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Analyses of inelastic proton and electron scattering to the 2^+ (4.43 MeV) state in ^{12}C

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Analyses of data from the inelastic scattering of 65 MeV protons and of 57 to 215 MeV electrons are used to test large basis (projected Hartree-Fock) model spectroscopies for the ground to 2_1^+ (4.43 MeV) transition in ^{12}C. From distorted wave approximation calculations of the proton scattering, reasonably good agreement with the empirical cross section was obtained, but no spectroscopy yields the distinctive structure of the recently measured analyzing power data. In like fashion, the longitudinal form factor extracted from the electron scattering data is well reproduced by calculations based upon the same projected Hartree-Fock models of spectroscopy. The predictions of the transverse form factor, however, are quite distinctive and only in one case, and that involving a strong spin orbit effect in the basis, are the data reproduced.

[NUCLEAR REACTIONS ^{12}C$(\vec{p},p')$ 2_1^+ $E=65$ MeV $\sigma(\theta)$, $A(\theta)$, DWA analyses using projected Hartree-Fock functions. Deduced longitudinal and transverse electron scattering form factors.]

I. INTRODUCTION

Inelastic proton and electron scattering analyses are useful and complementary means by which details of the structure of nuclear states may be probed. Electron scattering being a "weak" (Coulombic) probe can be adequately predicted by using a plane wave Born approximation (although a distorted wave approach is better for heavy nuclei), and thus gives information mainly about details of the proton distributions in nuclei in the form of the charge and charge current densities in those nuclei.

The neutron distributions in nuclei also influence electron scattering via magnetization density effects, but these are normally hidden by the strong charge effects. This is not the case for proton scattering from nuclei since, being mediated by the strong short ranged nuclear force, these reactions preferentially reflect the characteristics of the neutron distributions. Analyses of proton scattering data, however, often require higher order than the simplest Born approximation components in the reaction mechanism. Specifically, an appropriate two nucleon t matrix is required for which an effective interaction or pseudopotential representation is usually taken. Further, for a range of projectile energies, typically 20 to 40 MeV for most target nuclei, virtual excitation (doorway state effects) of giant resonances significantly influence reaction predictions.[1] To minimize the role of such higher order processes, therefore, analyses of data from the scattering of high-energy (60 MeV and above) protons are required.

Both the electromagnetic and nuclear force reactions, however, usually require core polarization corrections in their analyses. Such core polarization corrections, being correlated to effective charges,[2] detract from the ability to use the reaction analyses as tests of model spectroscopies. Of course, the size of any core polarization corrections is, by itself, a measure of inadequacy of the structure used in reaction analyses. Nevertheless, by using large basis prescriptions of nuclear states, with large bases projected Hartree-Fock states being useful examples, it is possible in some cases to require no core polarization corrections *per se*, since to match observed electromagnetic decay rates no polarization charges need be introduced into the calculations.

In thecase of ^{12}C, all the above ingredients are present. New inelastic scattering data, including the very sensitive analyzing power, has been taken with 65 MeV protons[3] and recent measurements[4] have been made of the longitudinal and transverse electron scattering form factors of the 2_1^+(4.43 MeV) state. Several large basis Hartree-Fock calculations have also been performed for ^{12}C, and we anticipate being able to investigate in some detail the $0_1^+ \rightarrow 2_1^+$ transition density.

II. INELASTIC PROTON SCATTERING

The results of distorted wave approximation (DWA) analyses of data[3] from the inelastic scattering of 65 MeV polarized protons leading to the 2_1^+ (4.43 MeV) state in ^{12}C are compared with the differential cross section data in Fig. 1 and with the analyzing power data in Figs. 2 and 3. Of these results all but one were obtained by using microscopic model wave functions to determine transition densities; the unique result being obtained by using a standard collective model description of the reaction process.[5] The DWA with microscopic model wave functions determine that the nuclear transition amplitudes can be evaluated from[1,6]

$$T_{if}=\sum_{j_1j_2m_1m_2IN}(-)^{j_1-m_1}(2J_f+1)^{-1/2}$$

$$\times\langle j_1j_2m_1-m_2|I-N\rangle\langle J_iIM_iN|J_fM_f\rangle S^{(x)}(j_1j_2;J_iJ_f;I)\langle\chi_f^{(-)}(0)\phi_{j_2m_2}(1)|t(01)|\mathfrak{a}_{01}\{\chi_i^{(+)}(0)\phi_{j_1m_1}(1)\}\rangle \tag{1}$$

