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Partially conserved axial-vector current and model chiral field theories in nuclear physics

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We comment on the relation between the two standard approaches to chiral symmetry–namely, the current algebra/partially conserved axial-vector current approach and the chiral Lagrangian method–in a manner intended to clarify recent and probable future applications of this symmetry in nuclear physics. Specifically, we show that in explicit chiral field theories the canonical πN scattering amplitude does not have the famed "Adler zero" unless partial conservation of axial-vector current holds as an operator equation. This implies that there are a number of familiar chiral models in which the "Adler self-consistency" condition does not apply to the canonical pion field. Among the problems of current interest for which our remarks are relevant are the studies of the pion-nucleus optical potential, pion condensation, and the attempts to formulate a model field theory having both reasonable nuclear saturation and good low energy pion phenomenology.

[NUCLEAR STRUCTURE Comparison of relation of pion field to axial-vector current in field theories of nuclear matter.]

The role of chiral symmetry in nuclear physics has recently been the subject of considerable study,[1-10] particularly in analyses of the pion optical potential in the contexts of both pion-nucleus scattering[2] and pion condensation.[3-8] As in the earlier particle physics applications, the consequences of chiral symmetry for nuclear physics problems have been examined by two basic approaches,[11] one[1,9,10] based on current algebra and the partially conserved axial-vector current (PCAC) relation of the divergence of the axial-vector current to the pion field[12]

$$\partial^\mu A^i_\mu(x) = m_\pi^2 f_\pi \pi^i(x)\,, \tag{1}$$

and the other [4-7] based on chiral symmetric Lagrangians modified by small explicit symmetry breaking terms. Both approaches have been useful historically and currently both are actively being pursued. Recently, for example, the current algebra/PCAC approach was applied to suggest that deep inelastic lepton processes could be used to measure, over certain kinematic ranges, the pion-nucleus optical potential,[9] and the chiral Lagrangian approach was used to clarify some subtleties involved in applying the ideas of chiral symmetry to pion condensation.[5,6]

Given these two different approaches, it is natural to study the extent to which they can be applied in combination consistently: that is, to answer the question: "Can one use PCAC and current algebra at one stage of a calculation and an explicit chiral model at another?" Such a combined approach could be especially useful in the study of the currently unsolved problem of constructing a model field theory of nuclear physics which has both saturation by a realistic mechanism[13] and reasonable low energy πN phenomenology.[14]

The present note provides a pedogogical explication of the answer to the above question, which answer, although obvious in retrospect, seems not to have been sufficiently appreciated. Briefly stated, the answer is as follows: In a given chiral model field theory with a specific choice of canonical fields, unless (1) holds as an operator equation, certain familiar results of PCAC will simply not be true, and attempting to enforce them "by hand" will lead to inconsistencies. In other words, if one has chosen an explicit model field theory, one *cannot* consistently add to it the conclusions of PCAC unless PCAC holds as a canonical equation, in which case the results can be derived directly from the theory anyway.

To illustrate this result we shall consider two familiar theorems of chiral symmetry or PCAC/current algebra. To state these theorems precisely, we begin by defining the quantities involved. First, the "pion-nucleon Σ term" $\Sigma(t)$, which is an important measure of chiral symmetry breaking, is defined by[11,15]

$$\Sigma(t) = \tfrac{1}{3}\sum_{i=1}^{3} \langle N(p')|\{{}^5Q^i(t),[{}^5Q^i(t),\mathcal{H}(0)]\}|N(p)\rangle\,, \tag{2}$$

where $\mathcal{H}(0)$ is the Hamiltonian density and

$${}^5Q^i(t) \equiv \int A^i_0(x)d^3x \tag{3}$$

is the axial-vector charge. Second, a particular combination of the standard πN invariant scattering amplitudes is defined by[15-17]

$$\tilde{D}^{(+)} \equiv A^{(+)} + \nu B^{(+)} - (A^{(+)} + \nu B^{(+)})_{PV\ \text{Born}}$$
$$\equiv D^{(+)} - (D^{(+)})_{PV\ \text{Born}}. \qquad (4)$$

