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Photodisintegration of the alpha particle

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I develop a formalism to calculate electric dipole transitions in the α particle, expanding both initial and final states in nine-dimensional hyperspherical harmonics. I calculate results using Ballot's ground state wave function for the Volkov potential and three different exchange mixtures, giving different potentials acting in the final state. Wigner exchange gives most of the absorption in a single line: The harmonic oscillator approximation is surprisingly good for this case. An exchange force gives a satisfactory value of the integrated photoeffect cross section (120 MeV mb) and a photoeffect curve in fair agreeement with Gorbunov, and Balestra *et al.* Extreme Born approximation is quite poor.

[NUCLEAR REACTIONS Photodisintegration of alpha, hyperspherical harmonics.]

I. INTRODUCTION

Several years ago Ballot, Beiner, and Fabre[1] (BBF) calculated the wave function of the ground state of the α particle using expansions in nine-dimensional hyperspherical harmonics (h.h.). Demin, Pokrovsky, and Efros[2] extended the calculation to treat tensor two-body forces. More recently Tjon[3] has used separable approximations to calculate the α particle. See Fiarman and Meyerhof[4] for a review of earlier work on the properties of the α particle.

The main problem in photoeffect calculations for the α particle lies then in finding good continuum wave functions. Recently Delsanto *et al.*[5] used their improved version of particle-hole calculations to find continuum wave functions and the cross section for electric dipole transitions. They limit their work to two-body breakup for photon energy up to 36 MeV.

Electric dipole transitions give continuum states with total orbital angular momentum of 1, negative parity, and isospin 1. These states can disintegrate by two-body, three-body, or four-body breakup. Gorbunov[6] and Balestra *et al.*[7] find experimentally that four-body breakup is a rather rare process. Nevertheless, I choose to emphasize the four-body character of the final state by expanding it also in nine-dimensional h.h. The presence of two-body (and three-body) breakup at a lower threshold is taken into account by evaluating the photon energy as the sum of the kinetic energy of the four-body system and the threshold energy for two-body breakup. I treated[8] a similar problem of competing two-body and three-body breakup for the trinucleon photoeffect with some success by a corresponding ansatz.

We are concerned with the serious problem of h.h. expansions for "mixed boundary conditions." In my earlier paper,[8] and in this paper, I assume that at small hyperradius the nucleon-nucleon interactions are strong enough to "wash out" any two-body (or three-body) structure that appears at large hyperradius. An alternative method of calculation would be to start with the wave functions for two-body or three-body breakup and use them (as is, or modified in some appropriate manner) to find the overlap integral and the photoeffect cross section. (This simplified model has been used successfully by Ballot and Fabre[9] for trinucleon breakup into a deuteron and a nucleon.)

Levinger and Fitzgibbon[10] state that their purpose is to extend the approximation of a truncated expansion of the trinucleon wave function in h.h. as far as they can, to see where it breaks down. In this paper I extend the h.h. method still further, to find out whether it will work for this case. It is clear[8] that my approximation will not give a complete solution to the problem of mixed boundary conditions, since I am unable to calculate the branching ratio for different decay modes. I wish to determine whether the h.h. method gives a reasonable result for the *total* cross section for photo absorption. The result will be tested in two different ways: (i) Comparison with experiments[6,7] and (ii) tests for internal consistency by comparison with sum rules. (See Myers *et al.*[11] and Maleki and Levinger[12] for analogous tests for the h.h. expansion for the trinucleon photoeffect.)

In the following section I introduce the formalism for h.h. in nine-dimensional space, appropriate for treatment of the ground or continuum states of the α particle. In Sec. III I calculate the cross section for electric dipole transitions using a single term in the h.h. expansion of the continuum wave function. Integrals over eight angles are evaluated analytically; the cross section is given in terms of a squared radial matrix element. In Sec. IV I present numerical results for a spin-independent Volkov[13] potential with three choices for its exchange character: Pure Wigner force, zero potential (an extreme Born approximation), and an exchange mixture (approximately a Serber force). In the last section, I compare the cross sections with an experiment and with sum rules and I discuss possible improvements in this approach.

II. HYPERSPHERICAL HARMONICS

I follow Fabre's[14] and Zickendraht's[15] notation for h.h. in the nine-dimensional space. I define three vectors (each in three dimensions) for the internal coordinates of the system

$$\vec{\xi}_i = \tfrac{1}{2}(\vec{x}_i + \vec{x}_4 - \vec{x}_j - \vec{x}_k)\,. \tag{1}$$

Here (i,j,k) are a cyclic permutation of $(1,2,3)$, and the vectors $\vec{x}_1, \ldots, \vec{x}_4$ refer to nucleon coordinates.