in which

$$S^{(x)}(j_1j_2;J_iJ_f;I)=\langle J_f\|[a_{j_2}^{\dagger}\times a_{j_1}]^{Ix}\|J_i\rangle \tag{2}$$

are spectroscopic amplitudes for the transition of a single proton ($x=\pi$) or neutron ($x=\nu$) connecting the initial and final nuclear states by an angular momentum transfer I. Thus the complete transition amplitude in this approximation is a weighted sum of two body matrix elements involving a nucleon in the continuum ($\chi^{\pm}$) and a bound nucleon (ϕ_{jm}). In these analyses the continuum wave functions were obtained from an optical model calculation; the potential parameters of which were those that gave a best fit to the elastic scattering.[3] The bound state wave functions, ϕ_{jm}, were chosen for simplicity to be harmonic oscillator wave functions associated with an oscillator energy of 19 MeV.

The two body matrix elements in Eq. (1) are antisymmetrized expectation values of a two nucleon transition interaction; a number of forms for which have been specified recently.[7] Of these we

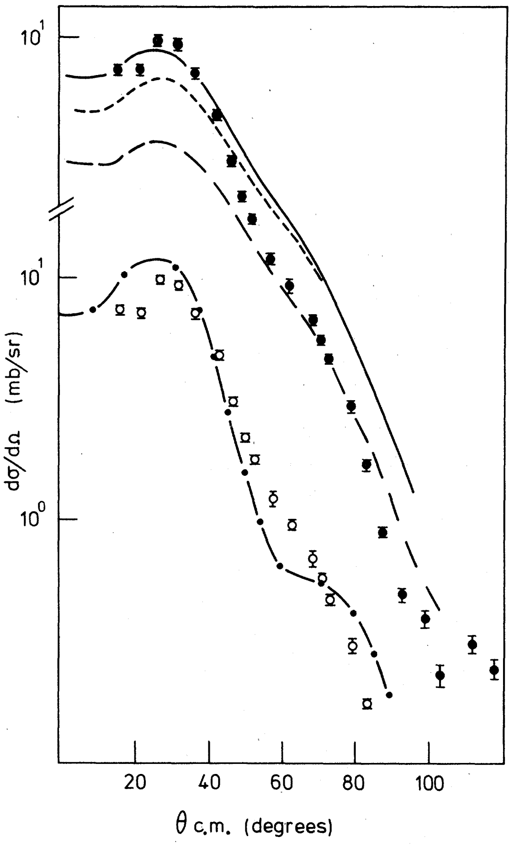

FIG. 1. Differential cross-section data and various DWA predictions from the inelastic scattering of 65 MeV protons exciting the 2_1^+(4.43 MeV) state in ^{12}C. The dash-dot curve is the standard collective model results. The top curves are the results found using microscopic spectroscopies with the solid curve being obtained with the PHFBA spectroscopy, the dashed curve being the result of using the PHFV′ spectroscopy and the broken curve being the shell model (s.m.) result.

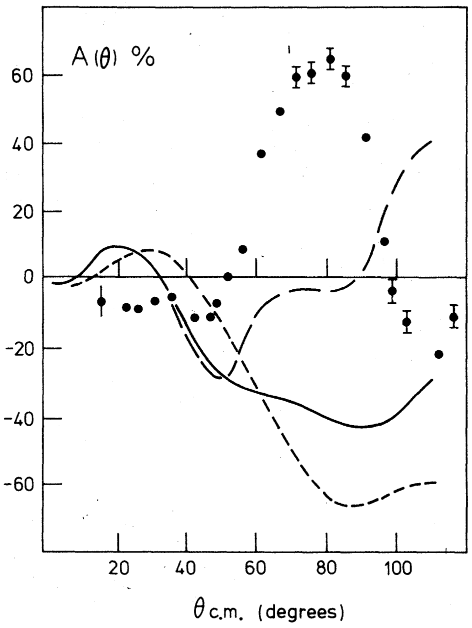

FIG. 2. DWA predictions compared with the 65 MeV analyzing power data from proton excitation of the 2_1^+(4.43 MeV) state in ^{12}C. The PHFV′ spectroscopy was used in the calculations and the direct, exchange (knock on), and total (direct plus exchange amplitudes) predictions are represented by the broken, small dashed, and solid curves, respectively.

