Energy dependence of the absorptive S-wave pion-nucleus optical potential

J. Chai and D. O. Riska

Department of Physics and Cyclotron Laboratory, Michigan State University, East Lansing, Michigan 48824

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It is shown that the simplest two-body pion rescattering model for pion absorption in nuclei leads to a rapid increase with pion energy for the absorptive part of the S-wave pion-nucleus optical potential proportional to ρ^2. The energy dependence is almost entirely due to the pion-nucleon rescattering amplitude and not to nuclear structure effects.

[NUCLEAR REACTIONS Absorptive part of S-wave pion nucleus optical potential calculated as a function of pion energy.]

I. INTRODUCTION

As pions cannot be absorbed on free nucleons nuclear-pion absorption involves at least a two-nucleon mechanism. This may either be viewed as absorption on one single but bound nucleon or as an explicit two-nucleon absorption mechanism. To reflect this situation the absorptive part of the S-wave pion-nucleus optical potential is commonly parametrized in a form which emphasizes the two-body aspect of the problem as[1]

$$2\omega U_{\rm opt} = -4\pi \mathrm{Im}B_0 \rho^2. \tag{1}$$

Here ω is the pion energy, ρ the nuclear density and $\mathrm{Im}B_0$ a potential parameter.

At threshold the value for $\mathrm{Im}B_0$ has been determined from the level widths of pionic atoms to be 0.042 μ^{-4}.[2] There has been some recent progress in attempting to understand this value of $\mathrm{Im}B_0$ in terms of elementary two-body rescattering mechanisms.[3,4] Hachenberg and Pirner[3] have considered a very sophisticated model for the rescattering operator, and Bertsch and Riska[4] have employed the simplest phenomenological model for the rescattering operator which fits the low energy pion-nucleon phase shifts. Both groups obtained very similar results for the threshold value of $\mathrm{Im}B_0$: 0.036 μ^{-4} (Ref. 3) vs 0.031 μ^{-4} (Ref. 4). The remaining 20–30 % discrepancy between the calculated threshold values for $\mathrm{Im}B_0$ and that extracted from pionic atom level widths may possibly be attributed to missing many-body absorption mechanisms which have not been taken into account in the parametrization of the optical potential. In addition one-nucleon absorption viewed as absorption on one nucleon bound in pair wavefunction could be of some importance.[1]

In this paper we address the question of the energy dependence of the optical model parameter $\mathrm{Im}B_0$ above threshold. The fact that recent phenomenological optical model fits to pion-nucleus scattering data seem to favor a rather marked increase of $\mathrm{Im}B_0$ with pion energy[5] makes this question particularly interesting. We consider the same two-body absorption operator that was considered recently by Bertsch and Riska[4] and much earlier by Woodruff[6] and Koltun and Reitan.[7] Since this operator can explain 70–80 % of the threshold pion absorption rates in nuclei we assume that it will also be adequate to the same extent at higher energies. The main development here therefore consists in the generalization of the calculational method of Ref. 4 to nonzero pion momenta.

The paper is divided into four sections. The second section is devoted to a brief description of the method employed and the model for the absorption operator. In the third section the matrix elements of the rescattering operator are evaluated in the Fermi gas model. The numerical results and a concluding discussion are presented in the last section.

II. THEORETICAL APPROACH

The lowest order contribution to the imaginary part of the pion-nucleus optical potential can be obtained from the multiple scattering expansion to have the form

$$\mathrm{Im}U_{\rm opt} = -\frac{\pi}{\Omega}\sum_f \delta(E_f - E_i - \omega)\,|T_{fi}|^2. \tag{2}$$

Here T_{fi} is the matrix element of the pion absorption operator between the initial (i) and final nuclear state (f) and the sum is taken over all final states allowed by energy conservation. We denote the energies of the nuclear states E_i and

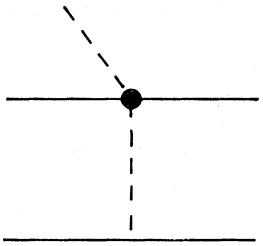

FIG. 1. Pion rescattering model for two-body pion absorption.

E_f and the nuclear volume Ω.

