$

where

$$b_0=\left(\frac{2}{1+\delta_{L_1L_2}\delta_{N_1N_2}}\right)^{1/2},\quad b_2=(2)^{1/2}$$

$$\zeta_0=\frac{1}{\sqrt{2}}\chi_1\eta_2,\quad \zeta_2=\frac{1}{\sqrt{2}}\chi_3\eta_2$$

and χ_3 is the $J=\frac{3}{2}$ three-nucleon spin wave function. The $C(N_1L_1N_2L_2)$ are the same coefficients as in Eq. (1) of Ref. 14, $\langle n_1l_1n_2l_2,\, l'\{|N_1L_1N_2L_2,\, l'\rangle$ are Brody-Moshinsky brackets, $|n_1l_1m_1\rangle$ and $|n_2l_2m_2\rangle$ are harmonic oscillator wave functions in the coordinates $\vec{\rho}$ and $\vec{r}$, respectively. The energy of the various terms in the above expansion is characterized by the number of oscillator quanta

$$Q=2n_1+l_1+2n_2+l_2.$$

In the present calculations the harmonic oscillator basis is truncated at $Q=50$. This wave function underbinds the triton by 1.8 MeV.

The various terms in Eqs. (13) and (14) which sum up to u_0 and u_2 were obtained by performing the overlap in the coordinate $\vec{\rho}$ between the expressions (15) and (26) and are shown in Fig. 1. As expected[2] we find that u_2^{20} is considerably smaller than u_2^{02}. The term u_2^{22} is likely to be even smaller than u_2^{20} and was not calculated. As regards u_0 we find that u_0^{22} is always about one order of magnitude smaller than u_0^{00}. In the present calculations we have not included the triton S'-state component due to its very small probability and because it contributes to u_2 only through the small overlap with the deuteron D state.

The total radial overlap functions u_0 and u_2 are represented in Fig. 2. From these we obtain the value $D_2=-0.113$ fm^2. The effect of the deuteron D state is to decrease D_2 by only about 10%. Neglecting u_0^{22} and u_2^{20} we obtain $D_2=-0.126$ fm^2. The decrease in D_2 results from the fact that u_2^{20} and u_2^{02} have opposite sign in the important asymptotic region while u_0^{00} and u_0^{22} have the same sign.

From u_0 we obtain $D_0=-177.6$ MeV fm$^{3/2}$ which is in good agreement with other recent calculations.[15] This value includes the small contribution from the overlap between the deuteron and triton D states which has not been estimated previously. Neglecting u_0^{22} we obtain $D_0=-165.0$ MeV fm$^{3/2}$.

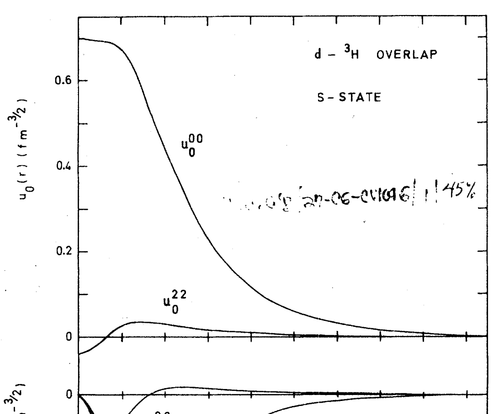

FIG. 1. Contributions to the radial wave functions u_0 and u_2 of the S- and D-state components of the d-^{3}H overlap as defined in Eqs. (13) and (14). u_0^{00} and u_0^{22} result from the overlap between the deuteron S state and triton D state and u_2^{20} results from the overlap between the deuteron D state and the triton S state. The total u_0 and u_2 are represented in Fig. 2.

