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Rotational-energy contributions to the kinetic energies of deep-inelastic reaction products

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The extraction of rotational contributions to the energies of deep-inelastic fragments from light systems is discussed. It is concluded that the data are consistent with the occurrence of scission at the critical radius as well as at the large radii previously suggested.

[NUCLEAR REACTIONS $^{35}Cl + ^{27}Al$, $^{20}Ne + ^{27}Al$, Calculated deep-inelastic fragment energies, extracted rotational contribution.]

For lighter heavy-ion systems, the contribution to the kinetic energies of deep-inelastic reaction products from rotation of the system has been the subject of some discussion in the recent literature.[1-4] Whereas it is not in doubt that such rotations do make significant contributions to the observed fragment energies, there seems to be some question regarding the validity of the methods used to infer the exact fraction of the final energies reflecting these rotations. It is the purpose of this communication to point out an alternative method of analysis of the data of Refs. 1–4 and also to illustrate some of the possible ambiguities in previous analyses.

In general, the total kinetic energy of a rotating heavy-ion system at scission is given by

$$E_f = V_{\mathrm{Coul}}(R) + V_{\mathrm{nuc}}(R) + \frac{L_f(L_f+1)\hbar^2}{2\mu R^2}, \qquad (1)$$

where the symbols have their usual meanings. The extraction of the rotational contribution from the experimentally measured kinetic energies therefore requires a knowledge of both the scission radius R and the value of the potential at that radius. In classical friction models for deep-inelastic scattering, it is usual to rewrite Eq. (1) as

$$E_f = V_{\mathrm{Coul}}(R) + V_{\mathrm{nuc}}(R) + f^2\,\frac{L_i(L_i+1)\hbar^2}{2\mu R^2}, \qquad (2)$$

where f is a numerical factor depending on the type of frictional force assumed. For final configurations corresponding to pure rolling motion or rigid rotation, $f=\frac{5}{7}$ or $\mu R^2/(\mu R^2 + \mathscr{I}_1 + \mathscr{I}_2)$ respectively.

The authors of Refs. 1 and 4, in their analysis of $^{20}Ne + ^{27}Al$ data at 120 MeV, make the reasonable assumption that the deep-inelastic collisions arise from incident partial waves just larger than those leading to fusion. This assumption is consistent with the integrated cross sections for the most strongly damped collisions which indicate the participation of at most three partial waves. With the further assumptions of rigid rotation and of a value for V_{nuc} taken from the potential of Bass,[5] Eq. (2) was then solved for the scission radius R which, for the symmetric fragmentation, was found to be 10.2 fm.

Braun-Munzinger *et al.*[2] point out some possible ambiguities in the above analysis and conclude that an unambiguous assessment of the rotational contribution is not possible when data exist only at a single bombarding energy. They further suggest that a measurement of the variation of deep-inelastic fragment energies with changing incident energy can lead to a nearly unambiguous resolution of this question. By differentiating Eq. (2) with respect to incident energy they obtained

$$\frac{\partial E_f}{\partial E_i} = \alpha \approx \frac{f^2(2L_i+1)\hbar^2}{\mu R^2}\,\frac{\partial L_i}{\partial E_i}, \qquad (3)$$

which was then used in an analysis of data for $^{35}Cl + ^{27}Al$ at several bombarding energies. Fair agreement with Eq. (3) was found for scission radii corresponding to a radius parameter of 1.62 fm. As before, it was assumed that partial waves just larger than those going to fusion are appropriate for deep-inelastic collisions. A more detailed analysis of the same data[3] leads to essentially the same conclusions, namely, scission of a rigidly rotating dinuclear system occurring at a large radius.

The results of these analyses have important physical consequences. The data for both $^{20}Ne + ^{27}Al$ and $^{35}Cl + ^{27}Al$ seem to indicate extremely large scission radii which, if the fragments are viewed as touching ellipsoids, require deformations of the order of $\beta = 0.4-0.5$. However, light nuclei such as those involved in these collisions are not easily deformable. In fact, estimates of the deformability using the energies of the giant resonances in these light nuclei give $\partial\beta_2/\partial E = 0.01\ \mathrm{MeV}^{-1}$, which implies that 30–40 MeV of energy per fragment is tied up in producing the deformations necessary for the large scission radii. Such a situation would have important consequences for the state of the fragments immediately following scission which might manifest itself in their decay properties. A further implication of the above treatments is that the radial and tangential frictional forces which dissipate energy and angular momentum extend out to the large radii deduced for scission.

In this communication, we present an analysis which does not require large scission radii and is perhaps also more in the spirit of classical friction models which are currently popular.

Starting from Eq. (2),

$$E_f = V_f(R) + \frac{f^2 L_i(L_i+1)\hbar^2}{2\mu_f R^2}, \tag{4}$$

where $V_f(R)$ is the sum of the nuclear and Coulomb potentials in the exit channel, we assume, as before, that the partial wave just greater than those which fuse (L_{cr}) gives rise to the deep-inelastic process. Following Glas and Mosel,[6] L_{cr} may be related to the fusion cross section by the expression

$$\sigma_{fus} = \frac{\pi\hbar^2}{2\mu_i E_i}(L_{cr}+1)^2 = \frac{\pi R_{cr}^2}{E_i}[E_i - V_i(R_{cr})], \tag{5}$$

which, substituting in Eq. (4), gives

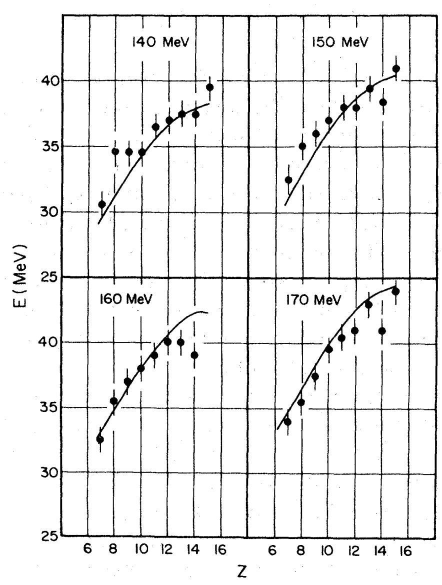

FIG. 1. Comparison of measured and calculated fragment energies for $^{35}Cl + {}^{27}Al$. The calculated curves assume $V_i(R_{cr}) = 17.3$ MeV and $R_{cr} = 7.33$ fm taken from fusion data.

