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Composite-particle structure of pion-nucleus amplitudes

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The structural properties of scattering amplitudes which involve composite particles are applied to pion absorption on and elastic scattering from nuclei. The reactive parts, as opposed to the initial and final state parts, of the amplitudes for these processes are identified and the constraints upon their structure are investigated. These constraints are used to determine the modifications, in various models for pionic interactions, which are necessary to avoid double-counting dynamics already accounted for in the nuclear states. The previously known decomposition of the elastic pion-nucleus scattering amplitude into the contributions from scattering and true absorption is derived in a general context. We go on to point out the structural features of both of these parts of the elastic amplitude which are required in order not to duplicate nuclear dynamics.

[NUCLEAR REACTIONS Pion absorption on nuclei. Pion elastic scattering from nuclei. Composite particle scattering theory.]

I. INTRODUCTION

The standard nonrelativistic potential scattering model of nuclear reactions below the threshold for pion production contains several simplifying features. The total number of elementary constituents (nucleons) remains fixed throughout the reaction process, these constituents are not involved in generating the interactions among themselves, and there exists a prescription for the calculation of any amplitude as the matrix element of a well-defined (transition) operator with respect to the wave functions of the initial and final nuclear fragments.

None of these features are realized to the same degree of accuracy in the description of pion-nucleus interactions. This circumstance can lead to interpretative difficulties if one wishes to meld the familiar nonrelativistic wave function picture of nuclear dynamics with the explicit field-theoretic characterization of the elementary π-N interaction. The interpretative problems intrinsic in such a merging are exemplified by the possibility of pion absorption— both real and virtual. For example, once an "external" pion begins to interact with a nuclear target, it is not always apparent how it can be distinguished from the pionic exchanges which contribute to the nuclear binding forces and the final-state interactions among the residual nuclear fragments. However, the possibility of such a distinction is often implicit in some models of the pion-nucleus interaction such as those which employ multiple-scattering algorithms taken over without modification from potential scattering.

Several of these difficulties were first appreciated in the study of the simplest pion-nucleus interactions,[1-7] viz.,

$$\pi + d \to \pi + d\,, \tag{1a}$$

$$\pi + d \to N + N\,. \tag{1b}$$

In the approximation in which the initial and final nuclear states are represented by nonrelativistic (NR) wave functions, the amplitudes associated with the processes (1) can be written as

$$M^{\rm el}_{\pi d} = \langle \vec{k}_f, \phi_d |\, \hat{T}\, | \vec{k}_i, \phi_d \rangle \tag{2a}$$

$$M^{\rm ab}_{\pi d} = \langle \psi^{(-)}_{NN} | J^{\pi}_d \,| \phi_d \rangle\,, \tag{2b}$$

respectively. Here ϕ_d and $\psi^{(-)}_{NN}$ represent the NR two-nucleon bound and scattering states, respectively, while $|\vec{k}_{i,f}\rangle$ correspond to the plane wave states which describe the π-d relative motion. $\hat{T}$ is a model transition operator for the $\pi NN \to \pi NN$ process, and J^{π}_d is an effective pionic absorption operator on deuterium which connects only two-nucleon states. An alternative form for (2b) is given in Ref. 8 which amounts to the transfer of part of J^{π}_d into the definition of the deuteron single-particle state.

We consistently regard Eqs. (2) as NR expressions involving NR two-nucleon wave functions. However, we are aware of the fact that with an appropriate interpretation of all quantities which enter into Eqs. (2) they can be taken as generally valid. For instance, in the general case, $|\psi^{(-)}_{NN}\rangle$ is a full "out state" corresponding to a definite state of only two nucleons in the infinite future. The Heisenberg state $|\psi^{(-)}_{NN}\rangle$ does not possess a single-component wave function representation as we have supposed for $\psi^{(-)}_{NN}(\vec{r})$ except in the NR limit.

Three distinct types of interpretative problems have arisen in connection with the description of (1) via the dynamical picture we have taken to be implicit in (2). The first concerns the consistent interpretation of the nucleon as a Fermi particle in early Faddeev models[1,5] for $\pi NN \rightarrow \pi NN$ and is not relevant to our subsequent discussion. The second involves the unambiguous identification of the effect of pionic absorption upon the elastic process (1a).[3-7] Finally, ambiguities can arise in given models for $\hat{T}$ and J_d^π as to the inclusion of dynamical effects already subsumed by the NR two-nucleon wave functions ϕ_d and $\psi_{NN}^{(-)}$.[3,5,9-11]

These last two types of interpretative ambiguities can be avoided by means of simple yet entirely general criteria. This is generally appreciated in the treatment of the absorptive contribution to elastic scattering.[3-7] On the other hand, there does not seem to be an equivalent degree of understanding for the problem of the separation of the dynamical effects contained in the nuclear states ϕ_d and $\psi_{NN}^{(-)}$ from those included in the specification of the pionic interaction processes in models for $\hat{T}$ and J_d^π. Part of the problem in this instance resides in the uncertainties which can enter into the definition of this separation, or factorization, in the course of passing to the NR limit of the composite nuclear states as they enter into the amplitudes $M_{\pi d}^{\mathrm{el}}$ and $M_{\pi d}^{\mathrm{ab}}$. Similar questions also arise in the consideration of relativistic and meson exchange current corrections to the deuteron form factor.[9,12-18]

Our primary concern is the specification of the constraints which must be imposed upon model operators such as J_d^π and $\hat{T}$ to ensure that dynamical effects already included in the initial and final nuclear states are not reproduced.[19,20] It is evidently imperative that one possess a precise idea of just what is represented by these states especially after the passage to a NR description.[16] This is particularly important when more complex targets than deuterium and final states with more than two nucleons are considered.

It would appear that, in order to settle these questions, it is necessary to treat all aspects of the problem on an equal footing, at least initially. This implies a consistent field-theoretical description of the composite nuclear states as well as of the pion-nucleon interaction. Fortunately, a description of composite-particle scattering processes sufficiently detailed to answer the questions we have posed and to resolve the ambiguities we have mentioned has been formulated.[21-24]

The objective of our investigation is to establish this last assertion in detail. To do this we apply the general structural properties of scattering amplitudes involving composite particles which have been developed in Refs. 21–24. One of these properties is that composite-particle states such as nuclei have a general, independent description in terms of Bethe-Salpeter amplitudes. These amplitudes, which contain the usual wave function representation of the composite states in the NR limit, enter into the scattering amplitudes, such as $M_{\pi d}^{\mathrm{el}}$ and $M_{\pi d}^{\mathrm{ab}}$, e.g., in a factorized manner. This property is analogous to the ordinary wave function factorization which is implicit in Eqs. (2). In general, it is found that the description of the composite-particle states can be segregated from what can be unambiguously identified as the reactive part of the amplitude for a process initiated by a pion on a nucleus. The specification of the class of Feynman graphs which enter into this reactive part then completes the description of the transition amplitude for the process.

