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Pion absorption by deuterons in field theory

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Using non-perturbative, field-theoretic techniques based on a generalization of the Low equation, an inhomogeneous, coupled, linear, integral equation is developed for the reaction matrix for $\pi d \rightarrow 2p$. The inhomogeneous term in this equation consists of a truncated, multiple-scattering expansion of the breakup process. The integral terms couple the reaction to elastic nucleon-nucleon and elastic pion-deuteron scattering. The inhomogeneous term is evaluated at the reaction threshold, with relativistic forms for the absorption and rescattering vertices. P-wave pion-nucleon rescattering is found to make a non-negligible contribution to the s-wave absorption process at threshold.

[NUCLEAR REACTIONS $d(\pi^+,pp)$, generalized Low equation, pion crossing, calculated threshold σ.]

I. INTRODUCTION

The pion-deuteron absorption process has been of considerable theoretical interest for over twenty years. In addition to its intrinsic interest as the simplest example of an inelastic pion-nucleus reaction, studies have indicated that the two-nucleon absorption process is the dominant mechanism for pion absorption by more complex nuclei. This reaction then plays a central role in the understanding of elastic and inelastic pion-nucleus scattering. It has also recently been suggested[1] that this reaction might be sensitive to the off-shell behavior of the pion-nucleon elastic scattering amplitude and thus serve as a testing place for our knowledge of the underlying pion-nucleon dynamics. Continued interest in this reaction seems ensured.

It is well understood that at least two basic mechanisms are needed for the description of this reaction: direct absorption, in which the pion is absorbed in a one-nucleon process, and two-nucleon absorption, in which the pion is first scattered from one nucleon, either forward or backward in time, and then absorbed by the second nucleon. In addition, however, the roles of the initial- and final-state interactions and the possible double counting of meson exchanges must be resolved before a reasonably complete understanding of the problem is achieved.

Problems involving the absorption and emission of mesons are best suited to the techniques of relativistic quantum field theory. Since perturbation theory is almost guaranteed not to converge for strong interactions, a nonperturbative approach such as that embodied in the Low equation seems more suitable. However, there are other difficulties with the Low equation (see Sec. II A): (1) The seagull terms which appear in this equation are not known in general, and (2) the equation itself is not manifestly antisymmetric with respect to the two nucleons.

Another alternative to a strict perturbative procedure is to construct an effective Hamiltonian which uses two-body scattering vertices in conjunction with absorption vertices rather than elementary couplings. While this procedure may obviate the convergence problems, it suffers from at least two other defects. The first is the difficulty in resolving the double-counting of meson exchanges mentioned above. The second is more fundamental and involves the rescattering and absorption vertices themselves. While it is true that any nonperturbative field-theoretic approach will ultimately make connection with the elementary three- and four-point functions of quantum field theory, it is necessary that the formalism itself uniquely specify these functions in order that the whole procedure make sense. There is simply too much latitude involved in the construction of the effective Hamiltonian.

In the present paper, we generalize the Low equation to a more suitable form. In Sec. II, we develop an operator identity for the two asymptotic, final-state, nucleon annihilation operators and the pion current. The matrix element of this operator identity provides the foundation for the generalization of the Low equation. The resulting expression is completely antisymmetric with respect to the two nucleons and contains terms which result from crossing the external pion leg separately with each external nucleon leg. This last feature is important since these crossing terms lead to the backward rescattering graphs which were earlier characterized as being an essential part of the two-nucleon absorption mechanism. In the final form, all seagull terms present are identifiable parts of two-body processes.

In Sec. III, we show that consideration of the low-lying intermediate state contributions leads naturally to an inhomogeneous, linear, integral equation for the reaction process. The driving term consists of a direct absorption term and both a forward and a backward rescattering term. The integral terms couple the breakup reaction to the elastic nucleon-nucleon and elastic pion-deuteron scattering channels. The relation of the potential to multiple-scattering approches is discussed.

In Sec. IV, we numerically investigate the potential at the reaction threshold. The reasoning which suggests the sensitivity of this reaction to the unphysical behavior of the pion-nucleon scattering amplitude also emphasizes the importance of nucleon recoil. In order to properly incorporate recoil, we do not make the static nucleon approximation, but begin with the fully relativistic form for the absorption and rescattering vertices. Recoil considerations manifest themselves in two ways: in the mixing of the pion-nucleon partial waves and in the kinematic features of the partial-wave amplitudes themselves. While many authors have dealt with the mixing of the pion-nucleon partial waves due to the nuclear form factors, the recoil mixing seems largely to have been ignored. This mixing, however, is significant at the reaction threshold. The important feature is that as the intermediate-state pion momentum $\vec{q}\,'$ changes, the Lorentz transformation from the pion-deuteron c.m. to the pion-nucleon c.m. changes. Consequently, both the initial and the final pion-nucleon relative momenta vary with $\vec{q}\,'$. Owing to this dependency on $\vec{q}\,'$ and the large momentum of the outgoing nucleons, the s-wave and p-wave pion-nucleon partial-wave amplitudes are comparable throughout much of the range of integration (over $\vec{q}\,'$). Thus p-wave rescattering cannot be ignored, even at the reaction threshold. Nucleon recoil also tends to decrease the sensitivity of the rescattering mechanism to the off-shell behavior of $T_{\pi N}$.

In Sec. V, we briefly conclude, comparing the content of our approach with that of other formalisms.

II. DEVELOPMENT OF THE BASIC THEORY

A. Introductory discussion

We consider the pion-deuteron absorption process

$$\pi^+(q)+d(p_d)\to p(p_1)+p(p_2)\,,$$

where q and p_d are the four-momenta of the pion and the deuteron respectively and p_1 and p_2 are the four-momenta of the final-state protons. In the following, we suppress spin and isospin indices other than where they are essential to the argument. The asymptotic pion creation operators $a_\pi(q,{}^{\rm in}_{\rm out})^\dagger$ are related by

$$a_\pi(q,\text{out})^\dagger - a_\pi(q,\text{in})^\dagger = -i\int d^4z\, e^{-iq\cdot z} j_\pi(z)\,, \tag{1}$$

with $j_\pi(z)$ the pion current. Applying Eq. (1) to the initial-state pion in the S matrix, we obtain the reaction matrix

$$_{\rm out}\langle p_1p_2|qp_d\rangle_{\rm in} = (2\pi)^4 i\,\delta^{(4)}(p_1+p_2-q-p_d) \times {}_{\rm out}\langle p_1p_2|j_\pi(0)|p_d\rangle\,. \tag{2}$$

Let $\psi(x)$ be the interpolating nucleon field operator and

$$J(x)=\overline{u}(p)(-i\gamma\cdot\partial+m)\psi(x)\,, \tag{3}$$

where $u(p)$ is a four-component Dirac spinor. We introduce the interpolating nucleon annihilation operator $a(t)$

$$\lim_{t\to\pm\infty} a(t)=a({}^{\rm out}_{\rm in}) \tag{4}$$

with[2]

$$a(t')-a(t)=i\int_t^{t'} dx_0\, \mathcal{J}(x_0)\,, \tag{5a}$$

$$\mathcal{J}(x_0)=\int d^3x\, e^{ip\cdot x}\overline{u}(p)(-i\gamma\cdot\partial+m)\psi(x)\,. \tag{5b}$$

To illustrate a technique which will prove useful in the following, consider the operator product $a(\text{out})\,j_\pi(0)$ in which only a single asymptotic nucleon operator is involved. Using Eq. (5a), we have

$$\begin{aligned} a(\text{out})j_\pi(0) &= a(0)j_\pi(0)+i\int_0^\infty dx_0\,\mathcal{J}(x_0)j_\pi(0)\\ &=[a(0),j_\pi(0)]+j_\pi(0)a(0)+i\int_0^\infty dx_0\,\mathcal{J}(x_0)j_\pi(0)\\ &=j_\pi(0)a(\text{in})+[a(0),j_\pi(0)]\\ &\quad+i\int_0^\infty dx_0\,\mathcal{J}(x_0)j_\pi(0)+i\int_{-\infty}^0 dx_0\, j_\pi(0)\mathcal{J}(x_0)\,, \end{aligned} \tag{6}$$

which may be rewritten in the more standard form

$$a(\text{out})j_\pi(0)=j_\pi(0)a(\text{in}) + i\int d^4x\, e^{ip\cdot x}\bar{u}(p)(-i\gamma\cdot\partial+m) \times T(\psi(x)j_\pi(0)). \qquad (7)$$

The Low equation for the reaction matrix may be obtained by taking the matrix element of Eq. (7) between the one-nucleon and deuteron states and inserting a complete set of intermediate states into the time-ordered product. The result is

$$_{\text{out}}\langle p_1p_2|j_\pi(0)|p_d\rangle=\langle p_1|\Gamma(p_2)|p_d\rangle-(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_n)}{p_{1_0}+p_{2_0}-p_{n_0}+i\epsilon}\langle p_1|J_2(0)|n\rangle_{\text{out}}\,{}_{\text{out}}\langle n|j_\pi(0)|p_d\rangle$$

$$+(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_2+\vec{p}_n-\vec{p}_d)}{p_{2_0}+p_{n_0}-p_{d_0}-i\epsilon}\langle p_1|j_\pi(0)|n\rangle_{\text{out}}\,{}_{\text{out}}\langle n|J_2(0)|p_d\rangle, \qquad (8a)$$

where

$$\Gamma(p_2)=\int d^4y\, e^{ip_2\cdot y}[\bar{u}(p_2)\gamma_2^0\psi_2(y),j_\pi(0)]\delta(y_0) \qquad (8b)$$

is the "seagull" term.

Equation (8) could serve as a starting point for a field-theoretic description of the absorption process. The Low equation has been used with considerable success in the study of pion-nucleon and pion-nucleus elastic scattering.[3] However, there is an important qualitative distinction between these elastic scattering applications and the reaction considered in the present paper that diminishes the attractiveness of the Low equation approach. In pion elastic scattering, the Bose statistics of the pion are the most important symmetry consideration and the Low equation provides a natural vehicle for the expression of this symmetry (i.e., the Low equation is explicitly crossing symmetric). In the breakup reaction $\pi d\to 2p$, the Fermi statistics of the final-state nucleons are similarly an important consideration. The difficulty with Eq. (8) is that this antisymmetry is not apparent, that is, the two nucleons seem to play quite different roles, even though both are on the mass shell. For example, the final term in Eq. (8a) results from "crossing" the external pion and nucleon $p(p_2)$ legs; there is no corresponding contribution from $p(p_1)$, nor is there any seagull term involving $p(p_1)$ [the matrix element of the seagull term in Eq. (8a), in fact, is not known].

In the next section, we generalize Eq. (7) to the case of two asymptotically free nucleons, obtaining a result which is explicitly antisymmetric with respect to the roles of the final-state nucleons.

B. The basic operator identity

In the following, we shall make repeated use of Eqs. (5) and (6). In addition, formal tricks involving the change of the order and range of the integrations are involved. These techniques are demonstrated in Appendix A.

