Low-energy $p + {}^3$He elastic scattering and the optical potential

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It is shown that the optical model potential with central, spin-orbit, and exchange terms can provide reasonably good fits to the cross section, polarization, and the polarization transfer coefficients $K_x^{x'}$ and $K_z^{x'}$ for $p + {}^3$He elastic scattering in the energy range 6.8–16.2 MeV. The measured values of the polarization transfer coefficient $K_y^{y'}$ are compared with those calculated by adding a spherical spin-spin interaction term to the potential. Both phenomenological and folding-model spin-spin terms are considered. Good agreement is obtained for the 16.2 MeV data but not for the data at lower energies. The effects of including a tensor spin-spin interaction are also studied.

[NUCLEAR REACTIONS $p + {}^3$He elastic scattering; optical model calculations.]

I. INTRODUCTION

The study of elastic scattering of protons on light nuclei has been a favorite subject with experimentalists during the past few decades. There now exists a large number of measurements of the differential cross section and polarization for $p + {}^3$He scattering at incident proton energies below 20 MeV. Because the target has nonvanishing spin, there has recently been a widening of the data base, by performing the more difficult measurements of the polarization transfer coefficients.[1,2] These measurements should be very useful in clarifying the complicated spin structure of the process.

Traditionally the data have often been interpreted using a phase-shift analysis.[3] However, it has proved difficult to extract unambiguous phase shifts from the data. In most cases, several sets of phase shifts were found at a given energy which fit the data equally well. The data have also been analyzed using the R-matrix formalism.[1,2,4] The results from these analyses are rather encouraging. On the microscopic level, resonating group calculations[5] have been performed for low-energy $p + {}^3$He scattering using a purely central nucleon-nucleon potential. Reasonable fits to the differential cross section data were obtained. The most important result of these calculations was the finding that the requirement of antisymmetry of the total wave function of the system causes the effective potentials between the projectile and target to be very different for l-even and l-odd states. As we shall see later, this result will have a bearing on the form of the potential used in the present analysis. Microscopic calculations with nucleon-nucleon potentials containing central, spin-orbit, and tensor terms have also been carried out.[6] The results of these calculations were in good agreement with cross section and polarization data in the energy range $E_p = 3$–8 MeV. There have also been some attempts to apply the optical model to the $p + {}^3$He system.[7-9] In an earlier note, Podmore and Sherif[8] have shown that one can use the optical model to fit the cross section and polarization data in the energy range $E_p = 5.51$–156 MeV. The presence of an exchange term[10] in the potential was confirmed in an analysis of the data at 30 MeV. Votta *et al.*[9] have used the optical potential to fit the elastic cross section data at 85 MeV. These initial successes encourage one to take a more serious look at the optical model potential as applied to the $p + {}^3$He system.

In the present paper, we apply an optical model analysis to the low-energy $p + {}^3$He data. These data include not only the differential cross section and polarization but also the measured polarization transfer coefficients. These latter coefficients, which have been measured at four energies in the past few years, should offer a more stringent test of the applicability of the model.

In Sec. II we give the form of the potential used to fit the cross section and polarization data at the four energies 6.82, 8.82, 10.77, and 16.2 MeV. We show how the polarization transfer coefficients $K_x^{x'}$ and $K_z^{x'}$ predicted by this potential agree with those measured at these energies. Section III is devoted to a discussion of the polarization transfer coefficient $K_y^{y'}$. In order to predict the observed deviations of $K_y^{y'}$ from unity one must include spin-spin interaction terms in the potential. Several possibilities are discussed and the $K_y^{y'}$ predicted using a distorted wave Born approximation (DWBA) calculation are compared with the experimental data.