This paper centers about the development of the implications of the preceding structural restrictions upon the types of graphs which can be included in the reactive parts of the amplitudes for pion absorption and elastic scattering on arbitrary nuclei. This structural analysis is carried out by means of graphical representations of the relevant amplitudes. Our principal results are stated as prescriptions concerning the types of graphs which can contribute to these amplitudes.

It is important to emphasize that these results, whether stated in equation or graphical form, represent guidelines which are to be used in the construction of consistent models for pionic interactions with nuclei; but do *not* in themselves comprise such models. For example, although we have not attempted at this time to develop practical integral equations involving the relevant pion-nucleus amplitudes, it is clear that the structural properties we have studied are highly relevant to such a development. Nevertheless, we do consider the applicability and the consequences of these results upon several models for pionic absorption[3,4,9-11] in which the nuclear states are represented by static NR wave functions. The question of the applicability of our results in the NR limit is one of the crucial aspects of the entire analysis and is discussed in some detail.

Let us outline the essential organization of this paper. The results of the composite-particle field theory which are relevant to pion absorption are stated in Sec. II and applied to pionic absorption on deuterium $(A = 2)$ as well as for targets with $A > 2$. In Sec. III the analysis is extended to pion elastic scattering in which the results already obtained in Sec. II are applied. A summary of our results and some concluding remarks constitute Sec. IV.

II. PION ABSORPTION

We apply the results of composite-particle field theory[21-24] first to the case of pion absorption.

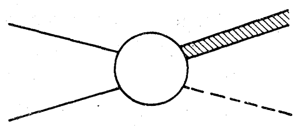

FIG. 1. Amplitude for $\pi + d \rightarrow N + N$. The dashed, solid, and cross-hatched lines, represent the pion, nucleon, and deuteron, respectively.

FIG. 2. (a) Graph reducible with respect to the final two nucleons. (b) Graph reducible with respect to the initial two nucleons. The square and oval interaction blobs represent connected graphs of the indicated type. The triangle represents a sum of dNN vertex graphs.

One of the reasons for doing so is that part of the pion-nucleus elastic scattering amplitude can be expressed in terms of what are essentially the absorption amplitudes without the final-state interactions. We remark also that our discussion can be applied with minor modifications to the time-reversed pion-production amplitudes.

We find it convenient to confine ourselves initially to the absorption process (1b) on deuterium. The amplitude for this process is the sum of all the connected Green's function-type graphs with the external particle lines indicated in Fig. 1, but with the free-particle propagators corresponding to these external lines stripped off.[25,26]

The attachment of the external pion to all the graphs contributing to Fig. 1 allows one to speak of two-fermion Bethe-Salpeter[22,23,27] reducibility both before and after the entry of the pion. Thus graphs of the form depicted in Fig. 2(a) are *reducible* in the final two fermions in that a vertical line intersecting only two internal fermion lines to the right of the external pion insertion and no other internal lines separates the graph into two connected graphs. In a similar way graphs of the form indicated in Fig. 2(b) are reducible in the initial two fermions. Graphs which are not reducible with respect to a particular pair of fermions (initial or final) are said to be *irreducible* in that pair. We note that graphs of the type displayed in Fig. 2(b) in which an interaction bubble can, figuratively speaking,[24] slide down into the bound state incorporate what are referred to as NN-reducible dNN vertex graphs.

We denote the transition amplitude corresponding to Fig. 1 as $T^{ab}[P_1, P_2 | k, q]$, where P, k, and q refer to the nucleon, pion, and deuteron four-momenta, respectively. (Throughout this article we suppress all spin and isospin indices.) The sum of all graphs contributing to this process which are irreducible in the final two nucleons is denoted as $T^{ab}[(P_1, P_2) | k, q]$, where the parenthesis around the four-momenta of the final two nucleons is used to signify the irreducibility property. Since the off-mass-shell NN transition amplitude, $\tau(P_1, P_2 | P_1', P_2')$, is the sum of all connected graphs with four external nucleon lines (with the free nucleon propagators stripped off) and therefore is the sum of all products of the connected irreducible graphs of this type, we infer that

$$T^{ab}[P_1, P_2 | k, q] = T^{ab}[(P_1, P_2) | k, q] + \int \frac{(d^4P_1')}{(2\pi)^4} \int \frac{(d^4P_2')}{(2\pi)^4} \tau(P_1, P_2 | P_1', P_2') S_F'(P_1') S_F'(P_2') T^{ab}[(P_1', P_2') | k, q], \quad (3)$$

where $S_F'(P)$ is the full single-fermion propagator. A graphical representation of Eq. (3) is provided by Fig. 3.

Equation (3) is a direct consequence of standard Bethe-Salpeter-type reasoning. Somewhat less familiar arguments are needed to obtain expressions for $T^{ab}[P_1, P_2 | k, q]$ and $T^{ab}[(P_1', P_2') | k, q]$ in terms of amplitudes irreducible in the initial two nucleons which emanate from the deuteron vertex. However, it is a direct consequence of composite-particle scattering theory[21-24] that $T^{ab}[P_1', P_2' | k, q]$, e.g., can be expressed in terms of an amplitude $T^{ab}_{ir}[(P_1', P_2') | k, (P_1'', P_2'')]$ which is irreducible in both the initial and the final pairs of nucleons, i.e.,

$$T^{ab}[(P_1', P_2') | k, q] = \int \frac{(d^4P_1'')}{(2\pi)^4} \int \frac{(d^4P_2'')}{(2\pi)^4} T^{ab}_{ir}[(P_1', P_2') | k, (P_1'', P_2'')] \chi_q(P_1'', P_2''). \quad (4)$$