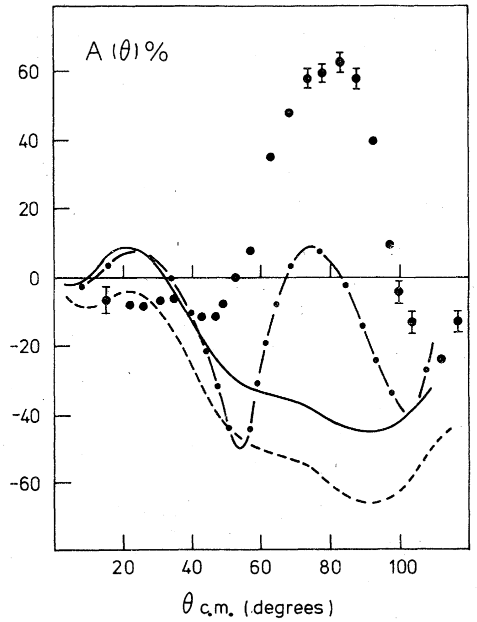

FIG. 3. A comparison with the analyzing power data from the inelastic scattering of 65 MeV protons leading to the 2_1^+(4.43 MeV) state in ^{12}C of DWA predictions in which PHFV′ spectroscopy (solid curve), shell model spectroscopy (dashed curve) and a simple collective phonon model spectroscopy (dot-dashed curve) were used.

choose to use the mixture of central and tensor forces that were used previously with some success in analyses of inelastic scattering data and with projected Hartree-Fock (PHF) wave functions.[6]

The results of these calculations are dominated by the central force transition amplitudes and, in particular, by those that involve neutron excitations. This is most evident from the differential cross-section magnitudes at the peak scattering angle and which are given in Table I. These results, which were obtained using the PHFB spectroscopy,[8] also reveal that proton excitations play a non-negligible role (as do also exchange amplitudes) in determining any resultant differential cross section since their transition amplitudes interfere coherently with those of the dominant direct neutron excitations with the result that the predicted magnitudes are trebled. The coherence of proton and neutron excitations in this case is further evidenced by the theoretical result given in the last line of Table I, being so close to those of the total result. It would be of interest then to seek data for simultaneous analyses of isoscalar and isovector states of given J^π in which may be destructive interference between proton and neutron excitation amplitudes. The 2^+ states in ^{12}C at 16 MeV excitation may be candidates for such analyses.

TABLE I. Peak magnitudes from DWA calculations made using the PHFB spectroscopy.

	$\theta_{\text{c.m.}}$	Direct	Exchange	Total
π central	27	0.301	0.026	0.460
ν central	17	2.369	0.260	3.898
π tensor	42	0.002	0.005	0.009
ν tensor	36	0.002	0.025	0.023
Total	27	4.347	0.444	6.949
$(\sqrt{\pi_c}+\sqrt{\nu_c})^2$	(27)	4.359	0.450	7.036

The methods described in Ref. 6 were used to determine spectroscopic amplitudes, and the results for four microscopic model spectroscopies are given in Table II. Presented therein are the dominant components (proton and neutrons being identical) for three PHF model spectroscopies that we label hereafter as PHFV′, PHFB, and PHFBA. The first two, and another PHF calculation labelled PHFB1, were taken from Ref. 8 (as were the mnemonics) while that which we have defined by PHFBA was obtained from the studies reported in Ref. 9. Additionally we have made a shell model calculation using the p-shell (8–162BME) matrix elements of Cohen and Kurath.[10] It is evident in Table II that, while p-shell amplitudes dominate, transition strengths associated with the f-p and s-d shell components in the intrinsic H-F states are important; the latter being most marked in the PHFV′ and PHFBA cases.

The resulting differential cross sections are shown in Fig. 1 from which it is evident that the peak magnitude of the shell model result lies well

TABLE II. Spectroscopic amplitudes for the 2_1^+ excitation (from ground) in ^{12}C.