The amplitude $D^{(+)}$ corresponds to the forward, spin-averaged, isospin even[18] scattering amplitude and $\tilde{D}^{(+)}$ has the *pseudovector* Born contribution subtracted. With q (q') the initial (final) pion four-momentum and p (p') the initial (final) nucleon four-momentum and defining the kinematic invariants

$$\nu \equiv \frac{(q+q')\cdot(p+p')}{4M} = \frac{s-u}{4M}$$

and

$$\nu_B \equiv \frac{-q\cdot q'}{2M} = \left(\frac{t-q^2-q'^2}{4M}\right),$$

one has[4,19]

$$(D^{(+)})_{PV\ \text{Born}} = \frac{g^2}{M}\frac{\nu_B^2}{\nu_B^2-\nu^2}. \qquad (5)$$

We can now give the heuristic statements and precise forms of the two theorems:

(1) The value of the pion nucleon scattering amplitude at the (on mass shell) Cheng-Dashen point[11,15,16]— $\nu=0$, $\nu_B=0$, $q^2=q'^2=m_\pi^2$, so $t=2m_\pi^2$— is equal, to first order in the chiral symmetry breaking, to the pion-nucleon Σ term. Explicitly,

$$\Sigma(t=2m_\pi^2) = f_\pi^2\tilde{D}^{(+)}(\nu=0,\ \nu_B=0;\ m_\pi^2,\ m_\pi^2) + \dots. \qquad (6)$$

(2) The pion nucleon scattering amplitudes should vanish at the (off mass shell) Adler point[11,15,20]— $\nu=0$, $\nu_B=0$, $q^2=m_\pi^2$, $q'^2=0$ so $t=m_\pi^2$— as a consequence of the Adler consistency condition. Explicitly,

$$0 = \tilde{D}^{(+)}(\nu=0,\ \nu_B=0;\ q^2=m_\pi^2,\ q'^2=0). \qquad (7)$$

We shall now show, by explicit calculation in chiral models with different forms of symmetry breaking, that the first theorem is true independent of the symmetry breaking,[21] whereas the second theorem is true only when the symmetry breaking is such that PCAC holds as a canonical operator equation.

Our calculations will be performed in the tree approximation[22] to the linear sigma model. The symmetric part of the Lagrangian is[23]

$$\mathcal{L}_0 = \bar{N}[i\gamma\partial - g(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]N$$
$$+ \tfrac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\vec{\pi})^2] - \lambda/4(\sigma^2+\pi^2-v^2)^2, \qquad (8)$$

where σ, π_i, and N are the sigma, pion, and nucleon fields and v is a positive constant. The nonsymmetric part of $\mathcal{L}$, $\mathcal{L}_{SB}$ contains three different types of explicit chiral symmetry breaking,

$$\mathcal{L}_{SB} = +\epsilon_1\sigma - \epsilon_2\vec{\pi}\cdot\vec{\pi} - \epsilon_3\bar{N}N, \qquad (9)$$

where $\epsilon_i>0$. The term $\epsilon_1\sigma$ is usually referred to as the "standard" symmetry breaking, but there is at present no conclusive experimental evidence indicating that it should be preferred to the other terms. An accurate measurement of the $\pi\pi$ scattering lengths could distinguish between $\epsilon_1\sigma$ and $\epsilon_2\pi^2$ breaking,[23,24] and a careful analysis of the Goldberger-Treiman relation can place limits on the $\epsilon_3\bar{N}N$ term.

The full Lagrangian $\mathcal{L}$ is equal to $\mathcal{L}_0+\mathcal{L}_{SB}$. $\mathcal{L}_0$ is invariant under chiral SU(2) $\times$ SU(2) transformations, which we display explicitly to establish our conventions. The (infinitesimal) transformations include isospin rotations with parameters α^j, $j=1,2,3$,

$$\sigma\to\sigma'=\sigma,$$
$$\pi^i\to\pi^{i\prime}=\pi^i+\epsilon^{ijk}\alpha^j\pi^k, \qquad (10a)$$
$$N\to N'=N-i\frac{\tau^i\alpha^i}{2}N,$$

and axial isospin rotations with parameters β^j, $j=1,2,3$,

$$\sigma\to\sigma'=\sigma+\beta^i\pi^i,$$
$$\pi^i\to\pi^{i\prime}=\pi^i-\beta^i\sigma, \qquad (10b)$$
$$N\to N'=N+\frac{i\tau^i\beta^i}{2}\gamma_5 N.$$