Consider the operator product

$$a_1(\text{out})a_2(\text{out})j_\pi(0)=a_1(\text{out})j_\pi(0)a_2(\text{in}) + a_1(\text{out})[a_2(0),j_\pi(0)]+i\int_0^\infty dy_0 a_1(\text{out})\mathcal{J}_2(y_0)j_\pi(0)+i\int_{-\infty}^0 dy_0 a_1(\text{out})j_\pi(0)\mathcal{J}_2(y_0). \qquad (9)$$

Using (see Appendix A)

$$a_1(\text{out})j_\pi(0)a_2(\text{in})=j_\pi(0)a_1(\text{in})a_2(\text{in})+[a_1(0),j_\pi(0)]a_2(\text{in}) + i\int_0^\infty dx_0\, \mathcal{J}_1(x_0)j_\pi(0)a_2(\text{in})+i\int_{-\infty}^0 dx_0\, j_\pi(0)\mathcal{J}_1(x_0)a_2(\text{in}), \qquad (10a)$$

$$a_1(\mathrm{out})[a_2(0),j_\pi(0)]=\{a_1(0),[a_2(0),j_\pi(0)]\}-[a_2(0),j_\pi(0)]a_1(\mathrm{in})$$
$$+i\int_0^\infty dx_0\mathcal{J}_1(x_0)[a_2(0),j_\pi(0)]-i\int_{-\infty}^0 dx_0[a_2(0),j_\pi(0)]\mathcal{J}_1(x_0)\,, \tag{10b}$$

(the curly brackets denote anticommutators)

$$i\int_0^\infty dy_0 a_1(\mathrm{out})\mathcal{J}_2(y_0)j_\pi(0)=-i\int_0^\infty dy_0\mathcal{J}_2(y_0)j_\pi(0)a_1(\mathrm{in})-i\int_0^\infty dy_0\mathcal{J}_2(y_0)[a_1(0),j_\pi(0)]$$
$$+\int_0^\infty dy_0\int_{-\infty}^0 dx_0\mathcal{J}_2(y_0)j_\pi(0)\mathcal{J}_1(x_0)+i\int_0^\infty dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}j_\pi(0)$$
$$-\int_0^\infty dx_0\int_0^{x_0}dy_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)+\int_0^\infty dy_0\int_0^{y_0}dx_0\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)j_\pi(0)\,, \tag{10c}$$

$$i\int_{-\infty}^0 dy_0 a_1(\mathrm{out})j_\pi(0)\mathcal{J}_2(y_0)=-i\int_{-\infty}^0 dy_0 j_\pi(0)\mathcal{J}_2(y_0)a_1(\mathrm{in})+i\int_{-\infty}^0 dy_0[a_1(0),j_\pi(0)]\mathcal{J}_2(y_0)$$
$$-\int_0^\infty dx_0\int_{-\infty}^0 dy_0\mathcal{J}_1(x_0)j_\pi(0)\mathcal{J}_2(y_0)+i\int_{-\infty}^0 dy_0 j_\pi(0)\{a_1(y_0),\mathcal{J}_2(y_0)\}$$
$$-\int_{-\infty}^0 dy_0\int_{y_0}^0 dx_0 j_\pi(0)\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)+\int_{-\infty}^0 dy_0\int_{-\infty}^{y_0}dx_0 j_\pi(0)\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)\,, \tag{10d}$$

we find

$$a_1(\mathrm{out})a_2(\mathrm{out})j_\pi(0)=j_\pi(0)a_1(\mathrm{in})a_2(\mathrm{in})+\{a_1(0),[a_2(0),j(0)]\}$$
$$+i\int_0^\infty dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}j_\pi(0)+i\int_{-\infty}^0 dy_0 j_\pi(0)\{a_1(y_0),\mathcal{J}_2(y_0)\}$$
$$+\left([a_1(0),j_\pi(0)]+i\int_0^\infty dx_0\mathcal{J}_1(x_0)j_\pi(0)+i\int_{-\infty}^0 dx_0 j_\pi(0)\mathcal{J}_1(x_0)\right)a_2(\mathrm{in})$$
$$-\left([a_2(0),j_\pi(0)]+i\int_0^\infty dy_0\mathcal{J}_2(y_0)j_\pi(0)+i\int_{-\infty}^0 dy_0 j_\pi(0)\mathcal{J}_2(y_0)\right)a_1(\mathrm{in})$$
$$+i\int_0^\infty dx_0\mathcal{J}_1(x_0)\left([a_2(0),j_\pi(0)]+i\int_0^{x_0}dy_0\mathcal{J}_2(y_0)j_\pi(0)+i\int_{-\infty}^0 dy_0 j_\pi(0)\mathcal{J}_2(y_0)\right)$$
$$-i\int_0^\infty dy_0\mathcal{J}_2(y_0)\left([a_1(0),j_\pi(0)]+i\int_0^{y_0}dx_0\mathcal{J}_1(x_0)j_\pi(0)+i\int_{-\infty}^0 dx_0 j_\pi(0)\mathcal{J}_1(x_0)\right)$$
$$-i\int_{-\infty}^0 dx_0[a_2(0),j_\pi(0)]\mathcal{J}_1(x_0)+\int_{-\infty}^0 dy_0\int_{-\infty}^{y_0}dx_0 j_\pi(0)\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)$$
$$+i\int_{-\infty}^0 dy_0[a_1(0),j_\pi(0)]\mathcal{J}_2(y_0)-\int_{-\infty}^0 dx_0\int_{-\infty}^{x_0}dy_0 j_\pi(0)\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)\,. \tag{11}$$

In Eq. (11), the antisymmetry of the two nucleons is apparent. However, some rearrangement of terms is still required. With

$$\int_0^\infty dx_0\int_0^{x_0}dy_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)=\int_0^\infty dx_0\int_0^\infty dy_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)-\int_0^\infty dy_0\int_0^{y_0}dx_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)\,, \tag{12}$$

the fifth and sixth lines may be written in the form

$$i\int_0^{\infty} dx_0 \mathcal{J}_1(x_0)\left(i\int d^4y\, e^{ip_2\cdot y}\overline{u}(p_2)\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)T(\psi_2(y)j_\pi(0))\right)+\int_0^{\infty} dy_0\int_0^{y_0} dx_0 \mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)\,. \tag{13}$$

Similarly, for the final two lines, we use

$$\begin{aligned}
&-i\int_{-\infty}^{0} dx_0[a_2(0),j_\pi(0)]\mathcal{J}_1(x_0)+\int_{-\infty}^{0} dy_0\int_{-\infty}^{y_0} dx_0 j_\pi(0)\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)\\
&\quad=-i\int_{-\infty}^{0} dx_0[a_2(0),j_\pi(0)]\mathcal{J}_1(x_0)-\int_0^{\infty} dy_0\int_{-\infty}^{0} dx_0 j_\pi(0)\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)+\int_{-\infty}^{0} dx_0\int_{x_0}^{\infty} dy_0 j_\pi(0)\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)\\
&\quad=-i\int_{-\infty}^{0} dx_0\left(i\int d^4y\, e^{ip_2\cdot y}\overline{u}(p_2)\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)T(\psi_2(y)j_\pi(0))\right)\mathcal{J}_1(x_0)\\
&\qquad+\int_{-\infty}^{0} dx_0\int_{-\infty}^{0} dy_0 \mathcal{J}_2(y_0)j_\pi(0)\mathcal{J}_1(x_0)+\int_{-\infty}^{0} dx_0\int_{x_0}^{\infty} dy_0 j_\pi(0)\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)\,.
\end{aligned} \tag{14}$$

The result is Eq. (15):

$$\begin{aligned}
&a_1(\text{out})a_2(\text{out})j_\pi(0)=j_\pi(0)a_1(\text{in})\,a_2(\text{in})+\{a_1(0),[a_2(0),j_\pi(0)]\}\\
&\quad+\left(i\int_0^{\infty} dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}+\int_0^{\infty} dy_0\int_0^{y_0} dx_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)-\int_0^{\infty} dx_0\int_0^{x_0} dy_0\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)\right)j_\pi(0)\\
&\quad+i\int_0^{\infty} dx_0\mathcal{J}_1(x_0)\left[i\int d^4y\, e^{ip_2\cdot y}\overline{u}(p_2)\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)T(\psi_2(y)j_\pi(0))\right]\\
&\quad-i\int_0^{\infty} dy_0\mathcal{J}_2(y_0)\left[i\int d^4x\, e^{ip_1\cdot x}\overline{u}(p_1)\left(-i\gamma\cdot\frac{\partial}{\partial x}+m\right)T(\psi_1(x)j_\pi(0))\right]\\
&\quad+i\int d^4y\, e^{ip_2\cdot y}\overline{u}(p_2)\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)\overline{T}(\psi_2(y)j_\pi(0))\left(-i\int_{-\infty}^{0} dx_0\mathcal{J}_1(x_0)\right)\\
&\quad-i\int d^4x\, e^{ip_1\cdot x}\overline{u}(p_1)\left(-i\gamma\cdot\frac{\partial}{\partial x}+m\right)\overline{T}(\psi_1(x)j_\pi(0))\left(-i\int_{-\infty}^{0} dy_0\mathcal{J}_2(y_0)\right)\\
&\quad+j_\pi(0)\left(i\int_{-\infty}^{0} dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}+\int_{-\infty}^{0} dx_0\int_{x_0}^{\infty} dy_0\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)-\int_{-\infty}^{0} dy_0\int_{y_0}^{\infty} dx_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)\right)\\
&\quad+i\int d^4x\, e^{ip_1\cdot x}\overline{u}(p_1)\left(-i\gamma\cdot\frac{\partial}{\partial x}+m\right)T(\psi_1(x)j_\pi(0))a_2(\text{in})\\
&\quad-i\int d^4y\, e^{ip_2\cdot y}\overline{u}(p_2)\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)T(\psi_2(y)j_\pi(0))a_1(\text{in})\\
&\quad+\int_{-\infty}^{0} dy_0\int_{-\infty}^{0} dx_0\mathcal{J}_2(y_0)j_\pi(0)\mathcal{J}_1(x_0)-\int_{-\infty}^{0} dx_0\int_{-\infty}^{0} dy_0\mathcal{J}_1(x_0)j_\pi(0)\mathcal{J}_1(y_0)\,,
\end{aligned} \tag{15}$$

where $\overline{T}$ denotes the anti-time-ordered product.

Equation (15) is the operator identity which serves as the basis for our treatment of the breakup reaction. With the exception of the seagull terms $\{a_i(t),\mathcal{J}_i(t)\}$, this expression is manifestly antisymmetric with respect to the final-state nucleons and contributions from crossing each external nucleon leg separately with the external pion leg are present. The equal-time commutators which break the antisymmetry of Eq. (15) involve the interaction of the two nucleons at the same space-time point and are presumed to vanish. This is true of any reasonable Lagrangian model.

Our derivation is far from unique. An alternate approach would involve the time-ordered product which results from contracting the two final-state nucleons with the pion current in the Lehmann-Symanzik-Zimmermann (LSZ) formalism

$$\int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)}\left(-i\gamma_2\cdot\frac{\partial}{\partial y}+m\right)\left(-i\gamma_1\cdot\frac{\partial}{\partial x}+m\right)\times T(\psi_2(y)\psi_1(x)j_\pi(0))\,.$$

Operating on the time-ordered product with the derivatives yields a result identical to ours. The virtue of the derivation presented in this section is that it allows a more direct connection to be made with the Low equation.