Our hyperspherical coordinates have one length, the hyperradius ξ, and eight angles, collectively designated by Ω. Six of these eight angles are given by the unit vectors $\hat{\xi}_1, \hat{\xi}_2, \hat{\xi}_3$. We use "spherical polar coordinates" ξ, θ, and ϕ to express the three lengths ξ_1, ξ_2, and ξ_3:

$$\begin{aligned} \xi_3 &= \xi \cos\theta/\sqrt{2}\,, \\ \xi_1 &= \xi \sin\theta \cos\phi/\sqrt{2}\,, \\ \xi_2 &= \xi \sin\theta \sin\phi/\sqrt{2}\,. \end{aligned} \tag{2}$$

Equations (1) and (2) are illustrated in Fig. 1.

The internal kinetic energy of the system is given by[14]

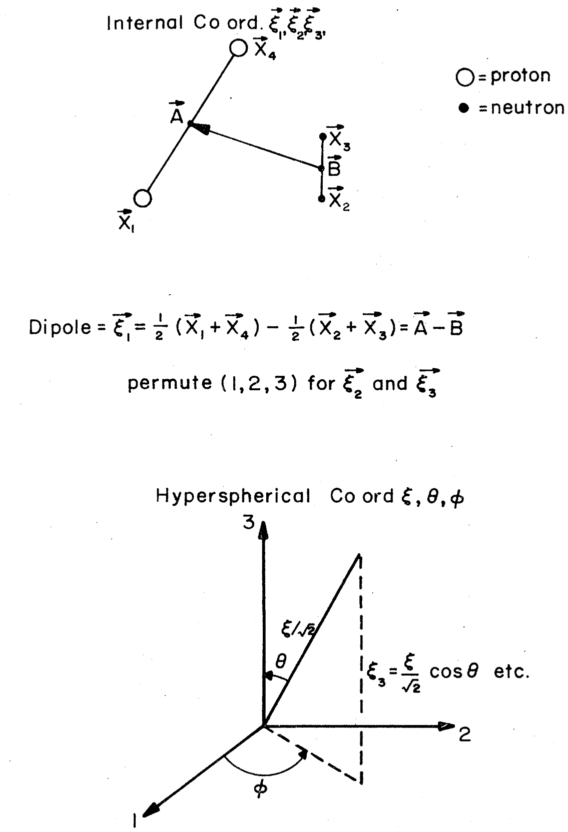

FIG. 1. Top: Internal coordinates, illustrating Eq. (1) for $\vec{\xi}_1$ and the dipole operator, Eq. (17). Bottom: Hyperspherical coordinates ξ, θ, ϕ.

$$\begin{aligned} T &= -(\hbar^2/M)(\nabla_{\xi_1}{}^2 + \nabla_{\xi_2}{}^2 + \nabla_{\xi_3}{}^2) \\ &= -(\hbar^2/M)[\partial^2/\partial\xi^2 + (8/\xi)(\partial/\partial\xi) + \Lambda/\xi^2]\,, \end{aligned} \tag{3}$$

$$\Lambda = \partial^2/\partial\theta^2 + (5\cot\theta - 2\tan\theta)\partial/\partial\theta$$
$$-l_3(l_3+1)/\cos^2\theta + \left[\partial^2/\partial\phi^2 + 4\cot 2\phi\,\frac{\partial}{\partial\phi} - l_1(l_1+1)/\cos^2\phi - l_2(l_2+1)/\sin^2\phi\right]/\sin^2\theta\,. \tag{4}$$

Here $\nabla_{\xi_i}{}^2$ is the Laplacian for the three-vector $\vec{\xi}_i$ $(i=1,2,3)$. The last expression in Eq. (3) gives the kinetic energy in terms of hyperspherical coordinates, where Λ is an operator that involves the eight angles in Ω. The quantum number l_i $(i=1,2,3)$ gives the orbital angular momenta associated with three vectors $\vec{\xi}_i$.