The vector and axial-vector currents, which are derivable from $\mathcal{L}$ and Eq. (10) by standard techniques,[12] are

$$V_\mu^i = \bar{N}\gamma_\mu\tfrac{1}{2}\tau^i N + \epsilon^{ijk}\pi^j\partial_\mu\pi^k \qquad (11a)$$

and

$$A_\mu^i = \bar{N}\gamma^5\gamma_\mu\tfrac{1}{2}\tau^i N + \pi^i\partial_\mu\sigma - \sigma\partial_\mu\pi^i. \qquad (11b)$$

By direct use of the canonical commutation relations— anticommutation relations for N— one can verify that these currents satisfy the usual SU(2) $\times$ SU(2) current algebra,[25] and that the charges ${}^5Q^i(t)$ and $Q^i(t)\equiv\int V_0^i d^3x$ do indeed generate the transformations (10b) and (10a), respectively.

When all the ϵ_i are zero, the axial-vector current is conserved. For nonzero ϵ_i, a standard calculation[12] gives

$$\partial^\mu A_\mu^i = \epsilon_1\pi^i + 2\epsilon_2\sigma\pi^i - i\epsilon_3\overline{N}\gamma_5\tau_i N. \qquad (12)$$

Note that only when $\epsilon_2=0=\epsilon_3$ does PCAC obtain as a canonical operator equation.

In the symmetric limit, minimizing the meson effective potential terms in (8) establishes explicitly that the $SU(2)\times SU(2)$ symmetry of $\mathcal{L}_0$ is realized in the Nambu-Goldstone mode:[23] the vacuum expectation value of σ, $\langle\sigma\rangle = v$ is nonzero, giving rise to a nucleon mass $M_0 = gv$, and the pion is a Goldstone boson, $m_\pi^2 = 0$. When $\epsilon_i \neq 0$, the minimum of the meson effective potential occurs at $\langle\sigma\rangle = f$, where

$$\lambda f(f^2 - v^2) = \epsilon_1 . \tag{13}$$

Thus, for small ϵ_1,

$$f = v + \frac{\epsilon_1}{2\lambda v^2} + \cdots . \tag{14}$$

Shifting the σ field by $\sigma = \tilde{\sigma} + f$, so that $\tilde{\sigma}$ has no vacuum expectation value, one finds the Lagrangian

$$\begin{aligned}\mathcal{L} = \bar{N}[i\gamma\cdot\partial - (gf+\epsilon_3) - g(\tilde{\sigma} + i\vec{\tau}\cdot\vec{\pi}\gamma_5)]N \\ + \tfrac{1}{2}[(\partial_\mu\tilde{\sigma})^2 - (\lambda/2)(3f^2 - v^2)\tilde{\sigma}^2] \\ + \tfrac{1}{2}\{(\partial_\mu\vec{\pi})^2 - [(\lambda/2)(f^2 - v^2) + \epsilon_2]\vec{\pi}^2\} \\ - \lambda f\sigma(\sigma^2 + \vec{\pi}^2) - (\lambda/4)(\sigma^2 + \vec{\pi}^2)^2 .\end{aligned} \tag{15}$$

Thus, the masses of the particles are, for the nucleon,

$$M = gf + \epsilon_3 \simeq gv + \frac{g\epsilon_1}{2\lambda v^2} + \cdots + \epsilon_3 , \tag{16a}$$

for the σ,

$$m_\sigma^2 = \lambda(3f^2 - v^2) \simeq 2\lambda v^2 + \frac{3\epsilon_1}{v} + \cdots , \tag{16b}$$

and for the pion

$$\begin{aligned}m_\pi^2 = \lambda(f^2 - v^2) + 2\epsilon_2 = \epsilon_1/f + 2\epsilon_2 \\ \simeq \epsilon_1/v + \cdots + 2\epsilon_2 .\end{aligned} \tag{16c}$$

The final approximate equalities in (16) hold to first order in the parameters ϵ_i.

