Analysis of $0^- \leftrightarrow 0^+$ beta decay and muon capture in the $A = 16$ nuclei

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The muon capture $\mu^- + {}^{16}O \rightarrow {}^{16}N^*(0^-) + \nu_\mu$ and the beta decay ${}^{16}N^*(0^-) \rightarrow {}^{16}O + e^- + \bar{\nu}_e$ are analyzed in the elementary particle approach supplemented by some minimal dynamical assumptions. Predictions are found to be consistent with experiment without the need for substantial meson exchange contributions.

NUCLEAR REACTIONS $^{16}O(\mu, \nu_{\mu})^{16}N^*(0^{-}),$ calculated capture rate; $^{16}N^*(0^{-}),$
calculated beta decay rate.

I. INTRODUCTION

Nuclear β decay and muon capture transitions between $J^P = 0^-$ and $J^P = 0^+$ levels have long been of interest. Originally they were suggested as a probe for the presence of a pseudoscalar coupling interaction in the weak Hamiltonian.¹ At the present time the absence 6f such a pseudoscalar term in the weak interaction is severely restricted by the rather precisely measured branching ratio between the $e^{-}\overline{\nu}_e$ and $\mu^{-}\overline{\nu}_\mu$ decay modes in pion decay.² Later 0^+ -0⁻ experiments were proposed as a measure of the size of the induced pseudoscalar form factor in the nucleon transition³ as a test of the partially conserved axial vector current (PCAC) hypothesis.⁴ Over the last several years evidence for the validity of PCAC in weak nuclear transitions has been obtained via systematic measurements in the β decays of ¹²N, ¹²B and the corresponding muon capture reaction in ${}^{12}C.5$ More recently these $0^{\text{+}}$ -0⁻ transitions have been suggested as a testing ground for "elementary particle" techniques' by Bottino, Ciocchetti, and Kim' and as a possible laboratory for study of meson exchange effects by Guichon, Giffon, and Samour. '

Our purpose in this note is to reanalyze the $0^{\text{+}}$ -0⁻ decay system by elementary particle techniques supplemented by some minimal dynamical assumptions in order to attempt to understand the relationship between recent measurements of the muon capture and β decay rates in the transitions, $9^{9,10}$ tions, 9,10

$$
^{16}\text{N}(0^{-};120 \text{ keV}) \rightarrow ^{16}\text{O} + e^{-} + \overline{\nu}_{e} ,
$$

$$
\mu^{-} + ^{16}\text{O} \rightarrow ^{16}\text{N}(0^{-};120 \text{ keV}) + \nu_{\mu} .
$$

In the next section we define notation and derive

a simple relationship between these transitions. In the final section we analyze this result, try to understand the connection with previous work, and present our conclusions.

II. DERIVATION

We begin with the most general matrix element
for the weak hadronic vertex of a $0^- \rightarrow 0^+$ transitic

$$
\langle 0^*_{\rho_2} | V_{\mu} | 0^-_{\rho_1} \rangle = 0 ,
$$

$$
\langle 0^*_{\rho_2} | A_{\mu} | 0^-_{\rho_1} \rangle = \frac{1}{2M} F_1(q^2) P_{\mu} + \frac{1}{2M} F_2(q^2) q_{\mu} ,
$$
 (1)

$$
P = p_1 + p_2, \quad q = p_1 - p_2 ,
$$

where $M = \frac{1}{2}(M_1 + M_2)$ is the nuclear mass and Δ $=M_1-M_2$ is the mass difference. Here p_1 and p_2 are, respectively, the momenta for parent and daughter nuclei.