II. ANALYSIS

The optical model potential used in this analysis consists of the Coulomb potential, a real central potential, a volume-type imaginary potential, a spin-orbit potential, and a real exchange potential. The latter is necessitated by the findings of resonating group method calculations[5] which showed that as a result of antisymmetrization, the potential for even l states is quite different from that for odd l states. The form of this term is taken to be the simple Majorana form suggested by Greenlees and Tang.[10] Thus the potential has the form

$$V(r) = V_c(r) - V_0 f(r, r_0, a_0) - iWf(r, r_i, a_i) + \left(\frac{\hbar}{m_\pi c}\right)^2 V_{so} \frac{1}{r} \frac{d}{dr} f(r, r_{so}, a_{so}) - V_{ex}(-)^l f(r, r_{ex}, a_{ex}) . \quad (1)$$

The Coulomb potential $V_c(r)$ is taken to be that due to a uniformly charged sphere of radius $R_c = 1.3\,A^{1/3}$ fm. The form factor function f has the usual Woods-Saxon shape. l is the orbital angular momentum quantum number.

The search for the best fit parameters involved only cross section and polarization data. However, reaction cross section data were also used to constrain the values of the imaginary part of the potential. The data considered here are the differential cross section and polarization data at the proton energies 6.82, 8.82, and 10.77 MeV,[11,12] and 16.2 MeV.[13,14] The computer program SEEK[15] was used in the search for the best parameters. Starting with the data at 16.2 MeV, the search was performed on all the parameters, but constraining the depth W such that the calculated reaction cross section is kept in line with those measured by Sourkes *et al.*[16] After obtaining the best fit parameters, the geometry parameters were then held fixed and the fits to the cross section and polarization data at the three lower energies were obtained by searching on the depth parameters of the various terms of the potential. The imaginary well depth W was constrained to be zero at the lowest two energies, and to produce a σ_R in the neighborhood of 10 mb at 10.77 MeV, as is suggested by extrapolation from existing data. The best fit parameters are shown in Table I. Figure 1 shows a comparison of the predicted cross sections and the corresponding measurements. We see that the agreement is excellent. The polarization fits are shown in Fig. 2. The data at 16.2 MeV are not shown here since they have yet to be published. Except for some differences near the polarization peak, and at the forward angles for the 6.82 MeV data, we find the agreement with experiment rather satisfactory.

TABLE I. Optical potential parameters for $p + {}^3$He scattering.

E_p (MeV)	V_0 (MeV)	W (MeV)	V_{so} (MeV)	V_{ex} (MeV)	σ_R (mb)
6.82	41.1	0	1.54	−4.41	···
8.82	36.2	0	1.93	−5.87	···
10.77	34.1	0.1	2.23	−5.69	7.8
16.2	33.2	0.44	2.75	−3.18	20.8

Geometry (in fm)
$r_0 = 1.489$ $r_i = 1.35$ $r_{so} = 1.049$ $r_{ex} = 1.32$
$a_0 = 0.144$ $a_i = 0.378$ $a_{so} = 0.289$ $a_{ex} = 1.19$

Because of the complex spin structure of the $p + {}^3$He scattering amplitude, more observables, besides cross section and polarization, must be interpreted before one can claim that the calculated amplitude is good enough. We can answer the question about the reliability of the potentials obtained above by testing them against polarization transfer coefficients.[17] The coefficients $K_x^{x'}$ and $K_z^{x'}$ have been measured[1,2] at the four energies

