Nuclear level density in the ^{4}He system

J. J. Bevelacqua*
Department of Physics, The Florida State University, Tallahassee, Florida 32306
(Received 9 August 1978; revised manuscript received 27 September 1978)

Level densities for the ^{4}He system are calculated within a $4\hbar\omega$ model space by utilizing a realistic nucleon-nucleon interaction. The fluctuations are smoothed with a least squares fitting procedure and exponential fits of the form $A\exp(E/T)$ are calculated for the density and total number of levels. The total number of levels is also compared with the predictions of the equidistant model. Systematics of level densities for even-even nuclei for $4 \leq A \leq 66$ and for nuclei in general for $4 \leq A \leq 240$ are discussed.

[NUCLEAR STRUCTURE ^{4}He, calculated $J^\pi = 0^+, 0^-, 1^+, 1^-, 2^+, 2^-, 3^+, 3^-, 4^+, 4^-, 5^+, 5^-$, and 6^+ levels up to 85 MeV excitation energy. Level density parameters are obtained for constant temperature and equidistant models.]

I. INTRODUCTION

Nuclear level density data are available for many nuclei, but most calculations are performed at neutron separation energies for nuclei with $A \geq 20$.[1] At the present time, most calculations utilize realistic single particle levels, hence shell effects are incorporated into these calculations. The only residual interaction is an approximate pairing force, and the calculations lead to approximate nuclear densities. The calculations usually consider excitation energies $E_x \leq 15$ MeV and level densities greater than $10-10^5$ levels/MeV. These approaches or other approximate calculations have not been extended to light nuclei. Such an extension is important because light nuclei, such as ^{4}He, represent an endpoint for density measurements, because the $A = 4$ system is the lightest system which has known continuum energy levels[2].

A second motivation for this paper is that the experimental determination of continuum levels in the ^{4}He system for $E_x \geq 20$ MeV is difficult, and to date only ten levels are known.[2] Two other possible levels have been suggested, but they are speculative at best.[3,4] In view of the experimental difficulties, a calculational technique, even if it is approximate, may serve as a guide to the ^{4}He continuum, until experimental studies confirm or refute the existence of the predicted levels.

II. CALCULATIONS IN A MODEL SPACE

The concept of a level density is not normally applied to ^{4}He, because there exist only a few resonances below 30 MeV. There is also very little known about continuum levels above this energy, but calculations suggest the levels become more dense as the energy increases.[5] The levels are also more narrow than expected. The calculations, predicting the levels, must be performed in a model space which adequately describes the energy region of interest. Since $4\hbar\omega$ represents a practical calculational limit, we will only investigate the energy region below which $5\hbar\omega$ or higher excitation becomes important. Preliminary calculations of 5 and $6\hbar\omega$ states suggest that they do not become a consideration until 80 MeV excitation. There may, of course, be isolated interloping states below 80 MeV, but these will not significantly affect the calculation. Our estimates suggest that a $4\hbar\omega$ model space, including states of $J^\pi = 0^+, 0^-, 1^+, 1^-, 2^+, 2^-, 3^+, 3^-, 4^+, 4^-, 5^+, 5^-$, and 6^+, is adequate to accurately determine the number of levels below 80 MeV excitation in the ^{4}He system.

Our discussion of the calculated properties of the ^{4}He system is based on the level scheme presented in Table I. The levels presented in Table I are more complicated than conventional shell model states because they include continuum effects of the $p + {}^3$H, $n + {}^3$He, and $d + {}^2$H channels.[6]

Before proceeding, it is important to discuss the spurious state problem. This problem is important because there are not an enormous number of states in Table I, and as such the spurious state problem can not be neglected. In most model spaces, there exist spurious states associated with center-of-mass motion.[7] If all particles are put in the lowest orbits allowed by the Pauli principle, the center of mass of the system is in a $0s$ state. But once particles are excited from the lowest allowed configurations, there may occur states in which the internal structure is identical to the internal structure of another state in the space, but the center of mass is in a different state. If these states are not eliminated from the space, the effects of the allowed states will be introduced more than once. This problem can be treated ex-

TABLE I. Tabulation of energy levels for ^{4}He.