Hyperspherical harmonics $H_{[L]}(\Omega)$ are eigenfunctions of the operator Λ, with eigenvalues $-L(L+7)$, where L is the "grand orbital":

$$\Lambda H_{[L]}(\Omega) = -L(L+7)H_{[L]}(\Omega)\,, \tag{5}$$

$$L = 2(n_2+n_3) + l_1 + l_2 + l_3\,. \tag{6}$$

Here n_2 and n_3 are non-negative integers. We see that the parity is $(-1)^L$. The symbol $[L]$ stands for the eight quantum numbers associated with the eight angular coordinates: L, L_1, l_i and m_i $(i=1,2,3)$. Here

$$L_1 = 2n_2 + l_1 + l_2 \tag{7}$$

$$H_{[L]}(\Omega) = {}^{(3)}P_L^{L_1, l_3}(\theta)\, {}^{(2)}P_{L_1}(\phi)\, Y_{l_1}^{m_1}(\hat{\xi}_1)\, Y_{l_2}^{m_2}(\hat{\xi}_2)\, Y_{l_3}^{m_3}(\hat{\xi}_3)\,. \tag{8}$$

The $Y_{l_i}^{m_i}(\hat{\xi}_i)$ are the standard spherical harmonics. The function ${}^{(2)}P_{L_1}(\phi)$ is that used for h.h. in six dimensions. They are tabulated, for example, by Fang.[16] The function

$$ {}^{(3)}P_L^{L_1, l_3}(\theta)=\{(L+\tfrac{7}{2})(n_3+L_1+2)!\,(2n_3+2L+2l_3+5)!!\,2^{-(L+1)}(n_3!)^{-1}[(2n_3+2l_3+1)!!]^{-1}\}^{1/2} \times[(L_1+2)!]^{-1}(\cos\theta)^{l_3}(\sin\theta)^{L_1}\,{}_2F_1(-n_3, n_3+L_1+l_3+\tfrac{7}{2}, L_1+3; \sin^2\theta)\,. \tag{9}$$

In general, one needs to follow the standard procedure[14] of finding appropriate linear combinations of h.h. to give (i) a specified total orbital angular momentum and its projection, (ii) a specified symmetry of the system on interchange of nucleonic spatial coordinates, and (iii) the optimal subset for a specified form of the nucleon-nucleon potential. In our work, using a single h.h. for the ground state and another single h.h. for the continuum, we can stop with $H_{[L]}(\Omega)$.

The hyperradial function ψ_L for a given grand orbital is conveniently written in terms of the hyperradial function $u_L(\xi)$ as follows:

$$\psi_L(\vec{\xi})=u_L(\xi)H_{[L]}(\Omega)\xi^{-4}\,, \tag{10}$$

The factor ξ^{-4} is chosen to eliminate the first derivative term in Eq. (3). For example, considering a free system with positive total energy $E=\hbar^2k^2/M$, the Schrödinger equation has the form

$$-d^2u_L/d\xi^2+(L+3)(L+4)\xi^{-2}u_L=k^2u_L\,. \tag{11}$$

[The coefficient $(L+3)(L+4)$ comes from the sum of $L(L+7)\xi^{-2}$ from Eq. (5) and $12\xi^{-2}$ from the transformation, Eq. (10).] We recognize the regular solution of the hyperradial Eq. (11) as a spherical Bessel function:

$$u_L(\xi)=k\xi j_{L+3}(k\xi)\,. \tag{12}$$

We shall use this normalization.

In general, we have a noncentral potential $V(\vec{\xi})$, which couples together states of different grand orbital. If we approximate $V(\vec{\xi})$ by its lowest hypermultipole $V_0(\xi)$, then the system of coupled differential equations reduces to Eq. (11) with an additional term involving $V_0(\xi)$,

$$-d^2u_L/d\xi^2+(L+3)(L+4)\xi^{-2}u_L+(M/\hbar^2)V_0(\xi)u_L=k^2u_L\,. \tag{13}$$

If at large hyperradius, V_0 decreases faster than ξ^{-2}, then at large ξ the solution of (13) must be a linear combination of regular and irregular spherical Bessel functions:

$$u_L(\xi)=k\xi[\cos\delta_L\, j_{L+3}(k\xi)-\sin\delta_L\, n_{L+3}(k\xi)]\,. \tag{14}$$

The phase shift $\delta_L(k)$ is for a continuum state of specified grand orbital L and specified wave number k. It could be used to find the cross section for four-body to four-body scattering, as a generalization of Fang's work[16] on three-body to three-body scattering.

The multipole $V_{2K}(\xi)$ can be found from the spin-independent two-body force $v(x_{ij})$ from the expression[14]

$$V_{2K}(\xi)=(48/\pi)\Gamma(\tfrac{9}{2})\Gamma(K+\tfrac{3}{2})[\Gamma(K+3)]^{-1} \times\int_0^1 v(u\xi)\,{}_2F_1(-K, K+\tfrac{7}{2}; \tfrac{3}{2}; u^2)(1-u^2)^2u^2du\,. \tag{15}$$

For a short range nucleon-nucleon potential, any hypermultipole does in fact fall off as ξ^{-3} at large hyperradius, thus justifying our use of spherical Bessel functions in Eq. (14).