We now make the assumptions that the absorption operator T is a two-body operator and that the independent particle model is adequate for the description of the nuclear wave functions. With the definition for the optical potential above these assumptions then lead to the following expression for the potential parameter $\mathrm{Im}B_0$ defined in Eq. (1) (Ref. 4):

$$\mathrm{Im}B_0=\frac{\omega}{2\rho^2\Omega}\sum_{f,i}\left|\int d(1)d(2)\psi_f^*(12)T(12)\psi_i(12)\right|^2 \times\delta(E_f-E_i-\omega). \tag{3}$$

In this expression ψ_i and ψ_f are the antisymmetric pair wave functions in the initial and final nuclear states with the sum taken over all of them. The coordinates (1) and (2) represent the spatial and spin-isospin coordinates of the pair nucleons.

We shall write the absorption operator T as a function of the fractional momenta k_1 and k_2 delivered to the pair nucleons:

$$T_{fi}=\int d^3r_1 d^3r_2\psi_f^*(\vec{r}_1,\vec{r}_2)\int\frac{d^3k_1}{(2\pi)^3}\int\frac{d^3k_2}{(2\pi)^3}e^{i\vec{k}_1\cdot\vec{r}_1+i\vec{k}_2\cdot\vec{r}_2}(2\pi)^3\delta(\vec{q}-\vec{k}_1-\vec{k}_2)T(\vec{k}_1,\vec{k}_2)\psi_i(\vec{r}_1,\vec{r}_2). \tag{4}$$

The δ function in Eq. (4) expresses overall momentum conservation, $\vec{q}$ being the momentum of the absorbed pion.

We assume that pion absorption takes place by one rescattering of the initial pion as illustrated in Fig. 1. It is important to point out that there is no possibility of double counting between the first iteration of this mechanism and the third order iteration of the one-nucleon optical potential (with an S-wave term and a P-wave Born term) since the iteration in the Schrödinger equation of the one-nucleon potential leaves the nucleus in the ground state while the absorption mechanism in Fig. 1 leads to highly excited states.

We describe the S-wave pion-nucleon rescattering vertex in Fig. 1 by the phenomenological zero range πN Hamiltonian employed by Woodruff[6] and recently by Bertsch and Riska[4]:

$$H=4\pi\frac{\lambda_1}{\mu}\bar{\psi}\phi\cdot\phi\psi+4\pi\frac{\lambda_2}{\mu^2}\bar{\psi}\tau\cdot\phi\times\pi\psi. \tag{5}$$

Here ψ and $\bar{\psi}$ are the nucleon field operators and ϕ the isovector pion field operator. The momentum operator conjugate to ϕ is denoted π. In (5) τ is the nucleon isospin operator and μ the pion mass. The two coupling constants λ are determined by a fit to the pion-nucleon S-wave scattering lengths to be[8]

$$\lambda_1=0.003\pm0.001, \tag{6}$$
$$\lambda_2=0.050\pm0.001.$$

With these values for the coupling constants λ the isospin $\frac{3}{2}$ S-wave pion-nucleon phase shift S_{31} is well reproduced to 250 MeV and the isospin $\frac{1}{2}$ phase shift S_{11} to 50 MeV pion kinetic energy in the laboratory system. The fit to the S_{11} phase shift can be improved at higher energies by allowing the λ's to be energy dependent as discussed in Sec. IV.

The absorption operator that corresponds to the Feynman diagram in Fig. 1 can now be constructed, and is for the case of a charged (here positive) pion found to be

$$T=-i\frac{8\pi}{\mu}f\frac{1}{\sqrt{\omega}}\frac{\vec{\sigma}^2\cdot\vec{k}_2}{\mu^2+k_2^2-\omega'^2}\times\left[\frac{\lambda_1}{\mu}\tau^2_+-i\frac{\lambda_2}{2\mu^2}(\omega+\omega')(\tau^1\times\tau^2)_+\right]. \tag{7}$$

Here $\vec{k}_2$ is the momentum delivered to the second nucleon by the virtual pion and ω' is the energy carried by the virtual pion. The energy of the initial pion is $\omega=(q^2+\mu^2)^{1/2}$. We shall denote the "effective" mass in the pion propagator in (7) $\mu^*\equiv(\mu^2-\omega'^2)^{1/2}$. In (7) f is the pseudovector coupling constant ($f^2/4\pi=0.08$). To this operator one should add the operator which corresponds to the situation where the rescattering takes place on the second rather than the first nucleon.