$j_1 \to j_2$	PHFV′	PHFB	PHFBA	s.m.
$0p_3 \to 0p_3$	0.835	0.753	0.549	0.503
$0p_3 \to 0p_1$	−0.837	−0.753	−1.086	−1.195
$0p_1 \to 0p_3$	0.837	0.753	0.804	0.704
$0p_3 \to 0f_7$	−0.190	−0.430	−0.197	
$0p_3 \to 0f_5$	0.079	0.175	0.088	
$0p_1 \to 0f_5$	−0.146	−0.328	−0.056	
$0p_3 \to 1p_3$	−0.081	−0.066	0.043	
$0s_1 \to 0d_5$	−0.266	−0.079	−0.234	
$0s_1 \to 0d_3$	0.327	−0.096	0.209	
$0d_5 \to 0s_1$	−0.161	−0.055	−0.128	
$0d_3 \to 0s_1$	−0.202	0.067	−0.114	
$0f_7 \to 0p_3$	−0.067	−0.198	−0.064	
$1p_3 \to 0p_1$	0.081	0.066	−0.054	
$0f_5 \to 0p_1$	−0.051	−0.151	−0.045	

below the data value. This is a direct result of the truncation of the basis space as comparison with the various PHF predictions reveals. But, while the PHF values all give reasonable peak magnitude predictions, the rate of decrease with scattering angles of data is not reproduced. A more rapid decrease can be obtained by reducing the oscillator energy of the bound state basis slightly. However, computational expense precludes any such variation in reaction analyses to seek a better fit to the data and, in any event, such an effort seems unwarranted since even at 90° scattering angle (for which the momentum transfer squared has a value near 6 fm^{-2}) the magnitude discrepancy between data and PHF predictions is but a factor of 2. Such a study is also irrelevant in view of the predictions of the analysing power data that are compared with the data in Figs. 2 and 3. In no case is the data reproduced.

In Fig. 2, the direct, exchange and total results found by using the PHFV′ spectroscopy are given. In the region of the large anomaly (60–100°) the exchange amplitudes dominate the spin dependence that determines the analyzing power. Even so the purely direct result does not resemble the data structure, hence the anomaly cannot be resolved by simply altering the relative strengths of direct and exchange amplitudes as, for example, one may achieve by varying the ranges of the forces.

The anomaly in the analyzing power is not simply a reflection of the spectroscopy either as the results given in Fig. 3 indicate. Even the best result, that from the purely collective model calculation, is far from being satisfactory, and concurs with the previous findings[11] in which the anomaly remained even with a coupled channels analysis.

In sum, it is most probably that reaction mechanism details such as the two-body spin orbit force and the complex nature of the t matrix will be necessary inclusions before such anomalies in analyzing power can be explained and then used to assist in delineation of the microscopic structure of nuclear transitions.

III. INELASTIC ELECTRON SCATTERING

For light nuclei, electron scattering form factors can be calculated by using a plane wave Born approximation,[12] including center of mass and finite proton size corrections.[13] These form factors are functions of momentum transfer q and angular momentum transfer I and can be expressed as

$$|F_\eta^I(q)|^2 = 4\pi/Z^2 |\langle J_f \| Q_\eta^I(q) \| J_i \rangle|^2/(2J_i+1), \tag{3}$$

in which η denotes longitudinal or transverse (L, T, respectively). The reduced matrix elements in Eq. (3) can be expressed in terms of the spectroscopic amplitudes ($S_{j_1 j_2}^{(x)}$) defined previously;

$$\langle J_f \| Q_\eta^I(q) \| J_i \rangle = \mathrm{trace}(SM)/(2I+1)^{1/4} \tag{4}$$

in which the single particle expectations

$$M_{12} = \langle j_2 \| Q_\eta^I(q) \| j_1 \rangle \tag{5}$$

are standard.[4,12,13]

The results presented in Figs. 4 and 5 were obtained by using the PHFV′, PHFB, and PHFB1 spectroscopic amplitudes. As is evident in Fig. 4, all these PHF models of structure yield good agreement with the longitudinal form factor, especially when it is possible to adjust the position of the first minimum in a 0.1 fm^{-1} range of momentum transfer accordingly as the oscillator length varies from 1.6 to 1.7 fm. Likewise, all calculated form factors overestimate the height of the second maximum (not shown in Fig. 4), and this is due in part to the neglect of distortion effects[13] and use of an harmonic oscillator basis.