C. Analysis of intermediate-state contributions

We are now ready to develop an integral equation for the reaction matrix by considering intermediate state contributions to the matrix elements of Eq. (15). The result will take a more transparent form, however, if we first notice that

$$\sum_n |n\rangle_{\text{out out}}\left\langle n\right|i\int d^4x\, e^{ip\cdot x}\bar{u}(p)(-i\gamma\cdot\partial+m)T(\psi(x)j_\pi(0))\left|p_d\right\rangle=\sum_n |n\rangle_{\text{out out}}\langle n|a(\text{out})j_\pi(0)|p_d\rangle$$
$$=\sum_{n'} a(\text{out})|n'\rangle_{\text{out out}}\langle n'|j_\pi(0)|p_d\rangle\,, \tag{16}$$

where $|n'\rangle_{\text{out}}=|n+N\rangle_{\text{out}}$. Taking the matrix elements of Eq. (15) between the vacuum and the deuteron state, introducing a complete set of intermediate states, and utilizing Eq. (16), we have

$$_{\text{out}}\langle p_1p_2|j_\pi(0)|p_d\rangle=\langle 0|\{a_1(0),[a_2(0),j_\pi(0)]\}|p_d\rangle$$
$$+\sum_n\left\langle 0\right|\Bigg(i\int_0^\infty dx_0\mathcal{J}_1(x_0)a_2(\text{out})-i\int_0^\infty dy_0\mathcal{J}_2(y_0)a_1(\text{out})+i\int_0^\infty dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}$$
$$+\int_0^\infty dy_0\int_0^{y_0}dx_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)-\int_0^\infty dx_0\int_0^{x_0}dy_0\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)\Bigg)\left|n\right\rangle_{\text{out out}}\langle n|j_\pi(0)|p_d\rangle$$
$$-i\sum_n\langle p_2|j_\pi(0)|n\rangle^c_{\text{out out}}\left\langle n\right|\int_{-\infty}^0 dx_0\mathcal{J}_1(x_0)\left|p_d\right\rangle$$
$$+i\sum_n\langle p_1|j_\pi(0)|n\rangle^c_{\text{out out}}\left\langle n\right|\int_{-\infty}^0 dy_0\mathcal{J}_2(y_0)\left|p_d\right\rangle$$
$$+\sum_n\langle 0|j_\pi(0)|n\rangle_{\text{out out}}\left\langle n\right|\Bigg(i\int_{-\infty}^0 dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}$$
$$-\int_{-\infty}^0 dy_0\int_{y_0}^0 dx_0\mathcal{J}_2(y_0)\mathcal{J}_1(x_0)+\int_{-\infty}^0 dx_0\int_{x_0}^0 dy_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)\Bigg)\left|p_d\right\rangle$$
$$-\sum_{m,n}\int_0^\infty dy_0\int_{-\infty}^0 dx_0\langle 0|\mathcal{J}_2(y_0)|m\rangle_{\text{out out}}\langle m|j_\pi(0)|n\rangle_{\text{out out}}\langle n|\mathcal{J}_1(x_0)|p_d\rangle$$
$$+\sum_{m,n}\int_0^\infty dx_0\int_{-\infty}^0 dy_0\langle 0|\mathcal{J}_1(x_0)|m\rangle_{\text{out out}}\langle m|j_\pi(0)|n\rangle_{\text{out out}}\langle n|\mathcal{J}_2(y_0)|p_d\rangle\,, \tag{17a}$$

$$=\langle 0|\Gamma'(p_1,p_2)|p_d\rangle+(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_2-\vec{p}_n)}{p_{1_0}+p_{2_0}-p_{n_0}+i\epsilon}\langle 0|T^{(+)}_{NN}(p_1,p_2)^\dagger|n\rangle_{\rm out\;out}\langle n|j_\pi(0)|p_d\rangle$$

$$-(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_n+\vec{p}_1-\vec{p}_d)}{p_{n_0}+p_{1_0}-p_{d_0}-i\epsilon}\langle p_2|j_\pi(0)|n\rangle^c_{\rm out\;out}\langle n|J_1(0)|p_d\rangle$$

$$+(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_n+\vec{p}_2-\vec{p}_d)}{p_{n_0}+p_{2_0}-p_{d_0}-i\epsilon}\langle p_1|j_\pi(0)|n\rangle^c_{\rm out\;out}\langle n|J_2(0)|p_d\rangle$$

$$+(2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_1+\vec{p}_2+\vec{p}_n-\vec{p}_d)}{p_{1_0}+p_{2_0}+p_{n_0}-p_{d_0}-i\epsilon}\langle 0|j_\pi(0)|n\rangle_{\rm out\;out}\langle n|T^{(-)}_{NN}(p_1,p_2)^\dagger|p_d\rangle$$

$$-(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_m-\vec{p}_2)}{p_{m_0}-p_{2_0}-i\epsilon}\;\frac{\delta^{(3)}(\vec{p}_n+\vec{p}_1-\vec{p}_d)}{p_{d_0}-p_{1_0}-p_{n_0}+i\epsilon}\langle 0|J_2(0)|m\rangle_{\rm out\;out}\langle m|j_\pi(0)|n\rangle_{\rm out\;out}\langle n|J_1(0)|p_d\rangle$$

$$+(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_m-\vec{p}_1)}{p_{m_0}-p_{1_0}-i\epsilon}\;\frac{\delta^{(3)}(\vec{p}_n+\vec{p}_2-\vec{p}_d)}{p_{d_0}-p_{2_0}-p_{n_0}+i\epsilon}\langle 0|J_1(0)|m\rangle_{\rm out\;out}\langle m|j_\pi(0)|n\rangle_{\rm out\;out}\langle n|J_2(0)|p_d\rangle\,. \tag{17b}$$

In the second step of Eq. (17) we have translated the operators and integrated. The superscript c on a matrix element denotes a connected matrix element. The operators T_{NN} and Γ appearing above are defined by

$$\Gamma'(p_1,p_2)=\{a_1(0),[a_2(0),j_\pi(0)]\}$$

$$=\int d^4x\,d^4y\,e^{i(p_1\cdot x+p_2\cdot y)}(-i\bar{u}(p_1)\gamma_1^0)(-i\bar{u}(p_2)\gamma_2^0)\{\psi_1(x),[\psi_2(y),j_\pi(0)]\}\delta(x_0)\delta(y_0) \tag{18}$$

and

$$T^{(+)}_{NN}(p_1,p_2)^\dagger=J_2(0)a_1(\text{out})-J_1(0)a_2(\text{out})$$

$$+i\int d^4y\,e^{ip_2\cdot y}J_1(0)J_2(y)\theta(y_0)-i\int d^4x\,e^{ip_1\cdot x}J_2(0)J_1(x)\theta(x_0)\,. \tag{19}$$

The operator $T^{(-)\dagger}_{NN}$ is obtained from $T^{(+)\dagger}_{NN}$ by the replacements $J_1 \leftrightarrow J_2$, and by neglecting the $a_i(\text{out})$. $T^{(+)}_{NN}$ can be shown to result from a careful application of the LSZ reduction technique to the product $a_1(\text{in})a_2(\text{in})$ (see Appendix B). The matrix elements $\langle 0|T^{(+)}_{NN}(p_1,p_2)^\dagger|n\rangle_{\rm out}$ and ${}_{\rm out}\langle n|T^{(-)}_{NN}(p_1,p_2)^\dagger|p_d\rangle$ are therefore related to the processes $p(p_1)+p(p_2)\to n$ and $D(p_d)\to\bar{p}(p_1)+\bar{p}(p_2)+n$.

In obtaining Eq. (17b) from (17a), we have dropped the seagull terms $\{a_i(0),\mathcal{J}_j(0)\}$, as discussed in Sec. IB. In the following, we likewise neglect the matrix element of $\Gamma'(p_1,p_2)$. This corresponds to diagrams where the pion and both final-state nucleons interact at the same space-time point. There is no evidence that either of the processes represented by these seagull terms contributes.

In Fig. 1, we depict the leading intermediate-state contributions to Eq. (17b). As a first approximation, we neglect nucleon-antinucleon pairs and multimeson intermediate states. Thus, of the first sum [Fig. 1(a)], we retain the two-nucleon, pion-two-nucleon, and pion-deuteron contributions (the deuteron does not contribute), while of the second and third sums [Fig. 1(b)], we retain the single-nucleon and pion-nucleon contributions. We neglect the fourth term in Eq. (17b) completely since the first contributing intermediate state in the sum is a nucleon-antinucleon pair [Fig. 1(c)]. In the remaining two terms [Fig. 1(d)], we examine only the m = pion-nucleon and n = single-nucleon contribution. The justification for the neglect of the $m=n=$ pion-nucleon contribution will be discussed later. Additional graphs will arise from disconnected diagrams involving the deuteron (Fig. 2), but these will all involve nucleon-antinucleon pairs and we do not consider them further here.

Equation (17), with the approximations outlined above, defines a linear, coupled, inhomogeneous, integral equation for the reaction matrix. The couplings of the reaction to the nucleon-nucleon and pion-deuteron elastic scattering channels involve integrals linear in the off-shell reaction matrix; the remaining contributions define the inhomogeneous part of this equation. It is to the further definition of this inhomogeneous term that we now address ourselves. In the following, we consider the single-nucleon, pion-two-nucleon, and pion-nucleon contributions in turn, reserving our discussion of the double intermediate-state sums in Eq. (17) until last. For brevity, we consider only those terms corresponding to absorption by $p(p_2)$, where possible. The remaining graphs follow from the antisymmetry of our result.

1. One-nucleon state

Inserting the single-nucleon phase space, we find

$$(2\pi)^3 \sum_N \frac{\delta^{(3)}(\vec{p}_N+\vec{p}_1-\vec{p}_d)}{p_{N_0}+p_{1_0}-p_{d_0}-i\epsilon} \langle p_2| j_\pi(0)| p_N\rangle^c_{\rm out}\ {}_{\rm out}\langle p_N| J_1(0)| p_d\rangle = \langle p_2| j_\pi(0)|p\rangle \frac{\langle p| J_1(0)| p_d\rangle}{p_0+p_{1_0}-p_{d_0}-i\epsilon} \frac{m}{p_0}\ , \tag{20}$$

$$\vec{p}=\vec{p}_d-\vec{p}_1\ .$$

In Eq. (20), $\langle p_2| j_\pi(0)|p\rangle$ is the pion-nucleon-nucleon vertex with the pion off the mass shell and $\langle p| J_1(0)|p_d\rangle$ is the neutron-proton-deuteron vertex with one nucleon off the mass shell. Thus, Eq. (20) is the direct absorption term (Fig. 3) in which the pion is absorbed in a single-nucleon process.