The Hamiltonian (5) is derived for the pion-nucleon center-of-mass frame. When constructing the contribution to the first order optical potential due to the S-wave pion-nucleon rescattering amplitude, the transformation to the pion-nucleus center-of-mass frame[9,10] introduces a multiplicative factor $\xi=(1+\omega/2m)/(1+\omega/2Am)$ in the optical potential[5] (m is the nucleon mass). For

consistency with the first order optical potential, we shall therefore also include the factor ζ with the amplitude (7). This corresponds to the choice of Miller[11] for the second order P-wave optical potential.

The transformation from the pion-nucleon center-of-mass frame to the pion-nucleus one also introduces a contribution of proportional $\nabla^2\rho$ in the S-wave first order optical potential which is due to P-wave pion-nucleon scattering. It is therefore natural that the second order P-wave rescattering mechanisms should also contribute a similar term to the second order (ρ^2) S-wave optical potential. We do not consider that term here, although at large pion energies it is far from unimportant, as it is not usually incorporated into the S-wave potential parameter B_0 but rather kept as a separate term in the optical potential because of its different origin.[5] In fact, it would be inconsistent to include such a term without including the P-wave optical potential simultaneously. The present results thus only relate to the ρ^2 term in the optical potential, but not to terms of the type $\nabla^2\rho$.

By carrying out the momentum integrals in Eq. (4) the configuration space expression of the symmetrized operator can be cast in the isospin-separated form

$$T(12) = t^1(\underset{\sim}{\tau}^1 + \underset{\sim}{\tau}^2)_+ + t^2(\underset{\sim}{\tau}^1 - \underset{\sim}{\tau}^2)_+ - it^3(\underset{\sim}{\tau}^1 \times \underset{\sim}{\tau}^2)_+ . \tag{8}$$

The amplitudes t are given as

$$t^1 = \frac{f}{\mu^2}\lambda_1\frac{\mu^{*2}}{\sqrt{\omega}}Y_1(\mu^* r)e^{i\vec{q}\cdot\vec{R}}[\cos(\vec{q}\cdot\vec{r}/2)(\vec{\sigma}^1 - \vec{\sigma}^2)\cdot\hat{r} - i\sin(\vec{q}\cdot\vec{r}/2)(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r}], \tag{9a}$$

$$t^2 = \frac{f}{\mu^2}\lambda_1\frac{\mu^{*2}}{\sqrt{\omega}}Y_1(\mu^* r)e^{i\vec{q}\cdot\vec{R}}[\cos(\vec{q}\cdot\vec{r}/2)(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r} - i\sin(\vec{q}\cdot\vec{r}/2)(\vec{\sigma}^1 - \vec{\sigma}^2)\cdot\hat{r}], \tag{9b}$$

$$t^3 = -\frac{f}{\mu^3}\lambda_2\frac{\mu^{*2}}{\sqrt{\omega}}Y_1(\mu^* r)(\omega + \omega')e^{i\vec{q}\cdot\vec{R}}[\cos(\vec{q}\cdot\vec{r}/2)(\vec{\sigma}^1 + \vec{\sigma}^2)\cdot\hat{r} - i\sin(\vec{q}\cdot\vec{r}/2)(\vec{\sigma}^1 - \vec{\sigma}^2)\cdot\hat{r}]. \tag{9c}$$

The function Y_1 has been taken to have the form

$$Y_1(x) = \left(1 + \frac{1}{x}\right)\frac{e^{-x}}{x}. \tag{10}$$

In Eqs. (9) the coordinate vector $\vec{r}$ is defined as $\vec{r}_1 - \vec{r}_2$ and the vector $\vec{R}$ as $\frac{1}{2}(\vec{r}_1 + \vec{r}_2)$. In the threshold limit $\vec{q}\to 0$ these expressions reduce to those in Ref. 4.

True S-wave absorption corresponds to the lowest order term in $\vec{q}\cdot\vec{r}$ in Eqs. (9). The higher order terms in $\vec{q}\cdot\vec{r}$ dependence lead to contributions to higher partial wave terms in the optical potential.