The full Lagrangian in (15) is described by six independent parameters, three of which are nonvanishing in the symmetric limit and three of which describe the symmetry breaking. For our later purposes it will be most useful to choose M, g, and m_σ as the former set, and m_π^2, ϵ_2, and ϵ_3 (Ref. 26) as the latter.

Note that the vacuum expectation value of σ, called f above, is equal to f_π, the pion decay constant. To see this in the tree approximation, recall that the definition of f_π,[12,23]

$$\langle 0|A_\mu^i|\pi^j(q)\rangle \equiv iq_\mu f_\pi \delta^{ij} , \tag{17}$$

implies that

$$\langle 0|\partial^\mu A_\mu^i|\pi^j(q)\rangle \equiv q^\mu q_\mu f_\pi \delta^{ij} = m_\pi^2 f_\pi \delta^{ij} . \tag{18}$$

Using the explicit form of $\partial^\mu A_\mu^i$ in (12) and shifting the σ field yields

$$\begin{aligned}\langle 0|(\epsilon_1\pi^i + 2\epsilon_2 f\pi^i) + (2\epsilon_2\tilde{\sigma}\pi^i - i\epsilon_3\bar{N}\gamma_5\tau_i N)|\pi^j(q)\rangle \\ = f_\pi m_\pi^2 \delta^{ij} .\end{aligned} \tag{19}$$

In the tree approximation, only the terms in the first bracket contribute—the others give rise to loops—and thus, one immediately obtains

$$\begin{aligned}(\epsilon_1 + 2\epsilon_2 f)\delta^{ij} = f(\epsilon_1/f + 2\epsilon_2)\delta^{ij} \\ \equiv m_\pi^2 f_\pi \delta^{ij} .\end{aligned} \tag{20}$$

Recalling the expression for m_π^2 in (16c), we see that $f = f_\pi$, as asserted.

With these preliminaries aside, we turn to the explicit evaluation of first the Σ term and then the amplitude $\bar{D}^+$. Since ${}^5Q^i(t)$ commutes with the symmetric part of the Hamiltonian density, the double commutator in (2) is, with $\mathcal{H}_{SB} = -\mathcal{L}_{SB}$,

$$\begin{aligned}[{}^5Q^i(t),[{}^5Q^i(t),\mathcal{H}]] = [{}^5Q^i(t),[{}^5Q^i(t),\mathcal{H}_{SB}]] \\ = [{}^5Q^i(t),[{}^5Q^i(t), -\epsilon_1\sigma \\ + \epsilon_2\pi^2 + \epsilon_3\bar{N}N]] .\end{aligned} \tag{21}$$

Inserting A_0^i from (11b) into (3), we obtain the explicit expression for ${}^5Q^i(t)$ in terms of the fundamental fields. Then by straightforward application of the appropriate canonical commutation/anticommutation relations, we find

$$\begin{aligned}[{}^5Q^i(t),[{}^5Q^j(t),\mathcal{H}_{SB}(x)]] = -\epsilon_1\sigma(x)\delta^{ij} \\ - 2\epsilon_2[\sigma^2(x)\delta^{ij} - \pi^i(x)\pi^j(x)] \\ + \epsilon_3\bar{N}(x)N(x)\delta^{ij} .\end{aligned} \tag{22}$$

To evaluate the Σ term in the tree approximation, it is easiest to use crossing to write[11,27]

$$\Sigma(t) = \tfrac{1}{3}\sum_{i=0}^{3} \langle 0|[{}^5Q^i(t),[{}^5Q^i(t),\mathcal{H}_{SB}]]|\bar{N}(-p')N(p)\rangle , \tag{23}$$

and to rewrite the double commutator in (22) in terms of the field $\tilde{\sigma}$, since this has no vacuum expectation value. It then follows that

$$\Sigma(t) = \tfrac{1}{3}\sum_{i=0}^{3}\langle 0|(-\epsilon_1\tilde{\sigma} - 4\epsilon_2 f\tilde{\sigma} + \epsilon_3\bar{N}N)\delta^{ii} + \{-\epsilon_1 f - 2\epsilon_2[(f^2 + \tilde{\sigma}^2)\delta^{ii} - \pi^i\pi^i]\}|\bar{N}(-p')N(p)\rangle . \tag{24}$$