III. ELECTRIC DIPOLE TRANSITIONS

The cross section σ_{if} for electric dipole absorption (integrated over the "line") from initial state i to final *discrete* state f is

$$\sigma_{if}=(4\pi^2/\hbar c)E_\gamma\langle i|D|f\rangle^2\,. \tag{16}$$

We choose particles 1 and 4 as protons, and we choose the photon polarization along the z direction. The dipole operator D is given by

$$D=\xi_{1z}=(4\pi^2/3\sqrt{105})e\,\xi H_{[1]}(\Omega)\,. \tag{17}$$

The h.h.

$$H_{[1]}(\Omega)=(3\sqrt{210}/8\pi^2)\sin\theta\cos\phi\cos\theta_1 \tag{18}$$

[see Eqs. (1), (2), (8), and (9)]. Here [1] stands for eight quantum numbers with the following values:

$$L=1,\quad L_1=l_1=1,\quad l_2=l_3=m_1=m_2=m_3=0\,, \tag{19}$$

The initial wave function

$$\psi_i(\xi,\Omega)=u_0(\xi)\xi^{-4}H_0(\Omega)\,, \tag{20}$$

where

$$H_0(\Omega)=(105/2)^{1/2}/(4\pi^2)\,. \tag{21}$$

The final wave function for quantum numbers given by (19) is

$$\psi_f(\xi,\Omega)=u_1(\xi)\xi^{-4}H_{[1]}(\Omega)\,. \tag{22}$$

We substitute (17), (18), (20), (21), and (22) in (16) and use the fact that $H_{[1]}(\Omega)$ is normalized to give

$$\sigma_{if}=(2\pi^2\alpha E_\gamma/9)(\xi_{if})^2\,, \tag{23}$$

the radial overlap integral

$$\xi_{if}=\int_0^\infty u_0(\xi)\xi u_1(\xi)d\xi\,. \tag{24}$$

For transitions to the continuum, we use Eq. (16) with the additional factor ρ_f for the density of final states. Consider a plane wave in nine dimensions, $\exp(i\vec{k}\cdot\vec{\xi})$, with

$$\rho_f=\tfrac{1}{2}(M/\hbar^2)k^7dk/(2\pi)^7d\Omega_k\,. \tag{25}$$

The final state wave function with quantum numbers (19) is found by expanding this plane wave in h.h.[14] I select the relevant term

$$\psi_f(\xi,\Omega)=i(2\pi)^{9/2}\sqrt{2/\pi}\,j_4(k\xi)(k\xi)^{-3}H_{[1]}(\Omega)H_{[1]}(\Omega_k)\,. \tag{26}$$

Substituting in (16) we find a differential cross section

$$d\sigma/d\Omega_k=(2\pi/9)\alpha(E_\gamma/k)(M/\hbar^2)(\xi_{if}^B)^2[H_{[1]}(\Omega_k)]^2\,. \tag{27}$$

Here the radial Born overlap integral

$$\xi_{if}^B=\int_0^\infty u_0(\xi)\xi k\xi j_4(k\xi)d\xi\,. \tag{28}$$

We find the total Born cross section by integrating over the eight angles in Ω_k and using the property that $H_{[1]}(\Omega_k)$ is normalized:

$$\sigma^B(E_\gamma)=(2\pi/9)\alpha(E_\gamma/k)(M/\hbar^2)(\xi_{if}^B)^2\,. \tag{29}$$

If we have a potential $V_0(\xi)$ acting in the final state, we change the final hyperradial wave functions $u_1(\xi)$ and therefore the radial overlap integral to ξ_{if}. We choose $u_1(\xi)$ as the regular solution of Eq. (13) with grand orbital unity and normalized according to Eq. (14). [If $\delta=0$, and $L=1$, this gives $k\xi j_4(k\xi)$ as in (28).] The differential cross section is given by the analog of (27), the total cross section by the analog of (29):

$$\sigma(E_\gamma)=(2\pi/9)\alpha(E_\gamma/k)(M/\hbar^2)(\xi_{if})^2\,, \tag{30}$$

$$\xi_{if}=\int_0^\infty u_0(\xi)\xi u_1(\xi)d\xi\,. \tag{31}$$

I check my photoeffect calculations by comparison with sum rule values for the moments σ_{-1} and σ_0: the bremsstrahlung-weighted and integrated cross sections, respectively.

Using closure for Eq. (23), the bremsstrahlung-weighted cross section is

$$\sigma_{-1}=(2\pi^2/9)\alpha\langle i|\,\xi^2\,|i\rangle\,. \tag{32}$$

This expression agrees with the relation between σ_{-1} and the root-mean-square radius of the α.[17] The value of σ_{-1} is independent of the potential chosen for the final state, just as in our trinucleon work.[10] I calculate it only to check numerical accuracy.