III. EXPLICIT EXPRESSIONS FOR THE MATRIX ELEMENTS

In this section we derive an explicit expression for the potential parameter $\mathrm{Im}B_0$ in symmetric nuclear matter starting from Eq. (3). It is convenient to evaluate the isospin matrix elements first using the separation (8). Since the operator $T(12)$ is symmetric in the coordinates of the two nucleons and since the final state energy does not depend on isospin, one may use closure for the isospin components of the final pair wave functions to obtain the result

$$\mathrm{Im}B_0 = \frac{\omega}{2\rho^2\Omega}\zeta^2\sum_{f,i}\delta(E_f - E_i - \omega) \times\left(2|t_{fi}^2 - t_{fi}^3|_{(+)}^2 + 4|t_{fi}^2|_{(-)}^2 + 2|t_{fi}^2 + t_{fi}^3|_{(-)}^2\right). \tag{11}$$

The matrix elements t_{fi} in (11) now involve the spin and spatial parts of the wave functions only. The subscripts $(+)$, $(-)$ in (11) indicate the symmetry of the initial spin-spatial pair wavefunction.

The sums over the spin parts of the pair wave functions can be performed by the same method as the isospin sums. For this purpose we write the spin dependence of the amplitudes explicitly as

$$t^i = \vec{U}^i\cdot(\sigma^1 + \sigma^2) + \vec{V}^i\cdot(\sigma^1 - \sigma^2). \tag{12}$$

The expressions for the vector operators $\vec{U}$ and $\vec{V}$ are then according to Eqs. (9)

$$\begin{aligned}
&\vec{U}_{fi}^1 = \vec{V}_{fi}^2 = 0,\\
&\vec{V}_{fi}^1 = \frac{f}{\mu^2}\lambda_1\frac{\mu^{*2}}{\sqrt{\omega}}\langle f|e^{i\vec{q}\cdot\vec{R}}Y_1(\mu^* r)j_0(\tfrac{1}{2}qr)\hat{r}|i\rangle,\\
&\vec{U}_{fi}^2 = \vec{V}_{fi}^1,\\
&\vec{U}_{fi}^3 = -\frac{f}{\mu^3}\lambda_2\frac{\mu^{*2}}{\sqrt{\omega}}(\omega + \omega')\langle f|e^{i\vec{q}\cdot\vec{R}}Y_1(\mu^* r)j_0(\tfrac{1}{2}qr)\hat{r}|i\rangle,\\
&\vec{V}_{fi}^3 = 0.
\end{aligned} \tag{13}$$

Here we have only retained the S-wave contribution. After carrying out the sums over the spin components of the initial and final pair wave functions we eventually find the result

$$\mathrm{Im}B_0=\frac{\omega}{2\pi\rho^2\Omega}\,16\,\xi^2\sum_{f,i}\delta(E_f-E_i-\omega)\{|\vec{V}^1_{fi}|^2_{(-)}+2\,|\vec{U}^1_{fi}|^2_{(-)}+|\vec{U}^2_{fi}+\vec{U}^3_{fi}|^2_{(-)}+\tfrac{1}{2}|\vec{V}_{fi}{}^2+\vec{V}_{fi}{}^3|^2_{(-)}$$
$$+|\vec{V}^1_{fi}|^2_{(+)}+\tfrac{1}{2}|\vec{V}^2_{fi}+\vec{V}^3_{fi}|^2_{(+)}+|\vec{U}^2_{fi}-\vec{U}^3_{fi}|^2_{(+)}+\tfrac{1}{2}|\vec{V}^2_{fi}-\vec{V}^3_{fi}|^2_{(+)}+\tfrac{1}{2}|\vec{V}^2_{fi}-\vec{V}^3_{fi}|^2_{(-)}\}. \quad (14)$$

Here the subscripts (+) and (−) indicate the symmetry of the initial spatial pair wave function.

For the evaluation of the matrix elements of the vector operators $\vec{U}$ and $\vec{V}$ in (14) we use Fermi gas wave functions to construct the pair wave functions

$$\psi_{mn}(\vec{r}_1,\vec{r}_2)=\frac{1}{\Omega\sqrt{2}}e^{i\vec{K}_{mn}\cdot\vec{R}}(e^{i\vec{k}_{mn}\cdot\vec{r}}\pm e^{-i\vec{k}_{mn}\cdot\vec{r}}). \quad (15)$$

Here the symmetry of the spatial pair wave function has been exhibited explicitly. The total and relative momentum variables $\vec{K}$ and $\vec{k}$ are defined as $\vec{K}_{mn}=\vec{k}_m+\vec{k}_n$ and $\vec{k}_{mn}=\frac{1}{2}(\vec{k}_m-\vec{k}_n)$, and the corresponding spatial variables as $\vec{R}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$, $\vec{r}=(\vec{r}_1-\vec{r}_2)$. The pair wave function is labeled by the momentum indices m and n.