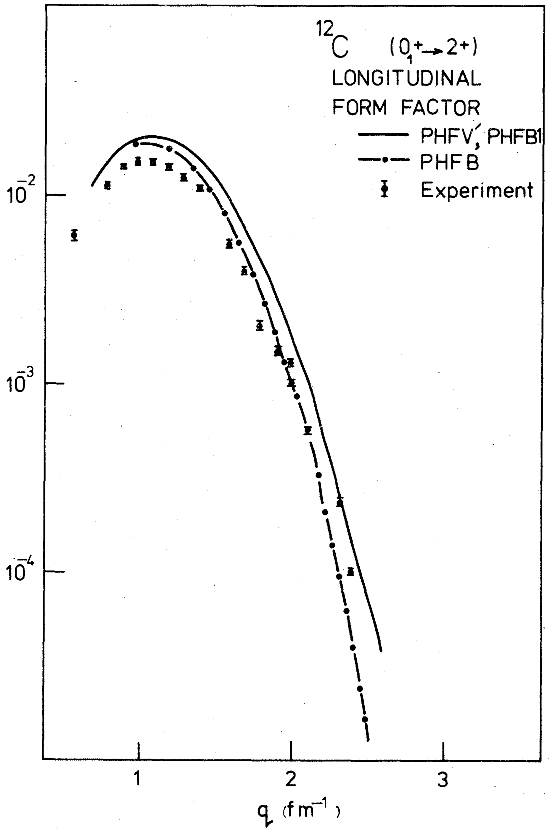

FIG. 4. The longitudinal form factor from the inelastic scattering of 135 MeV electrons leading to the 2_1^+(4.43 MeV) state in ^{12}C. All predictions were made using PHF wave functions with an harmonic oscillator basis defined by a length parameter value of 1.7 fm.

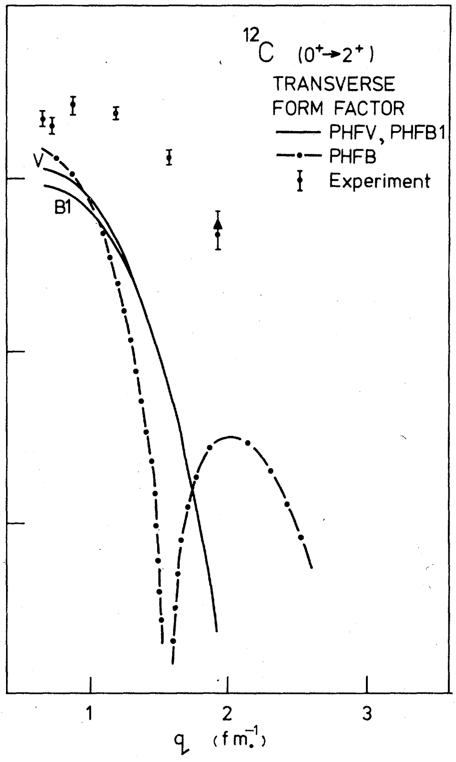

FIG. 5. The transverse form factor for the excitation of the 2_1^+(4.43 MeV) state in ^{12}C by the inelastic scattering of 135 MeV electrons. The predictions were obtained by using the same structure models that led to the results shown in Fig. 4.

The results of the transverse form factor are shown in Fig. 5. It is apparent that while the predicted values vary considerably according to which PHF model spectroscopy was used in the calculations, none of these predictions agree with the data. Specifically, all of these predictions decrease too rapidly with momentum transfer and cannot be corrected by varying the oscillator parameter '*b*.' Further, when the PHFB model of nuclear structure was used, a minimum in the form factor at 1.6 fm^{-1} resulted. There is no evidence for such a feature in the data.