In the absence of electromagnetic effects the weak transition amplitude is assumed to be given by

$$
T_0 = \frac{G}{\sqrt{2}} \cos \theta_C \langle 0^{\dagger}_{\ \rho_2} | (V_{\mu} + A_{\mu}) | 0^-_{\ \rho_1} \rangle l^{\mu} , \qquad (2)
$$

where $G\approx 10^{-5}{m_{\rho}}^{-2}$ is the weak coupling constant θ_c is the Cabibbo angle, and

$$
l^{\mu} = \bar{u}(p)\gamma^{\mu}(1+\gamma_{5})v(k)
$$
\n(3)

is the matrix element of the lepton current, where p and k are the respective electron and neutrino momenta. In the presence of electromagnetism Eq. (2) must be modified, and the correct generalization has been given as¹¹

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$$
T = \frac{G}{\sqrt{2}} \cos \theta_{\mathcal{C}} \int d^3 r \, \overline{\psi}_e(\vec{\mathbf{r}}, \vec{\mathbf{p}}) \gamma^{\mu} (1 + \gamma_5) v(k)
$$

$$
\times \int \frac{d^3 s}{(2\pi)^3} e^{i\vec{\mathbf{s}} \cdot \vec{\mathbf{r}}} \frac{1}{2} [\langle 0^{\dagger}_{\vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_1 - \vec{\mathbf{s}}} | (V_{\mu} + A_{\mu}) | 0^{-}_{\vec{\mathbf{p}}_1} \rangle + \langle 0^{\dagger}_{\vec{\mathbf{p}}_2} | (V_{\mu} + A_{\mu}) | 0^{-}_{\vec{\mathbf{p}}_1 - \vec{\mathbf{p}}_1 + \vec{\mathbf{s}}} \rangle],
$$
 (4)

where $\bar{\psi}_s(\vec{r}, \vec{p})$ is the solution to the Dirac equation in the presence of the nuclear Coulomb potential which where $\psi_e(\mathbf{r}, \mathbf{p})$ is the solution to the DIFac equation in the presence of the interest Coulomb potential with reduces to $\overline{u}(p)e^{-i\overline{p}\cdot\overline{r}}$ as $Z \to 0$. The initial and final hadronic states are prescribed to mass shells, and in addition we must make the substitution

 $q_u + q_u + e\phi(r)g_{u0}$, $P_u + P_u + (2Z - 1)e\phi(r)g_{u0}$,

where $\phi(r)$ is the nuclear Coulomb potential, in order to maintain gauge invariance.¹² Now

$$
e\phi(r) \sim \frac{Z\alpha}{R} \sim 4 \text{ MeV}, \text{ for } Z = 8.
$$
 (6)

Thus the gauge substitution is an important modification to q_μ since $q_0 \approx \Delta \approx 11$ MeV for ¹⁶N, ¹⁶O but is a negligible correction to the P_μ term since $P_0 \approx 2M \approx 16$ GeV. We thus omit the P_μ correction in subsequent formulas. Since

$$
(\vec{q} - \vec{p} + \vec{s})e^{i\vec{s} \cdot \vec{r}} = (\vec{k} - i\vec{\nabla})e^{i\vec{s} \cdot \vec{r}},
$$
\n(7)

we can integrate by parts to throw the gradient operator over to the electron wave function. We then have for the term in F_2 ,

$$
\overline{\psi}_e(\overline{\mathbf{r}},\overline{p})[\gamma_0(q_0+e\phi(r))-\overline{\mathbf{r}}\cdot(\overline{k}+i\overline{\nabla})](1+\gamma_5)v(k) = \overline{\psi}_e(\overline{\mathbf{r}},\overline{p})\cancel{(1+\gamma_5)v(k)} + \overline{\psi}_e(\overline{\mathbf{r}},\overline{p})[\gamma_0(E+e\phi(\overline{\mathbf{r}}))-\overline{\gamma}\cdot i\overline{\nabla}](1+\gamma_5)v(k) \n= m_e \overline{\psi}_e(\overline{\mathbf{r}},\overline{p})(1+\gamma_5)v(k),
$$
\n(8)

so that the only significant formal change from the plane wave result,

$$
q^{\mu}\bar{u}(p)\gamma_{\mu}(1+\gamma_{5})v(k)=m_{e}\bar{u}(p)(1+\gamma_{5})v(k), \qquad (9)
$$