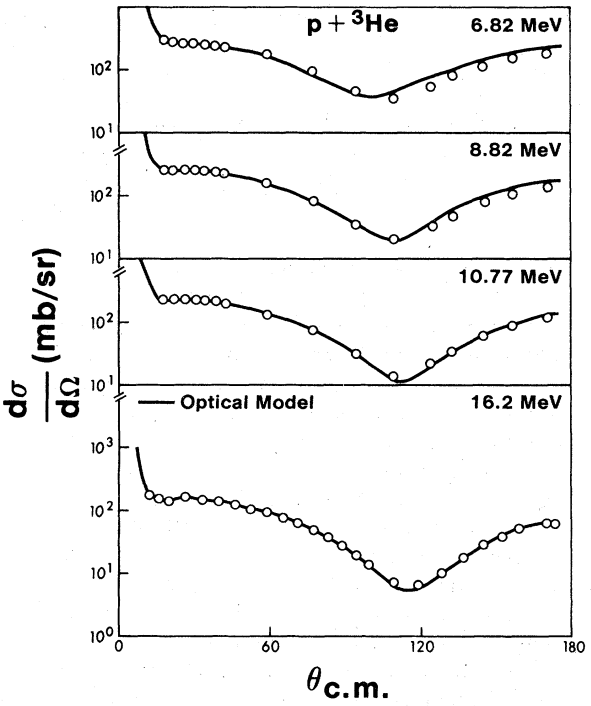

FIG. 1. Differential cross section for $p + {}^3$He elastic scattering at 6.82, 8.82, 10.77, and 16.2 MeV. The data are from Refs. 11 and 13. The solid curves are optical potential fits using the parameters of Table I.

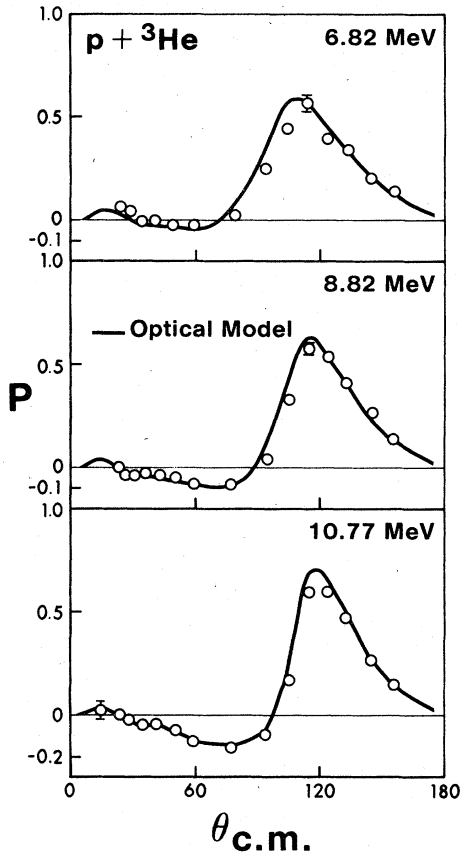

FIG. 2. Polarization in $p+{}^3$He elastic scattering at 6.82, 8.82, and 10.77 MeV. The data are from Ref. 12. The solid curves are optical potential fits using the parameters of Table I.

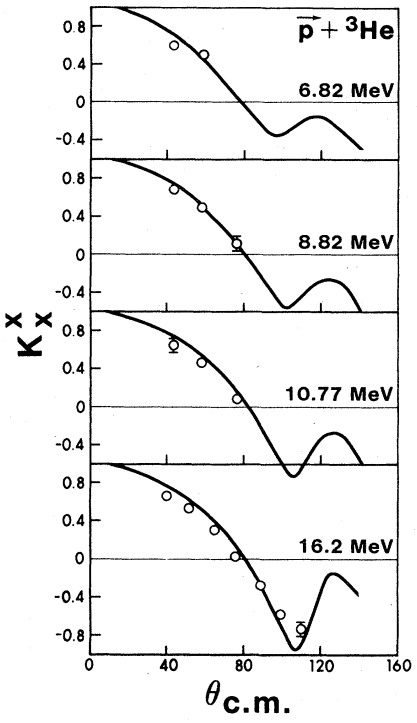

FIG. 3. The polarization transfer coefficient $K_x^{x'}$ in $\vec{p}+{}^3$He elastic scattering at 6.82, 8.82, 10.77, and 16.2 MeV. The data are from Refs. 1 and 2. The solid curves are optical model predictions using the parameters of Table I.