The matrix elements in Eq. (14) are of the type

$$\vec{V}=\int d^3r_1 d^3r_2\psi^*_{mn}e^{i\vec{q}\cdot\vec{R}}Y_1(\mu^*r)j_0(\tfrac{1}{2}qr)\hat{r}\psi_{kl}. \quad (16)$$

The initial pair wave function ψ_{kl} involves nucleons in occupied orbits and the final wave function ψ_{mn} nucleons in vacant ones. We shall use henceforth the abbreviation $f=(mn)$ and $i=(kl)$.

It is convenient to perform a multipole expansion of the pair wave functions in the integral. This leads to the expression

$$\vec{V}=\frac{2(4\pi)^2}{\Omega}\sum_{L'M'}\sum_{LM}(-i)^{L'}i^L Y_{L'M'}(\hat{k}_f)Y_{LM}(\hat{k}_i)\int d^3r_1 d^3r_2 e^{-i\vec{K}_f\cdot\vec{R}}e^{i\vec{q}\cdot\vec{R}}e^{i\vec{K}\cdot\vec{R}}$$
$$\times j_{L'}(k_f r)j_0(qr/2)j_L(k_i r)\hat{r}Y^*_{L'M'}(\hat{r})Y_{LM}(\hat{r}). \quad (17)$$

The sum over L involves only even or odd terms depending on whether the initial pair wave function is symmetric or antisymmetric.

In carrying out the radial integrals in (17) it is convenient to separate the integrations into one over the center-of-mass variable $\vec{R}$ and one over the relative variable $\vec{r}$. The center-of-mass integral leads simply to a Kronecker delta $\delta_{\vec{K}_f,\vec{K}_i+\vec{q}}$ which expresses momentum conservation. After evaluating the integral over the relative coordinate using standard methods we obtain the following result for the μth spherical component of the matrix element $\vec{V}$:

$$V_\mu=\frac{2(4\pi)^2}{\Omega}\delta_{\vec{K}_f,\vec{K}_i+\vec{q}}\sum_{L'M'}\sum_{LM}(-i)^{L'}i^L(-1)^\mu([L'][L])^{1/2}\begin{pmatrix}L'L1\\0\,0\,0\end{pmatrix}\begin{pmatrix}L'L1\\-M'M\mu\end{pmatrix}Y_{L'M'}(\hat{k}_f)Y_{LM}(\hat{k}_i)I_{LL'}(k_f,k_i,q) \quad . \quad (18)$$

Here the radial matrix element $I_{LL'}$ has been defined to be

$$I_{LL'}(k_f,k_i,q)=\int dr\,r^2 j_{L'}(k_f r)j_0(\tfrac{1}{2}qr)\times Y_1(\mu^*r)j_L(k_i r). \quad (19)$$

In (18) we have used the notation $[L]\equiv 2L+1$.

In the expression for the optical potential parameter (14) one needs the square of matrix elements of the type (18) summed over the momentum variables with account of the energy conservation δ function. The δ function can be written in terms of the Fermi gas momenta as

$$\delta(E_f-E_i-\omega)=\delta\left(\frac{1}{4m}(\vec{K}_f{}^2-\vec{K}_i{}^2)+\frac{1}{m}(\vec{k}_f{}^2-\vec{k}_i{}^2)-\omega\right), \quad (20)$$

where of course $\vec{K}_f=\vec{K}_i+\vec{q}$. The sum over momenta is complicated by the appearance of a term $\vec{q}\cdot\vec{K}_i$ in the δ function as a consequence of momentum conservation. To be able to proceed we therefore assume that it is a reasonable approximation to replace that term by its angle average value 0.