.is the replacement of the free particle spinor by the Coulomb wave function. Most of this is accounted for by use of a Fermi function $F(Z, E)$ as a multiplicative factor, where

$$
F_{BJ}(Z,E) = \bar{\psi}_e(0,\vec{p})\psi_e(0,\vec{p})/\bar{u}(p)u(p)
$$
\n(10)

 $F_{BJ}Z$, E) = ψ_e (0, p) ψ_e (0, p)/ u (*p*) u (*p*)
is the Fermi function defined by Behrens and Janecke.¹³ There are some small additional electromagnet corrections which arise from finite nuclear $size^{14}$ and from the standard radiative correction factor $\alpha g(E, E_o)/2\pi$ (Ref. 15) which we take into account but do not explicitly display here. For the β decay pro-

$$
\Gamma_{\beta} = \frac{G^2 \cos^2 \theta_C}{2\pi^3} |F_1(0)|^2 \int_{m_e}^{\Delta} pE(\Delta - E)^2 F_{\beta} (Z, E) dE.
$$
\n(11)

Similarly for the muon capture process the effects of the appropriate generalization of Eq. (4) and the gauge invariance substitution $[Eq. (5)]$ cancel against one another to yield

$$
\Gamma_{\mu} = \frac{G^2 \cos^2 \theta_C}{2\pi^2} \left(\frac{\alpha Z m_{\mu}}{1 + m_{\mu}/M} \right)^3 \frac{E_{\nu}^2}{1 + m_{\mu}/M} C_{\mu} |F_1(0)|^2 \left[\mathcal{F}_1(q^2 = -0.8 m_{\mu}^2) \right]^2 (1 + am_{\mu}/2M)^2 ,
$$
\n
$$
a = \frac{F_2(q^2 = -0.8 m_{\mu}^2)}{F_1(q^2 = -0.8 m_{\mu}^2)}, \quad C_{\mu} = 0.80, \quad E_{\nu} \approx m_{\mu} - \Delta \approx 95 \text{ MeV}, \quad \mathcal{F}_1(q^2) = \left[\frac{F_1(q^2)}{F_1(0)} \right].
$$
\n(12)

Here C_{μ} is a correction factor for the finite size of the nucleus.¹⁶ We have then

$$
\frac{\Gamma_{\mu}}{\Gamma_{\beta}} = \pi C_{\mu} \frac{E_{\nu}^{2}}{1 + m_{\mu}/M} \left(\frac{\alpha Z m_{\mu}}{1 + m_{\mu}/M}\right)^{3} \left[\mathfrak{F}_{1}(q^{2} = -0.8 m_{\mu}^{2})\right]^{2} \left(1 + \frac{m_{\mu}}{2M} a\right)^{2} / \int_{m_{e}}^{\Delta} p E(\Delta - E)^{2} F_{BJ}(Z, E) dE.
$$
 (13)

The standard elementary particle approach estimates $\mathfrak{F}_1(q^2 = -0.8m_\mu^2)$ by using a dipole form

$$
\mathfrak{F}_1(q^2) = \left(1 - \frac{q^2}{M_A^2}\right)^{-2}.\tag{14}
$$

The transition mean square radius

$$
\langle r^2 \rangle = \frac{12}{M_A^2} \tag{15}
$$

depends primarily on the sizes of the nuclei in-

(5)

volved and is quite insensitive in general to other nuclei properties. Using the experimental value

$$
M_A^2 = 2.2 m_\pi^2 \tag{16}
$$

from electron scattering data on 16 O we find

$$
\mathfrak{F}_1(q^2 = -0.8m_\mu^2) = 0.69 \,. \tag{17}
$$

Then

$$
\frac{\Gamma_{\mu}}{\Gamma_{\beta}} = 0.414 \times 10^3 \left(1 + \frac{m_{\mu}}{2M} a \right)^2.
$$
 (18)