Range of excitation energy (MeV) [a]	0^+	0^-	1^+	1^-	2^+	2^-	3^+	3^-	4^+	4^-	5^+	5^-	6^+
0–1	1	0	0	0	0	0	0	0	0	0	0	0	0
22–23	0	1	0	0	0	0	0	0	0	0	0	0	0
27–28	0	0	0	0	0	1	0	0	0	0	0	0	0
29–30	0	0	0	0	0	1	0	0	0	0	0	0	0
30–31	1	1	0	0	0	0	0	0	0	0	0	0	0
31–32	0	0	0	1	0	0	0	0	0	0	0	0	0
33–34	0	0	0	2	0	0	0	0	0	0	0	0	0
36–37	0	0	1	0	0	0	0	0	0	0	0	0	0
38–39	1	0	0	0	1	0	0	0	0	0	0	0	0
40–41	0	0	0	0	1	0	0	0	0	0	0	0	0
41–42	0	0	1	0	0	0	0	0	0	0	0	0	0
42–43	0	0	1	0	0	0	1	0	0	0	0	0	0
43–44	1	0	0	0	1	0	0	0	1	0	0	0	0
45–46	0	0	1	0	1	0	0	0	0	0	0	0	0
46–47	0	0	1	0	2	0	1	0	0	0	0	0	0
47–48	0	0	0	0	0	0	1	0	0	0	0	0	0
48–49	2	0	2	0	0	0	0	0	0	0	0	0	0
49–50	0	0	0	0	0	0	1	0	0	0	0	0	0
50–51	0	0	0	0	1	0	1	0	0	0	0	0	0
51–52	0	0	0	0	2	0	0	0	0	0	0	0	0
52–53	0	0	1	0	1	0	0	0	0	0	0	0	0
53–54	1	0	1	0	2	1	0	0	0	0	0	0	0
54–55	0	0	2	1	1	0	0	0	0	0	0	0	0
55–56	0	0	1	0	0	0	0	0	0	0	0	0	0
56–57	1	0	0	0	0	0	0	0	0	1	0	0	0
57–58	1	1	0	0	0	0	0	2	0	0	0	0	0
58–59	0	0	0	0	0	1	0	1	0	0	0	0	0
59–60	0	0	0	0	0	1	0	1	0	0	0	0	0
60–61	0	2	0	1	0	1	0	0	0	0	0	0	0
62–63	0	0	0	1	0	0	0	0	0	0	0	0	0
63–64	0	0	0	1	0	1	0	1	0	0	0	0	0
64–65	0	1	0	1	0	2	0	0	1	1	0	0	0
65–66	1	1	0	1	0	1	0	1	0	2	0	1	0
66–67	0	0	0	2	0	2	0	2	0	0	0	0	0
67–68	1	0	0	0	0	1	0	1	1	0	1	0	0
68–69	0	0	1	2	1	1	2	1	2	0	0	0	0
69–70	0	0	0	1	1	2	1	1	0	2	0	0	0
70–71	0	0	2	2	1	1	1	0	1	1	0	0	0
71–72	0	1	1	1	1	2	0	2	1	0	0	0	1
72–73	1	0	1	0	2	0	0	0	0	0	1	0	0
73–74	1	2	1	2	3	4	0	1	1	1	0	0	0
74–75	1	0	1	1	0	1	3	1	3	0	1	0	0
75–76	1	1	2	0	3	1	2	1	0	1	0	0	0
76–77	0	1	3	1	5	2	3	2	2	1	1	0	0
77–78	1	0	2	1	3	1	2	1	2	0	1	0	0
78–79	2	0	3	3	4	2	3	1	4	0	1	0	0
79–80	0	1	3	2	2	2	3	0	1	0	2	0	1
80–81	2	1	2	1	5	2	4	2	1	0	0	0	1
81–82	0	0	3	1	3	0	3	0	3	0	1	0	0
82–83	2	0	2	0	4	1	4	1	1	0	1	0	0
83–84	1	0	3	0	3	1	6	0	2	0	1	0	0
84–85	1	0	2	1	5	0	3	0	3	0	0	0	0
85–86	1	1	2	2	3	1	3	0	3	0	0	0	0