The Bethe-Salpeter amplitude $\chi_q(P_1'', P_2'')$ is the Fourier transform of the propagation function $\langle 0 | T[\psi(x_1)\psi(x_2)] | q \rangle$, where $|q\rangle$ is the single-particle deuteron state. Equation (4) is displayed graphically in Fig. 4. We recall that χ_q is directly related to the proper dNN vertex function by means of nucleon propagators.[13,28] If we insert the expression (4) into the right-hand side of Eq. (3) we obtain

$$T^{ab}[P_1,P_2|k,q]=\prod_{i=1}^{4}\int\frac{(d^4P_i')}{(2\pi)^4}\{\delta(P_1-P_1')\,\delta(P_2-P_2')+\tau(P_1,P_2|P_1',P_2')S_F'(P_1')S_F'(P_2')\}$$
$$\times T_{ir}^{ab}[(P_1',P_2')|k,(P_3',P_4')]\chi_q(P_3',P_4')\,. \quad (5)$$

The representation for the absorption amplitude is the general expression to which Eq. (2b) with its NR interpretation constitutes an approximation. Nevertheless, there appears to be a strong similarity in the structural forms of the two equations.

The relativistic counterpart of the NR wave function $\psi_{NN}^{(-)}(\vec{r})$ is contained within the curly brackets on the right-hand side of (5). We note that these terms involving τ and S_F' have a meaning which is independent of this specific pionic absorption process.

The single-particle deuteron state is defined in terms of the Bethe-Salpeter amplitude $\chi_q(P_3',P_4')$ which is related to the ordinary static deuteron wave function in the NR limit.[28,29] Some care must be exercised in handling the negative-energy components of the deuteron[29] as one passes to this limit,[15] as we shall comment upon later. The definition of $\chi_q(P_3',P_4')$ is independent of this particular process.

The only part of (5) which is directly linked to the details of the absorptive process is T_{ir}^{ab}. This amplitude can thus be clearly identified as the reactive part of T^{ab}. Evidently, T_{ir}^{ab} is the counterpart of J_d^{τ} which appears in Eq. (2b). The choice of a model for the absorption process then is equivalent to an assumption about the form of T_{ir}^{ab}. We note that such dynamical assumptions can be prescribed in a manner independent of the description of the initial and final nuclear states.

The type of graphs which contribute to T_{ir}^{ab} is clear from its definition. In this regard, it is of considerable interest to examine some aspects of the structure of T_{ir}^{ab} in connection with some recently proposed models for pion absorption on deuterium.[3,4,9-11,30-33] The simplest contributions to T_{ir}^{ab} are the disconnected impulse graphs (Fig. 5) for pion absorption on a single nucleon.

While the impulse graphs make a nonnegligible contribution to T^{ab} at moderate energies, a dominant feature of the pion absorption reaction appears to be the two-nucleon mechanism of pion rescattering (Fig. 6), where the off-mass shell πN elastic scattering is expected to evolve principally through the Δ-isobar intermediate state.[6-11]

As we have drawn the pion rescattering graph in Fig. 6, it would appear that it is a piece of T_{ir}^{ab}. However, this is not exactly true, since Fig. 6 can be decomposed into parts which are reducible with respect to the pairs of nucleons to the right or to the left of the external pion insertion. This follows from the fact that the pion-nucleon amplitude, t, can itself be expressed as a sum of direct-channel (B_D) and crossed-channel (or nucleon exchange) (B_C) nucleon-pole terms plus a remainder t_R (Fig. 7).

The amplitude t_R is distinguished from B_D and B_C by the fact that by a cutting of any single internal nucleon line of any graph contributing to t_R, one does not disconnect the graph into two nontrivial components. In this sense we say that t_R is the sum of all one-nucleon *proper* πN connected graphs. Obviously, the two Born terms can be disconnected into nontrivial parts by cutting the intermediate nucleon line. This last property should not be confused with Bethe-Salpeter reducibility.

However, this one-nucleon improper structure of B_D and B_C is promoted to two-nucleon reducibility, as we have defined this concept, when the contributions of these terms to the pion rescattering graph (Fig. 6) are examined. We see from Fig. 8 that B_D and B_C generate graphs reducible in the final and initial pairs of nucleons, respectively. This is because if these graphs were included as part of T_{ir}^{ab} in the first instance, we would have in T^{ab} a single-pion exchange line which can slide into the final-state NN interaction, while in the second case the pion exchange can slide back into the dNN vertex (Fig. 2). We conclude that the pion rescattering part of T_{ir}^{ab} is given by the graph of Fig. 6, but with t replaced by t_R. Namely, *both* Born terms should be omitted from the off-mass shell πN amplitude in Fig. 6 to obtain the part of this graph which belongs to

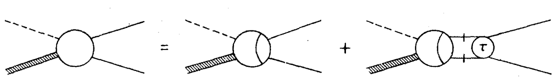

FIG. 3. The graphical representation of Eq. (3). The internal arcs enclosing the final two nucleon lines signify the irreducibility property. The full single-nucleon propagator, S_F', is represented as a solid line crossed by a small solid perpendicular line.

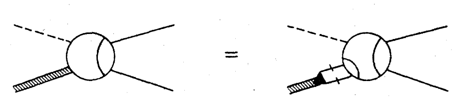

FIG. 4. Graphical representation of Eq. (4). The internal arcs indicate irreducibility as in Fig. 3. The darkened triangle represents the proper dNN vertex function.

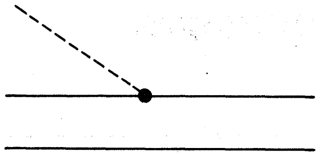

FIG. 5. Impulse graph. The darkened circle represents the proper πNN vertex.

T^{ab}_{ir}.

The prescription that the contribution corresponding to the graph of Fig. 8(a) must be omitted in the calculation of the pion rescattering part of T^{ab} seems to be in accord with recent work on the topic.[3-7,11] However, the concomitant exclusion of the contribution of the graph of Fig. 8(b) has not been proposed in these investigations. Although in the discussion to follow we will specifically address the Feynman graph of Fig. 8(b), our considerations will apply to the graph of Fig. 8(a) as well.

These studies[3-7,11] utilize a static NR wave function description of the deuteron bound state. This is employed in connection with the prescriptions implicit in (2b) to calculate both the impulse and pion rescattering parts of T^{ab}. When the calculation is done in this way the pertinent question is whether the graph of Fig. 8(b) (when also treated in the NR limit) leads to effects already included in the contribution of the impulse graph.