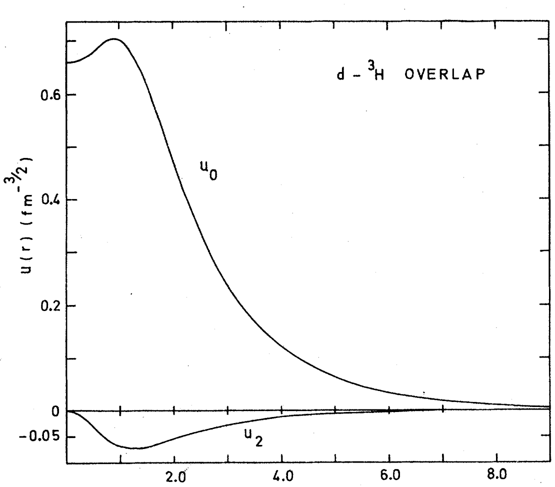

FIG. 2. Total radial wave functions u_0 and u_2 of the S- and D-state components of the d-^{3}H overlap calculated with the Strayer and Sauer triton wave function.

C. Comparison of D_2 for ^{3}H and ^{3}He

The difference between D_2 for ^{3}H and ^{3}He is probably mainly determined by the different binding energy of the two nuclei. Using Eq. (10) we obtain

$$\frac{D_2(^3\text{He})}{D_2(^3\text{H})} \cong 1.138 \frac{\epsilon(^3\text{He})}{\epsilon(^3\text{H})} . \tag{27}$$

If we take into account that asymptotically the overlap function d-^{3}He behaves as a Whittaker function, instead of a Hankel function, the multiplicative constant in Eq. (27) is slightly reduced[4] to 1.089.

When comparing the above ratio with values of D_2 extracted from DWBA analysis of (d, t) and $(d, {}^3\text{He})$ reactions we have to consider that the definition of the reaction form factor may not be the same in the two cases. In the conventional form of the DWBA matrix element[16] for $(d, {}^3\text{He})$ reactions the Coulomb part of the d-p interaction should be omitted from Eq. (1) when the distorted wave in the exit channel is generated with the total Coulomb interaction between ^{3}He and the residual nucleus. The main effect on D_2, because of its sensitivity to the tail of the overlap, is due to the long range part in r of the Coulomb d-p interaction. With this approximation the function $F(\vec{r})$ becomes

$$F(\vec{r}) \cong -\left(T+B+\frac{e^2}{r}\right)G(\vec{r}), \tag{28}$$

where $G(\vec{r})$ is the overlap between the deuteron and ^{3}He. Considering, as in Eq. (10), that the asymptotic region has a dominant role we obtain

$$\frac{D_2(^3\text{He})}{D_2(^3\text{H})} \cong 1.085 \frac{\epsilon(^3\text{He})}{\epsilon(^3\text{H})} . \tag{29}$$

The ratio decreases since the numerator in Eq. (6), because of the r^4 factor, is less sensitive than the denominator to the Coulomb term. The effect of omitting the Coulomb part of the d-p interaction is therefore predicted to be smaller in D_2 than in D_0. The multiplicative constant in Eq. (29) decreases to 1.035 when the Hankel functions are replaced by Whittaker functions.

To determine $D_2(^3\text{He})$ from Eq. (29) we would also have to estimate $D_2(^3\text{He})/D_2(^3\text{H})$. This quantity is probably close to one since ϵ is mainly determined by the intermediate and long range tensor force and therefore it is not very sensitive to Coulomb effects. This implies that $D_2(^3\text{He})$ is larger than $D_2(^3\text{H})$. However, the precise determination of $D_2(^3\text{He})$ requires the use of a reliable ^{3}He wave function.

III. DISCUSSION

The values of D_2 extracted from a DWBA analysis of the tensor analyzing powers of (d, t) reactions vary within the range -0.22 fm^2 to -0.30 fm^2. For the $^{118}\text{Sn}(d, t)\,^{117}\text{Sn}$ and $^{208}\text{Pb}(d, t)\,^{207}\text{Pb}$ Knutson *et al.*[2] obtain $D_2 = -0.24$ fm^2. Further calculations for the $^{208}\text{Pb}(d, t)\,^{207}\text{Pb}$ reaction lead to a slightly smaller value[17] of -0.22 fm^2. For the $^9\text{Be}(d, t)^8\text{Be}$ reaction[18] the agreement between theory and experiment is not good enough to enable the determination of D_2. More recent data[3] for the $^{64}\text{Zn}(d, t)\,^{63}\text{Zn}$ favors a larger value of $D_2 = -0.30$ fm^2.