For a Wigner force, the same in initial and final states, the integrated cross section is given by the Thomas-Reiche-Kuhn sum rule[17]

$$\sigma_0=(2\pi^2)\alpha(\hbar^2/M)=59.7\text{ MeV mb}\,. \tag{33}$$

If we choose potentials for initial and final states that differ by ΔV, the integrated cross section is changed by an amount proportional to the ground state expectation value of $\Delta V\xi^2$. For instance, if we use potential $V_0(\xi)$ for the initial state and, in the Born approximation, zero potential for the final state, the integrated Born cross section is

$$\sigma_0^B=(2\pi^2)\alpha(\hbar^2/M)-(4\pi^2/27)\alpha\langle i|\,V_0\xi^2\,|i\rangle\,. \tag{34}$$

IV. RESULTS FOR THE VOLKOV POTENTIAL

BBF[1] used Volkov's[13] spin-independent potential to calculate the ground state of the α particle by an h.h. expansion. The expansion converges rapidly: (i) A single term gives the energy to within 2 MeV; (ii) the lowest term contains 99% of the probability for the ground state wave function.

In this section I use BBF's ground state wave function $u_0(\xi)$ for the Volkov potential together with three different choices of the potential for the final state: (i) The potential $V_0(\xi)$, which is the dominant term in the ground state potential; (ii) zero potential, or extreme Born approximation; (iii) an exchange mixture for the two-body force that gives a potential $\frac{1}{2}(V_0+V_2)$ for the final state.

The Volkov potential $V(x_{ij})$ in MeV for a pair of nucleons separated by a distance x_{ij} in fm is given by the sum of two Gaussians

$$v(x_{ij})=144.86\exp[-(x_{ij}/0.82)^2] -83.4\exp[-(x_{ij}/1.60)^2]\,. \tag{35}$$

I substitute in Eq. (15) to find the lowest hypermultipole.

$$\begin{aligned} V_0(\xi)&=11\,410f(x)-6570f(x')\,,\\ f(x)&=(1/4x^4-15/8x^6)\exp(-x^2)\\ &\quad+\tfrac{1}{2}\pi^{1/2}(1/2x^3-3/2x^5+15/8x^7)\mathrm{erf}(x)\,,\\ x&=\xi/0.82,\quad x'=\xi/1.60\,. \end{aligned} \tag{36}$$

For grand orbital of zero, $(M/\hbar^2)$ times the nuclear potential (36) is combined with the "centrifugal potential," $12/\xi^2$ of Eq. (13). The sum of these two terms is plotted as a solid curve against hyperradius in Fig. 2. I compare with the dashed curve for the (nine-dimensional) simple harmonic oscillator (HO) approximation replacing $V_0(\xi)$. Here

$$(M/\hbar^2)V_{\rm HO}(\xi)=0.070\,\xi^2-3.1\,. \tag{37}$$

The two curves agree reasonably well in the range

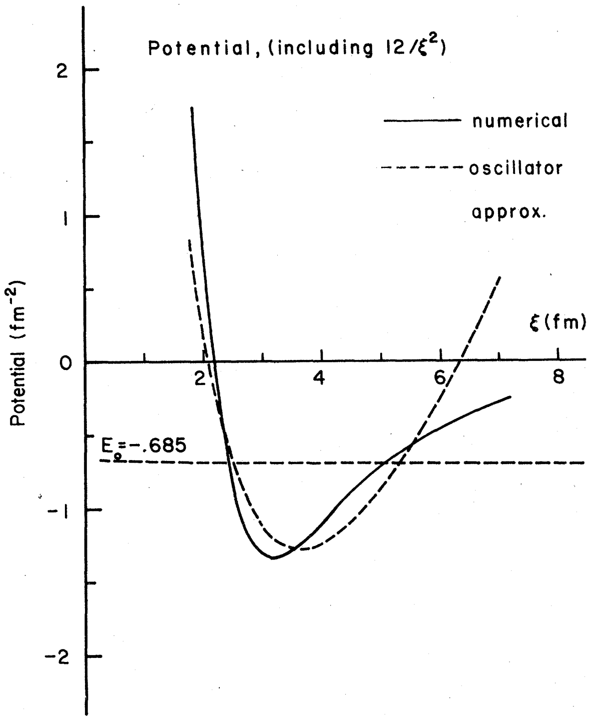

FIG. 2. Potential in fm^{-2} vs hyperradius in fm. The solid curve shows the Volkov and centrifugal potentials $V_0(\xi)+12/\xi^2$ [see Eqs. (13) and (36)]; the dashed curve shows the oscillator approximation, Eq. (37). The horizontal dashed line shows the ground state energy of the α particle.