From the experimental values^{9,10}

$$
\Gamma_{\beta} = (0.43 \pm 0.10) \, \text{s}^{-1} \,, \tag{19}
$$

$$
\Gamma_{\mu} = 1560 \pm 170 \text{ s}^{-1}, \qquad (22)
$$

we determine

$$
\left(\frac{\Gamma_{\mu}}{\Gamma_{\beta}}\right)^{\exp} = (3.63 \pm 0.93) \times 10^{-3}, \tag{20}
$$

so that

$$
\frac{m_{\mu}}{2M}a^{\exp}=1.96\pm0.37, -3.96\pm0.37. \tag{21}
$$

Thus far our analysis has been purely phenomenological. What is of more interest here are the attempts to understand the situation theoretically. The standard approach is to utilize the nuclear impulse approximation. Defining for the nucleon

$$
\langle p_{p_2} | A_\mu | n_{p_1} \rangle = \overline{u}(p_2) [g_A(q^2) \gamma_\mu \gamma_5 + g_P(q^2) q_\mu \gamma_5] u(p_1)
$$
\n(22)

and temporarily omitting electromagnetic corrections we find

$$
F_1(q^2) = g_A(q^2) \langle \gamma_5 j_0(qr) \rangle
$$

\n
$$
-g_A(q^2) \frac{q_0}{3} \langle i\vec{\sigma} \cdot \hat{r} \frac{3}{q} j_1(qr) \rangle,
$$

\n
$$
F_2(q^2) = g_A(q^2) \langle \gamma_5 j_0(qr) \rangle
$$

\n
$$
+ A \left(\frac{2m_N}{3} g_A(q^2) + \frac{\vec{q}^2}{6m_N} g_P(q^2) \right) \langle i\vec{\sigma} \cdot \hat{r} \frac{3}{q} j_1(qr) \rangle.
$$

\n(23)

Using the PCAC prediction'7

$$
g_P(q^2) = \frac{(2m_N)^2 g_A(q^2)}{q^2 - m_*^2} \tag{24}
$$

and defining

$$
2m_{N}R(q) = \langle \gamma_{5}j_{0}(qr) \rangle / \langle i\vec{\sigma} \cdot \hat{r} \frac{3}{q}j_{1}(qr) \rangle \qquad (25)
$$

we predict

$$
\frac{F_1(q^2)}{F_2(q^2)} = \frac{-q_0/6m_N + R(q)}{\frac{1}{3}A(m_n^2 - \Delta^2)/(m_n^2 - q^2) + R(q)}.
$$
 (26)

In the absence of electromagnetic effects we have

$$
\langle \gamma_{5} j_{0}(qr) \rangle = -\frac{1}{2m_{N}} \langle \left\{-i\vec{\sigma} \cdot \vec{\nabla}, j_{0}(qr) \right\} \rangle \tag{27}
$$

and, ignoring velocity-dependent nuclear forces¹⁸

$$
R(q=0) \approx \frac{\Delta}{2m_N}.
$$
 (28)

Turning on electromagnetism there are two major changes. First, $R(0)$ is modified to become¹⁹

$$
R(0) = \frac{\Delta}{2m_N} + \Lambda \frac{\alpha Z}{4m_N R},
$$
\n(29)

where

$$
\Lambda \simeq \frac{12}{5} + \frac{2R}{\alpha Z} (m_{\rho} - m_{\eta}) \approx 1.7 \,. \tag{30}
$$

In addition q_0 in Eq. (23) is modified by the gauge invariance substitution to become

$$
q_0 + q_0 + \kappa \frac{3}{2} \frac{\alpha Z}{R} , \qquad (31)
$$

where

$$
\kappa = \frac{2R}{3\alpha Z} \frac{\langle i\vec{\sigma} \cdot \vec{\mathbf{r}} e \phi(r) \rangle}{\langle i\vec{\sigma} \cdot \vec{\mathbf{r}} \rangle} \approx 0.7 \,.
$$
 (32)