2. Pion-two-nucleon state

Consider the matrix element

$$\langle 0| T^{(+)}_{NN}(p_1,p_2)^\dagger | q' p_2' p_1'\rangle_{\rm out}\ .$$

An important feature of $T^{(+)\dagger}_{NN}$ is the possibility of disconnected contributions where, in this case, one nucleon propagates freely and the other emits a pion. When the four-momentum constraints of physical pion production are imposed, such processes are not possible. In Eq. (17), however, only three-momentum conservation is required and these disconnected contributions are the lowest order approximation to the production process $NN\to NN\pi$. Accordingly, we take

$$\langle 0| T^{(-)}_{NN}(p_1,p_2)| q' p_2' p_1'\rangle_{\rm out} \simeq \langle 0| J_2(0)|q' p_1'\rangle_{\rm out} \langle p_1| p_2'\rangle - \langle 0| J_2(0)| q' p_2'\rangle_{\rm out} \langle p_1| p_1'\rangle - \{1\leftrightarrow 2\}\ . \tag{21}$$

The other matrix element involved in this contribution ${}_{\rm out}\langle q' p_1' p_2'| j_\pi(0)| p_d\rangle$, we evaluate in the single-scattering approximation (see Appendix C) obtaining

$${}_{\rm out}\langle q' p_1' p_2'| j_\pi(0)| p_d\rangle \simeq \frac{1}{(2\pi)^3}\ {}_{\rm out}\langle q' p_2'| j_\pi(0)| p\rangle \frac{\langle p| J_1'(0)| p_d\rangle}{p_{d_0}-p_{1_0}'-p_0+i\epsilon} \frac{m}{p_0} - \{1'\leftrightarrow 2'\}\ , \tag{22}$$

$$\vec{p}=\vec{p}_d-\vec{p}_1'\ ,$$

where ${}_{\rm out}\langle q' p_2'|j_\pi(0)|p\rangle$ is the rescattering vertex. Combining Eqs. (21) and (22), we find

$$\frac{(2\pi)^3}{2!} \sum_{\pi NN} \frac{\delta^{(3)}(p_1+p_2-q'-p_1'-p_2')}{p_{1_0}+p_{2_0}-q_0'-p_{1_0}'-p_{2_0}'+i\epsilon} \langle 0| T^{(+)}_{NN}(p_1,p_2)^\dagger | q' p_2' p_1'\rangle_{\rm out}\ {}_{\rm out}\langle q' p_1' p_2'| j_\pi(0)| p_d\rangle$$

$$\simeq \frac{1}{(2\pi)^3} \int \frac{d^3q'}{2q_0'} \frac{m}{p_{1_0}'} \frac{m}{p_0} \frac{\langle 0| J_2(0)|q' p_1'\rangle_{\rm out}\ {}_{\rm out}\langle q' p_1| j_\pi(0)| p\rangle}{p_{2_0}-q_0'-p_{1_0}'+i\epsilon} \frac{\langle p| J_1'(0)| p_d\rangle}{p_{d_0}-p_{1_0}'-p_0+i\epsilon}$$

$$-\frac{1}{(2\pi)^3} \int \frac{d^3q'}{2q_0'} \frac{m}{p_{2_0}'} \frac{m}{p_0} \frac{\langle 0| J_2(0)| q' p_2'\rangle_{\rm out}\ {}_{\rm out}\langle q' p_2'| j_\pi(0)| p\rangle}{p_{2_0}-q_0'-p_{2_0}'+i\epsilon} \frac{\langle p| J_1(0)| p_d\rangle}{p_{d_0}-p_{1_0}-p_0+i\epsilon} - \{1\leftrightarrow 2\}\ . \tag{23}$$

The first term in Eq. (23) is the "forward rescattering" contribution [Fig. 4(b)], the first order correction to the direct absorption term in a multiple-scattering series expansion of the absorption process. The second term [Fig. 4(a)], in which the pion is first scattered and then absorbed by the same nucleon, is canceled by the m = pion-nucleon, n = nucleon intermediate-state contribution from the final two terms in Eq. (17).

3. Pion-nucleon state

Inserting the appropriate phase-space elements, we have

$$(2\pi)^3 \sum_{\pi N} \frac{\delta^{(3)}(\vec{q}\,' + \vec{p}_2\,' + \vec{p}_1 - \vec{p}_d)}{q_0' + p_{2_0}' + p_{1_0} - p_{d_0} - i\epsilon} \langle p_2 | j_\pi(0) | q' p_2' \rangle_{\text{out}\ \text{out}}\langle q' p_2' | J_1(0) | p_d \rangle$$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3 q'}{2q_0'} \frac{m}{p_{2_0}'} \frac{\langle p_2 | j_\pi(0) | q' p_2' \rangle_{\text{out}\ \text{out}}\langle q' p_2' | J_1(0) | p_d \rangle}{q_0' + p_{2_0}' + p_{1_0} - p_{d_0} - i\epsilon}, \quad (24)$$

$\vec{p}_2\,' = \vec{p}_d - \vec{p}_1 - \vec{q}\,'$.

In Appendix D, we show that to lowest order

$${}_{\text{out}}\langle q' p_2' | J_1(0) | p_d \rangle = \langle q' | J_2'(0) | p' \rangle \frac{\langle p' | J_1(0) | p_d \rangle}{p_0' - q_0' - p_{2_0}' - i\epsilon} \frac{m}{p_0'} + \langle q' | J_1(0) | p \rangle \frac{\langle p | J_2'(0) | p_d \rangle}{p_{d_0} - p_{2_0}' - p_0 + i\epsilon} \frac{m}{p_0}, \quad (25)$$

where

$\vec{p} = \vec{p}_d - \vec{p}_2\,'$,

$\vec{p}\,' = \vec{p}_d - \vec{p}_1$.

When inserted into Eq. (24), Eq. (25) leads to two terms

$$(2\pi)^3 \sum_{\pi N} \frac{\delta^{(3)}(\vec{q}\,' + \vec{p}_2\,' + \vec{p}_1 - \vec{p}_d)}{q_0' + p_{2_0}' + p_{1_0} - p_{d_0} - i\epsilon} \langle p_2 | j_\pi(0) | q' p_2' \rangle_{\text{out}\ \text{out}}\langle q' p_2' | J_1(0) | p_d \rangle$$

$$= \frac{1}{(2\pi)^3} \int \frac{d^3 q'}{2q_0'} \frac{m}{p_{2_0}'} \frac{m}{p_0} \frac{\langle p_2 | j_\pi(0) | q' p_2' \rangle_{\text{out}} \langle q' | J_1(0) | p \rangle}{q_0' + p_{2_0}' + p_{1_0} - p_{d_0} - i\epsilon} \frac{\langle p | J_2'(0) | p_d \rangle}{p_{d_0} - p_{2_0}' - p_0 + i\epsilon}$$

$$+ \frac{1}{(2\pi)^3} \int \frac{d^3 q'}{2q_0'} \frac{m}{p_{2_0}'} \frac{m}{p_0'} \frac{\langle p_2 | j_\pi(0) | q' p_2' \rangle_{\text{out}} \langle q' | J_2'(0) | p' \rangle}{q_0' + p_{2_0}' + p_{1_0} - p_{d_0} - i\epsilon} \frac{\langle p' | J_1(0) | p_d \rangle}{p_0' - q_0' - p_{2_0}' - i\epsilon}. \quad (26)$$

The first term [Fig. 5(b)] is the "crossed" version of the first order rescattering correction mentioned in conjunction with Eq. (23). The second term [Fig. 5(a)] constitutes a correction to the direct absorption vertex. For a physical nucleon, graphs in which the pion is rescattered backward (or forward) and then absorbed (by the same nucleon) are included in the definition of the absorption vertex. Indeed, if we replace the deuteron by two asymptotically free "in" nucleons in Eq. (24) there is no disconnected part where $p(p_1)$ propagates freely. In this context, we also note the cancellation of such terms as mentioned in our discussion of Eq. (23) to the order of approximation considered there. In Eq. (24), however, we are probing the pion cloud of the deuteron, not the nucleon, and it should not be surprising that this cloud requires a modification of the direct absorption vertex. For reactions with small momentum transfer,

$$\frac{p_{d_0} - p_{2_0}' - p_0'}{k_0 - q_0' - p_{2_0}'} \simeq \frac{\text{deuteron binding energy}}{\text{pion mass}},$$

and this correction will be relatively unimportant. For the large momentum transfer involved in the breakup reaction considered here, the ratio is much closer to 1 and this correction cannot be automatically disregarded.

4. Double intermediate-state sums

$$-(2\pi)^6\sum_{m,n}\frac{\delta^{(3)}(\vec{p}_m-\vec{p}_2)}{p_{m_0}-p_{2_0}-i\epsilon}\,\frac{\delta^{(3)}(\vec{p}_n+\vec{p}_1-\vec{p}_d)}{p_{d_0}-p_{1_0}-p_{n_0}+i\epsilon}\langle 0|J_2(0)|m\rangle_{\text{out}}\;{}_{\text{out}}\langle m|j_\pi(0)|n\rangle_{\text{out}}\;{}_{\text{out}}\langle n|J_1(0)|p_d\rangle\,. \tag{27}$$

As mentioned previously, the m = pion-nucleon, n =nucleon contribution to Eq. (27) cancels the forward rescattering graph in Eq. (23) in which the pion is first scattered and then absorbed by the same nucleon. The $m=n$ =pion-nucleon contribution is of the same order as inelastic effects neglected in the single-scattering evaluation of $\pi d\to\pi NN$ [Eq. (22)]. We therefore neglect this contribution here as well.

Collecting the results of the foregoing analysis, our approximation to the inhomogeneous term V becomes

$$\begin{aligned}V(p_1,p_2;p_d)=&-\langle p_2|j_\pi(0)|p\rangle\frac{\langle p|J_1(0)|p_d\rangle}{p_0+p_{1_0}-p_{d_0}-i\epsilon}\,\frac{m}{p_0}\\&-\frac{1}{(2\pi)^3}\int\frac{d^3q'}{2q_0'}\,\frac{m}{p_{2_0}'}\,\frac{m}{p_0}\,\frac{\langle p_2|j_\pi(0)|q'p_2'\rangle_{\text{out}}\langle q'|J_2'(0)|p\rangle}{q_0'+p_{2_0}'+p_{1_0}-p_{d_0}-i\epsilon}\,\frac{\langle p|J_1(0)|p_d\rangle}{p_0-q_0'-p_{2_0}'-i\epsilon}\\&-\frac{1}{(2\pi)^3}\int\frac{d^3q'}{2q_0'}\,\frac{m}{p_{1_0}'}\,\frac{m}{p_0}\,\frac{\langle 0|J_2(0)|q'p_1'\rangle_{\text{out}}\;{}_{\text{out}}\langle q'p_1|j_\pi(0)|p\rangle}{p_{2_0}-q_0'-p_{1_0}'+i\epsilon}\,\frac{\langle p|J_1'(0)|p_d\rangle}{p_{1_0}'+p_0-p_{d_0}-i\epsilon}\\&-\frac{1}{(2\pi)^3}\int\frac{d^3q'}{2q_0'}\,\frac{m}{p_{1_0}'}\,\frac{m}{p_0}\,\frac{\langle p_1|j_\pi(0)|q'p_1'\rangle_{\text{out}}\langle q'|J_2(0)|p\rangle}{q_0'+p_{1_0}'+p_{2_0}-p_{d_0}-i\epsilon}\,\frac{\langle p|J_1'(0)|p_d\rangle}{p_{1_0}'+p_0-p_{d_0}-i\epsilon}-\{p_1\langle-\rangle p_2\}\,,\end{aligned}\tag{28}$$

where it is to be understood that three-momentum is conserved at each vertex.

Equation (28), exclusive of the correction to the direct absorption vertex, resembles the standard, truncated, multiple-scattering expansion of the absorption process obtained by a number of authors (to within the neglect of the final-state interaction).[4-7] However, there are a number of subtle differences between our result and that obtained by others. For example, in the multiple-scattering approach, the intermediate-state pion is taken to be unphysical. In Eq. (28), the intermediate-state pion is a physical pion; it is the mass-shell constraint associated with the pion current j_π which must be relaxed in order to satisfy the kinematical constraints of the rescattering vertex. These are two distinctly different methods of relating the rescattering vertex to the πN scattering amplitude. There is also a definite prescription for obtaining the pion absorption vertex; for the direct absorption term, it is the pion which is off its mass shell, while for the rescattering corrections, the nucleon is off the mass shell. Other approaches have not differentiated between these possibilities.

The physical nature of the intermediate-state pions in Eq. (28) has yet another consequence. In the multiple-scattering formalism, the "backward rescattering" graph is obtained through the insertion of negative energy pions into the intermediate-state sum and the rescattering vertex is interpreted in terms of $\pi^+p\to\pi^+p$ and $\pi^+n\to\pi^0p$.[8] In Eq. (28), this graph occurs as a natural consequence of the crossing relations between the pion and a final-state nucleon. This leads to the interpretation of the rescattering vertex in terms of the Hermitian conjugate of the amplitudes for $\pi^0n\to\pi^-p$ and $\pi^-p\to\pi^-p$. Consequently the isospin combination of $T_{\pi N}$ which appear are different in the two formalisms. We also note in this context that the relative phase of the forward and backward rescattering integrals is fixed by these crossing requirements.