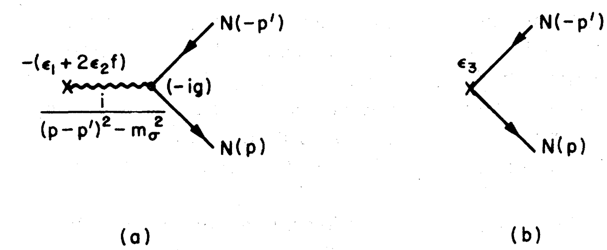

FIG. 1. The Feynman diagrams contributing to the Σ term in the tree approximation. The solid lines are nucleons and the wiggly line is a σ.

Either from Feynman diagrams[23] (see Fig. 1) or from a dispersion calculation[27] based on inserting a complete set of states between the operators and the state $|\bar{N}(-p')N(p)\rangle$, one can see that in the tree approximation, the contributions to $\Sigma(t)$ come only from the operators in the first bracket in (24). Evaluating the Feynman diagrams in Fig. 1 leads to the result

$$\Sigma(t)=\frac{g(\epsilon_1+4\epsilon_2 f)}{m_\sigma^2-t}+\epsilon_3 \tag{25}$$

for the Σ term in the tree approximation. In terms of g, M, m_σ^2 and $m_\pi^2, \epsilon_2, \epsilon_3$, using $f=f_\pi$, this becomes

$$\begin{aligned}\Sigma(t)&=\frac{g f_\pi}{m_\sigma^2-t}(\epsilon_1/f_\pi+2\epsilon_2+2\epsilon_2)+\epsilon_3\\&=\frac{M-\epsilon_3}{m_\sigma^2-t}(m_\pi^2+2\epsilon_2)+\epsilon_3\,.\end{aligned} \tag{26}$$

For $t=2m_\pi^2$, to first order in the symmetry breaking parameters, this gives

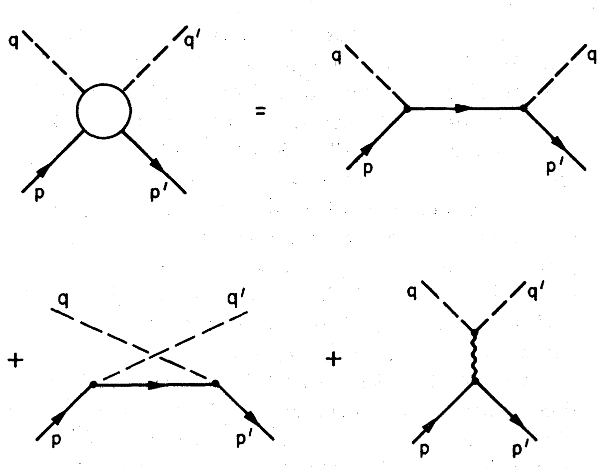

FIG. 2. The Feynman diagrams contributing to πN scattering in the tree approximation to the σ model. The solid lines are nucleons, the dashed lines are pions, and the wiggly line is a σ.

$$\Sigma(t=2m_\pi^2)\simeq M\left(\frac{m_\pi^2+2\epsilon_2}{m_\sigma^2}\right)+\epsilon_3+\cdots\,. \tag{27}$$