The answer to the last question evidently depends critically upon the interpretation of the NR limit of the graph of Fig. 8(b). In this limit it suffices for us to confine ourselves to forward-propagating nucleons, so that the Feynman graph Fig. 8(b) reduces to the sum of the three NR perturbation theory diagrams of Fig. 9 corresponding to the different relative time orderings of the pion insertions.

It has been understood for some time[9,14] that the diagrams of Figs. 9(a) and 9(b) represent effects already accounted for in the contribution of the impulse graph calculated according to the prescription (2b) in agreement with our conclusions. However, the so-called recoil emission graph of Fig. 9(c) seems to represent something distinct from anything one would expect to be included in

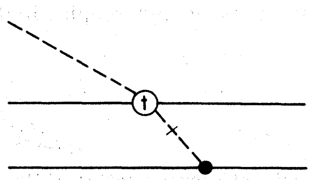

FIG. 6. Pion rescattering graph. The dashed internal line crossed by a small solid perpendicular line represents a full single-pion propagator Δ_F'.

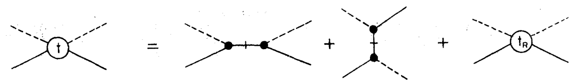

FIG. 7. Pion-nucleon amplitude expressed as a sum of its nucleon-pole terms plus a one-nucleon-proper remainder.

the contribution of an impulse graph when calculated with a static deuteron wave function.[9,14]

The interpretation of the role of Fig. 9(c) is a matter of some subtlety. In brief, depending upon how one handles other aspects of the problem including the positive- and negative-energy components of the deuteron, it appears that one can regard Fig. 9(c) alternatively as a relativistic correction to the contribution of the impulse graph or as being already included in that contribution.[12-18] In either case, it is obvious that our prescription regarding the consistent treatment of the pion rescattering term (Fig. 6) remains intact. Namely, one should *always* use t_R rather than t in the pion rescattering term (Fig. 6) in order to obtain a consistent assessment of the contribution of this term to the absorptive process. Under those circumstances where Fig. 9(c) assumes the role of a relativistic correction to the impulse graph its contribution should be calculated separately; however, the inclusion of B_C in the πN scattering amplitude part of Fig. (6) yields not only Fig. 9(c) but also the graphs Figs. 9(a), 9(b) which duplicate effects arising from the impulse graph.

The work of Thompson and Heller[12] and Woloshyn[13] would appear to indicate that in accord with our Feynman graph analysis of the role of Fig. 8(b), the recoil emission term Fig. 9(c) is part of the impulse graph contribution. It is shown in Refs. 12 and 13 that all three graphs of Fig. 9 conspire to yield a static Yukawa exchange mediated by a Lippmann-Schwinger propagator when one considers the NR limit of a graph such as that of Fig. 8(b), but folded in on the left by a dNN vertex. Such a structure is the correct NR limit of a one-pion exchange graph between the initial nucleons and thereby identical to effects already included in the contributions of Fig. 5.

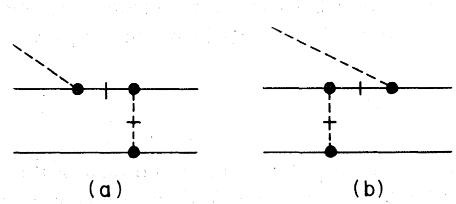

FIG. 8. (a) Contribution of the direct nucleon-pole term B_D to Fig. 6. (b) Contribution of the nucleon-exchange pole term B_C to Fig. 6.

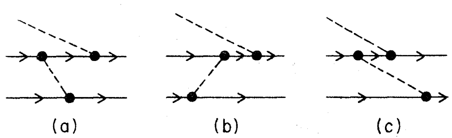

FIG. 9. Nonrelativistic limit of the Feynman graph of Fig. 8(b). The arrows on the nucleon lines denote forward propagation in time. Figs. 9(a)-9(c) correspond to the different time orderings of the pion exchange insertions.

Thompson and Heller[12] and Woloshyn[13] counsel against the inclusion of recoil emission terms in the calculation of static NR matrix elements such as (2b). This is in accord with our interpretation of the impulse graph matrix element as the implicit repository of all of the graphs of Fig. 9 in the NR limit.

This seemingly unequivocal conclusion concerning the role of the recoil emission terms is apparently at variance with recent work on meson-exchange current and relativistic corrections to the deuteron form factor in the impulse approximation.[9,14,15] This refers to the consideration of Fig. 5, but with the external pion line replaced by a photon line and with *both* the initial and the final pairs of nucleons folded into *dNN* vertices. Gross[15] has made a detailed comparison ot the low-order corrections to the static NR limit of this amplitude developed using NR perturbational and relativistic Feynman graph formulations.

Gross has shown that an approach which develops low-order corrections to the impulse graph for the deuteron form factor when calculated with static NR deuteron wave functions which arise from effects such as the depicted in Fig. 9(c) is equivalent to the NR limit plus lowest-order corrections of a Feynman diagram approach. In the latter method the deuteron is, of course, represented by the Bethe-Salpeter amplitude χ_q. However, to the order of approximation considered, Gross finds the equivalence between the two methods is achieved only with the retention in the Feynman diagram approach of the negative-energy components of the deuteron in addition to its positive-energy parts. Again to the degree of approximation considered, the negative-energy components are determined by the positive-energy ones in a straightforward manner.

However, the results of Ref. 15 cannot be directly applied to the case at hand because of the considerable difference in structure of the matrix elements corresponding to the impulse graph in the two cases. Thus the implications of this work upon the present problem are unclear and would have relevance in any case only to corrections to the strict static limit. We conclude, then, that in the NR approximation implicit in the use of (2b) the recoil emission graph Fig. 9(c) can be ignored.

The preceding conclusion concerning the role of the recoil emission terms is also apparently at variance with the work of Johnson[16] in which the so-called folded-diagram technique[17] is used to relate *energy-independent* nucleon-nucleon potentials to meson exchange diagrams below the inelastic threshold. Johnson[16] shows that the folded-diagram technique provides a systematic framework for explicitly determining how the effective exchange current operator depends upon the choice of nucleon-nucleon potential. With regard to the recoil emission terms, Johnson demonstrates that they must be included as corrections to the exchange current operator if an energy-independent potential is used for determining the deuteron wave function. In *addition* to each recoil emission term, however, Johnson also finds a "model correcting" term which, in fact, tends to negate the recoil emission terms. Hence, the net effect is approximately the same as if the recoil emission terms were omitted. It can then be argued that Johnson's results are in accord with the recommendations of Thompson and Heller[12] and of Woloshyn.[13] We hasten to point out, however, that this agreement is an approximate one and the exact connection between our technique and Johnson's remains to be established. This relationship will be explicated elsewhere.