In summary, our intermediate-state analysis of the absorption process yields a linear, inhomogeneous, integral equation

$$T(\pi d\to NN)=V-\int'\frac{T(NN\to N'N')^+T(\pi d\to N'N')}{E(NN)-E(N'N')+i\epsilon}-\int'\frac{T(NN\to\pi'd')^+T(\pi d\to\pi'd')}{E(NN)-E(\pi'd')+i\epsilon}\tag{29}$$

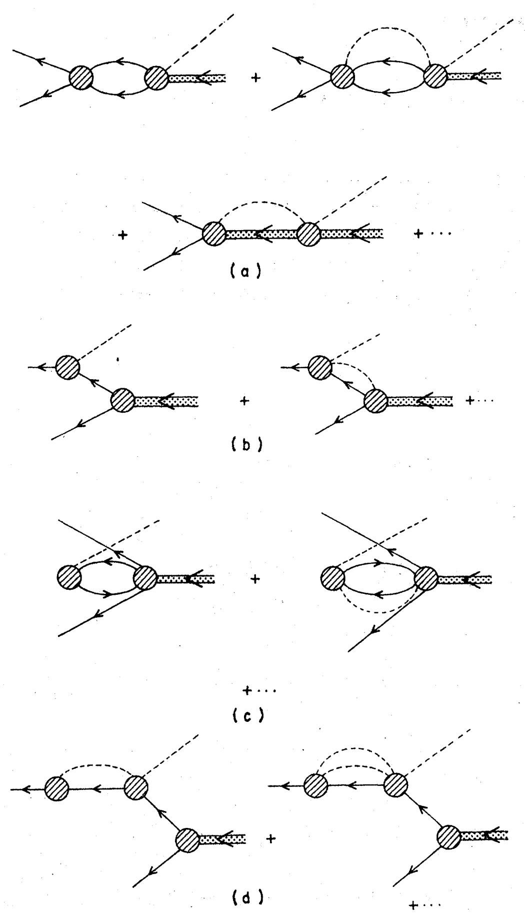

FIG. 1. The diagrammatic representation of Eq. (17); (a) the first intermediate-state sum, (b) the second (or third) intermediate-state sum, (c) the fourth intermediate-state sum, and (d) the double intermediate-state sums.

for the reaction matrix. The potential V, which is essentially a multiple-scattering expansion of the absorption process, involves the complete neglect of both the final-state interaction and the distortion of the incident pion wave. Our basic philosophy is that the integral terms in Eq. (29) provide the appropriate vehicle for the introduction of these interactions. In the present paper, we shall neglect these integral contributions. In the next section, we deal with the further development of the inhomogeneous term and its analysis at the reaction threshold.

III. EVALUATION OF THE POTENTIAL

In Sec. III B, we present the results of a threshold calculation of V. However, some further elucidation of the details of the matrix elements in Eq. (28) is first required.

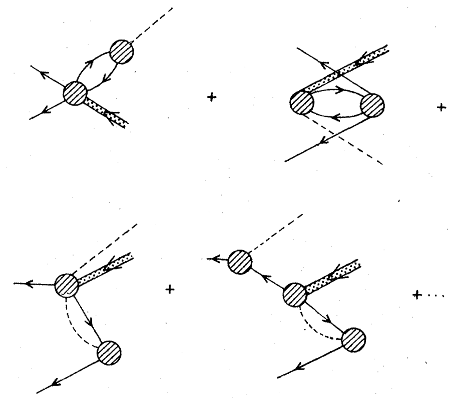

FIG. 2. The lowest order graphs from Eq. (17) involving disconnected contributions from the deuteron.

A. Discussion of elementary vertices

Consider first the rescattering vertex. Arguments based on Lorentz covariance and translation invariance require that the matrix element ${}_{\text{out}}\langle k'p'|j_\pi(0)|p\rangle$ define a half-off-mass-shell amplitude associated with the elastic scattering process

$$\pi(k)+N(p)\to\pi(k')+N(p'),$$
$$p'^2=m^2,\quad p=m^2,$$
$$k'^2=m_\pi^{\,2},\quad k^2\equiv(p'+k'-p)^2.$$

In the πN c.m. frame, where

$$\begin{aligned}
&k'=(k_0',\vec{l}_2),\quad k_0'=(m_\pi^{\,2}+\vec{l}_2^{\,2})^{1/2},\\
&p'=(p_0',-\vec{l}_2),\quad p_0'=(m^2+\vec{l}_2^{\,2})^{1/2},\\
&p=(p_0,-\vec{l}_1),\quad p_0=(m^2+\vec{l}_1^{\,2})^{1/2},\\
&W=k_0'+p_0',
\end{aligned}\tag{30}$$

this matrix element may be expressed in terms of the half-off-mass-shell partial-wave amplitudes $f_\nu(l_2,l_1)$, where

$$f_\nu(l_2,l_2)=\frac{\eta_\nu e^{2i\delta_\nu}-1}{2il_2},\tag{31}$$

in the normal fashion. In Eq. (28), however, we are in the πd c.m., not the πN c.m. In fact the

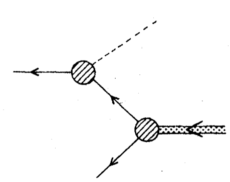

FIG. 3. The direct absorption graph.

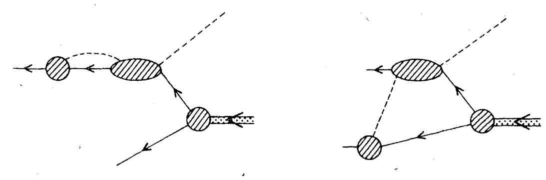

FIG. 4. The graphical representation of Eq. (23). The second graph is the forward rescattering contribution and the first is a part of the direct absorption graph.

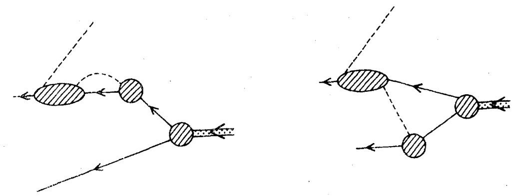

FIG. 5. The graphical representation of Eq. (26). The first diagram represents the modification of the direct absorption graph due to the pionic cloud of the deuteron and the second, the backward rescattering graph.

πN c.m. system changes with the intermediate-state pion momentum $\vec{q}\,'$. To effect the transformation from the πN to the πd c.m. system we first write the rescattering vertex in terms of the Lorentz scalars A and B:

$$_{\rm out}\langle k'p'|j_\pi(0)|p\rangle = \bar{u}(p')(A+\gamma\cdot k'B)u(p)\,. \qquad (32)$$

A and B may then be expressed in terms of quantities which are most easily calculated in the πN c.m. system [i.e., $f_\nu(l_2, l_1)$]:

$$A = \frac{4\pi(W+m)}{[(p_0+m)(p_0'+m)]^{1/2}} \times\left(f_1 - \frac{W-m}{W+m}\,\frac{(p_0+m)(p_0'+m)}{l_2 l_1}\,f_2\right),$$

$$B = \frac{4\pi}{[(p_0+m)(p_0'+m)]^{1/2}} \times\left(f_1 + \frac{(p_0+m)(p_0'+m)}{l_2 l_1}\,f_2\right),$$

$$f_1 = \sum_{\nu=0}^{\infty} f_{\nu^+}(l_2, l_1)P'_{l+1}(\hat{l}_2\cdot\hat{l}_1) - \sum_{\nu=2}^{\infty} f_{\nu^-}(l_2, l_1)P'_{l-1}(\hat{l}_2\cdot\hat{l}_1)\,, \qquad (33)$$

$$f_2 = \sum_{\nu=1}^{\infty} (f_{\nu^-}(l_2,l_1) - f_{\nu^+}(l_2,l_1))P'_l(\hat{l}_2\cdot\hat{l}_1)\,,$$

completing the transformation. The magnitudes of the relative momenta $|\vec{l}_2|$ and $|\vec{l}_1|$ and their scalar product $\vec{l}_2\cdot\vec{l}_1$ may be related to quantities in the πd c.m. system through the use of the Lorentz scalars

$$s = (p'+k')^2\,, \quad t = (p'-p)^2\,, \quad u = (k'-p)^2\,. \qquad (34)$$

We find

$$|\vec{l}_2| = [\tfrac{1}{4}s - \tfrac{1}{2}(m^2+m_\pi^2) + (m^2-m_\pi^2)^2/4s]^{1/2}\,,$$
$$|\vec{l}_1| = [(3m^2+m_\pi^2-t-u)^2/4s - m^2]^{1/2}\,, \qquad (35)$$
$$\vec{l}_2\cdot\vec{l}_1 = \tfrac{1}{2}t - m^2 + (m^2+\vec{l}_2^{\,2})^{1/2}(m^2+\vec{l}_1^{\,2})^{1/2}\,.$$

The above remarks obtain equally for all rescattering vertices. However, a few additional remarks are in order regarding the backward rescattering graphs. The backward rescattering term in Eq. (28) implicitly contains the δ function, energy-denominator combination [see Eq. 17(b)]

$$\frac{\delta^{(3)}(\vec{p}_n+\vec{p}_2-\vec{p}_d)}{p_{n_0}+p_{2_0}-p_{d_0}-i\epsilon} = \frac{\delta^{(3)}(\vec{q}\,'+\vec{p}_1'+\vec{q}-\vec{p}_1)}{p_{1_0}'+q_0'+q_0-p_{1_0}-i\epsilon}\,, \qquad (36)$$

where the equality holds only if all external particles in the reaction process are on the mass shell. Notice that the four-momentum associated with the pion current in the rescattering vertex is not q^μ, the four-momentum of the incident pion in the absorption process. All that Eq. (36) requires is three-momentum conservation—there is no prescription for inserting a negative energy for either pion associated with the rescattering vertex. In general, the fourth component of the momentum of the unphysical pion in Eq. (32),

$$k_0 = k_0' + p_0' - p_0\,, \qquad (37)$$

may take on either positive or negative values, dependent of course on the relative size of $|\vec{l}_2|$ and $|\vec{l}_1|$. In this detail, we differ with most workers in the field.

In the present work, we use the s-wave pion-nucleon amplitudes of Banerjee and Cammarata[9] and the classic Chew-Low p-wave amplitudes as determined by Salzman and Salzman.[10] Higher partial waves are ignored.

Consider the pion absorption vertices in Eq. (28). For the matrix element of the pion current between nucleon states, we take

$$\langle p'|j_\pi(0)|p\rangle = i\bar{u}(p')[g_\pi((p'-p)^2)\gamma_5]\tau_\alpha u(p)\,. \quad (38)$$

This choice is the most general form consistent with the requirements of Lorentz covariance.