All the HF results were obtained in the *LS* limit, ignoring any effect of the nuclear spin-orbit force. To ascertain the effect of including a spin-orbit force, form factors were generated from the (j–j) shell model amplitudes and are displayed in Fig. 6. The longitudinal form factor is overestimated due to the arbitrary polarization charge ($e=0.5$) assigned to the nucleons, but of more immediate consequence is the drastic change of shape of the predicted transverse form factor which has the correct hehavior at large '*q*.' With this encouragement, form factors were generated using the PHFBA results which included spin-orbit effects and required no core polarization corrections. Overall agreement with experiment is good with the calculated transverse form factor, apart from a factor of 2 in magnitude, displaying the correct shape and '*q*' dependence of the data (Fig. 6). Thus it appears that not only is a large basis calculation necessary to eliminate the masking effect of core polarization corrections, but that inclusion of spin-orbit effects in the nuclear states is essential for any calculation of the transverse form factor.

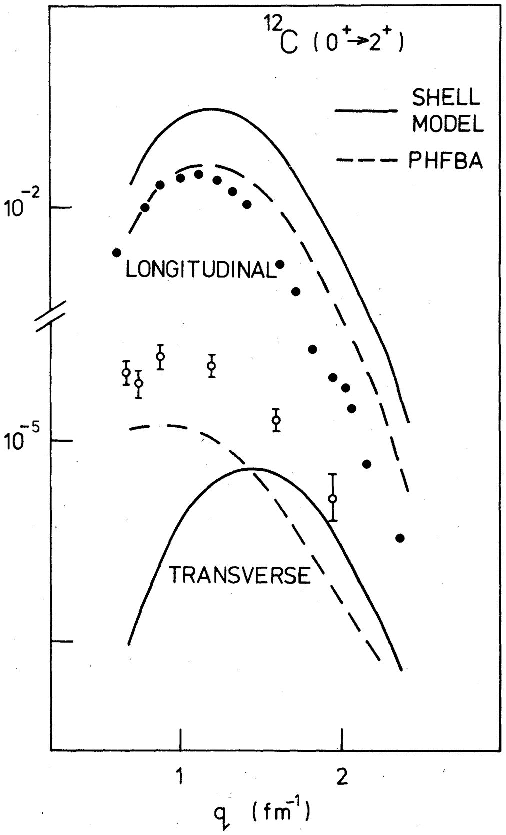

FIG. 6. Longitudinal and transverse form factors from the inelastic scattering of 135 MeV electrons leading to the 2_1^+(4.43 MeV) state in ^{12}C compared with the PHFBA and shell model predictions. The shell model predictions include the effect of an overall polarization charge of 0.5e.

IV. CONCLUSIONS

The complementary tests of nuclear structure, analyses of electron and proton inelastic scattering, have been made for the 2_1^+(4.43 MeV) state excitation in ^{12}C. Various large basis projected Hartree-Fock models of the nuclear spectroscopy of this transition as well as that of a *p*-shell model were used to evaluate the longitudinal and transverse form factors ascertained from the inelastic scattering of 57 to 215 MeV electrons and to evaluate the direct interaction differential cross-section and analyzing power for the inelastic scattering of 65 MeV protons.

All spectroscopies yield good agreement with the longitudinal form factor and differential cross-section data, albeit that the shell model calculations require a polarization charge in accord with the basis truncation. There is some disagreement with data at higher momentum transfer but slight variation in details of the basis states can account for most, if not all, of that disagreement.

None of the calculations could reproduce the very striking structure of the analyzing power data and thus there appears to be an inadequacy as yet in the detailed prescription of the reaction mechanism.

Such is not the case for the transverse form factor from inelastic electron scattering for which only one PHF model of nuclear structure investigated herein gave plausible results. Evidently a pertinent description of the nuclear structure of ^{12}C required not only a big basis calculation, but also a basis in which spin-orbit splittings are not negligible. This shows that the inclusion of magnetization current density effects (and hence neutron distributions) which are identically zero in the LS limit, is essential even in the case of such a light nucleus as ^{12}C if transverse form factors are to be calculated.

Furthermore, convection current effects in transverse form factors not only vary markedly with details of large basis spectroscopic models (as displayed by the LS limit predictions shown in Fig. 5) but also, although they are large, do not dominate magnetization effects even at the low momentum transfers in the range 0.5 to 1.0 fm^{-1}. Thus, not only is the procedure recently used[4] to estimate the nuclear convection current for ^{12}C from the transverse form factor suspect but also the conjecture[14] that one might observe the nuclear velocity (per its relationship to the nuclear convection current) seems most improbable.

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