It is useful to summarize the preceding discussion concerning the handling of the contributions of Fig. 8(b) and Fig. 9 to the pion rescattering term. The usual approach [where Fig. 8(b) or all of Figs. 9 are included] incorporates effects [Figs. 9(a), 9(b)] already included in the impulse term. Also, the recoil emission term [Fig. 9(c)] plays an ambiguous role in the specification of the initial bound state and of the reactive process. Finally, this approach carries with it a disturbing asymmetry in the assignment of dynamical effects between the initial and the final *NN* states; namely, the contribution of Fig. 8(a) is regarded as already included in the impulse term, while that of Fig. 8(b) is not. By way of contrast, the method we have described, which has a fully relativistic starting point, is both unequivocal and symmetrical in the specification of the dynamical content of the initial and final nuclear states. It is this circumstance which leads to an unambiguous structural analysis. These properties also define the NR limit of these states in a consistent manner. We comment in further detail on some of these points later in this section when we examine some specific models[11] for pion absorption upon deuterium.

The pion rescattering graph of Fig. 6 is a special case of the two graphs contained in Fig. 10.

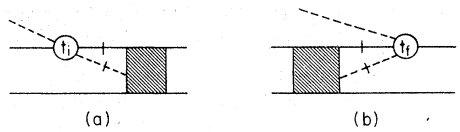

FIG. 10. General pion rescattering graphs corresponding to initial (a) and final (b) pion entries.

Evidently the graph of Fig. 10(a) is reducible in the final two fermions unless t_i is the amplitude,

$$t_D \equiv t - B_D, \tag{6a}$$

obtained by deleting the direct nucleon-pole Born term from the πN transition amplitude. Similarly, the graph of Fig. 10(b) is reducible in the initial two nucleons unless t_f is given by

$$t_C \equiv t - B_C, \tag{6b}$$

which is the πN amplitude without its crossed Born term. In the special case in which the square blobs of Figs. 10 are simply proper πNN vertices on the lower nucleon line, we obtain the ordinary pion rescattering graph contribution to $T_{\rm ir}^{\rm ab}$. In this instance *both* of the subtractions implied by Eqs. (6) must be imposed and we obtain $t_i = t_f = t_R$ as before.

In addition to the preceding constraints upon t_i and t_f, the structures of the square blobs of Figs. 10 are still restricted by the demands of irreducibility. For example, let us consider a multiple-scattering model where the square blobs in Fig. 10 are replaced by a Faddeev-type amplitude for $\pi NN \to \pi NN$ with one pion line connected directly to t_i or t_f while the other ends in a πNN vertex. Then all of the intermediate πN multiple scatterings must be mediated by t_D, rather than the full πN transition amplitude. No restrictions need be placed upon any intermediate NN scatterings mediated by τ. Some examples are displayed in Fig. 11.

Although we have restricted ourselves up to this point to an $A=2$ target, it is evident that this general line of development extends itself to more complex targets. The concommitant multiplicity of final states in the absorption process would appear to bring in a variety of reducibility criteria. For example, for $A=3$ we have

$$\pi^- + {}^3\text{He} \to n + n + p, \tag{7a}$$

$$\pi^- + {}^3\text{He} \to n + d, \tag{7b}$$

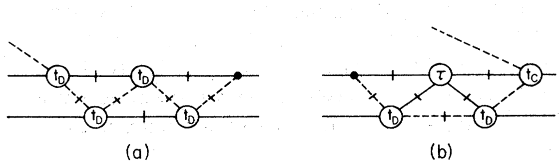

FIG. 11. Completely irreducible multiple scattering examples of Fig. 10.

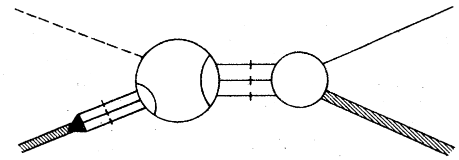

FIG. 12. Pion absorption on an $A=3$ nucleus leading to a nucleon-deuteron final state. The subgraph with two internal arcs corresponds to $T_{\rm ir}^{\rm ab}$. The final-state interaction subgraph represents the three-nucleon amplitude for formation of a nucleon and a deuteron. The darkened triangle denotes the appropriate nuclear vertex function.

namely two- and three-particle final states. Nonetheless, both of the processes (7) can be expressed in terms of a single amplitude, $T_{\rm ir}^{\rm ab}$, which is irreducible in all three final nucleons as well as all three initial nucleons. The structure of the amplitude for the process (7b) is illustrated in Fig. 12. The amplitude for the reaction (7a) can also be expressed in terms of the same $T_{\rm ir}^{\rm ab}$ which leads, in turn, to a graphical representation similar to that realized in Figs. 3, 4 except that τ is replaced by a $3N$-to-$3N$ scattering amplitude.

The two-nucleon pion rescattering absorption process contribution to $T_{\rm ir}^{\rm ab}$ for $A=3$ is simply the disconnected graph of Fig. 13(a). However, things become more complicated for similar processes involving all three nucleons. One can easily see that the only double rescattering contribution to $T_{\rm ir}^{\rm ab}$ which involves all three nucleons is the graph of Fig. 13(b). For $A>2$ the impulse and irreducible pion rescattering graphs are given by Fig. 5 and Fig. 13(a), but with $(A-1)$ and $(A-2)$ disconnected nucleon lines, respectively. Models for absorption for $A>2$ are considered in Ref. 33.

It is pertinent to note that similar (but not identical) prescriptions for the subtraction of direct and crossed Born terms from subsidiary πN amplitudes are used by Coon *et al.*[34] in their interesting work concerning the three-nucleon force in nuclear matter. In Ref. 34 only the forward-propagating nucleon parts of the Born terms are subtracted out, in order to avoid double counting the one-pion exchange parts of the two-nucleon *potential* in nuclear matter. This bound-state situation is not quite the same as the scattering

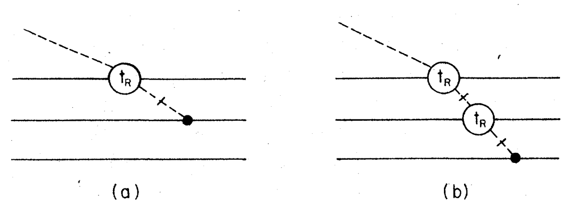

FIG. 13. (a) Two-nucleon pion rescattering process for $A=3$. (b) Three-nucleon pion rescattering process for $A=3$.