For the matrix element of the nucleon current between the vacuum and the pion-nucleon state, Lorentz covariance is not quite so limiting. We have

$$\bar{u}(p')\langle 0|(-i\gamma\cdot\partial+m)\psi'(0)|qp\rangle_{\rm out}$$
$$=i\bar{u}(p')[g_1((q+p)^2)\gamma_5+g_2((q+p)^2)\gamma\cdot P\gamma_5]\tau_\alpha u(p)\,,$$
$$P=p'-p-q\,. \tag{39}$$

The possibility of the second form factor g_2 when the nucleon is off the mass shell seems to have first been mentioned by Banerjee and Levinson.[11] Little is known of g_2, and for the present we take $g_2=0$. There have been several theoretical analyses of g_1;[12] unfortunately they are model dependent and in contradiction with each other. In the present paper, we shall ignore the distinction between g_1 and g_π, that is, we take

$$g_1=g_\pi\,. \tag{40}$$

Applying a similar logic, we choose

$$\langle q'|J'(0)|p\rangle=i\bar{u}(p')[g_\pi((p-q')^2)\gamma_5]\tau_\alpha u(p)\,. \tag{41}$$

For the form factor g_π, we use an expression similar to that of Banerjee and Cammarata.[13] They take

$$g_\pi(x)=g_\pi(0)\left(1+\frac{x(x-4m^2)}{4m^2m_0^{\,2}}\right)^{-1},\quad x\leqslant 0$$
$$g_\pi(x)=g_\pi(4m^2)\left(1+\frac{x-4m^2}{4m_0^{\,2}}\right)^{-1},\quad x\geqslant 0 \tag{42}$$

where $g_\pi(0)=12.7$, $g_\pi(4m^2)=11.7$, and $m_0=8.6\,m_\pi$. In the second of Eqs. (42), the quantity $(x-4m^2)/4$ is the relative momentum of the pion molecule in the rest frame of the molecule.[14] Thus, for the form factor g_1 defined in Eq. (39), we replace this expression with

$$[(p\cdot q)^2-m^2m_\pi^{\,2}]\times\frac{1}{(p+q)^2}\,, \tag{39a}$$

the relative momentum of the pion-nucleon molecule in its rest frame. For the form factor defined in Eq. (41), we simply take $q\to -q$ in Eq. (39a). For $0\leqslant x\leqslant 4m^2$, we use a linear extrapolation between $g_\pi(0)$ and $g_\pi(4m^2)$. This parametrization ignores the fact that $g_\pi(x)$ may be complex for $x\geqslant 0$. We note in this context that the analysis of Epstein's[13] suggests that g_1 has only a small imaginary part and is approximately constant for some range of its argument. Thus, our ansatz, Eq. (40), may not be as inappropriate as it at first seems, given our parametrization of g_π.

The deuteron-two-nucleon vertex may be related to the nonrelativistic wave function

$$\phi(\vec{k})=\left(u(k)+\frac{3}{2\sqrt{2}}\,w(k)(\vec{\sigma}_1\cdot\hat{k}\vec{\sigma}_2\cdot\hat{k}-\tfrac{1}{3}\right)Y_0^0(\hat{k}) \tag{43}$$

by

$$\frac{\langle p|J'(0)|p_d\rangle}{p_0+p_0'-p_{d_0}-i\epsilon}=(p_0-p_0'-p_{d_0})\frac{\langle p|J'(0)|p_d\rangle}{(p_d-p)^2-m^2+i\epsilon}$$
$$\simeq 2(2\pi)^{3/2}\left(\frac{p_{d_0}p_0p_0'}{m^2}\right)^{1/2}$$
$$\times\langle\tfrac{1}{2},\tfrac{1}{2};m,m'|\phi(\tfrac{1}{2}(\vec{p}-\vec{p}\,'))|1,\lambda\rangle\,, \tag{44}$$

where $|1,\lambda\rangle$ is the spin state of the deuteron, and $|\frac{1}{2},\frac{1}{2};m,m'\rangle$ represents the Pauli spinors of the two nucleons. For the deuteron wave function, we choose the convenient analytic parametrization of Gourdin *et al.*,[15]

$$u(k)=(2/\pi)^{1/2}N\sum_i C_i(k^2+\alpha_i^{\,2})^{-1}\,,$$
$$W(k)=-2(2/\pi)^{1/2}\frac{\rho N}{\alpha_1^{\,2}}\sum_i C_ik^2(k^2+\alpha_i^{\,2})^{-1}\,, \tag{45}$$

where N is the normalization, ρ is the asymptotic ratio of the radial s- and d-wave parts of the deuteron wave function, and the α_i and C_i have been obtained from a fit to photodisintegration data.

B. Threshold absorption

We present here the results of an analysis of the potential $V(p_1,p_2;p_d)$ at the reaction threshold. We first define the angular momentum decomposition of V with

$$V(p_1,p_2;p_d)=\left\langle\tfrac{1}{2}\tfrac{1}{2};m_1m_2\right|\sum_{J,L,L'}\Lambda^J_{L'L}(\hat{p}_f,\hat{q}_i)\,V^J_{L'L}(p_f,q_i)\left|1,\lambda\right\rangle\,, \tag{46}$$

where $\Lambda^J_{L'L}$ is the angular momentum projection operator, L and L' the initial and final orbital angular momentum respectively, J the total angular momentum, and where $\vec{p}_f$ and $\vec{q}_i$ are the final and initial relative momenta in the πd c.m..

The use of proper relativistic kinematics at the absorption and the rescattering vertices complicates the evaluation of $V^J_{L'L}$ considerably. However, both the coefficients of $f_{\nu\pm}$ and the relative momenta $\vec{l}_1$ and $\vec{l}_2$ themselves have a strong angular $(\hat{p}\cdot\hat{q})$ dependence (see Table I and the discussion later in this section). Thus, any treatment which is to accurately reflect the unphysical content of $T_{\pi N}$ must deal with this angular variation. In Appendix E, we outline the method of calculation. Our result may be written

$$V^1_{10}=2(f^{1'}_{da}+f^1_{fr}+f^1_{br})\,, \tag{47}$$

where f^1_{fr}, f^1_{br}, and $f^{1'}_{da}$ are the contributions from

TABLE I. The relative momenta $|l_2|$ and $|l_1|$ and the pion-nucleon partial-wave coefficients [see Eq. (48)] as functions of $\hat{p}_f \cdot \hat{q}'$ at the reaction threshold in units of $m_\pi = 1$.

	$q'=2m_\pi$		$q'=10m_\pi$		$q'=20m_\pi$		$q'=2m_\pi$		
$\hat{p}\cdot\hat{q}'$	l_2	l_1	l_2	l_1	l_2	l_1	C_0	C_1	C_2
1	2.2	0.2	6.4	3.6	9.4	7.2	2.91	4.68	3.32
0.6	2.0	0.6	5.9	3.8	8.7	7.0	2.77	2.31	0.57
0.2	1.8	0.7	5.5	3.8	8.2	6.8	2.65	0.20	−1.65
−0.2	1.6	0.8	5.1	3.7	7.6	6.5	2.53	−1.66	−3.39
−0.6	1.3	0.8	4.6	3.6	7.0	6.1	2.41	−3.29	−4.65
−1	1.0	0.7	4.0	3.3	6.2	5.5	2.30	−4.68	−5.44

forward rescattering, backward rescattering, and the direct absorption correction defined in Appendix E, respectively. The combinations of the pion-nucleon isospin amplitudes which enter into the calculation are $(-4f_{\nu\pm}^{I=3/2}+f_{\nu\pm}^{I=1/2})/3$ (forward rescattering), $f_{\nu\pm}^{I=1/2}$ (backward rescattering), and $f_{\nu\pm}^{I=1/2}$ (direct absorption correction). The integrals were numerically evaluated using Gaussian quadrature. The individual contributions are given in Table II. The total, in units of $m_\pi=1$, is

$$V_{10}^1 = 7.84 - 11.62i\,.$$

The experimental number most often quoted is α, defined by

$$\lim_{\vec{q}\to 0}\sigma(p+p\to\pi^+ +d)=\alpha\,\frac{|\vec{q}\,|}{m_\pi}\,.$$

Using detailed balancing and V_{10}^1 as a first order approximation to the reaction matrix at threshold, we obtain $\alpha = 123\ \mu$b. Experimental estimates of α range from $\alpha = 138\pm 15\ \mu$b[16] to $\alpha = 240\pm 20\ \mu$b.[17] When comparing our estimate of α with experiment, one should recall that both the initial- and the final-state interactions have been ignored and that our crude model for the absorption vertex with the nucleon off the mass shell clearly needs improvement. A much more realistic theoretical estimate of α entails solving the coupled integral equation, Eq. (29), with an improved model for g_1 and g_2 [see Eq. (39)]. Work on this problem is now underway.

From Table II, we see that $f_{da}^{1'}$ is considerably smaller than f_{fr}^1 and f_{br}^1. This is due in large part to the extra energy factor

$$(2E(0)-p_{d_0})/(E(0)-\omega'-E(\vec{q}\,')-i\epsilon)\,,$$

which occurs in the integrand of $f_{da}^{1'}$ [see Eq. (E15)]. Notice also the important contribution that p-wave, pion-nucleon rescattering makes to the real part of V_{10}^1. It follows that a more realistic model for $f_{1\pm}(l_1,l_1)$ could change our results significantly.

The origin of p-wave rescattering in the s-wave absorption process lies in the consideration of nucleon recoil. This may be conveniently illustrated by rewriting Eq. (32) in the form

$$_{\text{out}}\langle p_2,q'\,|j_\pi(0)\,|p\rangle$$

$$\simeq\left\langle\frac{1}{2}m_2\right|\left[C_0 f_{0+}+C_1\frac{(f_{1-}-f_{1+})}{|\vec{l}_2|\,|\vec{l}_1|}+C_2\frac{f_{1+}}{|\vec{l}_2|\,|\vec{l}_1|}\right]\left|\frac{1}{2}m\right\rangle, \qquad (48)$$

where C_0, C_1, and C_2 are complicated functions of $\vec{p}_f$ and $\vec{q}\,'$. Their exact form is unimportant; their behavior as a function of $\vec{p}\cdot\vec{q}\,'$ for $|\vec{q}\,'| = 2m_\pi$ is given in Table I. Notice that $|C_1|$ and $|C_2|$ are maximum when nucleon recoil is maximum and zero near minimum nucleon recoil (i.e., $\vec{p}\cdot\vec{q}\,'=\vec{q}\,'^2$), as expected. This behavior is typical over much of the range of integration, although the cancellation between C_2 and C_1 (the total coefficient of f_{1+} is C_2-C_1) is not so complete as in Table I. We note in this context that the Lagrangian of Koltun and Reitan,[6] which has gained widespread acceptance for the description of threshold absorption, has only an s-wave re-

TABLE II. Contributions to the potential V_{10}^1 at the reaction threshold in units of $m_\pi = 1$.

	πN s wave	πN p wave
$\mathrm{Im}(f_{da}^{1'})$	−0.38	−0.18
$\mathrm{Im}(f_{fr}^1)$	−13.33	1.59
$\mathrm{Im}(f_{br}^1)$	5.77	0.72
$\mathrm{Re}(f_{da}^{1'})$	−0.01	−0.09
$\mathrm{Re}(f_{fr}^1)$	−4.45	9.39
$\mathrm{Re}(f_{br}^1)$	−0.77	−0.15

scattering mechanism. Thus, the non-negligible character of p-wave rescattering is particularly noteworthy. It is also instructive to consider the behavior of the relative momenta $|\vec{l}_2|$ and $|\vec{l}_1|$ for large values of $|\vec{q}'|$ (see Table I). Notice in particular that the difference of the average values of these momenta is never large, even for $|\vec{q}'|=20m_\pi$. Thus the rescattering mechanism is not sampling the behavior of the $f_{\nu\pm}$ as far off-shell as one would naively expect, and so it is far from clear that the threshold absorption mechanism should display much sensitivity to the off-shell behavior of $f_{\nu\pm}$.[18] These considerations should be valid above threshold and may explain the success of calculations which have ignored off-shell effects. In view of the above discussion, it is doubtful that the simple parametrization of the rescattering vertex in terms of cutoff functions of the form $V(\vec{q}^2)$ and $V(\vec{q}'^2)$, which seems to have become standard practice, has any clear relationship to the off-shell behavior of the partial-wave amplitudes.