With this approximation the sum over initial momenta (i) in the expression (14) may be immediately replaced by integrals as[4]

$$\sum_i=\tfrac{1}{32}\Omega^2\rho^2\int_0^1 d\left(\frac{k_i}{k_F}\right)P\left(\frac{k_i}{k_F}\right)\int\frac{d\hat{k}_i}{4\pi}, \quad (21)$$

with ρ being the nuclear density and k_F the Fermi momentum. The function P is a normalized finite geometry weight function:

$$P(x)=24(1-\tfrac{3}{2}x+\tfrac{1}{2}x^3)x^2. \quad (22)$$

The sum over final pair wave functions can be converted to an integral as well:

$$\sum_f \delta(E_f - E_i - \omega) \simeq \tfrac{1}{2}\Omega \int \frac{d^3k_f}{(2\pi)^3}\,\delta\left(\frac{q^2}{4m} + \frac{1}{m}(k_f^2 - k_i^2) - \omega\right)$$

$$= \frac{mk_F x'\Omega}{4(2\pi)^3}\int d\hat{k}_f\,. \tag{23}$$

here x' is the final relative momentum divided by k_F:

$$x' = [x^2 + \omega m(1 - q^2/4\omega m)/k_F^2]^{1/2}. \tag{24}$$

The variable x is defined as $x = k_i/k_F$.

Using (21) and (22) to evaluate the momentum sums and exploiting the orthonormality and completeness of the spherical harmonics we find for the summed squares of the vector operator matrix elements (12) the expression

$$\sum_{f,i} \delta(E_f - E_i - \omega)\,|\vec{\mathcal{V}}|^2 \simeq \tfrac{1}{4}\rho^2 m k_F \Omega \sum_{LL'} J_{LL'}. \tag{25}$$

The quantity $J_{LL'}$ has been defined as

$$J_{LL'} = [L][L']\int_0^1 dx\, F(x) x' I_{LL'}^2(x'k_F, xk_F, q) \times \begin{pmatrix} L' L 1 \\ 0\ 0\ 0 \end{pmatrix}^2. \tag{26}$$

By now substituting the result (25) for the vector operator matrix elements into (14) we obtain the final result for the optical potential parameter $\mathrm{Im}B_0$ as

$$\mathrm{Im}B_0 = 2f^2\left(\frac{\mu^*}{\mu}\right)^4 m k_F \xi^2 \left\{ \sum_L{}^{+}\sum_{L'} \left[\lambda_1^2 + \left(\lambda_1 + \lambda_2 \frac{\omega+\omega'}{\mu}\right)^2\right] J_{LL'} + \sum_L{}^{-}\sum_{L'} \left[\lambda_1^2 + \left(\lambda_1 - \lambda_2 \frac{\omega+\omega'}{\mu}\right)^2\right] J_{LL'} \right\}. \tag{27}$$

Here the superscripts + and − on the summation symbols indicate whether even or odd values of the summation variable are to be included.

The choice of the value ω' for the energy transfer of the rescattered virtual pion is not completely obvious. Considering the process depicted in the Feynman diagram in Fig. 1 one has

$$\omega' = E_2' - E_2 = \frac{k_2'^2}{2m} - \frac{k_2^2}{2m}, \tag{28}$$

with k_2, k_2' being the initial and final momenta of nucleon 2. If one assumes that the momentum of the knocked-out nucleon is much larger than that of the initial one, $|\vec{k}_2'| \gg |\vec{k}_2|$, momentum conservation at the vertex leads to $\omega' = k^2/2m$ with $\vec{k}$ being the momentum of the virtual pion. This prescription was used in the recent study of the reaction $\pi^+ d \to pp$ in Ref. 8 to argue that the energy transfer should be small compared to the pion mass.

On the other hand, one can also express ω' in terms of the energy of the initial pion as

$$\omega' = \frac{\omega}{2} + \frac{1}{2m}(\vec{K}_f\cdot\vec{k}_f - \vec{K}_i\cdot\vec{k}_i). \tag{29}$$

The most common ansatz for ω' is the one obtained from (29) by neglecting the Fermi motion terms and keeping only $\omega/2$. Setting $\omega' = \omega/2$ leads to a singularity in the pion propagator when $q = \sqrt{3}\,\mu$,[8] which is an artificial consequence of neglecting Fermi motion and not related to any physical threshold and thus undesirable. This ansatz does, however, give a much larger value for $\mathrm{Im}B_0$ at threshold than the other one (28) and results for the imaginary parts of the pion-nucleus scattering lengths which are in better agreement with empirically obtained values.[4] We shall therefore use this latter value here, i.e., we set $\omega' = \omega/2$.