Then we have

$$
\frac{F_1(q^2)}{F_2(q^2)} \approx \frac{\Delta}{M} \left[1 + \frac{3}{2}\rho + \frac{3\alpha Z}{4\Delta R} \left[\Lambda \left(1 + \rho \right) - \kappa \right] \right] \frac{m_\pi^2 - q^2}{m_\pi^2 - \Delta^2},\tag{33}
$$

where we have defined

$$
\frac{R(q)}{R(0)} \equiv 1 + \rho \tag{34}
$$

This corresponds to the prediction

$$
\frac{m_{\mu}}{2M} a = \frac{m_{\mu}}{2\Delta} \frac{m_{\pi}^{2} - \Delta^{2}}{m_{\pi}^{2} - q^{2}} \left[1 + \frac{3}{2}\rho + \frac{3}{4} \frac{\alpha Z}{\Delta R} (\Lambda (1 + \rho) - \kappa) \right]^{-1}
$$

$$
\approx 3.25/(1.26 + 1.94\rho).
$$
 (35)

Now even if $\rho = 0$, which is the conventional elementary particle approach assumption, we are consistent with experiment at the two standard deviation level. However, there is reason to believe that the situation is somewhat better than this, based on a simple wave function calculation. If one takes the state ¹⁶O as the effective vacuum state, it is to be expected that the shell model will provide a rather accurate and simple description of the state $^{16}N(0^-)$ as resulting from the creation of a hole in the $1p$ proton shell and a particle in the $(2s, 1d)$ neutron shell. Due to the requirement that the hole and particle couple to zero total angular momentum only two such hole-particle con-

figurations can contribute, giving the wave function

$$
|^{16}N(0^-)\rangle = (1-\gamma^2)^{1/2}|1\bar{p}_{1/2}, 2s_{1/2}\rangle + \gamma |1\bar{p}_{3/2}, 1d_{3/2}\rangle .
$$
\n(36)

Since the energy required to effect a $1p_{3/2}$, $1d_{3/2}$ transition. is nearly 10 MeV more than that required for a $1p_{1/2}$, $2s_{1/2}$ transition, we expect that

$$
|\gamma| \ll 1. \tag{37}
$$

For a simple estimate then we assume $\gamma = 0$. Using a simple harmonic oscillator model with oscillator parameter $b = 1.76$ fm we find

$$
\left\langle {}^{16}O\left|i\vec{\sigma}\cdot\hat{r}\frac{3}{q}j_1(qr)\right|{}^{16}N\right\rangle = i\sqrt{2}b\,e^{-x^2/4}(1-\frac{1}{4}x^2)\,,\tag{38}
$$

$$
\frac{1}{2m_N}\langle {}^{16}\text{O}|\{i\bar{\sigma}\cdot\vec{\nabla},j_0(qr)\}\,|{}^{16}\text{N}\rangle = \frac{i\sqrt{2}}{m_Nb}e^{-x^2/4}(1+\frac{1}{12}x^2)\,,
$$

where $x = qb = 0.84$. Then we find in this simple impulse approximation picture

$$
R(0) = \frac{1}{2m_{N}^{2}b^{2}} \approx 7.1 \times 10^{-3},
$$
\n(39)

which agrees reasonably well with the estimate

$$
R(0) = 7.6 \times 10^{-3}
$$
 (40)

obtained from Eq. (29) with $\Lambda = 1.7$. However, a significant difference from our earlier assumptions is that, whereas we assumed

$$
R(q) = R(0) \tag{41}
$$

our impulse approximation result yields

$$
R(q) = R(0) \frac{1 + \frac{1}{12}x^2}{1 - \frac{1}{4}x^2} \approx 1.29R(0) \ . \tag{42}
$$