To compare this result to $\tilde{D}^{(+)}$ at the Cheng-Dashen point, we must evaluate the tree approximation to πN scattering in the σ model.[23] The three contributing Feynman graphs are shown in Fig. 2. Recalling that the pion has *pseudoscalar* coupling in the σ model, we see that the diagrams in Figs. 2(a) and 2(b) yield the standard *pseudoscalar* Born pole[19,23]

$$(D^{(+)})_{(a)+(b)}=\frac{g^2}{M}\frac{\nu^2}{\nu_B^2-\nu^2}\,. \tag{28a}$$

The diagram in Fig. 2(c) contributes

$$(D^{(+)})_{(c)}=\frac{2\lambda f g}{m_\sigma^2-t}\,. \tag{28b}$$

Using $f=f_\pi$ and

$$\begin{aligned}2\lambda f_\pi&=\frac{1}{f_\pi}(m_\sigma^2-m_\pi^2+2\epsilon_2)\\&=\frac{1}{f_\pi}(m_\sigma^2-t+t-m_\pi^2+2\epsilon_2)\end{aligned}$$

allows us to rewrite (28b) as

$$(D^{(+)})_{(c)}=\frac{g^2}{M-\epsilon_3}+\frac{g^2}{M-\epsilon_3}\left(\frac{t-m_\pi^2+2\epsilon_2}{m_\sigma^2-t}\right)\,. \tag{28c}$$

Thus,

$$\begin{aligned}\tilde{D}^{(+)}&=\frac{g^2}{M-\epsilon_3}+\frac{g^2}{M-\epsilon_3}\left(\frac{t-m_\pi^2+2\epsilon_2}{m_\sigma^2-t}\right)\\&\quad+\frac{g^2}{M}\frac{\nu^2}{\nu_B^2-\nu^2}-\frac{g^2}{M}\frac{\nu_B^2}{\nu_B^2-\nu^2}\\&=\left(\frac{g^2}{M-\epsilon_3}-\frac{g^2}{M}\right)+\frac{g^2}{M-\epsilon_3}\left(\frac{t-m_\pi^2+2\epsilon_2}{m_\sigma^2-t}\right)\,.\end{aligned} \tag{29}$$

To verify the first theorem we must multiply by f_π^2, set $t=2m_\pi^2$, express the result in terms of g, M, m_σ^2 and m_π^2, ϵ_2, ϵ_3, and expand to first order in the symmetry breaking parameters. We obtain

$$f_\pi^2\tilde{D}^+(t=2m_\pi^2)=g^2 f_\pi^2\left(\frac{1}{M-\epsilon_3}-\frac{1}{M}\right)+\frac{g^2 f_\pi^2}{M-\epsilon_3}\left(\frac{m_\pi^2+2\epsilon_2}{m_\sigma^2-2m_\pi^2}\right) \tag{30a}$$

$$\simeq\epsilon_3+\frac{M}{m_\sigma^2}(m_\pi^2+2\epsilon_2)+\cdots\,, \tag{30b}$$

which agrees exactly with the Σ term in (27).

To study the second theorem, we must define the amplitude $\tilde{D}^{(+)}$ with the pions off mass shell. Since we have a definite field theory with an explicit choice of canonical fields, this off-shell amplitude is well defined and depends on q^2 and q'^2 in addition to the usual kinematic invariants

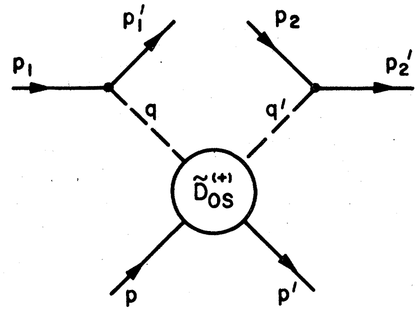

FIG. 3. A schematic representation of the off-shell πN amplitude, $\tilde{D}_{OS}^{(+)}$.

s and t or ν and ν_B. The off-shell amplitude is perhaps easiest to picture as the set of all Feynman diagrams contributing to the bubble labeled "$\tilde{D}_{OS}^{(+)}$" in Fig. 3. In the tree approximation, only the three diagrams in Fig. 2 contribute. If we express the off-shell amplitude in terms of ν, ν_B, q^2, and q'^2, then it is clear that, independent of q^2 and q'^2, the contribution of the diagrams in Figs. 2(a) and 2(b) is given by (28a). Further, to take the diagram in Fig. 2(c) off-shell in terms of these variables we need only replace t by $4m\nu_B + q^2 + q'^2$. Hence, in the tree approximation,

$$\tilde{D}_{OS}^{(+)}(\nu,\nu_B;\ q^2,q'^2)=\left(\frac{g^2}{M-\epsilon_3}-\frac{g^2}{M}\right)+\frac{g^2}{M-\epsilon_3}\times\left(\frac{4M\nu_B+q^2+q'^2-m_\pi^2+2\epsilon_2}{m_\sigma^2-4M\nu_B-q^2-q'^2}\right). \tag{31}$$