under consideration and will be discussed here (the $K_y^{y'}$ coefficient requires the addition of spin-spin interactions to the potential and will be discussed in Sec. III below). Using the amplitudes corresponding to the parameters of Table I, we have calculated the coefficients $K_x^{x'}$ and $K_z^{x'}$. The results are shown in Figs. 3 and 4. Noting that the transfer coefficients were not included in the search for the best fit parameters, we find the agreement with experiment rather impressive. The present results are of the same quality as those obtained from R-matrix calculations and are superior to those calculated using matrix elements based on microscopic calculations.[1,2] The success of the simple optical model potential used here in predicting the observed values for $K_x^{x'}$ and $K_z^{x'}$ indicates that these two coefficients are not sensitive to the finer details of the interaction and that the spin-orbit interaction plays the dominant role in causing the deviations of $K_x^{x'}$ and $K_z^{x'}$ from the values they would assume in the absence of spin-dependent forces.

III. THE $K_y^{y'}$ COEFFICIENT AND THE SPIN-SPIN INTERACTION

The optical model potential used in the preceding section does not depend on the spin of the ^{3}He target. It follows that the polarization transfer coefficients $K_y^{y'}$ calculated from this potential are strictly equal to 1. Deviations of $K_y^{y'}$ from unity are related to the nonvanishing spin of the target and thus offer a handle on determining the size of the spin-spin interactions between projectile and target. Following a suggestion by Feshbach,[18] one may add to the optical potential a spherical spin-spin interaction of the form

$$U_{ss}(r) = -V_{ss} F_s(r) \vec{\sigma} \cdot \vec{I} , \tag{2}$$

where $\vec{\sigma}$ is the Pauli spin operator for the nucleon and $\vec{I}$ is the target spin operator.

There have been several attempts in the past aimed at determining the strength of the spin-spin interaction. Unfortunately these efforts have been rather inconclusive.[19] Calculations by Satchler[20]

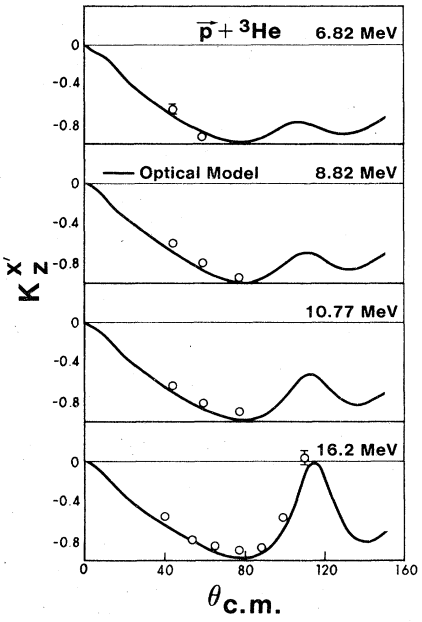

FIG. 4. The polarization transfer coefficient $K_z^{x'}$ in $\vec{p}+{}^3He$ elastic scattering at 6.82, 8.82, 10.77, and 16.2 MeV. The data as taken from Refs. 1 and 2. The solid curves are optical model predictions using the parameters of Table I.

