Nuclear charge dependent interactions in the 1p shell

R. D. Lawson
Argonne National Laboratory, Argonne, Illinois 60439
(Received 5 June 1978; revised manuscript received 12 March 1979)

A shell model analysis of the isobaric mass multiplets in the 1p shell shows that the isotensor charge dependent contribution (the coefficient of T_z^2 in the multiplet relationship) is, in all cases save one, underestimated when only electromagnetic interactions [including $(v/c)^2$ corrections, vacuum polarization, the effect of short range correlations, and finite neutron and proton size] are considered. Furthermore, the discrepancy is nearly twice as large for the seniority zero levels as for the higher seniority states, indicative of the omission of a short range charge dependent interaction. Since the free nucleon-nucleon scattering data indicate that the (np) interaction is $\sim 2\%$ more attractive than the (nn), the effect of including such a charge dependent nuclear interaction on this coefficient is examined. When a Yukawa potential with range given by the π-meson Compton wavelength is taken for this potential it is found that theory and experiment can be brought into much better agreement.

[NUCLEAR STRUCTURE Shell model calculation of isotensor coefficient in mass-multiplet relationship. 1p shell studied; need for isotensor nucleon-nucleon interaction demonstrated.]

I. INTRODUCTION

In this paper we show that an analysis of isobaric multiplets in the 1p shell leads to the conclusion that electromagnetic interactions alone cannot explain the observed energy differences. This is demonstrated by examining the c coefficient in the equation relating the energies of states in a given multiplet[1]

$$E = a + bT_z + cT_z^2 . \tag{1}$$

Several methods are discussed for calculating the Coulomb interaction and in all cases the computed value of c is found to be smaller than the experimental one.

In first order perturbation theory, provided isospin is a good quantum number, c depends only on the isotensor part of the residual interaction. If in addition to the Coulomb interaction one includes the isotensor part of the $(v/c)^2$ corrections to the electromagnetic interaction, the effects of short range correlations, vacuum polarization, and the finite size of the neutron and proton, theory and experiment are brought into better agreement. However, even when all these effects are included in every case, except one, the experimental value of c is greater than the theoretical prediction. Moreover, the rms error in c is about a factor of 2 larger for the seniority zero states than for the higher seniority levels—for the two seniority zero states it is 56.7 keV whereas for the nine higher seniority levels it is 28.1 keV. Such an effect is exactly what one would expect if a short range isotensor interaction had been neglected.

Since the free nucleon-nucleon scattering data indicate that the $T=1$ (np) interaction is approximately[2] 2% more attractive than the (nn) potential this is a possible source of the short range interaction. We have, therefore, examined what effect an increase of V_{np} over V_{nn} or V_{pp} would have on the calculated value of c. Although the difference between V_{np} and V_{nn} can arise from both an isovector and isotensor interaction, the former does not contribute to c in lowest order perturbation theory and consequently only the latter is considered. If we take an isotensor interaction V_{CD} which makes V_{np} more attractive than V_{nn} by 2% in the $T=1$ spin-singlet state the rms error in c is reduced by almost a factor of 2. Without the charge dependent nuclear force the rms error in c is 35.1 keV, whereas when it is included the error becomes 18.1 keV. Furthermore, the rms error in the seniority zero values becomes almost the same as that for the higher seniority levels, 21 keV for the former and 17.4 keV for the latter. Consequently, the addition of an isotensor interaction of the magnitude required by the free nucleon-nucleon scattering data materially improves the fit to c. However, in view of the uncertainties in calculating c one cannot deduce a reliable value for $\{|V_{np}| - |V_{nn}|\}$—all one can say is that the data imply that V_{np} is more attractive than V_{nn} or V_{pp}.

II. DATA AND RESULTS

In column 4 of Table I we list the experimental values of c in keV. In general these have been obtained by use of the binding energy tables of

TABLE I. Values of the coefficient c, Eq. (1), for the $1p$ shell. The Coulomb results were calculated using Woods-Saxon eigenfunctions with the binding energy prescription of Eq. (5). Other electromagnetic contributions, $(a)-(e)$ discussed in Sec. II B, are included in column 6. These were calculated using oscillator wave functions with $\alpha = 0.48$ fm^{-1} for $A = 8$ and 9, $\alpha = 0.53$ fm^{-1} for $A = 10$, 12, and 13, and $\alpha = 0.575$ fm^{-1} for $A = 14$ [$R_{1p} \sim r \exp(-\frac{1}{2}\alpha^2 r^2)$]. (These values of α more or less reproduced the rms radii of the Woods-Saxon eigenfunctions.) The last two columns give the theoretical values of c when the nuclear isotensor potential of Eq. (14) is included and evaluated with the values of α cited above. The rms error has been evaluated on the assumption that the experimental value for the $A = 8$ 3^+ state is 223 keV.

			c in keV				
						Charge dependent nuclear interaction	
					Coulomb	$\tilde{V} = 0.3$ MeV	
A	I	T	Expt.	Coulomb	+(a)−(e)	$Q = 1$	$Q = -\frac{5}{9}$
	1	1	208	201	198	225	201
8	2	1	223	190	197	216	199
	3	1	≤223	214	216	258	239
9	$\frac{1}{2}$	$\frac{3}{2}$	240	205	207	243	224
	$\frac{3}{2}$	$\frac{3}{2}$	264	210	212	247	229
10	0	1	363	287	296	385	379
	2	1	297	251	260	324	314
12	1	1	246	216	221	255	240
	2	1	208	204	213	238	201
13	$\frac{3}{2}$	$\frac{3}{2}$	256	226	234	278	255
14	0	1	338	282	294	378	363
rms error in keV				40.7	35.1	23.6	18.1

Wapstra and Bos[3] together with the compilations of Ajzenberg-Selove.[4] There are two exceptions: First, the value of c for the $A=9$ states was taken from the work of Kashy *et al.*[5] Second, the states in ^{8}Be have mixed isospin and since we are interested in the c coefficient for the $T=1$ states we must take this into account. The best available data on the 2^+ states indicate that the 16.627 and 16.911 MeV levels are each ~50% $T=0$ and $T=1$ so that the unperturbed $T=1$ state was taken to lie at 16.769 MeV. As far as the 1^+ states are concerned, Oothoudt and Garvey[6] find that the 18.154 MeV state is about 4.5% $T=1$ and this has been included in making the entry in Table I. Finally, Oothoudt and Garvey assign $T=1$ as the dominant component of the 19.06 3^+ state with an undetermined amount of $T=1$ mixed into the 19.22 MeV level. For this reason only, a limit is placed on c for the 3^+ level.