$$E_f = V_f(R) + f^2 \frac{\mu_i R_{cr}^2}{\mu_f R^2}[E_i - V_i(R_{cr})], \tag{6}$$

which is similar to an expression derived in Ref. 2. We now, however, assume that scission effectively occurs at $R = R_{cr}$, the strong-damping radius, and evaluate f and V_f at this radius. The assumption here is that either the range of the frictional forces is short or that the relatively low velocities of the separating fragments result in only weak dissipative forces in the exit channel.

The calculations of the final kinetic energies for $^{35}Cl + {}^{27}Al$ and $^{20}Ne + {}^{27}Al$ were performed using values for $V_i(R_{cr})$ and R_{cr} extracted from the fusion data quoted in Refs. 1–4. The exit channel potential V_f was calculated using the entrance channel value $V_i(R_{cr})$ and the A dependence of the nuclear potential suggested by Bass,[5] Randrup *et al.*,[7] and Ngô *et al.*,[8]

$$V_{nuc} \propto \frac{A_1^{1/3} A_2^{1/3}}{A_1^{1/3} + A_2^{1/3}}, \tag{7}$$

together with a Coulomb potential due to two overlapping charged spheres each of radius $1.25\ A^{1/3}$ fm. The value of f used assumed rigid rotation of two rigid spheres of radius $1.25\ A^{1/3}$ fm with centers separated by R_{cr}. In some sense, therefore, there are no free parameters in this calculation although it is certainly model dependent.

Figure 1 shows a comparison of the $^{35}Cl + {}^{27}Al$ data with the calculated final energies. The overall agreement is excellent, at least as good as in previous analyses. The results for ^{20}Ne

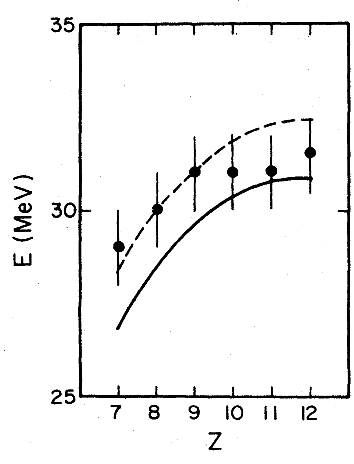

FIG. 2. Comparison of measured and calculated fragment energies for $^{20}Ne + {}^{27}Al$. The solid curve is a calculation assuming $V_i(R_{cr}) = 7.6$ MeV and $R_{cr} = 5.80$ fm taken from fusion data. The dashed curve was calculated with $R_{cr} = 6.0$ fm.

$+^{27}$Al are shown in Fig. 2. The solid curve shows the calculated energies using the parameters extracted from the fusion data. The agreement is less satisfactory than that obtained for ^{35}Cl $+^{27}$Al. However, an increase in the critical radius of less than 5% produces much better agreement, as shown by the dashed curve. The agreement for these two cases is at least as good as that obtained in previous analyses.

We therefore see that equally consistent methods of analysis can lead to quite different values for the scission radius. In order to understand this ambiguity it is necessary to examine each of the ingredients in the calculation of the final energies. In Eq. (2), it is clear that any calculation of the final energies involves three inputs: a choice of V_{nuc}, a choice of L_i, and a choice of R. The authors of the earlier papers choose R to be large, essentially outside the range of nuclear forces, thus tacitly avoiding the choice of V_{nuc}. The choice of angular momenta has been mentioned earlier. On the other hand, we choose R to be small but with a value at which we claim to know V_{nuc}. The value chosen for V_{nuc} comes from measured fusion data but is also closely equal to that given say by the Bass potential[5] calculated with a_s=17 MeV and d=1 fm. Other choices of R and V_{nuc} are also possible but are less firmly anchored to the two points at which it might be claimed that V_{nuc} is known. In fact, it is possible to generate a "reasonable" nuclear potential for which Eq. (2) gives the same final energy almost independent of scission radius (Woods-Saxon $V=-30$ MeV, r_0=1.44 fm, a =0.73 fm). A study of Eq. (2), as has been pointed out, is therefore insufficient for the unambiguous determination of final fragment energies.

The analyses of Refs. 2 and 3 are similarly ambiguous. In Ref. 2 *fair* agreement is obtained with the measured final fragment energies assuming, as above, that L_i is the partial wave just greater than those going to fusion. The more sophisticated analysis of Ref. 4 improves the agreement between theory and data by increasing the value of L_i used from the above value to one midway between fusion and grazing. Our analysis on the other hand seems to give at least as good agreement with the data, improvement over Ref. 2 being obtained by reducing the scission radius which then influences f and thus $\partial E_f/\partial E_i$.

We conclude therefore that the analyses of deep-inelastic fragment energies as presented are ambiguous and are insensitive to the choice of scission radius. We note, however, that all three analyses seem consistent with the assumption of rigid molecule-like rotation of the two fragments and differ only in the radius of this rotation. Further experimental work will be necessary to elucidate this point.

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