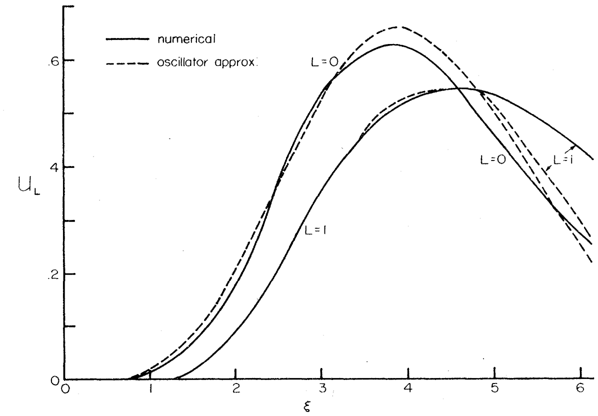

FIG. 3. Hyperradial wave functions $u_L(\xi)$ vs hyperradius ξ, in fm, for grand orbit $L=0$ (ground state) and $L=1$ (dipole state). The solid curves are for the Volkov potential $V_0(\xi)$; the dashed curves are for the harmonic oscillator approximation.

of hyperradius from 3 to 6 fm. (Of course, they differ radically at small hyperradius and at large hyperradius. The difference of nuclear potentials at small hyperradius is "masked" by the very large centrifugal potential.) The coefficient of the quadratic term in (37) gives $\hbar\omega = 11$ MeV, in Fabre's[14] notation. Fabre earlier estimated $\hbar\omega = 10$ MeV from his harmonic oscillator fit to the experimental α particle form factor. The energy of the ground state in the oscillator approximation is

$$E_0 = -3.1(\hbar^2/M) + 9\hbar\omega = -30 \text{ MeV}. \qquad (38)$$

Equation (38) agrees well with either numerical calculation using the uncoupled Eq. (13) or with their result for seven coupled differential equations. (The former gives $E_0 = -28.582$ MeV, the latter -30.376 MeV.)

In the harmonic oscillator approximation, the dipole oscillator strength is localized in a δ function at energy $E = 2\hbar\omega$, or 22 MeV, using our fit (37). We would then anticipate a discrete excited state with grand orbital one, some 22 MeV higher than the ground state. The cross section integrated over the line is calculated using Eq. (23) and compared with the TRK sum rule (33).

I solve the differential Eq. (13) numerically for the two values zero and unity for the grand orbital and determine eigenvalues and eigenfunctions. I find $E_0 = -28.4$ MeV and $E_1 = -7.8$ MeV. (The former result checks well with Ballot, providing confirmation of the accuracy of my numerical work.) The difference $E_1 - E_0$ is within 10% of the 22 MeV value given by the harmonic oscillator approximation. I compare in Fig. 3 the normalized wave functions $u_0(\xi)$ and $u_1(\xi)$ found numerically, with those given by the harmonic oscillator approximation.[14] Again the agreement is satisfactory in the range of hyperradius from 3 to 6 fm which gives the predominant contribution both to normalization integrals and to the overlap integral (24). My numerical wave functions give

$$\sigma_{if} = 55 \text{ MeV mb}. \qquad (39)$$

That is, the line contains more than 90% of the integrated cross section of 59.7 MeV mb.

The remaining cross section is found by calculating continuum wave functions from (13) with grand orbital one with normalization (14). The cross sections found from (30) give a continuum contribution of 0.1 mb to σ_{-1} and a contribution of 3 MeV mb to the integrated cross section. The moments σ_{-1} and σ_0 are compared in Table I with sum rule values (32) and (33). The former gives $\sigma_{-1} = 2.9$ mb. The moments found from the cross sections are in satisfactory agreement with sum rule values.

I turn now to the other extreme of a free particle wave function for the continuum. I designate this as the "Born approximation," but the reader will recognize that the same term is used for other approximations, e.g., a product of a triton and a proton wave function, with no triton-proton inter-

TABLE I. Moments of α photoeffect.