A. Coulomb effects

To estimate c we have made a conventional shell model calculation in which the sum of the nuclear two-body residual interaction and the Coulomb repulsion between two protons was diagonalized using all possible $1p$-shell basis states and assuming $^4_2$He$_2$ is an inert core. For $A>9$ the (8−16)2BME of Cohen and Kurath[7] were used for the residual nucleon-nucleon interaction and for $A \leq 9$ the Kumar interaction,[8] which was specifically designed to reproduce the data at the beginning of the shell, was used. [There is little difference if the (6−16)2BME of Cohen and Kurath are used instead of the Kumar ones.] The single particle wave functions used to calculate the two-body Coulomb matrix elements were eigenfunctions of

$$H\psi = E\psi \tag{2}$$

with

$$H = p^2/2m - V_0/\{1+\exp[(r-R)/a]\} + \bar{V}_c\,, \tag{3}$$

where

$$\bar{V}_c = \frac{Ze^2}{2R}[3-(r/R)^2] \quad \text{for } r \leq R$$

$$= \frac{Ze^2}{r} \quad \text{for } r > R\,. \tag{4}$$

The parameters of the Woods-Saxon potential were taken to be $a=0.65$ fm, $R=1.2A^{1/3}$ fm (the

effect of changing a and r_0 will be subsequently discussed), and the value of V_0 was adjusted to give the "correct" binding energy. For the results listed in column 5 of Table I the correct binding energy was assumed to be

$$\epsilon = -\left(\frac{E_B({}^A_Z X_N(I,T)) - E_B({}^4_2\mathrm{He}_2)}{(A-4)}\right), \tag{5}$$

where $E_B({}^A_Z X_N(I,T))$ is the binding energy of the nucleus ${}^A_Z X_N$ in the state (I,T)—in other words, each of the valence p-shell nucleons was assumed to have the same binding energy. Thus, for example, in calculating the two-body Coulomb matrix elements for the $I=2$ state in ${}^8_4\mathrm{Be}_4$, solutions of Eq. (2) were used with $\epsilon = -2.859$ MeV, whereas for the $I=2$ state in ${}^8_5\mathrm{B}_3$, $\epsilon = -2.360$ MeV. [We shall show later that using ϵ given by Eq. (5) gives values of c slightly larger, but not appreciably different, from those obtained with other reasonable prescriptions.]

A comparison of columns 4 and 5 shows that c is always underestimated by the shell model calculation. One's first reaction is to ask if this discrepancy can be cleared up if a different prescription is used to obtain ϵ. Before discussing that, however, we would like to make some comments about the $A=14$ system.

The $A=14$ nuclei were treated as two holes and the single-hole energies used in the calculation were taken from the spectra ${}^{15}_7\mathrm{N}_8$ and ${}^{15}_8\mathrm{O}_7$. The energy of the two proton holes in ${}^{14}_6\mathrm{C}_8$ relative to ${}^{16}_8\mathrm{O}_8$ is 22.335 MeV so that the Coulomb interaction was calculated using eigenfunctions for which $\epsilon = -11.1675$ MeV. The Cohen-Kurath matrix elements were used in all three nuclei. However, since the proton and neutron wave functions are different one should really change the nuclear matrix elements as one goes across the multiplet.[9] To estimate what effect this would have, we assume a δ function for the residual nucleon-nucleon interaction and evaluate its matrix elements using different Woods-Saxon eigenfunctions for neutrons and protons. With the δ-function potential the interaction matrix elements are given by products of Clebsch-Gordan coefficients multiplied by $\int R_\nu{}^4 r^2 dr$, $\int R_\nu{}^2 R_\pi{}^2 r^2 dr$, and $\int R_\pi{}^4 r^2 dr$ for the (n,n), (n,p), and (p,p) systems, respectively, where R_ν (R_π) is the neutron (proton) radial wave function. Thus the value of say the (n,n) interaction compared to the (n,p) would be given simply by the ratio $\int R_\nu{}^4 r^2 dr / \int R_\nu{}^2 R_\pi{}^2 r^2 dr$. We assume the Cohen-Kurath matrix elements are the appropriate (n,p) ones and that they scale in this manner. When $\epsilon = -11.1675$ MeV for protons and -14.4443 MeV for neutrons (half the ${}^{14}_8\mathrm{O}_6$ binding relative to ${}^{16}_8\mathrm{O}_8$),

$$\frac{\int R_\nu{}^4 r^2 dr}{\int R_\nu{}^2 R_\pi{}^2 r^2 dr} = 1.0289\,,$$

$$\frac{\int R_\pi{}^4 r^2 dr}{\int R_\nu{}^2 R_\pi{}^2 r^2 dr} = 0.9727\,.$$

When these numerical results are used, for example, in conjunction with the Cohen-Kurath $\langle (p_{3/2})_0{}^2 | V | (p_{3/2})_0{}^2 \rangle$, one finds that this matrix element should have the values $-3.285\,60$, $-3.193\,42$, and $-3.106\,28$ MeV for the (n,n), (n,p), and (p,p) systems, respectively. Similar changes occur for the other two $I=0$ matrix elements and when these are computed the resulting value of c becomes 276 keV—a decrease of 6 keV from the Coulomb value.