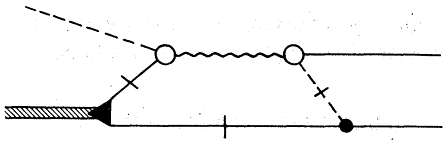

FIG. 14. Δ-dominated pion rescattering process for $A=2$. The wavy line indicates the Δ propagator; the open circles represent $\Delta N\pi$ vertices.

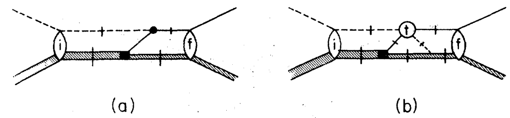

FIG. 15. (a) Pionic stripping process. The ovals correspond to initial- and final-state interactions. The cross-hatched lines represent nuclei and the darkened square the appropriate nuclear vertex function. (b) Generalization of pion rescattering which includes initial-state interactions.

problems addressed in this paper, particularly with regard to the description of the nuclear medium and therefore the double-counting or, equivalently, reducibility criteria are slightly different.

We now investigate what these concepts imply for some specific treatments of the absorption process. The work of Brack *et al.*[11] serves as a representative example of our present level of understanding of the pion absorption process on deuterium.[30-31] In the model of Ref. 11 the pion rescattering contribution to $\pi+d \rightarrow N+N$ is taken to correspond to the graph of Fig. 14. In this case the direct Born term is automatically excluded, but the nucleon exchange or crossed Born term is not. The effect of this last term is small in the neighborhood of the πN resonant energy for nearly on-shell scattering since the Δ enhancement is built up from higher-order terms. However, this is not necessarily the case away from resonance and/or far off shell. Therefore, some discrepancies can be expected between the standard interpretation of Fig. 14 and the one proposed here, where both Born terms are omitted.

In addition to the impulse (Fig. 5) and the Δ-dominated pion rescattering (Fig. 14) graphs, the model of Ref. 11 includes a Δ-mediated ρ-meson exchange process. This is represented by Fig. 14 but with the internal pion line replaced by a ρ propagator. The amplitude which this graph represents interferes destructively with the amplitude corresponding to Fig. 14. However, we note that this model for the ρ-exchange process includes a reducible part which is the counterpart of Fig. 8(b) with the internal pion line replaced by a ρ propagator. The importance of this reducible term, which must be omitted so as to exclude effects already included in the impulse amplitude, is not apparent. This example serves to illustrate how double-counting ambiguities can arise in other than one-pion-exchange situations.

For absorption on complex nuclei a generalization of the single-nucleon absorption process (or impulse approximation) which includes an initial pion-nucleus interaction is the so-called pionic stripping process which is pictured in Fig. 15(a).[35] In this model a pion is absorbed on a target of mass number A, after an initial state interaction, to give a final state consisting of a nucleon plus a bound $(A-1)$ nucleus. The amplitude which Fig. 15(a) represents is coherent with the amplitude corresponding to Fig. 15(b). The diagram Fig. 15(b) is the generalization of the pion rescattering process suggested in Ref. 35. Clearly the B_D part of t in Fig. 15(b) reproduces effects already induced in Fig. 15(a). The effect of B_C is not as easy to discern because the reducibility criteria with respect to initial groups of nucleons are obscured by the inclusion of an initial-state interaction. Nonetheless, B_C produces a pion line attached to the nucleon and the $(A-1)$ composite-particle leg of the nuclear vertex (darkened square) in Fig. 15(b). Thus this graph must reproduce effects already included in that vertex and hence in the amplitude of Fig. 15(a). We conclude in this case as well that the graph of Fig. 15(b) should be calculated with t replaced by t_R if the amplitudes represented by Figs. 15 are included together in the calculation of the absorption process.

The initial-state pion-nucleus interactions in Figs. 15 typically possess a distorted-wave structure and therefore have a disconnected and a connected part. The former piece is simply the noninteracting pion-nucleus state, while the latter is an off-mass shell pion-nucleus elastic scattering amplitude with propagators attached to the outgoing lines. If this elastic pion-nucleus amplitude includes *all* the diagrams normally included in the physical amplitude for this process ambiguities can arise in the interpretation of Figs. 15 and other such model graphs with an initial-state interaction. For example, the elastic pion-nucleus amplitude certainly contains the full impulse graph of Fig. 16. However, if this graph is included in the connected part of the initial-state interaction in Fig. 15(a), then it is easily seen that the direct Born term of t reproduces part of the contribution associated with Fig. 15(a) which arises from the disconnected (free) part of the initial interaction. In this instance, if one were to employ a multiple-scattering model for the elastic pion-nucleus amplitude to be used to generate the initial-state dis-

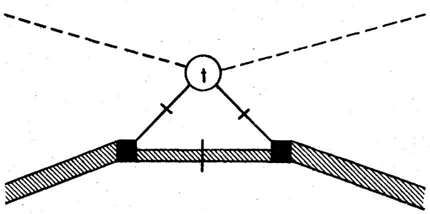

FIG. 16. Impulse graph for pion-nucleus elastic scattering.

torted wave, one would have to employ t_D rather than t for all of the intermediate pion-nucleon scatterings. As another example, if the initial and final distorted waves which enter into Figs. 15 are calculated using optical potentials obtained from the appropriate elastic processes, then some double-counting ambiguities are necessarily introduced. A more detailed analysis of distorted-wave prescriptions involving pions is clearly called for and we intend to explore this elsewhere.

III. ELASTIC PION SCATTERING

We next apply methods of the scattering theory of composite particles[21-24] to elastic pion-nucleus scattering. For simplicity we confine ourselves entirely to an $A = 2$ target, specifically the process (1a).

The elastic amplitude, T^{el}, has two external pion insertions. Thus, it is again possible to speak of reducibility in the initial and final pairs of nucleons, where these concepts have the same meaning as in the preceding section.