Substitution into Eq. (35) yields then

$$
\frac{m_{\mu}}{2M}a = 1.79,\t(43)
$$

which is in very good agreement with the experimental value

$$
\frac{m_{\mu}}{2M}a^{\exp}=1.96\pm0.37\,.
$$
 (44)

Although we have used the impulse approximation with the simple wave function of equation 36 in evaluating the ratio $R(q^2)/R(0)$, use of such a simple model may not be appropriate for calculating the absolute q^2 dependence of $F_1(q^2)$; the muoncapture rate depends sensitively on the q^2 -dependence of $F_1(q^2)$ but not on the ratio $R(q^2)/R(0)$. The prediction of $\mathfrak{F}_1(q^2)$ in this naive picture is²⁰

$$
[\mathfrak{F}_1(q^2)]^2 = e^{-x^2/2} \left[1 + \frac{1}{12} x^2 \frac{1+\tau}{1-\frac{1}{3}\tau} \right]^2, \tag{45}
$$

where

$$
\tau = \left(\Delta + \kappa \frac{3}{2} \frac{\alpha Z}{R}\right) m_N b^2 \approx 1.12.
$$

Numerically

$$
[\mathfrak{F}_1(q^2)]^2 \approx 1.01 \tag{46}
$$

which is twice as large as the "empirical" value of equation 17. This suggests that for a careful analysis of the problem, more realistic wave functions with the strong ground state correlation in O^{16} be used. Such realistic models²¹ indeed reduce the q^2 -dependence of $F_1(q^2)$ significantly so as to reproduce the observed q^2 dependence of E, $F_1(q^2)$ in the A = 16 system, in particular 2⁻ -0⁺
and 0⁻ -0⁺ transitions.²² and $0^{-}+0^{+}$ transitions.²²

III. CONCLUSIONS

Similar analyses of this 0^- - 0^+ system have been undertaken previously by Bottino, Ciocchetti, and Kim (BCK), by Guichon, Giffon, and Samour (GGS), Kim (BCK), by Guichon, Giffon, and Samour (
and by Donnelly and Walecka (DW).²³ GGS have concluded that a substantial meson exchange amplitude is needed in order to produce experimental agreement, while DW claim consistency with experiment within the context of the impulse approximation. However, the conclusion of DW was based on a value of Γ_u which differs by a factor of 2 from the presently accepted value, and thus this analysis should be reexamined. Our techniques are similar to those of BCK. However, our use of the gauge invariance substitution and Coulomb wave function relation in Eq. (8) is a simplification and improvement over the Feynman diagram techniques employed in Ref. (7} and gives a correct Γ_{β} (without large Coulomb correction). Another significant difference from BCK is our relation (30) between reduced matrix elements. The previous work omitted the term in the nuclear mass difference, as is appropriate for typical high Z transitions wherein the Coulomb energy term dominates, but this is certainly incorrect for the $^{16}N-^{16}O$ case.²⁴ Hence the conclusions reached by BCK must be superceded by the present work.

Finally, we point out that the conclusion obtained by GGS was based on the use of the simple harmonic oscillator model without the ground state correlation. A recent calculation by Koshigiri, Ohtsubo and Morita²² (KOM) demonstrates that rate formula including forbidden matrix elements indeed reduces the ratio $(\Gamma_{\mu}/\Gamma_{\beta})$ obtained by GGS by a factor of two, leading to the conclusion that there is no need for a large meson exhange com-

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ponent. This conclusion is the same as ours.

In conclusion then we obtain good agreement with the experimental β decay and muon capture results on the $A = 16$ nuclei using only simple impulse approximation calculations. There is no need for either sizable meson exchange currents or a substantial renormalization of the induced pseudo-

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scalar coupling constant either upwards or downwards. Of course, a conspiratorial combination of these two effects cannot be ruled out.

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- ²⁴Note for $Z=8$, $\lambda \alpha Z/2R \sim \Delta/3$.