Thus, although the change in matrix elements due to different neutron and proton wave functions is quite substantial, the net effect on c is small. The reason for this may be simply seen if one writes the change in the matrix element in terms of isospin tensors of rank zero, one, and two. If we define the change in $\langle (p_{3/2})_0{}^2 | V | (p_{3/2})_0{}^2 \rangle$ to be $\Delta E_0(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})$ this quantity may be written as

$$\frac{\Delta E_0(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})}{E_0(\frac{3}{2}\frac{3}{2};\frac{3}{2}\frac{3}{2})} = a_0 + a_1[\tau_z(1) + \tau_z(2)] + a_2 T_{12}\,, \tag{6}$$

where

$$T_{12} = \tau_z(1)\tau_z(2) - \tfrac{1}{3}\vec{\tau}(1)\cdot\vec{\tau}(2) \tag{7}$$

is an isospin tensor of rank two and has the value $+\frac{2}{3}$ when operating on an (nn) or (pp) state and $-\frac{4}{3}$ when applied to an (np) eigenfunction. From the numerics given in the preceding paragraph it follows that

$$a_0 = 0.000\,53\,,$$

$$a_1 = 0.014\,05\,,$$

$$a_2 = 0.000\,40\,.$$

Consequently, the size change gives an almost isovector contribution and this will not effect the coefficient of $T_z{}^2$ in lowest order unless the nuclear wave functions have mixed isospin. In particular, for the $T=1$ states, which comprise most of our data, only a $T \geq 2$ admixture can give a contribution to c and since we deal with yrast $T=1$ states this is not likely to be important. For the $T=\frac{3}{2}$ states in the $A=9$ and 13 systems, the isovector part of this operator could give a contribution proportional to $T_z{}^2$ if some $T=\frac{1}{2}$ is mixed in. However, there is no experimental evidence for significant isospin mixing in these levels and consequently the difference in neutron and proton wave functions will be neglected from now on.

Another worry is the effect of $2\hbar\omega$ excitations which Goldhammer[10] has found to be important in the mixing of the $T=0$ and $T=1$ levels in the $1p$ shell. To estimate this effect in the $A=14$ nuclei we have used the Kuo matrix elements[11] for the other-than-p-shell part of the residual nucleon-nucleon interaction and have calculated the Coulomb matrix elements assuming $\alpha=0.575$ fm^{-1} [$R_{1p}\sim r\exp(-\frac{1}{2}\alpha^2r^2)$]. The reason for this choice of α is that it reproduces the rms radius given by the Woods-Saxon eigenfunctions. To simplify the calculation we have assumed that the 0^+ state in the $A=14$ system is $(1p_{1/2})^{-2}$ (this comprises more than 85% of the actual wave function of this state). In this case the possible $2\hbar\omega$ excitations are as follows:

(a) excitation of a pair of $1s_{1/2}$ particles to the $1p_{1/2}$ shell,
(b) excitation of a single $1s_{1/2}$ particle to the $(2s, 1d)$ shell,
(c) excitation of a single $1p$-shell nucleon to the $(2p, 1f)$ shell,
(d) excitation of two $1p$ particles to the $(2s, 1d)$ shell.

A perturbation theory estimate of these quantities leads to the conclusion that

$$\Delta c[(1s_{1/2})^2\rightarrow(1p_{1/2})^2]=0.185\,66/2\hbar\omega$$
$$=6.78\text{ keV}$$

when $\hbar\omega=13.7$ MeV, the value appropriate to $\alpha=0.575$ fm^{-1}. For the second type of excitation one finds that

$$\Delta c[1s_{1/2}\rightarrow(2s, 1d)]=-0.153\,97/2\hbar\omega$$
$$=-5.62\text{ keV}.$$

The contribution to c that arises when a single p-shell nucleon is excited to the $(2p, 1f)$ shell is

$$\Delta c[1p\rightarrow(2p, 1f)]=-0.442\,46/2\hbar\omega$$
$$=-16.15\text{ keV},$$

with by far the largest contribution coming from excitation of a $1p_{3/2}$ particle to the $2p_{3/2}$ orbit. The final $2\hbar\omega$ excitation involving two p-shell nucleons excited to the $(2s, 1d)$ shell will be discussed in the next section under the heading of short-range correlations.

Thus,

$$\Delta c(a+b+c)=-14.99\text{ keV}$$

is much smaller than the isovector contribution which mixes the $T=0$ and $T=1$ states in the $1p$ shell.[10] Furthermore, the effect of these $2\hbar\omega$ excitations is to decrease c and hence bring theory and experiment into even greater disagreement.

Finally we consider what influence changes in a and r_0 have on the computed value of c. First, if we try to reproduce the observed value by keeping a fixed, r_0 must be reduced from 1.2 to 0.8 fm when $\epsilon=-11.1675$ MeV. That one needs such an abnormally small value to explain c by changes in r_0 alone has already been noted for the $A=9$ system by Auerbach *et al.*[12] One can, of course, obtain the same spatial confinement of the nucleons by keeping r_0 fixed and increasing the binding energy. To reproduce experiment in this way, ϵ must be -34 MeV. Alternatively, a somewhat less diffuse potential slightly increases the value of c. For example, if we take $a=0.5$ fm, $r_0=1.2$ fm, and $\epsilon=-11.1675$ MeV, $c=294$ keV. Thus although different choices of r_0 and a can change c, no reasonable juggling of the well parameters can account for experiment, and it seems clear that something other than the Coulomb interaction is needed in the $A=14$ system—a result already noted by Altman and MacDonald.[13]

For other than two-particle or two-hole nuclei one has more flexibility in the way the Coulomb energy can be calculated and we shall now discuss some other alternatives.

1. Different binding for neutrons and protons

Because of the Coulomb repulsion between a valence proton and the core, a proton will be less tightly bound than a neutron. For example, in the $A=5$ system this difference is 1.07 MeV. If this is taken into account in Eq. (5) the correct value of ϵ will be smaller in absolute value. As a consequence, the Coulomb matrix elements calculated with these wave functions will be smaller, hence c will be smaller.