T^{el} can be expressed in terms of an amplitude T^{el}_{ir} which is irreducible in the initial and in the final pairs of nucleons.[21-24] Explicitly, we have

$$T^{el}(k_f,q_f|k_i,q_i)=\left(\prod_{i=1}^{4}\int\frac{(d^4P_i)}{(2\pi)^4}\right)\bar{\chi}_{q_f}(P_3,P_4)\,T^{el}_{ir}[k_f,(P_1,P_2)|k_i,(P_3,P_4)]\,\chi_{q_i}(P_3,P_4)\,, \qquad (8)$$

where the conjugate Bethe-Salpeter amplitude $\bar{\chi}_q(P_3,P_4)$ is the Fourier transform of the propagation function $\langle q_f|T[\bar{\psi}(x_3)\bar{\psi}(x_4)]|0\rangle$. Figure 17 is a graphical representation of Eq. (8). We next study some of the implications that the irreducibility requirements have upon the structure of T^{el}_{ir}.

Faddeev models which represent a summation of multiple scattering diagrams are often used to describe elastic π-d scattering.[1,2,5-7] In this regard, we note that while the entire single-scattering or impulse graph (Fig. 16) contributes to T^{el}_{ir} the double-scattering diagrams of Fig. 18 are not irreducible in either the initial or the final pairs of nucleons. In this particular instance, the reducibility comes about because of the πN Born terms. However, the general occurrence of such ambiguities is related to the contribution of the (virtual) pion absorption process to the elastic scattering amplitude. It is only because of the possibility of virtual pion absorption that one can, figuratively speaking, disengage an internal pion line and slide it down either the initial or final pairs of nucleons. Because of this circumstance it turns out that precisely the same difficulties encountered in Sec. II also appear in the elastic case. This will become manifest after the absorptive substructure of T^{el}_{ir} is explicated.

The set of all graphs which comprise T^{el}_{ir} decomposes into two classes. The first consists of all diagrams irreducible in the initial and final pairs of nucleons which are, nevertheless (Bethe-Salpeter) reducible with respect to a vertical cutting of the two internal nucleon lines which does not cross any other line internal or external. A graph of this type is drawn in Fig. 19. The second class consists in all those graphs which are irreducible in this sense. We call the sum of all graphs of the latter type T^{el}_{IR} and represent this amplitude by the diagram of Fig. 20.

The sum of those graphs of the form of Fig. 19 can be expressed in terms of T^{ab}_{ir} and its counterpart for the process $N+N\rightarrow\pi+d$ which designate as T^{pr}_{ir}. We find then that

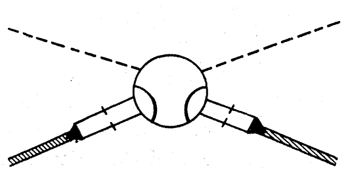

FIG. 17. Elastic pion-deuteron scattering in terms of an amplitude irreducible in the initial and in the final pairs of nucleons.

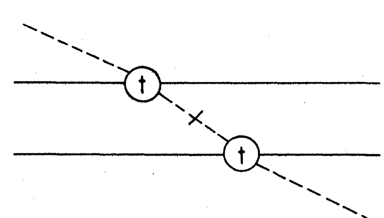

FIG. 18. Reducible double-scattering graph.

$$
\begin{aligned}
T^{\rm el}_{\rm ir}[k_f,(P_1,P_2)\,|\,k_i,(P_3,P_4)] = {} & T^{\rm el}_{\rm IR}[k_f,(P_1,P_2)\,|\,k_i,(P_3,P_4)] \\
& + \left(\prod_{i=1}^{4}\int\frac{(d^4P_i')}{(2\pi)^4}\right) T^{\rm ab}_{\rm ir}[k_f,(P_1,P_2)\,|\,(P_1',P_2')]S_F'(P_1')S_F'(P_2') \\
& \times \{\tau(P_1',P_2'\,|\,P_3',P_4')S_F'(P_3')S_F'(P_4') + \delta(P_1'-P_3')\,\delta(P_2'-P_4')\}\,T^{\rm pr}_{\rm ir}[(P_3',P_4')\,|\,k_i,(P_3,P_4)]\,,
\end{aligned}
\tag{9}
$$

which is displayed in Fig. 21. Equation (9) generalizes the work of Refs. 3–7 so as to give a precise and general specification of the constituent amplitudes, but it expresses essentially the same basic content as the corresponding results found in these investigations. When the expression (9) for $T^{\rm el}_{\rm ir}$ is inserted into Eq. (8), we obtain a decomposition of $T^{\rm el}$ in which an explicit contribution from pion absorption is separated off. This last is the Brueckner term for absorption.[36] We now see how some of the same ambiguities which were encountered in Sec. II can also occur for elastic scattering. This can come about because there are a class of graphs contributing to $T^{\rm el}_{\rm ir}$ which explicitly involve $T^{\rm ab}_{\rm ir}$ and $T^{\rm pr}_{\rm ir}$ and their approximate specification can bring in some of the ambiguities discussed in Sec. II.

The second type of interpretative difficulty mentioned in the Introduction concerns the unambiguous identification of the effect of pionic absorption upon elastic scattering. This refers to the constraints which must be imposed upon $T^{\rm el}_{\rm IR}$. The advantage of our approach to this problem is that the exact nature of these constraints is elucidated. In the particular case where one chooses a Faddeev model for $T^{\rm el}_{\rm IR}$, it is obvious that all subsidiary πN scattering amplitudes must be replaced by t_D in order to satisfy the intermediate-pair irreducibility criterion. If this were not done some effects already present in the absorptive piece of $T^{\rm el}_{\rm ir}$ would be reproduced, which then creates a double-counting situation. These remarks are in complete accord with prior work relevant to this point.[3-7]

$T^{\rm el}_{\rm IR}$ contains still another class of diagrams which can be identified as arising from intermediate absorption. We refer to the sum of all *two-nucleon improper* graphs remaining in $T^{\rm el}_{\rm IR}$. These are graphs of the $T^{\rm el}_{\rm IR}$ type which disconnect into two nontrivial diagrams if *only* the two internal nucleon lines are cut. The various irreducibility constraints already imposed upon $T^{\rm el}_{\rm IR}$ require that each piece which results from such a disconnection has one external pion insertion. Moreover, the Bethe-Salpeter internal two-nucleon irreducibility requirement upon $T^{\rm el}_{\rm IR}$ implies further that the external pion lines bear a crossed relationship to one another. Thus the sum of these improper graphs amount to the crossed-pion version of the Brueckner term (Fig. 22).[6,37] An estimate of the magnitude of the contribution of this crossed-pion absorption term to the π-d scattering length has been made by Mizutani and Mukhopadhyay.[38]

The extension of the preceding discussion to elastic scattering from more complex targets is straightforward.[7] In particular, the separation of the two types of absorptive contributions goes through as before, but the off-shell NN amplitude τ in the intermediate states is replaced by an off-shell (A-nucleon)-to-(A-nucleon) amplitude. The multiplicity of final nuclear states which are possible end products of the pion absorption upon the target nucleus is reflected in the contributions of the various pole terms of this A-to-A amplitude.