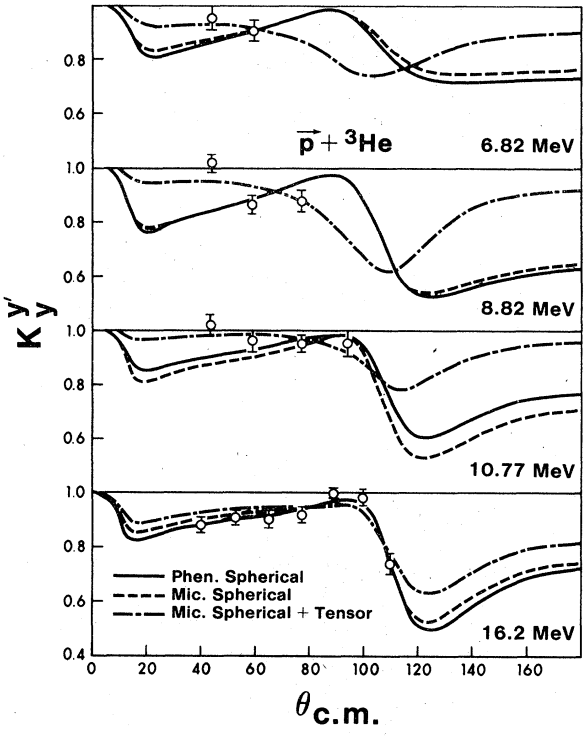

FIG. 5. The polarization transfer coefficient $K_y^{y'}$ in $\vec{p}+{}^3He$ elastic scattering at 6.82, 8.82, 10.77, and 16.2 MeV. The data are taken from Refs. 1 and 2. The curves are the results of introducing spin-spin interactions whose effects are calculated using DWBA. The solid curves correspond to a phenomenological spherical interaction (see Table II). The dashed curves result from a microscopic folding-model spherical interaction. The dash-dot curves result from a combination of microscopic folding-model spherical and tensor interactions.

and by Dabrowski and Haensel[21] showed that the interaction is weak for heavier targets. The analysis of experiments on the scattering of polarized neutrons from polarized targets was often hampered by the role played by compound nuclear effects.[22] Deviations of $K_y^{y'}$ from unity for targets with spin $I > \frac{1}{2}$ were shown to suffer either from these same difficulties[23] or to be mainly due to quadruple spin-flip effects.[24] In this respect the ^3He nucleus offers a unique case since the latter effects do vanish for a spin-$\frac{1}{2}$ target. Moreover, ^3He, being a light nucleus, should have a stronger spin-spin interaction since the strength of this interaction should, in an approximate sense, have an A^{-1} dependence on the target mass number.

Our first attempt at interpreting the $K_y^{y'}$ data involved treating the interaction of Eq. (2) completely phenomenologically. The radial dependence of the interaction was taken to be of the same shape as that of the real central potential. The effect of this potential on the scattering amplitude is calculated, to first order, using the distorted-wave Born approximation.[25] The depth V_{ss} was treated as a free parameter to be determined from a fit to the data. The results of these calculations are shown by the solid curves in Fig. 5. The values of V_{ss} corresponding to these fits are shown in Table II. The quality of the fits at the two lower energies is marginal, but improves at $E_p = 10.77$ MeV, and for incident proton energy of 16.2 MeV the fit is indeed excellent. Noting that the data for this latter case do cover a wider angular range, one should not be too discouraged by the lack of fit at 6.82 MeV, where the measurements consist of only two data points. We should also point out that the present fit to the 16.2 MeV data is slightly better than the one obtained using R-matrix calculations[1] and that our fits to the 8.82 and 10.77 MeV data are comparable to those using phase-shift analysis, R-matrix, or microscopic calculations.[2]

A folding-model calculation was also carried out for the spin-spin interaction. In this calculation the first-order potentials are obtained by folding the appropriate terms in the nucleon-nucleon potential into the target ground state wave function.[20,26] For example, the spherical spin-spin

TABLE II. The strength of the phenomenological spherical spin-spin potential [a] obtained from $K_y^{y'}$ data.

E_p (MeV)	V_{ss} (MeV)
6.82	4.5
8.82	7.0
10.77	6.0
16.2	7.0

[a] The geometry is that of the real central potential (Table I).

interaction of Eq. (2) is written as

$$U_{ss} = \langle \Phi_H | V_{0n}(\vec{\sigma}, \vec{\sigma}_n, \vec{r}_{0n}) | \Phi_H \rangle , \quad (3)$$

where Φ_H is the internal wave function of ^{3}He and V_{0n} is the nucleon-nucleon spin-spin interaction (the subscripts 0 and n refer to the incident proton and the unpaired neutron in the ^{3}He target, respectively). This interaction was taken from the work by Eikemeier and Hackenbroich[27] and the single Gaussian wave function of Reichstein *et al.*[5] was used for Φ_H. The depth of the resulting potential was left as a free parameter to be determined from the data. The fits obtained from this procedure were found to be essentially the same as those using the phenomenological interaction [Eq. (2)]. This is shown by the dashed curves in Fig. 5. We note, however, that both fits miss the very forward point at the energies 6.82, 8.82, and 10.77 MeV. It is seen that the forward data point near $\theta_{c.m.} = 40°$, at these three energies, is difficult to reproduce using a spherical spin-spin interaction. This is in contrast to the situation at 16.2 MeV where the corresponding data show a larger deviation of $K_y^{y'}$ from unity and are in good agreement with the calculations.