[a] The values of the range of excitation energies represent the 1 MeV bin end points. Omitted values, such as 1–2, 4–5, and 23–24, indicate that the bin is empty.

actly in the case when the eigenstates of the harmonic oscillator are used as a single partical basis, and in that case only if all possible configurations up to a given $N\hbar\omega$ excitation are used. The method of extraction involves performing a projection from the original oscillator basis to a smaller space in which there are no spurious states. The Hamiltonian is calculated and diagonalized in this truncated space and then, when necessary, the eigenvectors can be transformed back to the original basis. This complicated procedure becomes very difficult to perform as N increases. It is partly because of the spurious state problem that more than $2\hbar\omega$ excitations are not included in shell model spaces. Suppose for example, $4\hbar\omega$ excitations, such as $(0s)(0p)^2(0d)$ or $(0s)^2(0d)^2$, were included in the calculation, then one would also have to include configurations such as $(0s)^3(2s)$ or $(0s)^2(0p)(1p)$ if one wanted to eliminate the spurious state problem. This leads to a calculation beyond the scope of many existing shell model codes. As discussed previously, the spurious state problem can not be neglected and a more practical method of eliminating the center-of-mass motion must be found in order to make our calculations meaningful.

As an alternative approach, we perform a complete $4\hbar\omega$ calculation by choosing internal coordinates to describe the ^{4}He system and avoid the center-of-mass problem entirely. The internal coordinates carry the full $4\hbar\omega$ and the center-of-mass coordinate is forced into an acceptable $0\hbar\omega$ of oscillator excitation. The internal coordinates and basis states are discussed in detail in Ref. 6. Once the center-of-mass problem is eliminated and a suitable potential is determined, level density estimates can be obtained.

The calculated level densities, obtained from our effective Sussex interaction[6,8], exhibit considerable fluctuations. Some of these fluctuations are real, and some result from the approximate nature of our effective interaction. The calculated density can be obtained from the partition function $Z_c(B)$, which is defined in terms of the calculated levels E_i:

$$Z_c(B) = \sum_i \exp(-BE_i). \tag{1}$$

The level density $\rho(E)$ is related to Z_c by

$$\rho(E) = \frac{1}{2\pi i} \int_{-i\infty}^{+i\infty} Z_c(B) \exp(BE) dE. \tag{2}$$

This integral can be used to smooth the level density by employing standard saddle point methods.[1] As an alternative, the smoothing process may be performed by a least squares fit of the calculated levels. The remainder of this paper will consider

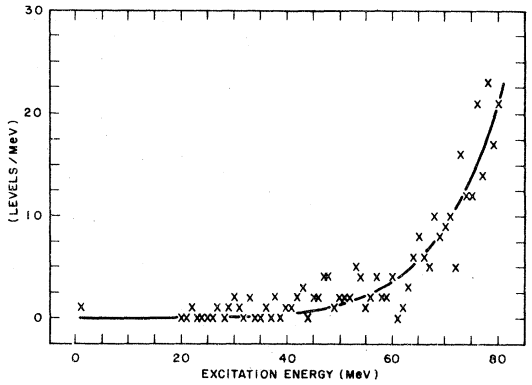

FIG. 1. Energy level density for ^{4}He using the constant temperature model. The "×" represents the state density for a 1 MeV bin. The solid curve is a least squares fit to the form $A \exp(BE)$ with $A = 3.492 \times 10^{-5}$ and $1/T = 0.5554$.

fits to various models. Particular attention will be paid to models which can provide insight to heavier nuclei as well as aid in the understanding of the ^{4}He system.

III. CONSTANT TEMPERATURE FORMULA

Experimental data involving level densities are often analyzed with the constant temperature formula[9]

$$\rho(E) = A \exp(E/T), \tag{3}$$

where $\rho(E)$ is the total number of levels per MeV, and A and T are constants which are determined from a least squares fit to the levels of Table I. It should be emphasized that these simple level density approximations may not reproduce very well the level density of a nucleus which has marked structure in its single particle levels. As such, the investigation of level density parameters as a function of mass leads to a better understanding of level density systematics as well as individual differences between nuclei.