In conclusion, we find that relativistic effects associated with the rescattering vertex are not negligible at the reaction threshold. If the absorption reaction is to serve as a probe of the unphysical behavior of pion-nucleon scattering, these effects must be dealt with realistically.

IV. DISCUSSION

We have used field-theoretic techniques to develop a coupled, linear, inhomogeneous integral equation for the pion-deuteron absorption reaction. Our method is based upon the intermediate-state analysis of the matrix element of the operator identity Eq. (15) between the vacuum and the deuteron state. The virtue of this method is that it obviates the difficulty encountered in constructing phenomenological Lagrangians for the pion-two-nucleon systems: the need to differentiate in a unique and consistent way between the pion-nucleon and the nucleon-nucleon scattering processes. The pion absorption vertex is an integral part of both the elastic pion-nucleon and nucleon-nucleon scattering amplitudes and this is a source of considerable difficulty.[19] In our approach, we achieve what we believe is a clear and consistent separation of the final-state interaction, the initial-state interaction, and the rescattering mechanism; we find no need to subtract the nucleon pole from the rescattering vertex to avoid double counting of meson exchanges.

Our approach is closely allied to the Low equation, although as one can see from Eq. (10b), a careful treatment of the seagull term is required. Banerjee, Levinson, Shuster, and Zollman[20] evaluate $\Gamma(p_2)$ in the soft-pion limit using a current-field algebra identity. These authors also consider dispersive corrections to the soft-pion limit but do not examine rescattering contributions in any detail. In related work, Banerjee and Yang[21] have done considerable work on an analysis of the disconnected contributions to the Low equation. In neither case, however, do their results have the full manifest antisymmetry exhibited by Eq. (17), due to their incomplete appreciation of the role of the matrix element of $\Gamma(p_2)$. We disagree with Alberg, Henley, and Miller's[22] strict interpretation of the matrix element of $\Gamma(p_2)$ as the Born graph (see Fig. 3). This term plays an important role in both the forward and backward rescattering contributions. In fact, it is difficult to see how these authors obtain both forward and both backward rescattering graphs with their interpretation of $\Gamma(p_2)$ and lack of crossing in $p(p_1)$ [see Eq. (8a)], nor do they obtain the correction to the direct absorption vertex. This effect, which is due to the pionic cloud of the deuteron, seems to be unique to our approach.

Another approach based on ideas similar to those of the Low equation is that of Lazard, Ballot, and Becker.[7] Our work differs from theirs in a number of essential details. Their neglect of pion crossing terms necessitates the introduction of negative energy intermediate-state pions (see discussion in Sec. II C) and their rescattering vertices seem to require that a nucleon be off the mass shell. It is worth emphasizing that the rescattering vertices and absorption vertices in our approach are all objects which are currently under theoretical investigation, and that while partial conservation of axial-vector current (PCAC) may serve as a guide for the continuation of $T_{\pi N}$ off the pion mass shell, there is no equivalent dynamical statement for a continuation in the nucleon four-momenta. This would seem to be a fundamental difficulty with Lazard *et al.* It is worth noting that the physical nature of the nucleons involved in the rescattering vertices does not imply that we are neglecting the effects of nuclear binding; binding effects are included insofar as the deuteron-two-nucleon vertex $\langle p' | J(0) | p_d \rangle$ is known.

In our final result, both dipion exchanges and nucleon-antinucleon pairs have been neglected. Brack, Riska, and Weise[23] find that ρ exchange is important. We feel that this point is unclear, but dipion exchanges may be included. Further investigations into the role of recoil and channel couplings are currently underway.

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APPENDIX A

In this appendix, we present an example of the techniques used in deriving Eqs. (10). We consider Eq. (10c). Using (5a), we have

$$i\int_0^\infty dy_0 a_1(\text{out})\mathcal{J}_2(y_0)j_\pi(0)=i\int_0^\infty dy_0 a_1(y_0)\mathcal{J}_2(y_0)j_\pi(0)-\int_0^\infty dy_0\int_y^\infty dx_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)\,. \tag{A1}$$

The order of integrations in the second term may be interchanged if the limits of the variables of integration are also appropriately modified, i.e.,

$$\int_0^\infty dy_0\int_y^\infty dx_0 f(x_0,y_0)=\int_0^\infty dx_0\int_0^{x_0} dy_0 f(x_0,y_0)\,. \tag{A2}$$

Utilizing Eq. (A2) and anticommuting $a_1(y_0)$ and $\mathcal{J}_2(y_0)$ in the first term in Eq. (A1), we find

$$\begin{aligned} i\int_0^\infty dy_0 a_1(\text{out})\mathcal{J}_2(y_0)j_\pi(0)&=i\int_0^\infty dy_0\{a_1(y_0),\mathcal{J}_2(y_0)\}j_\pi(0)-i\int_0^\infty dy_0\mathcal{J}_2(y_0)a_1(y_0)j_\pi(0)\\ &\quad-\int_0^\infty dx_0\int_0^{x_0} dy_0\mathcal{J}_1(x_0)\mathcal{J}_2(y_0)j_\pi(0)\,. \end{aligned} \tag{A3}$$

Finally, using

$$\begin{aligned} a_1(y_0)j_\pi(0)&=a_1(0)j_\pi(0)+i\int_0^{y_0} dx_0\mathcal{J}_1(x_0)j_\pi(0)\\ &=[a_1(0),j_\pi(0)]+j_\pi(0)a_1(\text{in})+i\int_{-\infty}^0 dx_0 j_\pi(0)\mathcal{J}_1(x_0)+i\int_0^{y_0} dx_0\mathcal{J}_1(x_0)j_\pi(0)\,, \end{aligned} \tag{A4}$$

for the second term in Eq. (A3), we obtain Eq. (10c).

APPENDIX B

Consider the operator product $a_1(\text{in})a_2(\text{in})$. Using

$$a_2(\text{in})=a_2(\text{out})-i\int d^4y\, e^{ip_2\cdot y}J_2(y)\,, \tag{B1}$$

we have

$$\begin{aligned} a_1(\text{in})a_2(\text{in})&=a_1(\text{in})\left(a_2(\text{out})-i\int d^4y\, e^{ip_2\cdot y}J_2(y)\right)\\ &=a_1(\text{out})a_2(\text{out})-i\int d^4x\, e^{ip_1\cdot x}J_1(x)a_2(\text{out})\\ &\quad+i\int d^4y\, e^{ip_2\cdot y}J_2(y)a_1(\text{out})-\int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)}\overline{T}(J_1(x)J_2(y))\,, \end{aligned} \tag{B2}$$

where we have used

$$-i\int d^4y\, e^{ip_2\cdot y}(a_1(\text{in})J_2(y)+J_2(y)a_1(\text{out}))=-\int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)}\overline{T}(J_1(x)J_2(y))\,, \tag{B3}$$

(we assume $\{\psi_i(x),J_j(y)\}\delta(x_0-y_0)=0$), and where $\overline{T}$ denotes the anti-time-ordered product. Taking the matrix elements of Eq. (B2) between the vacuum and a general "out" state n, and using

$$\int d^4x\, d^4y\, e^{i(p_1\cdot x+p_2\cdot y)}\langle 0|T(J_1(x)J_2(y))|n\rangle_{\rm out} = (2\pi)^4\delta^{(4)}(p_1+p_2-p_n)\left(\int d^4y\, e^{ip_2\cdot y}\langle 0|J_1(0)J_2(y)|n\rangle_{\rm out}\,\theta(y_0) - \int d^4x\, e^{ip_1\cdot x}\langle 0|J_2(0)J_1(x)|n\rangle_{\rm out}\,\theta(x_0)\right), \quad ({\rm B4})$$

we find

$$_{\rm in}\langle p_1p_2|n\rangle_{\rm out} = \delta_{fi} + (2\pi)^4 i\delta^{(4)}(p_1+p_2-p_n)\langle 0|T^{(+)}_{NN}(p_1,p_2)^\dagger|n\rangle_{\rm out}\,, \quad ({\rm B5})$$

$$T^{(+)}_{NN}(p_1,p_2)^\dagger = J_2(0)a_1({\rm out}) - J_1(0)a_2({\rm out}) + i\int d^4y\, e^{ip_2\cdot y}J_1(0)J_2(y)\theta(y_0) - i\int d^4x\, e^{ip_1\cdot x}J_2(0)J_1(x)\theta(x_0)\,. \quad ({\rm B6})$$

The first term in Eq. (B5) is the totally disconnected part of the S matrix, in which both nucleons propagate freely. The first line in Eq. (B6) corresponds to the "semidisconnected" part of the S matrix in which one nucleon propagates freely, while the second line in Eq. (B6) corresponds to the fully connected part. The development of $T^{(-)\dagger}_{NN}$ proceeds along similar lines.

APPENDIX C

We evaluate

$$_{\rm out}\langle q'p_1'p_2'|j_\pi(0)|p_d\rangle$$

in the single-scattering approximation through the judicious use of Eq. (15). The relevant terms in Eq. (15) are

$$i\int d^4y\, e^{ip_2'\cdot y}\bar u(p_2')\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)T(\psi_2'(y)j_\pi(0))\left(-i\int_{-\infty}^{0}dx_0\mathcal{J}_1'(x_0)\right) - \{1\leftrightarrow 2\}\,. \quad ({\rm C1})$$

Taking the matrix elements of Eq. (C1) between the pion and deuteron states and considering only the single-nucleon intermediate state, we have

$$_{\rm out}\langle q'p_1'p_2'|j_\pi(0)|p_d\rangle \simeq \sum_N \left\langle q'\left|i\int d^4y\, e^{ip_2'\cdot y}\bar u(p_2')\left(-i\gamma\cdot\frac{\partial}{\partial y}+m\right)T(\psi_2'(y)j_\pi(0))\right|N\right\rangle \times\left\langle N\left|-i\int_{-\infty}^{0}dx_0\mathcal{J}_1'(x_0)\right|p_d\right\rangle - \{1\leftrightarrow 2\}\,,$$

$$= {}_{\rm out}\langle q'p_2'|j_\pi(0)|p\rangle\frac{\langle p|J_1'(0)|p_d\rangle}{p_{d_0}-p_{1_0}'-p_0+i\epsilon}\,\frac{m}{p_0}\times\frac{1}{(2\pi)^3} - \{1\leftrightarrow 2\}\,, \quad ({\rm C2})$$

$\vec{p} = \vec{p}_d - \vec{p}_1'$.