2. Parentage problems

In perturbation theory the Coulomb energy associated with the n-particle state (I, T) is given by the expression

$$E_c=\frac{n(n-1)}{2}\sum_{JJ_3\tau\gamma jj'j''j'''}\langle(n-2)J\tau\gamma;(jj')J_3T_3=1\,|\}IT\rangle\langle(n-2)J\tau\gamma;(j''j''')J_3T_3=1\,|\}IT\rangle$$
$$\times(\tau1\,T_z+1-1\,|\,TT_z)^2\langle(jj')_{J_3T_3=1}\,|\,V_c\,|\,(j''j''')_{J_3T_3=1}\rangle,\qquad(8)$$

where $\langle|\}\rangle$ is the two-particle parentage coefficient, $(\tau 1\, T_z+1-1\,|\,T T_z)$ is the isospin Clebsch-Gordan coefficient, and $(jj')_{J_3T_3}$ stands for the antisymmetric wave function of two protons in the states j and j' coupling to (J_3T_3). In all but the two-particle or two-hole systems the probability that the two singled-out protons couple to the ground state of the neighboring nucleus is less than one. Consequently, one might take a different binding energy for each singled-out pair. That is, when the proton pair is coupled to the parent state ${}^{A-2}_{Z-2}Y_N(J\tau)$, the energy ϵ to be used in calculating

$$\langle (jj')_{J_3T_3=1}\,|\,V_c\,|\,(j''j''')_{J_3T_3=1}\rangle$$

would be

$$\epsilon=-\tfrac{1}{2}\{E_B({}^{A-2}_{Z-2}Y_N(J\tau))-E_B({}^{A}_{Z}X_N(IT))\}.$$

(Such a prescription has already been considered by Blomqvist[14] and Wilkinson[15] in the calculation of β-decay matrix elements.) Using different values of ϵ is likely to be most important when the valence particles are loosely bound to the core. Consequently, we have investigated the effect this will have on the values of c for the $A=8$ system. Wherever possible we have used experimental values of ϵ; when this was not possible the theoretically predicted values were used. The change in c, Δc, that comes about when this scheme is used instead of Eq. (5) is $\Delta c=-2$ keV, -4 keV, and -7 keV for the 1^+, 2^+, and 3^+ states, respectively. Thus this prescription leads to a slightly smaller value of c, increasing the discrepancy between theory and experiment. [It should be noted that the value of c obtained using perturbation theory and ϵ determined by Eq. (5) is within 1 keV of the value obtained from a full diagonalization of the Hamiltonian.]

3. *Nilsson-type model*

One might imagine a situation in which a group of valence nucleons form a very stable structure and the last nucleon(s) are rather loosely bound.

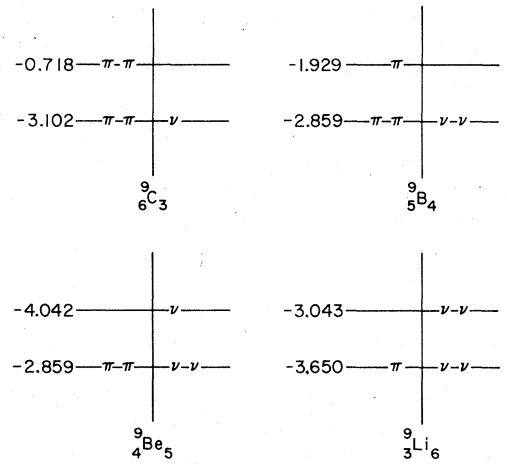

FIG. 1. Nilsson model for the $A=9$ quartet. The numbers to the left of each level are the values of ϵ in MeV for the $I=\frac{3}{2}^-$ state.

To estimate the effect this might have on the value of c we have considered a Nilsson-type model[16] where at most two neutrons and two protons can occupy a level. The situation for the $A=9$ quartet is shown in Fig. 1. The numbers beside the levels correspond to the binding energy (value of ϵ) for each particle in the orbit. For example, in ${}^9_6C_3$ one finds from the binding energy tables that the energy of the last two protons relative to the ${}^7_4Be_3$ core is -1.436 MeV and the energy of the nucleons in ${}^7_4Be_3$ relative to ${}^4_2He_2$ is -9.306 MeV. If we assume each nucleon in a given level has the same energy, the numbers shown in the figure are obtained. The Coulomb interaction is now incorporated into the shell-model calculation in the following way: In ${}^9_6C_3$ there is one Coulomb interaction involving two protons bound by 0.718 MeV, one involving particles with $\epsilon=-3.102$ MeV, and four in which one is bound by 0.718 MeV and the other by 3.102 MeV. Thus in carrying out the shell-model calculation for ${}^9_6C_3$ we use the average Coulomb matrix elements

$$\langle (jj')_J\,|\,V_c\,|\,(j''j''')_J\rangle=\tfrac{1}{6}\{\langle (jj')_J\,|\,V_c\,|\,(j''j''')_J\rangle_{0.718;\,0.718}+\langle (jj')_J\,|\,V_c\,|\,(j''j''')_J\rangle_{3.102;\,3.102}$$
$$+4\langle (jj')_J\,|\,V_c\,|\,(j''j''')_J\rangle_{0.718;\,3.102}\},$$

where $\langle (jj')_J\,|\,V_c\,|\,(j''j''')_J\rangle_{\epsilon;\,\epsilon'}$ is the Coulomb matrix element evaluated when one of the particles is bound by ϵ and the other by ϵ'. When c is calculated in this way its value for the $I=\frac{3}{2}$ $T=\frac{3}{2}$ state is 196 keV—14 keV smaller than given in Table I and again in the wrong direction to reconcile theory and experiment.

From the foregoing it would appear that other reasonable methods of calculating the Coulomb interaction give somewhat smaller values of c.

Consequently, one must look to other effects if one is to bring theory and experiment into agreement.

B. Other electromagnetic effects

In this section we shall discuss other electromagnetic effects that will influence the value of c. Since these are quite small an adequate approximation is to evaluate matrix elements of the requisite operators by use of harmonic oscillator wave functions. We shall first discuss each one individually in the context of the $A=14$ system where the results are given for the oscillator constant $\alpha = 0.575\ \mathrm{fm}^{-1}$ [$R_{1p} \sim r \exp(-\frac{1}{2}\alpha^2 r^2)$].