Within the constant temperature model the functional form for the level density is expected to be a simple exponential from experimental measurements on $A = 36$–66 even-even nuclei.[9] The information of Table I can be fitted to the form of Eq. (3). The values $A = 3.492 \times 10^{-5}$ and $1/T = 0.5554$ are obtained from the fit shown in Fig. 1. As expected there is considerable fluctuation, and this can be removed by considering the total number of levels $N(E)$.

The total number of levels $N(E)$ can be deduced from analysis of emergent particle spectra from (p, p') or (p, α) reactions. The data conform to an exponential distribution of spacings. The mass

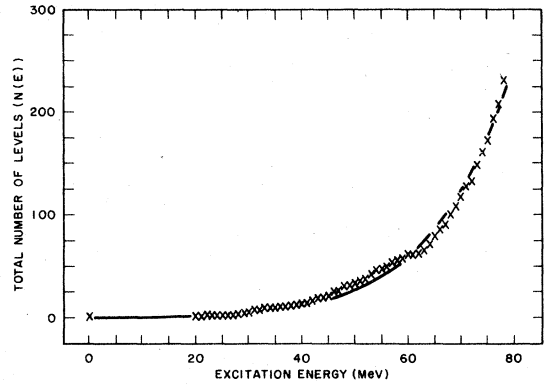

FIG. 2. Number of energy levels as a function of energy for the ^{4}He system using the constant temperature model. The "×" represents the total number of levels $N(E)$ up to an energy E. The solid curve is a least squares fit to the form $A\exp(BE)$ with $A = 5.520\times10^{-3}$ and $B = 0.3579$. The parameter $d = 1/B = 2.79$.

36–66 levels were fitted to the functional form[9]

$$N(E) = c\exp(E/d), \tag{4}$$

where the values of the parameter d obtained by Huizenga[9] increase as A decreases.

When the levels $N(E)$ of Table I are fitted to the functional form of Eq. (4), the curve shown in Fig. 2 is obtained. The parameters $c = 5.520\times10^{-3}$ and $d = 2.79$ are obtained.

The calculated value for d in the ^{4}He system represents an end-point value and may be compared with the values of Huizenga,[9] summarized in Table II. When the values of Ref. 9 are fitted to a linear equation

$$d = mA + b, \tag{5}$$

the values $m = -2.421\times10^{-2}$ and $b = 2.622$ are obtained. If the value of d, calculated by our ^{4}He model, is to carry any credibility, it should be

TABLE II. Parameters for even-even nuclei.

Nucleus	d (MeV)
^{4}He	2.79 [a]
^{36}Ar	1.87
^{38}Ar	1.47
^{40}Ca	1.73
^{50}Cr	1.29
^{52}Cr	1.43
^{54}Cr	1.22
^{54}Fe	1.40
^{56}Fe	1.40
^{58}Fe	1.31
^{66}Zn	0.90

[a] This work; all others Reference 9.

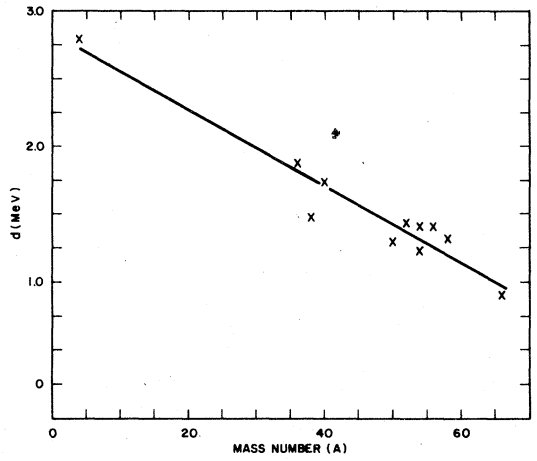

FIG. 3. Energy level systematics for $4 \leq A \leq 66$ using the constant temperature model. The "×" represents the experimental values of Ref. 9 and the value for ^{4}He calculated herein. The parameter d is fitted to the functional form $mA + b$ with $m = 2.822\times10^{-2}$ and $b = 2.831$.

at least in qualitative agreement with the results of Ref. 9. Figure 3 illustrates a plot of $d(A)$ which includes the values of Huizenga and those of our model. The inclusion of the $A = 4$ value leads to linear parameters of $m = -2.822\times10^{-2}$ and $b = 2.831$ which are representative of the expected values established by Huizenga's data.