	Bremsstrahlung-weighted		Integrated	
	From cross section	Sum rule	Cross section	Sum rule
Wigner exchange	2.8 mb	2.9 mb	58 MeV mb	60 MeV mb
Exchange mixture	2.8 mb	2.9 mb	120 MeV mb	...
Born approximation	2.6 mb	2.9 mb	200 MeV mb	180 MeV mb
Experiment	2.5 mb		100 MeV mb	

action. The cross section is found from Eq. (29). I use Ballot's wave function $u_0(\xi)$ in finding the overlap integral σ_{if}^B of Eq. (28). My Born cross section is plotted (dashed curve) against photon energy in Fig. 5. For this purpose I evaluate the photon energy as $E_\gamma = \hbar^2k^2/M + 30.4$ MeV, i.e., I use the threshold for four-body breakup for Ballot's solution of seven coupled differential equations. Born approximation gives a very slow rise near the high threshold of 30.4 MeV and a very slow decrease of the cross section at high photon energy. The moments $\sigma_{-1}^B = 2.6$ mb and $\sigma_0^B = 200$ MeV mb are compared in Table I with sum rule results $\sigma_{-1} = 2.9$ mb and $\sigma_0 = 180$ MeV mb. The latter number comes from the evaluation of Eq. (34).

Finally, I consider an exchange mixture, with the potential acting in the final state given by

$$U_1^{(1)}(\xi) = \tfrac{1}{2}[V_0(\xi) + V_2(\xi)]\,. \tag{40}$$

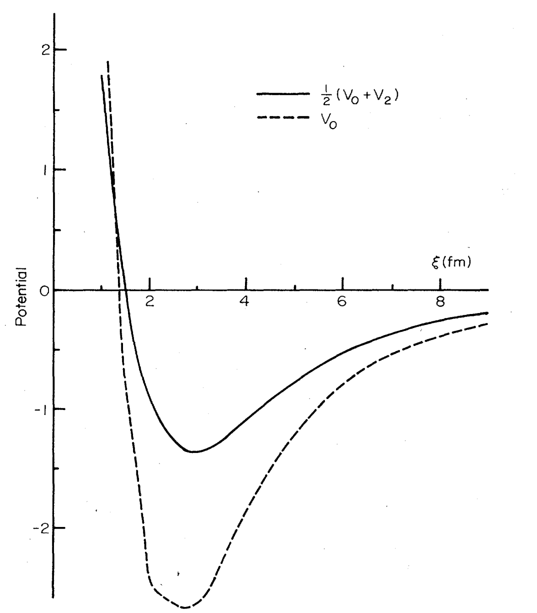

FIG. 4. Comparison of potentials in fm^{-2} vs hyperradius ξ. The dashed curve is $V_0(\xi)$, Eq. (36); the solid curve is $\frac{1}{2}(V_0 + V_2)$ used for an exchange force [Eqs. (40) and (41)].

$U_1^{(1)}(\xi)$ replaces $V_0(\xi)$ in Eq. (13). [In future work I plan to derive the potential $U_1^{(1)}(\xi)$ from some assumed Majorana exchange mixture in the two-body force. In the absence of this derivation, the ansatz (40) is made, by analogy to the same result for a two-body Serber mixture for the trinucleon.[18]] The hypermultipole $V_2(\xi)$ is found from Fabre's formula, (15) for $K=1$.

$$V_2(\xi) = 11\,410\,g(x) - 6570\,g(x')\,,$$
$$g(x) = (1/8x^4 + 315/32x^8)\exp(-x^2) + \tfrac{1}{2}\pi^{1/2}(1/4x^3 - 15/8x^5 + 105/16x^7 - 315/32x^9)\,\text{erf}\,(x)\,; \tag{41}$$

x and x' are defined in Eq. (36).

At small hyperradius V_2 is proportional to ξ^2. At large hyperradius both V_0 and V_2 fall off as ξ^{-3}, but V_2 is half as large as V_0. Figure 4 illustrates $V_0(\xi)$ and $\frac{1}{2}(V_0 + V_2)$.

With the potential given by (40) used in the differential Eq. (13) for grand orbital one, there is no bound state. The numerical solutions are normalized, and used to find the overlap integral ξ_{if} with BBF's ground state u_0 and the cross section $\sigma(E_\gamma)$. Here I follow my ansatz,[8] discussed briefly

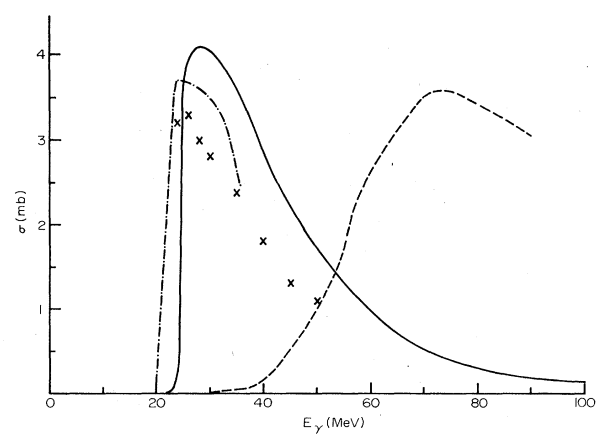

FIG. 5. Total cross section σ in mb for α photoeffect vs photon energy in MeV. The solid curve is for exchange force, the dashed curve for Born approximation, the dash-dot curve by Delsanto *et al.*, Ref. 5. The "x" shows Gorbunov's measurements, Ref. 6.