(a) $(v/c)^2$ corrections to the electromagnetic interaction

The $(v/c)^2$ corrections to the Coulomb interaction are conveniently calculated from the formula given by Close and Osborn.[17] There are three terms to be considered. First, the Breit-Darwin interaction which results from unretarded one-photon exchange.

$$H_{\mathrm{BD}} = \frac{1}{32}\left(\frac{e}{mc}\right)^2 \left(\vec{p}_{12}\frac{1}{r_{12}^3}\cdot\vec{p}_{12} + \vec{p}_{12}\cdot\vec{r}_{12}\frac{1}{r_{12}^3}\vec{r}_{12}\cdot\vec{p}_{12} + \vec{P}_{12}\frac{1}{r_{12}}\cdot\vec{P}_{12} + \vec{P}_{12}\cdot\vec{r}_{12}\frac{1}{r_{12}^3}\vec{r}_{12}\cdot\vec{P}_{12}\right) T_{12}\,, \tag{9a}$$

where $\vec{r}_{12}=\vec{r}_1-\vec{r}_2$, $\vec{p}_{12}=\vec{p}_1-\vec{p}_2$, $\vec{P}_{12}=\vec{p}_1+\vec{p}_2$, and T_{12} is the isotensor operator of Eq. (7). There are also isoscalar and isovector contributions to this operator, but as already discussed they give vanishing contributions to c in first order and as a consequence have been neglected.

There are two $(v/c)^2$ corrections that take the form of a spin-orbit interaction, namely the electromagnetic spin-orbit coupling with the Thomas factor and the interaction between the magnetic moment of one particle and the field produced by the motion of the other. When these are combined, the resulting operator whose matrix elements we have to evaluate is

$$H_{ls} = \frac{1}{2}\left(\frac{e\hbar}{2mc}\right)^2 \left\{(g_N - g_P + \tfrac{1}{2})\frac{\vec{S}\cdot\vec{l}_{12}}{r_{12}^3} + \frac{1}{4}\frac{1}{r_{12}^3}(\vec{\sigma}_1-\vec{\sigma}_2)\cdot[\vec{r}_{12}\times\vec{P}_{12}]\right\} T_{12}\,, \tag{9b}$$

where g_N and g_P are the nuclear g factors and have the values -3.82 and 5.58, respectively, $\vec{S}$ is the total spin operator, and $\hbar\vec{l}_{12}=\vec{r}_{12}\times\vec{p}_{12}$. Once more, only the isotensor part has been written down.

The third contribution comes from the dipole-dipole interaction and this gives rise to a tensor force

$$H_{dd} = -\frac{1}{16}\left(\frac{e\hbar}{2mc}\right)^2 (g_N - g_P)^2 \frac{1}{r_{12}^5} \times (3\vec{\sigma}_1\cdot\vec{r}_{12}\vec{\sigma}_2\cdot\vec{r}_{12} - \vec{\sigma}_1\cdot\vec{\sigma}_2 r_{12}^2) T_{12}\,. \tag{9c}$$

In addition to these three terms there are two δ-function contributions, one in the dipole-dipole interaction and the other from the effect of zitterbewegung. These will be considered in parts (d) and (e) of this section.

Explicit expressions for the $1p$-shell matrix elements of the spin-orbit and tensor force can be found in the literature.[18] The remaining terms can be evaluated by straightforward Racah algebra and use of the fact that

$$\vec{r}\cdot\vec{p} = -i\hbar r\frac{d}{dr}\,.$$

When all three of these $(v/c)^2$ corrections are added to the Coulomb potential the resulting value of c for the $A=14$ system is $c=283$ keV, a 1 keV increase over the bare Coulomb value.

(b) Vacuum polarization

The virtual emission and absorption of an electron-positron pair modifies the usual Coulomb interaction by adding to it a term[19]

$$V_{\mathrm{pol}} = \left(\frac{2e^2}{3\pi\hbar c}\right)\left(\frac{e^2}{r_{12}}\right)\left\{\tfrac{5}{6} - \gamma + \left|\ln(Kr_{12})\right| + \frac{3\pi}{4}Kr_{12} + O((Kr_{12})^2) + \cdots\right\}, \tag{10}$$

where $\gamma=0.5772$ is Euler's constant and $K^{-1}=\hbar/m_e c = 386.2$ fm. Since this increases the Coulomb repulsion it will increase the calculated value of c. However, the magnitude obtained by use of oscillator wave functions with $\alpha=0.575$ fm^{-1} is small and only increases c by 3 keV for the $A=14$ nuclei.

(c) Short-range correlations

In the preceding section the Coulomb interaction was calculated using uncorrelated wave functions. Although most $2\hbar\omega$ excitations were considered at that time, one was left out—namely, the one in which a p-shell nucleon is promoted to the $(2s, 1d)$ shell. Such $2\hbar\omega$, and greater, excitations give rise to short-range correlations in the relative wave function of the two interacting nucleons. McCarthy and Walker[20] and Bertsch and Shlomo[21] have estimated that these correlation effects give only a slight change in the Coulomb energy. In Table I of the McCarthy-Walker paper the Talmi integrals appropriate to the Coulomb force are given for os-

cillator wave functions and for wave functions obtained by solving the Bethe-Goldstone equation with the nucleon-nucleon interaction taken to be the Hamada-Johnstone potential. If we use as the "starting energy" $\omega_r = -24.3$ MeV the change brought about by correlations gives a slight decrease in c and for $A=14$, $c=277$ keV.

(d) Finite proton and neutron size

Electron scattering experiments lead to the conclusion that the proton and neutron are not point particles but instead both have an electric charge and magnetic moment distribution. Sauer and Walliser,[22] using Sachs form factors for the nucleons, have given explicit expressions for the effect of finite nucleon size plus zitterbewegung. The appropriate formulas are Eqs. (A-2)–(A-4) of their paper with the coefficient of $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ set equal to zero. If one evaluates matrix elements of these operators using harmonic oscillator wave functions with $\alpha = 0.575$ fm^{-1}, one finds a very substantial decrease in the value of c. When this effect alone is added to the static Coulomb interaction one finds $c=262$ keV.