IV. THE EQUIDISTANT MODEL

In the equidistant model,[10] the single particle levels are equidistant and nondegenerate. The total state density for a system composed of neu-

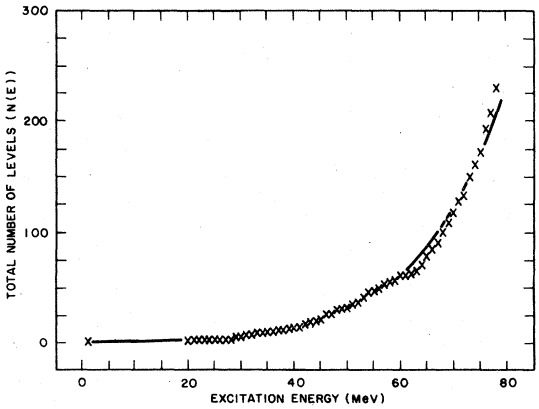

FIG. 4. Number of energy levels as a function of energy for the ^{4}He system using the equidistant model. The "×" represents the total number of levels $N(E)$ up to an energy E. The solid curve is a least squares fit to the form of Eq. (6) and leads to a level density parameter $a = 0.50$.

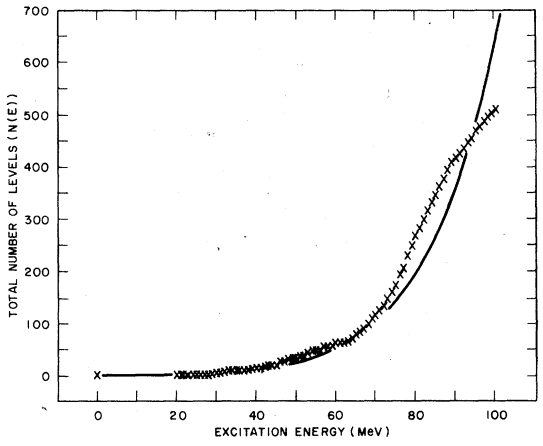

FIG. 5. Number of energy levels as a function of energy up to 100 MeV excitation in the ^{4}He system using the equidistant model. The "×" represents the total number of levels $N(E)$ up to an energy E. The solid curve is a least squares fit to the form of Eq. (6) and leads to a level density parameter $a = 0.49$.

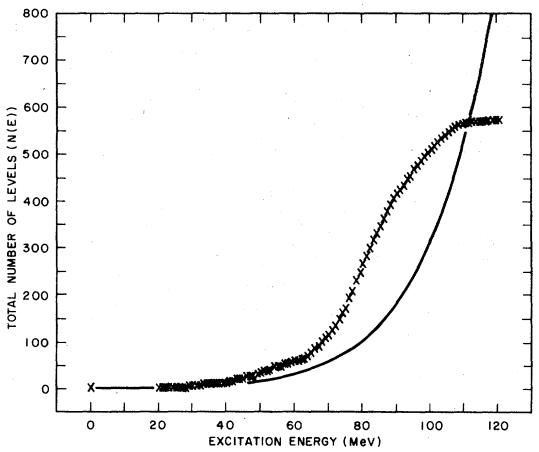

FIG. 6. Number of energy levels as a function of energy up to 120 MeV excitation in the ^{4}He system using the equidistant model. The "×" represents the total number of levels $N(E)$ up to an energy E. The solid curve is a least squares fit to the form of Eq. (6) and leads to a level density parameter $a = 0.44$.

trons and protons is given by

$$N(E) = \frac{\sqrt{\pi}}{12} \frac{\exp(2\sqrt{aE})}{E^{5/4}\, a^{1/4}}, \tag{6}$$

where a is a level density parameter which has been determined[11] to have the value $A/8$ and A is the atomic mass. The formula is sometimes described as a Fermi gas level density expression but actually only represents the zeroth order approximation to the level density of a Fermi gas. We consider the expression because the level density parameter has been tabulated for $20 \leq A \leq 240$. These tabulated values suggest that a value of 0.5 for a is the appropriate end-point value for ^{4}He.