APPENDIX D

In this appendix, we express the pion-two-nucleon-deuteron vertex $_{\rm out}\langle q'p_2'|J_1(0)|p_d\rangle$ in terms of the more elementary two-nucleon-deuteron vertex $\langle p|J_1(0)|p_d\rangle$ to lowest order. Using the LSZ reduction formalism, we have

$$_{\rm out}\langle q'p_2'|J_1(0)|p_d\rangle = i\int d^4x\, e^{ip_2'\cdot x}\langle q'|T(J_2'(x)J_1(0))|p_d\rangle$$

$$= (2\pi)^3\sum_n\frac{\delta^{(3)}(\vec{p}_n-\vec{p}_2-\vec{q}\,')}{p_{n_0}-q_0'-p_{2_0}'-i\epsilon}\langle q'|J_2'(0)|n\rangle_{\rm out}\;{}_{\rm out}\langle n|J_1(0)|p_d\rangle + (2\pi)^3\sum\frac{\delta^{(3)}(\vec{p}_d-\vec{p}_2'-\vec{p}_n)}{p_{d_0}-p_{2_0}'-p_{n_0}+i\epsilon}\langle q'|J_1(0)|n\rangle_{\rm out}\;{}_{\rm out}\langle n|J_2'(0)|p_d\rangle\,, \quad ({\rm D1})$$

where, as mentioned in the main text, we have assumed that the equal-time anticommutator $\{\psi_i, J_j\}$ vanishes. In Eq. (D1), we consider only the single-nucleon intermediate-state contribution. To this order of approximation,

$$_{\rm out}\langle q'p_2'|J_1(0)|p_d\rangle = \langle q'|J_1(0)|p\rangle\frac{\langle p|J_2'(0)|p_d\rangle}{p_{d_0}-p_{2_0}'-p_0+i\epsilon}\,\frac{m}{p_0} + \langle q'|J_2'(0)|p'\rangle\frac{\langle p'|J_1(0)|p_d\rangle}{p_0'-q_0'-p_{2_0}'-i\epsilon}\,\frac{m}{p_0'} \quad ({\rm D2})$$

$\vec{p} = \vec{p}_d - \vec{p}_2'\,,\quad \vec{p}\,' = \vec{p}_d - \vec{p}_1\,.$

APPENDIX E

In this appendix, we present the method of calculation of the threshold potential. To be specific, we consider the forward rescattering contribution with absorption at the $p(p_1)$ vertex. The computation of the remaining terms is similar.

For the following, it is convenient to reduce all matrix elements to two-component form. Let $|\frac{1}{2},m\rangle$ be the two-component Pauli spinor representing a nucleon with $s_z=m$. At the reaction threshold, we have

$$\langle 0|J_1(0)|q'p_1'\rangle_{\text{out}} = -i\frac{g_\pi((p_1'+q')^2)}{2m}\left(\frac{p_{1_0}+m}{p_{1_0}'+m}\right)^{1/2}\left\langle\frac{1}{2}m_1\right|\left[\vec{\sigma}_1\cdot\vec{q}'+\frac{(p_{1_0}'-p_{1_0})}{(p_{1_0}+m)}\vec{\sigma}_1\cdot\vec{p}_1\right]\left|\frac{1}{2}m_1'\right\rangle \tag{E1}$$

and

$$_{\text{out}}\langle q'p_2|j_\pi(0)|p\rangle \simeq \langle\tfrac{1}{2}m_2|(C_aA+C_bB)|\tfrac{1}{2}m\rangle\ , \tag{E2}$$

where

$$C_a=\left[\frac{(p_{2_0}+m)(p_0+m)}{4m^2}\right]^{1/2}\left[1-\frac{(p_{2_0}-m)}{(p_0+m)}\right],$$
$$C_b=\left[\frac{(p_{2_0}+m)(p_0+m)}{4m^2}\right]^{1/2}\left[q_0'\frac{(p_0+p_{2_0})}{(p_0+m)}-\frac{\vec{q}'^2}{p_0+m}+2\frac{\vec{p}\cdot\vec{q}'}{p_{2_0}+m}\right]. \tag{E3}$$

In Eq. (E2), we have neglected terms of order

$$\frac{pq'}{(p_{2_0}+m)^2}\times(A,q_0'B)\ .$$

Using the above in conjunction with Eq. (44), we rewrite the pertinent rescattering contribution as

$$\frac{1}{(2\pi)^3}\int\frac{d^3q'}{2q_0'}\frac{m}{p_0}\frac{m}{p_{1_0}'}\frac{\langle 0|J_1(0)|q'p_1'\rangle_{\text{out}}\ {}_{\text{out}}\langle q'p_2|j_\pi(0)|p\rangle}{p_{1_0}-q_0'-p_{1_0}'+i\epsilon}\frac{\langle p|J_1'(0)|p_d\rangle}{p_{1_0}'+p_0-p_{d_0}-i\epsilon}$$

$$=-i\left\langle\frac{1}{2}\frac{1}{2};m_1m_2\right|\int d^3q'\mathfrak{A}(\vec{p}_f,\vec{q}')\left[\vec{\sigma}_1\cdot\vec{q}'+\frac{(E(\vec{q}')-E(0))}{(E(0)+m)}\vec{\sigma}_1\cdot\vec{p}_f\right]\left[u(|\vec{k}|)+\frac{3W(|\vec{k}|)}{\sqrt{2}\,\vec{k}^2}((\vec{S}\cdot\vec{k})^2-\tfrac{2}{3}\vec{k}^2)\right]\left|1,\lambda\right\rangle, \tag{E4}$$

with

$$k=(E(\vec{q}'),\vec{p}_f-\vec{q}')\,,$$
$$E(\vec{q}')=(m^2+(\vec{p}_f-\vec{q}')^2)^{1/2}\ ,$$
$$\omega'=(m_\pi^{\ 2}+\vec{q}'^2)^{1/2}\ , \tag{E5}$$

and

$$\mathfrak{A}(\vec{p}_f,\vec{q}')=\frac{g_\pi((k+q')^2)}{2\sqrt{2}\,(2\pi)^2\omega'}\left[\frac{p_{d_0}(E(0)+m)}{E(\vec{q}')^2(E(\vec{q}')+m)}\right]^{1/2}\frac{C_aA+C_bB}{E(0)-\omega'-E(\vec{q}')+i\epsilon}\ . \tag{E6}$$

Here we have used the fact that we are in the πd c.m. and have defined $\vec{p}_f=\vec{p}_1=-\vec{p}_2$ as the relative momenta of the final-state nucleons.

With a little effort, Eq. (E4) can be rewritten in the form

$$\langle\tfrac{1}{2}\tfrac{1}{2};m_1m_2|f_{fr}^1\Lambda_{10}^1(\hat{p}_f,\hat{q}_2)|1,\lambda\rangle\ ,$$

where

$$\Lambda_{10}^1(\hat{p}_f,\hat{q}_2)=-\frac{1}{4\pi}(\tfrac{3}{2})^{1/2}\vec{S}\cdot\hat{p}_f\ ,$$
$$\vec{S}=\tfrac{1}{2}(\vec{\sigma}_1+\vec{\sigma}_2) \tag{E7}$$

is the $S\to{}^3P_1$ projection operation and where

$$f_{fr}^1=(4\pi)^2 i(\tfrac{3}{2})^{1/2}\int_0^\infty \vec{q}'^2 dq'[\tfrac{2}{9}|\vec{q}'|\alpha_{S1}+\tfrac{2}{3}|\vec{p}_f|\alpha_{S0}^0 - \tfrac{10}{27}|\vec{p}_f|\vec{q}'^2\alpha_{D0}^1+\tfrac{2}{27}|\vec{q}'|(\vec{p}_f^{\,2}+\vec{q}'^2)\alpha_{D1}^1-\tfrac{2}{3}\tfrac{1}{45}|\vec{p}_f|\vec{q}'^2\alpha_{D2}^1 +\tfrac{2}{9}\vec{p}_f^{\,3}\alpha_{D0}^0-\tfrac{4}{27}\vec{p}_f^{\,2}|\vec{q}'|\alpha_{D1}^0+\tfrac{2}{45}|\vec{p}_f|\vec{q}'^2\alpha_{D2}^0]. \tag{E8}$$

The quantities α in Eq. (E8) are defined by

$$\begin{aligned}\alpha_{Sl}^0&=\frac{2l+1}{2}\int_{-1}^1 dZ\,P_l(Z)u(|\vec{k}|)\alpha(\vec{p},\vec{q}')\frac{(E(\vec{q}')-E(0))}{(E(0)+m)}\,,\\ \alpha_{Sl}^1&=\frac{2l+1}{2}\int_{-1}^1 dZ\,P_l(Z)u(|\vec{k}|)\alpha(\vec{p},\vec{q}')\,,\\ \alpha_{Dl}^n&=\alpha_{Sl}^n\left\{u(|\vec{k}|)-\frac{w(|\vec{k}|)}{\vec{k}^2}\times\frac{3}{\sqrt{2}}\right\}\,,\end{aligned} \tag{E9}$$

where $Z=\hat{p}_f\cdot\hat{q}'$ and where $P_l(Z)$ is the Legendre polynomial of order l. The relative πN momenta are to be computed using Eqs. (34) and (35).

The evaluation of the backward rescattering graph is almost identical. To obtain f_{br}^1, one merely replaces $\alpha(\vec{p}_f,\vec{q}')$ with

$$\alpha^c(\vec{p}_f,\vec{q}')=\frac{g_\pi(X)}{2\sqrt{2}\,(2\pi)^2\omega'}\left[\frac{p_{d_0}(E(0)+m)}{E(\vec{q}')^2(E(\vec{q}')+m)}\right]^{1/2}\frac{C_a^cA^{c*}+C_b^cB^{c*}}{\omega'+E(\vec{q}')+E(0)-p_{d_0}-i\epsilon}\,, \tag{E10}$$

where $X=(E(\vec{q}')-\omega')^2-\vec{p}_f^{\,2}$ and

$$\begin{aligned}C_a^c&=\left[\frac{(E(\vec{q}')+m)(E(0)+m)}{4m^2}\right]^{1/2}\left[1-\frac{E(0)-m}{E(\vec{q}')+m}\right],\\ C_b^c&=\left[\frac{(E(\vec{q}')+m)(E(0)+m)}{4m^2}\right]^{1/2}\left[\omega'\left(1+\frac{E(0)-m}{E(\vec{q}')+m}\right)+\frac{|\vec{q}'|^2}{E(\vec{q}')+m}-\frac{2\vec{p}_f\cdot\vec{q}'}{E(0)+m}\right].\end{aligned} \tag{E11}$$

Now, however, the relative πN momenta are to be computed using

$$\begin{aligned}s_c&=(\omega'+E(\vec{q}'))^2-\vec{p}_f^{\,2}\,,\\ t_c&=(E(\vec{q}')-E(0))^2-\vec{q}'^2\,,\\ u_c&=(E(0)-\omega')^2-(\vec{p}_f-\vec{q}')^2\,,\end{aligned} \tag{E12}$$

where $p_{f_0}=(m^2+\vec{p}_f^{\,2})^{1/2}$, in obvious notation.

For the correction to the direct absorption vertex, we have

$$f_{da}^{1'}=(4\pi)^2 i(\tfrac{3}{2})^{1/2}\int_0^\infty q'^2dq'\alpha'\left[u(|\vec{p}_f|)+\frac{1}{\sqrt{2}}w(|\vec{p}_f|)\right]\,, \tag{E13}$$

where

$$\alpha=\tfrac{1}{3}\int_{-1}^1 dZ\left[-q'P_1(Z)-\frac{(E(\vec{q}')-E(0))}{(E(0)+m)}p_f\right]\alpha'(\vec{p}_f,\vec{q}')\,, \tag{E14}$$

$$\alpha'(\vec{p}_f,\vec{q}')=\frac{g_\pi(X')}{2\sqrt{2}\,(2\pi)^2\omega'}\left[\frac{p_{d_0}(E(0)+m)}{E(\vec{q}')^2(E(\vec{q}')+m)}\right]^{1/2}\frac{2E(0)-p_{d_0}}{E(0)-\omega'-E(\vec{q}')-i\epsilon}\,\frac{C_a^cA^{c*}+C_b^cB^{c*}}{\omega'+E(\vec{q}')+E(0)-p_{d_0}-i\epsilon}\,, \tag{E15}$$

and $X'=(E(0)-\omega')^2-(\vec{p}_f-\vec{q}')^2$.

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$$a(t)=\int d^3x\, e^{i(p_0t-\vec{p}\cdot\vec{x})}\bar{u}(p)\gamma^0\psi(\vec{x},t).$$

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