The repulsive core in the nucleon-nucleon interaction cuts down the probability of two nucleons being close to each other and consequently, since the finite size correction is short ranged, one overestimates its value when harmonic oscillator wave functions are used. To investigate the effect that the repulsive core has on the computed decrease in c, we have redone the calculation including a Jastrow correlation function appropriate to the standard hard-core potential of Moszkowski and Scott,[23]

$$f(r) = 0 \quad \text{for } r \leq 0.4 \text{ fm}$$
$$= \{1 - \exp[-2.12(r-0.4)]\}\{1 + 1.276\exp[-2.12(r-0.4)]\} \quad \text{for } r > 0.4 \text{ fm}. \tag{11}$$

When this Jastrow function is used one finds that $c=276$ keV. If the nucleon-nucleon interaction is described by a soft-core potential one would obtain a value between these two extremes.

(e) Spin-spin contact term

The final term in the $(v/c)^2$ reduction of the Hamiltonian is proportional to $\vec{\sigma}_1 \cdot \vec{\sigma}_2$, and when the finite size of the nucleon is taken into account the operator for this is given by the $\vec{\sigma}_1 \cdot \vec{\sigma}_2$ terms in Eqs. (A-2)–(A-4) of the Sauer-Walliser paper.[22] We first evaluate the matrix elements using harmonic oscillator wave functions with the Jastrow function of Eq. (11) set equal to 1 for all values of r. Under this assumption, one obtains a 34 keV increase over the static Coulomb limit so that $c=316$ keV for the $A=14$ system.

Of course, as already discussed, $f(r)=1$ gives an overestimate for the change in c. As an alternative we have carried out the calculation using the Jastrow function of Eq. (11) and obtain $c=299$ keV. Clearly a soft-core potential will give a value intermediate between these two.

If we include all five corrections enumerated in this section and add them to the static Coulomb energy, we find that for the $A=14$ system

$$c = 294 \text{ keV when } f(r) = 1, \text{ for all } r$$

and

$$c = 290 \text{ keV when } f(r) \text{ is given by Eq. (11)}. \tag{12}$$

In column 6 of Table I we list the values of c for all the nuclei when $f(r)=1$. Since the nucleon-nucleon interaction does have a repulsive core, these entries provide upper limits for the theoretical values of c that can be obtained from the static Coulomb interactions plus the corrections (a) to (e) of this section. Moreover, the effects discussed in Sec. A, i.e., the use of different prescriptions for calculating the static Coulomb interaction and the $2\hbar\omega$ excitations discussed there, are not included in the tabulated results. Since these tend to make c even smaller, column 6 almost certainly gives upper limits for c and even these upper limits are, in all cases except one, smaller than experiment.

It is clear that some of the c values could be brought into line with experiment with only minor modifications in the wave functions used to calculate the Coulomb matrix elements. However, it is also obvious that this is not always the case. The two seniority zero 0^+ levels have an rms error 56.7 keV compared to 28.1 keV for the other nine states (in calculating the latter number the experimental value for the $A=8$ 3^+ state was taken to be 223 keV). Thus the rms error in the seniority zero states is twice as large as that encountered in the higher seniority levels. Such an effect is exactly what one would expect if a short-range isotensor interaction had been omitted. In the next section we examine the effects such an added potential would have.

III. CHARGE DEPENDENT NUCLEAR FORCES

The free nucleon-nucleon data indicate that in the spin-singlet state the (np) interaction is[2,24] about 2% more attractive than the (nn) potential. This difference can be brought about by an interaction of the form

$$V_{SB}=V_{SB}(r)\{\tau_z(1)+\tau_z(2)\} \tag{13}$$

or

$$V_{CD}=V_{CD}(r)T_{12},$$

where T_{12} is the isotensor operator of Eq. (7). The first of these, which implies that the (nn), (pp), and (np) interactions are all different, is a vector in isospin space and unless the wave functions describing the nuclear states in question have mixed isospin it will not contribute to c. On the other hand V_{CD} is an isospin tensor of rank two and consequently its matrix elements, even between states with good isospin, will have a T_z^2 dependence. We have, therefore, examined the effect on c of a potential of the form

$$V_{CD}=\tilde{V}\frac{e^{-\mu r_{12}}}{\mu r_{12}}\{P_0+QP_1\}T_{12}, \tag{14}$$

where μ is the inverse of the π-meson Compton wavelength, P_0 and P_1 are the singlet and triplet spin projection operators, respectively, and $\tilde{V}$ and Q are numbers. When

$$\tilde{V}=0.3 \text{ MeV} \tag{15}$$

the (np) interaction due to Eq. (14) is 2% stronger than the (nn) potential.

One could, of course, vary $\tilde{V}$ and Q so as to best reproduce the experimental values of c. However, in view of the uncertainties in the calculation of c, we have merely examined what happens when $\tilde{V}$ is held fixed at 0.3 MeV and Q is allowed to take on two different values. (Once Δc is known for two values of Q one can extrapolate to any other values of $\tilde{V}$ and Q since in perturbation theory Δc depends linearly on $\tilde{V}$ and $\tilde{V}Q$.) In all cases matrix elements of V_{CD} were evaluated using harmonic oscillator wave functions with the values of α quoted in the table caption. If V_{CD} is spin independent ($Q=1$) the results listed in column 7 of Table I are obtained when the Coulomb interaction is calculated using the binding energy prescription Eq. (5), and the corrections (a)–(e) discussed in the preceding section are included. In all cases, c is increased by the addition of this term and as a consequence the theoretical values become closer to the experimental ones.

As an alternative we have examined what happens when V_{CD} has a Rosenfeld spin dependence, $Q=-\frac{5}{9}$. The results of this calculation, listed in column 8 of Table I, show that the rms error is now reduced to 18.1 keV and, as might be expected, the most marked improvement is in the seniority zero states; their rms error has been reduced by 35.7 keV and now stands at 21 keV. For the higher seniority states there is also some improvement—a decrease in their rms error by 10.7 keV to 17.4 keV. Thus the rms error for the seniority zero states is about the same as for all others, and, with the possible exception of the 3^+ states in the $A=8$ system, the addition of V_{CD} improves the fit to the data in every case.