Finally, we carried out a calculation to study the effect of a tensor-type spin-spin interaction on the fits to the $K_y^{y'}$ data. This potential was calculated using the folding-model procedure outlined above and the nucleon-nucleon tensor interaction of Eikemeier and Hackenbroich.[27] The effect of this potential on the scattering amplitude was again calculated using DWBA.[25] It was found that this tensor interaction alone does not yield good fits to the $K_y^{y'}$ data. A combination of folding-model spherical and tensor terms was tried (their strengths were treated as free parameters). The results are shown by the dash-dot curves in Fig. 5. We find that a good fit to the 6.82 MeV data is obtained and that the fit to the 10.77 MeV data is noticeably improved. It is not possible, however, to improve drastically the fit to the 8.82 MeV data. Moreover, it is found that the fit to the 16.2 MeV data obtained using both spherical and tensor terms is inferior to the one obtained using the spherical term alone.

The inclusion of the spin-spin potentials into the calculations also has some effect on the other observables. These effects are generally small. We find that when the spin-spin terms are included the calculated cross sections are increased by about 10% at back angles. This results in a slight deterioration in the fit at these angles. The calculated polarizations are also slightly different, but the quality of the fits remains very nearly the same. The effects of the spin-spin terms on the polarization transfer coefficients $K_x^{x'}$ and $K_z^{x'}$ are such that there is generally a slight improvement in the agreement between the calculated and the observed values of these coefficients.

IV. CONCLUSIONS

It has been shown in this work that the ordinary optical model potential does provide reasonably good fits to the cross section and polarization data in $p + {}^3$He elastic scattering in the energy range 6.8–16.2 MeV. The reaction cross sections predicted at the two higher energies are reasonable in the light of available data. The potential contains a simple Majorana exchange term which is important in obtaining good fits to the data. Without this term, a forced fit would require unreasonably large values for the absorptive part of the potential. The exchange term is closely connected with the requirement of full antisymmetrization of the total wave function.[10] It is interesting to note that the volume integrals per nucleon for the real central potentials obtained in the present work agree well with the corresponding values one obtains[28] for much heavier targets such as ^{208}Pb and ^{120}Sn. Because of the limited energy range studied, it is not possible to comment here on the energy dependence of these volume integrals.

It is most encouraging to find that not only does the optical potential provide reasonably good fits to the cross section and polarization data, but it also yields values for the polarization transfer coefficients $K_x^{x'}$ and $K_z^{x'}$, which are in good agreement with experiment. From this one concludes that the spin-orbit interaction is responsible for most of the effects of spin-dependent forces on these two observables. The measured values of the polarization transfer coefficient $K_y^{y'}$ for 16.2 MeV protons are in very good agreement with those calculated by introducing a spherical spin-spin interaction. Both phenomenological and microscopic folding-model potentials give similar results. However, the agreement between the calculated and observed values of $K_y^{y'}$ deteriorates

as the energy decreases. This seems to be related to marked differences between the two sets of measurements near forward angles. Including a tensor interaction leads to improvements in the fits for the 6.82 MeV and the 10.77 MeV data, but would worsen the agreement at 16.2 MeV. It is not likely that the fact that the spin-spin effects are calculated to first order only is responsible for the above difficulties, since these same problems are also present in other types of analysis.[1,2] However, we stress here that the spin-spin depths given in Table II should be regarded with caution since they result from first-order calculations. A better calculation would involve solving appropriate coupled equations in which the spin-spin terms (as well as the rest of the potential) are treated exactly. This effort, however, must await the availability of more detailed polarization transfer data.

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