Thus, at the Adler point[11,15,20] $\nu=\nu_B=0$, $q^2=0$, $q'^2=m_\pi^2$, we have

$$D_{OS}^{(+)}\Big|_{\substack{\text{Adler}\\ \text{point}}}=\left(\frac{g^2}{M-\epsilon_3}-\frac{g^2}{M}\right)+\left(\frac{g^2}{(M-\epsilon_3)}\,\frac{2\epsilon_2}{(m_\sigma^2-m_\pi^2)}\right), \tag{32}$$

which is *not* zero unless *both* ϵ_2 and ϵ_3 are zero, that is, unless PCAC holds in the theory as a canonical operator equation. Thus, for example, if in a chiral model with $\epsilon_2 \neq 0 \neq \epsilon_3$, one took the on-shell πN scattering amplitude, expanded it in powers of $(q^2-m_\pi^2)$ and $(q'^2-m_\pi^2)$, and enforced by hand the Adler zero, one would be simply incorrect; such models do *not* have πN amplitudes which vanish at the Adler point. To clarify one further possible point of confusion, let me stress that there is absolutely nothing wrong with the Adler self-consistency condition.[20] Indeed, there *is* an amplitude in all these theories which does contain the Adler zero: namely, the amplitude for "$\partial^\mu A_\mu^i$" $-N$ scattering, that is, the nucleon matrix element of the product of two divergences of the axial current.[28] Our result merely reflects the obvious fact that, when ϵ_2 or ϵ_3 is nonzero, this $\partial^\mu A_\mu^i - N$ scattering amplitude is *not* simply related to the canonical π-N amplitude, because canonical PCAC does not hold. Said another way, the proof that the π-N amplitude has the Adler zero breaks down at step one: One cannot replace π^i by $\partial^\mu A_\mu^i/(m_\pi^2 f_\pi)$.

One final comment should precede our conclusion. One occasionally hears the "folk" theorem that "the Σ term is equal to the shift in the nucleon mass when chiral symmetry breaking is turned on."[11] Our calculation shows immediately when this is actually the case. The nucleon's mass in the absence of chiral symmetry breaking is, as noted above,

$$M_0 = g v\,, \tag{33a}$$

so that the change in the nucleon's mass when the symmetry breaking is added is

$$\Delta M \equiv M - M_0 = g f_\pi + \epsilon_3 - g v \simeq g v + \frac{\epsilon_1 g}{2\lambda v^2} + \cdots + \epsilon_3 - g v\,. \tag{33b}$$

But $\epsilon_1 = m_\pi^2 f_\pi - 2\epsilon_2 f_\pi$, and, to first order in the symmetry breaking, $g f_\pi \simeq M$ and $2\lambda v^2 \simeq m_\sigma^2$, so

$$\Delta M \simeq M\left(\frac{m_\pi^2-2\epsilon_2}{m_\sigma^2}\right)+\epsilon_3\,. \tag{34}$$

This equals the Σ term in (27) *only* if $\epsilon_2 = 0$.[29] The reason for this is clear. The shift in the nucleon mass to first order in the symmetry breaking is just the nucleon matrix element of $\mathcal{H}_{SB}$, whereas the Σ term involves the nucleon matrix element of the double commutator $[{}^5Q^i(t), [{}^5Q^i(t), \mathcal{H}_{SB}]]$. Thus, unless

$$[{}^5Q^i(t),\ [{}^5Q^i(t),\ \mathcal{H}_{SB}]] = \mathcal{H}_{SB}\,, \tag{35}$$