IV. DISCUSSION

Eight of the eleven pieces of data considered in this analysis involve yrast $T=1$ triads. The actual wave functions of these states may have admixtures involving $T=0$ (when $T_z=0$) and $T=2$, so that the eigenfunction describing a physical state would have the form

$$\psi=\psi_{T=1}+\alpha\psi_{T=0}+\beta\psi_{T=2}. \tag{16}$$

It is possible that α could be fairly large for the $T_z=0$ nucleus since the yrast $T=1$ level is embedded in a sea of $T=0$ states. On the other hand, the $T=2$ states lie about 10 MeV above the yrast $T=1$ of given spin and because of this large energy difference β should be small. Thus, aside from small effects dependent on β, the change in energy ΔE brought about by V_{SB} and V_{CD} would be given by

$$\Delta E=\langle\psi_{T=1}+\alpha\psi_{T=0}|\,V_{SB}+V_{CD}\,|\psi_{T=1}+\alpha\psi_{T=0}\rangle. \tag{17}$$

The main terms in ΔE involve matrix elements of V_{SB} and V_{CD} between the purely $T=1$ states. The first of these, which involves the isospin vector V_{SB}, will give a contribution proportional to T_z and the second, which depends on the isospin tensor V_{CD}, is proportional to T_z^2. The term linear in α vanishes for V_{CD} and is independent of T_z for V_{SB}. In addition, the terms proportional to α^2 vanish. Thus the only admixtures into the $T=1$ states that might be large give no contribution proportional to T_z^2 and as a consequence c depends almost entirely on V_{CD}.

In view of the above arguments, it follows that for the majority of the cases given in Table I V_{SB} does not affect c directly. It does, however, indirectly, in that one only knows that the (np) interaction is more attractive than the (nn) by about 2%. Part of this difference can be attributed to a nonvanishing V_{SB}, and the remainder to V_{CD}. Since V_{CD} does not contribute to the difference in the (nn) and (pp) interactions, V_{SB} should be ob-

tainable if the (pp) and (nn) scattering lengths are known. However, the separation of the Coulomb and specifically nuclear (pp) scattering lengths is extremely sensitive to the very short range part of the nucleon-nucleon interaction[25] and as a consequence the free particle data determine neither a_{pp} the specifically nuclear proton-proton scattering length, nor V_{SB}. Thus $\tilde{V}$, the interaction strength of Eq. (14), is uncertain, but undoubtedly is only a few percent of the isoscalar potential strength which would be 30 MeV for a Yukawa potential.

On theoretical grounds one would expect a short range isotensor component in the nucleon-nucleon interaction and there are two aspects of the theoretical values of c that indicate such a term is needed:

(1) In Sec. II A we discussed a variety of ways of calculating the Coulomb interaction and found that in all cases our binding energy prescription, Eq. (5), gave the largest value of c. The results listed in column 6 of Table I are for the Coulomb interaction calculated using this binding energy prescription plus corrections (a)–(e) of Sec. II B. The $2\hbar\omega$ excitations discussed in Sec. II A, which tend to decrease c, were not included. Thus the results listed in column 6 should be upper limits for c which in all cases, except one, are smaller than the experimental values. The inclusion of the isotensor potential Eq. (14) with $\tilde{V}=0.3$ MeV increases the calculated values of c thus bringing theory and experiment closer together.

(2) The values of c given in column 6 of Table I show that the rms error in the seniority zero states is a factor of 2 worse than that for the other levels—56.7 keV compared to 28.1 keV. In calculating the electromagnetic interaction energies (in contrast to nuclear energies), one is dealing with matrix elements of well-known operators whose expectation values for the tightly bound seniority zero states do not depend sensitively on the value of ϵ, Eq. (5). For example, when $A=14$ we have seen that for $a=0.65$ fm and $r_0=1.2$ fm one must take ϵ about 23 MeV more attractive than given by Eq. (5) if one is to obtain agreement with experiment for c. Furthermore, this same result, that the worst discrepancy for the Coulomb energy occurs in the seniority zero states, has also been observed in the $A=18$ nuclei,[26] in the $d_{3/2}$ shell,[27] and perhaps even in the $f_{7/2}$ region.[28] The inclusion of the short-range isotensor interaction V_{CD} of Eq. (14) significantly improves the fit to the $1p$-shell c values, particularly for the seniority zero states. The rms error in these two c values becomes 21 keV compared to 17.4 keV for the remaining nine higher seniority levels.

Even after the charge dependent interaction of Eq. (14) has been introduced there is still one datum which stands out from the rest. When $\tilde{V}=0.3$ MeV and $Q=-\frac{5}{9}$, the value of c for the $I=\frac{3}{2}^-$ $T=\frac{3}{2}$ state in the $A=9$ nuclei is 35 keV smaller than experiment, and this exceeds the rms difference of 15.5 keV for the remaining ten values by more than a factor of 2. In this case the $T=\frac{3}{2}$ level in the $T_z=\pm\frac{1}{2}$ nuclei is embedded in a sea of $T=\frac{1}{2}$ states and the analog of Eq. (17) is

$$\Delta E'=\langle\psi_{T=3/2}+\alpha\psi_{T=1/2}|\,V_{SB}+V_{CD}\,|\psi_{T=3/2}+\alpha\psi_{T=1/2}\rangle\,. \tag{18}$$