In the nonrelativistic description of the nuclear states the $\hat{\tau}$ insertions in Figs. 21 and 22 along with the one-body propagators attached to the external legs simply reduce to $G = (E + i0 - H)^{-1}$, where H is the nonrelativistic two-nucleon Hamiltonian.[7] Exactly the same remark holds for the A-nucleon-target generalizations of Figs. 21 and 22 except that in these cases H is the nonrelativistic A-nucleon Hamiltonian. The insertion of a complete set of "in" states which diagonalizes G then yields an expression for the absorption terms as sums over the accessible A-nucleon states after absorption.

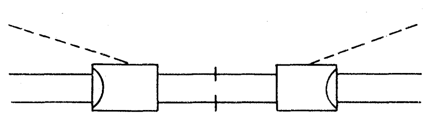

FIG. 19. $T^{\rm el}_{\rm ir}$ graph reducible in the intermediate nucleons.

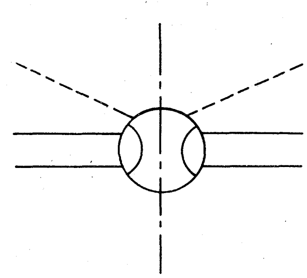

FIG. 20. Graph corresponding to $T^{\rm el}_{\rm IR}$ which is irreducible in the initial, intermediate, and final pairs of nucleons. Intermediate pair irreducibility is designated by the vertical dash-dot line.

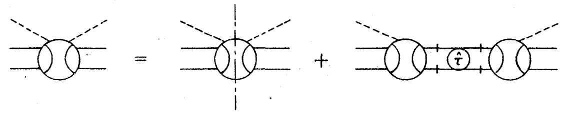

FIG. 21. Decomposition of T_{ir}^{el} into reducible and irreducible parts with respect to the internal nucleons. Here $\hat{\tau} = \tau + (S_F')^{-1}(S_F')^{-1}$.

IV. SUMMARY AND CONCLUSIONS

We have applied the general structural properties of scattering amplitudes which involve composite particles to pion absorption by nuclei (Sec. II) and elastic scattering from nuclei (Sec. III). The motivation for so doing derives from the fact that pions form part of the composite structure of nuclear states. Thus, it is important to be able to separate off the dynamical effects which are typically represented by a wave function description of these nuclear states from those effects which can be identified with the reactive process. To do so unequivocally, one must have a precise idea of the dynamical content of the nuclear states. Since this content is not manifest in the standard wave function description of these states, it would be remarkable if one were to achieve this separation of dynamical effects in a scattering formalism which represents the nuclear states in this way throughout.

An important feature of our argument is the fact that bound composite-particle states such as nuclei possess an independent and general characterization in terms of appropriate Bethe-Salpeter amplitudes. It is from these amplitudes that one can extract a static NR wave function description when the internal dynamics of the composite system is such that a nonrelativistic kinematical situation results along with the identification of a definite number of constituent particles. However, the *initial representation* of the bound nuclear states by means of the Bethe-Salpeter amplitudes allows us to separate off what can then be unambiguously identified as the reactive parts of the various pion-nucleus amplitudes.

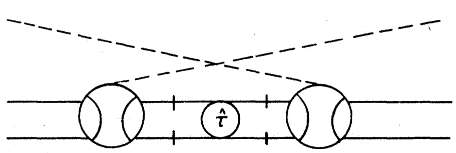

FIG. 22. Crossed-pion absorptive contribution to T_{IR}^{el}.

In most practical applications, of course, the Bethe-Salpeter amplitudes for nuclei will be approximated in terms of some model NR wave functions. The dynamical content of such wave functions may not be explicit. From the point of view taken in the present work, the use of NR nuclear wave function presumes, as part of the total set of model assumptions involved in a calculation of a pion-nucleus process, that this wave function is related to a Bethe-Salpeter amplitude in a well-defined manner. The physical significance of any proposed NR nuclear wave function which does not satisfy this criterion of direct relationship to the appropriate Bethe-Salpeter amplitude appears obscure, particularly with regard to the appearance of these wave functions in scattering and reaction amplitudes.

The principal results obtained in Sec. II and III are consequences of the structural properties of the graphs which can contribute to these reactive parts without duplication of dynamical effects already accounted for in the bound or continuum nuclear states. Several models for pion absorption on nuclei are considered and the specific modifications, necessary in order to avoid double counting nuclear-state dynamics, are pointed out. We also indicate the requisite alterations in the representation of the propagation of a Δ in a nucleus. We recover some known results concerning the general overall structure of the pion elastic scattering amplitudes with regard to contributions from what can be called "true pion absoprtion." However, we go on to identify the explicit contributions to the elastic amplitudes of the same reactive amplitudes which enter into pion absorption. As a consequence of this identification, we show how two types of double-counting problems can thereby enter into approximations to the elastic scattering amplitudes. Finally, we argue that the various ambiguities which are connected with the passage from the Bethe-Salpeter to the static NR wave function description of the bound nuclear states do not alter the overall results of our structural analysis.

We conclude that it is expedient to use a completely general representation of pion-nucleus interactions to obtain information which is applicable to various approximate calculations of these processes. Indeed, for some aspects which relate to the dynamical content of the nuclear states such a description, at least in the initial stages of the problem, seems indispensible. The methods used in this paper can be extended straightforwardly to other pion-initiated processes on nuclei or to the time-reversed production reactions. These structural considerations are also of evident importance in the development of systems of scattering integral equations for these processes although we have not discussed this aspect here.

ACKNOWLEDGMENTS

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