above, where the photon energy is given by

$$E_\gamma = \hbar^2 k^2/M + 21.9 \text{ MeV}. \tag{42}$$

Here 21.9 is the difference of BBF's binding energy for the α and for the triton. [Instead of 21.9 MeV in (42), I could use the experimental threshold for proton-triton breakup but that refinement is an unimportant detail in the present primitive calculation.] The cross sections are plotted as a solid curve in Fig. 4. I also show Gorbunov's experimental values[6] for the α photoeffect cross section and Delsanto's calculation[5] (dash-dot) for two-body ($p - {}^3$H or $n - {}^3$He) breakup. (Balestra's[7] data lie close to Gorbunov's.)

As a by-product of calculating the normalized continuum wave function, I obtain the phase shift δ_1 for four-body to four-body scattering. I show this phase shift in Fig. 6 for the exchange potential (40). The phase shift becomes larger than 180°, even though there is no bound state.

The cross section for this exchange potential gives the moment values in Table I: 2.8 mb for the bremsstrahlung-weighted cross section, as compared with Gorbunov's value of 2.5 mb, and 120 MeV mb for the integrated cross section, as compared with Gorbunov's 100 MeV mb.

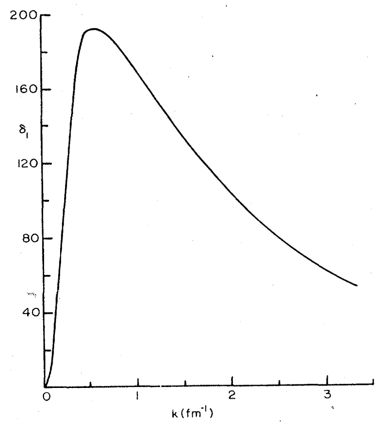

FIG. 6. Phase shift δ_1 in degrees (for four-body to four-body scattering) vs wave number k in fm^{-1}, for an exchange mixture: See Eqs. (14) and (40).

V. DISCUSSION

I use two criteria for the success of h.h. expansions in the calculation of continuum wave functions. First, the calculation needs to be internally consistent. Second, if we use a reasonable nucleon-nucleon potential, we should get satisfactory agreement with experimental results. As noted above, I am trying to push the h.h. expansion farther and farther, to determine the limits of a useful application of this method. (The limits cannot be found by fiat, or by a "back of an envelope" calculation.)

In this paper, I test internal consistency for the choice of a Wigner exchange nucleon-nucleon force, as was done by Myers[10] for the trinucleon photoeffect with the same force. In each case, we obtain agreement within 10% with the model-independent Thomas-Reiche-Kuhn sum rule for the integrated cross section. See Fang *et al.*[19] for the small correction in the trinucleon case due to use of a second grand orbital in the continuum.

I also explore the good agreement between results for a two-body Volkov potential with Wigner exchange and for a harmonic oscillator approximation. The agreement is remarkably good for wave functions, for energy of excitation, and for properties of the photoeffect cross section. This result is a generalization of the agreement between the independent particle model and the collective model of the giant dipole resonance. But it is more difficult to calculate for an exchange force for which the giant dipole resonance moves up to the continuum.

In the case of our exchange mixture [Eq. (40], we are concerned with the second criterion of agreement with experiment. My sum rule results are just a bit outside Gorbunov's quoted errors. My solid curve in Fig. 5 is in fair but not excellent agreement with experiment. (Delsanto's dash-dot curve agrees somewhat better with experiment than my curve.)

The Born approximation, treating the final state as four free particles, is very poor. This is shown by Fig. 5 for the cross sections and Fig. 6 for the four-body phase shifts, which are *not* small.

The calculations in this paper could be improved in two different ways. First, the approximations of this paper could be continued, with certain refinements: (i) Use of a specified two-body Majorana exchange to give the exchange potential $U_1^{(1)}$ and the sum rule value for the integrated cross section; (ii) use of a spin-dependent or a tensor two-body potential and treatment of Coulomb forces; (iii) use of two or more coupled partial waves in the continuum. Second, the problem of mixed boundary conditions in the continuum needs further study. Future work might (i) justify the present ansatz (42), or (ii) replace it by a better approximate method using h.h., or (iii) cast doubt on the utility of h.h. expansion for these continuum calculations.

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