If one neglects terms proportional to α the entire T_z^2 dependence of $\Delta E'$ is given by V_{CD}. On the other hand, in this case the terms linear in α can have a T_z^2 and higher dependence from both V_{SB} and V_{CD}. If we denote the contribution proportional to α by $\Delta E'_\alpha$ we find

$$\begin{aligned}\Delta E'_\alpha &= 2\alpha[(\tfrac{1}{2}1T_z0\,|\,\tfrac{3}{2}T_z)\langle\psi_{T=3/2}||\,V_{SB}\,||\psi_{T=1/2}\rangle+(\tfrac{1}{2}2T_z0\,|\,\tfrac{3}{2}T_z)\langle\psi_{T=3/2}||\,V_{CD}\,||\psi_{T=1/2}\rangle]\\ &= 2\alpha(9-4T_z^2)^{1/2}[\langle\psi_{T=3/2}||\,V_{SB}\,||\psi_{T=1/2}\rangle/\sqrt{12}+T_z\langle\psi_{T=3/2}||\,V_{CD}\,||\psi_{T=1/2}\rangle/\sqrt{5}]\,,\end{aligned}$$

where $(\frac{1}{2}TT_z0\,|\,\frac{3}{2}T_z)$ is the isospin Clebsch-Gordan coefficient, whose explicit value has been put into the second line of this equation, and the reduced matrix elements are independent of T_z. Although we have evaluated $\Delta E'_\alpha$ for V_{SB} and V_{CD}, there can also be a contribution proportional to α due to the nuclear wave function size effect of Eq. (6) which can be large and is mainly isovector. Consequently, it would act in the same way as V_{SB} and could substantially change c even when α is small. This type of mixing would also manifest itself by adding additional terms to the mass multiplet relationship Eq. (1), for example, a dT_z^3 term. For the yrast $I=\frac{3}{2}^-$ $T=\frac{3}{2}$ level in the $A=9$ nuclei, a nonvanishing value of d has been found[5] and in fact it is larger than for other light nuclei, $d=5.8$ keV. Although this value of d is small, it is larger than normal and this may be related to the fact that c is badly estimated for this state. At first sight one might hope to learn something about the charge dependence of nuclear forces from a study of the d coefficient. Bertsch and Kahana[29] have estimated a value of 1 keV for d from second-order Coulomb effects

and charge dependent nuclear forces. However, one could obtain an even larger value from Coulomb effects alone if a $T=\frac{1}{2}$ state lies very close in energy to the yrast $I=\frac{3}{2}^-$ $T=\frac{3}{2}$ state. Furthermore, even the sign of d depends on whether the nearby $T=\frac{1}{2}$ state lies just above or just below the $T=\frac{3}{2}$ level. Thus, until one knows the position of a possible nearby $T=\frac{1}{2}$ $I=\frac{3}{2}^-$ state, which is predicted by both the Kumar and Cohen-Kurath interactions, one cannot hope to learn anything about charge dependent nuclear forces from a study of the d coefficient in this nucleus.

In conclusion, one can say that a study of the coefficient of T_z^2 in the mass multiplet relationship for the $1p$ shell leads to the unambiguous conclusion that the specifically nuclear (np) interaction is of the order of a few percent more attractive than the (nn). However, the size of this difference and its spin dependence cannot be accurately determined from this study because of the uncertainties in calculating the static Coulomb energy, particularly for loosely bound systems.

The author would like to thank G. Garvey and D. Kurath for interesting discussions and particularly S. Coon for pointing out an error in the original manuscript. This work was performed under the auspices of the U. S. Department of Energy.

[1]See, for example, G. T. Garvey, Ann. Rev. Nucl. Sci. 19, 433 (1969); J. Jänecke in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 297.
[2]See, for example, E. M. Henley, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 15.
[3]A. H. Wapstra and K. Bos, At. Data Nucl. Data Tables 19, 175 (1977).
[4]F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. A227, 1 (1974); F. Ajzenberg-Selove, *ibid.* A248, 1 (1975); A268, 1 (1976).
[5]E. Kashy, W. Benenson, D. Mueller, R. G. H. Robertson, and D. R. Goosman, Phys. Rev. C 11, 1959 (1975).
[6]M. A. Oothoudt and G. T. Garvey, Nucl. Phys. A284, 41 (1977).
[7]S. Cohen and D. Kurath, Nucl. Phys. 73, 1 (1965).
[8]N. Kumar, Nucl. Phys. A225, 221 (1974).
[9]R. D. Lawson, Phys. Lett. 78B, 371 (1978).
[10]Paul Goldhammer, Phys. Rev. C 11, 1422 (1975).
[11]T. T. S. Kuo, Nucl. Phys. A103, 71 (1967) and D. J. Millener, private communication.
[12]N. Auerbach, A. Lev, and E. Kashy, Phys. Lett. 36B, 453 (1971).
[13]A. Altman and W. M. MacDonald, Nucl. Phys. 35, 593 (1962).
[14]J. Blomqvist, Phys. Lett. 35B, 375 (1971).
[15]D. H. Wilkinson, Phys. Rev. Lett. 27, 1018 (1971).
[16]S. G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk 29, No. 16 (1955).
[17]F. E. Close and H. Osborn, Phys. Rev. D 2, 2127 (1970).
[18]R. D. Lawson, *Theory of the Nuclear Shell Model* (Oxford University Press, N. Y., 1979), Chap. 4.
[19]See, for example, N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. 44, 48 (1972).
[20]R. J. McCarthy and G. E. Walker, Phys. Rev. C 9, 809 (1974).
[21]G. F. Bertsch and S. Shlomo, Phys. Rev. C 10, 931 (1974).
[22]P. U. Sauer and H. Walliser, J. Phys. G 3, 1513 (1977).
[23]D. A. Chakkalakal, C. H. Yang, and J. W. Clark, Nucl. Phys. A271, 185 (1976).
[24]J. C. Alder *et al.*, in *Nucleon-Nucleon Interactions*, Proceedings of the Second International Conference on Nucleon-Nucleon Interactions, Vancouver, 1977, edited by H. Fearing, D. Measday, and A. Strothdee (AIP, New York, 1978), p. 535.
[25]P. U. Sauer, Phys. Rev. Lett. 32, 626 (1974).
[26]S. Kahana, Phys. Rev. C 5, 63 (1972).
[27]R. Sherr and I. Talmi, Phys. Lett. 56B, 212 (1975).
[28]R. Sherr, Phys. Rev. C 16, 1159 (1977).
[29]G. Bertsch and S. Kahana, Phys. Lett. 33B, 193 (1970).