The levels of Table I have been fitted to the functional form of Eq. (6). A least squares fit to the calculated levels is shown in Fig. 4. The fit is obtained for the value $a = 0.50$ which is in agreement with the anticipated result.

It is important to note that the constant density formula given in Eq. (6) is an approximation which is valid only for energies which are low compared to the energy of the deepest hole that can be made in the nucleus. Thus, as stated in the introduction, most investigations consider excitation energies of less than 15–20 MeV. That is, Eq. (6) is an asymptotic expression which is valid for an infinite number of occupied levels—i.e. infinite atomic mass A. For finite A, there is a limit to how deep the single-hole energy can be. For the ^{4}He system, this limit should occur at an energy of about $4\hbar\omega$, since four holes can be created. Once the four hole energy is exceeded, the level density should begin to level off as the excitation energy reaches $4\hbar\omega$ and not continue to climb as rapidly as suggested by Fig. 1. In order to illustrate this point, the level calculations are extended to 100 and 120 MeV excitation. Figures 5 and 6 illustrate this leveling-off at higher energies. Part of the leveling is attributed to the hole effect with the remainder coming from the neglect of levels with $5\hbar\omega$ or more oscillator excitation.

V. CONCLUSIONS

The $4\hbar\omega$ calculations predict a series of levels which are adequate to represent the ^{4}He system up to 80 MeV excitation energy. The level scheme is fitted equally well by both constant temperature and equidistant models. Comparison of the calculated results is consistent with mass 36–66 even-even nuclei calculations if the constant temperature model is utilized. The equidistant model, which has been utilized from mass 20–240, predicts a level density parameter which is consistent with our calculated values. The agreement of our model with both the constant temperature and equidistant models suggests that the ^{4}He system is reasonably well described by both models up to 80 MeV excitation. The results of this calculation also provides an end point for level density parameters of the constant temperature and equidistant models.

This work is based on studies initiated at the Florida State University and is related to the author's Ph.D. dissertation. The author is indebted to Professor R. J. Phillpott for his interest and numerous helpful discussions during the initial phase of this work.

*Present address: Planning Division, United States Department of Energy, P.O. Box E, Oak Ridge, Tennessee 37830.

[1]B. C. Smith, F. N. Choudhury, and S. Das Gupta, Phys. Rev. C 17, 318 (1978) and references therein.
[2]S. Fiarman and W. E. Meyerhof, Nucl. Phys. A206, 1 (1973).
[3]P. W. Lisowski, T. C. Rhea, R. L. Walter, C. E. Busch, and T. B. Clegg, Nucl. Phys. A259, 1 (1976).
[4]J. J. Bevelacqua, Bull. Am. Phys. Soc. 22, 994 (1977); and Phys. Rev. C 16, 1782 (1977).
[5]J. J. Bevelacqua, Phys. Rev. C 18, 591 (1978).
[6]J. J. Bevelacqua and R. J. Philpott, Nucl. Phys. A275, 301 (1977).
[7]T. T. S. Kuo and J. B. McGrory, Nucl. Phys. A134, 633 (1969).
[8]J. P. Elliott, A. D. Jackson, H. A. Mavromatis, E. A. Sanderson, and B. Singh, Nucl. Phys. A121, 241 (1968).
[9]J. R. Huizenga and A. A. Katsanos, Nucl. Phys. A98, 614 (1967).
[10]H. Bethe, Phys. Rev. 50, 332 (1936); Rev. Mod. Phys. 9, 69 (1937); Phys. Rev. 53, 675 (1938).
[11]J. R. Huizenga, in *Statistical Properties of Nuclei*, edited by J. B. Garg (Plenum, New York, 1972), p. 425.