$\Delta M \neq \Sigma$.[29] Our explicit calculations in (22) show that Eq. (35) holds for $\epsilon_1\sigma$ and $\epsilon_3\bar{N}N$ symmetry breaking, but not for $\epsilon_2\vec{\pi}^2$. Technically,[11,23] only if the symmetry breaking belongs to the $(\frac{1}{2}, \frac{1}{2})$ representation of SU(2) × SU(2)—as the terms $\epsilon_1\sigma$ and $\epsilon_3\bar{N}N$ do—is $\Delta M = \Sigma$.[29]

To conclude, we reiterate our central caveat: One cannot naively combine the results of an explicit chiral Lagrangian model with the formal consequences of PCAC and current algebra unless, within the model, PCAC holds as an operator equation. Particularly in nuclear physics applications, such as the search for a model field theory with both reasonable nuclear saturation and chiral symmetry or the study of the pion optical potential, must this caveat be borne in mind.

It is a pleasure to thank Roger Dashen for a number of valuable discussions. This work was supported by the Department of Energy.

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[13]Theories producing nuclear saturation by the realistic mechanism of balancing scalar meson attraction with vector meson repulsion have been discussed, for example, in J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974), and S. A. Chin and J. D. Walecka, Phys. Lett. 52B, 24 (1974). However, these theories do not attempt to incorporate reasonable low energy pion phenomenology.

[14]To obtain reasonable low energy pion phenomenology, a theory must be approximately invariant under chiral SU(2)×SU(2). A familiar example is the linear σ model, some aspects of which we discuss later in detail. This model, however, does not lead to realistic nuclear forces; for example, as discussed by S. Barshay and G. E. Brown, Phys. Rev. Lett. 34, 1106 (1975), the nuclear three-body force is far too large in this model.

[15]For an introduction to chiral symmetry breaking see R. F. Dashen, in *Developments in High-Energy Physics*, Proceedings of the International School of Physics "Enrico Fermi," Course LIV, edited by R. Gatto (Academic, New York, 1973).

[16]T.-P. Cheng and R. F. Dashen, Phys. Rev. Lett. 26, 594 (1971).

[17]L. Brown, W. Pardee, and R. Peccei, Phys. Rev. D 4, 2801 (1971).

[18]The "isospin even" amplitude corresponds to $I = 0$ in the t channel.

[19]For the explicit forms of both the pseudoscalar and pseudovector Born poles, see, for example, G. Höhler, H. P. Jakob, and R. Strauss, Nucl. Phys. B39, 237 (1972).

[20]S. L. Adler, Phys. Rev. 139, B1638 (1965).

[21]Since we are interested only in illustrating the problems arising from a naive combination of PCAC/current algebra with explicit chiral models, we do not attempt to prove these results as theorems. Rather, we simply show that the first result *is* true for a variety of symmetry breaking terms whereas the second holds only when canonical PCAC holds.

[22]This approximation is known to respect the low energy theorems of chiral symmetry. See, for example, R. F. Dashen and M. Weinstein, Phys. Rev. 183, 1261 (1969).

[23]For a review of the σ model, including the Feynman rules, see B. W. Lee *Chiral Dynamics* (Gordon and Breach, New York, 1972).

[24]S. Weinberg, Phys. Rev. 166, 1568 (1968).

[25]M. Gell-Mann, Phys. Rev. 125, 1067 (1962). For a review, see S. L. Adler and R. F. Dashen, *Current Algebras and Applications to Particle Physics* (Benjamin, New York, 1968).

[26]From the expression for m_π^2 in (16c), it is clear that (m_π^2, ϵ_2, and ϵ_3 can be used in place of the obvious choice (ϵ_1, ϵ_2, and ϵ_3) to describe the symmetry breaking.

[27]H. Pagels and W. J. Pardee, Phys. Rev. D 4, 3335 (1971).

[28]Of course, as usual one must be careful to extract the singular Born pole contributions. See Refs. 11, 15, and 20.

[29]Expert readers will notice the technical point that the matrix element giving ΔM should be evaluated with the nucleons at rest, implying $t = 0$. In the tree approximation, $\Sigma(t = 0)$ and $\Sigma(t = 2m_\pi^2)$ are equal to first order in the symmetry breaking. The important question of loop corrections to this result is discussed in Ref. 27.