The reliability of this information on D_2 is of course limited by the accuracy of the DWBA theory and of the approximations involved in its application. We note, however, that the predictions for reactions that are carried out at energies below the Coulomb barrier are considerably more reliable than for reactions where the transfer can take place deep inside the nucleus. It is significant that the agreement between theory and experiment is considerably better for the heavier targets. D_2 between -0.22 fm^2 and -0.24 fm^2 is therefore likely to be the most reliable present estimate which can be obtained from (d, t) reactions. They are in reasonable agreement with the value $D_2 = -0.20$ fm^2 calculated with the triton wave function of Jackson and Riska.

For the $^{27}\text{Al}(d, {}^3\text{He})^{26}\text{Mg}$ reaction the best fit to the data is obtained[4] with $D_2 = -0.22$ fm^2 while for the $^{64}\text{Zn}(d, {}^3\text{He})^{63}\text{Cu}$ it is[3] $D_2 = -0.37$ fm^2. As the tensor analyzing powers for the mirror reaction have also been measured,[3] it is therefore possible to deduce directly from experiment that for this pair of reactions $D_2(^3\text{He})/D_2(^3\text{H}) = 1.27 \pm 0.13$, which is qualitatively in agreement with the theoretical predictions.

The rather small value obtained for D_2 with the Strayer and Sauer wave function can be a consequence of the fact that u_0 and u_2 do not have the correct asymptotic behavior. Owing to the nature of the harmonic oscillator functions generated in an infinite potential well, the convergence of the expansion is very slow in the asymptotic region. In fact the calculated u_0 and u_2 are not proportional to the Hankel functions $h_0(i\alpha r)$ and $h_2(i\alpha r)$, respectively, for very large r. Furthermore, we have to consider that the value of α corresponds to a triton binding energy of 6.7 MeV instead of the experimental value of 8.48 MeV.

In order to estimate the effect on D_2 of the anomalous behavior of u_0 and u_2 in the asymptotic region we have assumed that in this region they are proportional to h_0 and h_2. For α we take the value that corresponds to a triton binding energy of -7.3 MeV which is the minimum extrapolated value[19] that can be obtained with the Reid soft-core potential. By varying C and ϵ in Eqs. (7) and (8) we can match the calculated u_0 and u_2 to $u_{0\,\mathrm{asym}}$ and $u_{2\,\mathrm{asym}}$. The matching points for the S and D states occur respectively at 3.5 and 5.0 fm where only the long range part of the central and tensor force acts. This may be interpreted as an indication that the calculated u_0 and u_2 provide an accurate description of the overlap function at smaller distances.

With the wave functions corrected in the asymptotic region we obtain $D_2=-0.17\ \mathrm{fm}^2$. Thus, assuming that these wave functions give a reliable representation of the most accurate overlap which can be obtained with the Reid soft-core potential, we conclude that this potential overestimates D_2 by about 20%.

It is a rather unexpected result since D_2 is essentially a measure of the relative strength of the intermediate and long range tensor force relative to the central force and the Reid soft-core potential has a strong tensor component giving a deuteron D-state probability of 6.47%.

Full finite range calculations for the tensor analyzing powers of (d,t) and $(d,{}^3\mathrm{He})$ reactions are necessary to determine whether the disagreement between theory and experiment, particularly for the light targets, is a consequence of the LEA and therefore determine the reliability of the values of D_2 which have been extracted from experiment. The present radial overlap wave functions can be used in full finite range (d,t) and $(d,{}^3\mathrm{He})$ calculations provided that they are corrected in the asymptotic region.

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