When setting $\omega' = \omega/2$ the form of the integral $I_{LL'}$ defined in Eq. (19) changes to

$$\tilde{I}_{LL'}(k_f, k_i, q) = \int dr\, r^2 j_{L'}(k_f r) j_0(\tfrac{1}{2}qr) \times [ij_1(\kappa r) - y_1(\kappa r)] j_L(k_i r) \tag{30}$$

with $\kappa = (\omega'^2 - \mu^2)^{1/2}$ being the effective pion mass. In the expression (26) for the integral $J_{LL'}$ the square of the radial integral has to be replaced by

$$I_{LL'}^2 \to (\mathrm{Re}\tilde{I}_{LL'})^2 + (\mathrm{Im}\tilde{I}_{LL'})^2. \tag{31}$$

Otherwise the results (27) are unmodified.

IV. RESULTS AND CONCLUSIONS

We have calculated the values for the optical potential parameter $\mathrm{Im}B_0$ from Eq. (27) as a function of pion momentum and show the results in Fig. 2. As can be seen $\mathrm{Im}B_0$ is a rapidly increasing function of pion momentum (or energy). At threshold our value 0.029 μ^{-4} is somewhat smaller than the value 0.031 μ^{-4} reported in Ref. 4, a difference we explain by our use of a more accurate numerical integration method. The threshold value is thus ~35% smaller than the standard value 0.042 μ^{-4} obtained from the level widths of pionic atoms,[1] but that value also has

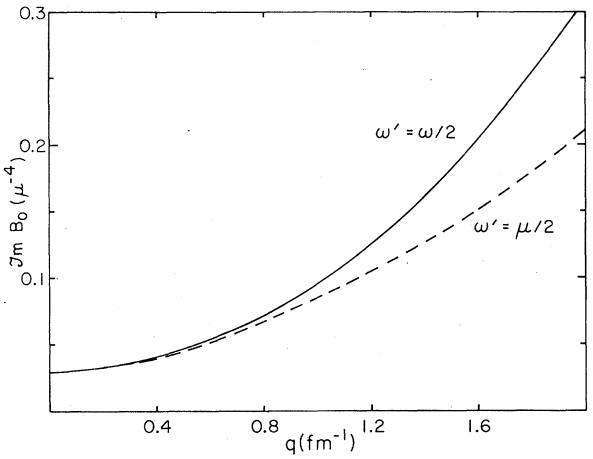

FIG. 2. The optical potential parameter $\mathrm{Im}B_0$ as a function of pion momentum. The solid curve is obtained with $\omega' = \omega/2$ and the dashed curve with $\omega' = \sqrt{3}\mu/2$ (i.e., threshold value).

an uncertainty of ~20%. The remaining discrepancy is probably due to one-body and/or many-body absorption mechanisms not considered here. The increase with pion energy that we find for $\mathrm{Im}B_0$ is, however, strong enough to make $\mathrm{Im}B_0 \simeq 0.040$ already at 25 MeV, an energy where the zero-range pion-nucleon interaction (5) still gives adequate values for the S-wave pion-nucleon phase shifts.

It is interesting to note that the result for $\mathrm{Im}B_0$ is rather insensitive to the choice of the value for the energy transfer ω' of the virtual pion except at large energies. The solid curve in Fig. 2 for $\mathrm{Im}B_0$ was obtained with the standard prescription $\omega' = \omega/2$. On the other hand, the dashed curve was obtained when keeping the value of ω' at the threshold value $\mu/2$ with $\mu^* = \sqrt{3}\mu/2$. While the matrix elements $J_{LL'}$ are indeed rapidly varying with the value of ω', this variation is almost completely compensated for by the change in the value for the factor $(\mu^*)^4$ in front of the expression (27). The net result is the relative insensitivity to the value for ω'. This insensitivity, noted already in Ref. 8, is fortunate since it shows that the artificial singularity of the pion propagator at $q = \sqrt{3}\mu$ obtained when setting $\omega' = \omega/2$ is of little practical importance.

If one, on the other hand, sets $\omega' = 0$ and employs the static pion propagator as was done in Ref. 8 the results for $\mathrm{Im}B_0$ turn out to be $\simeq 20\%$ smaller than those obtained with $\mu^* = \sqrt{3}\mu/2$. Since the value for $\mathrm{Im}B_0$ is anyway somewhat too small this method would make the discrepancy between the calculated and phenomenologically obtained value for $\mathrm{Im}B_0$ worse. The results for the cross section for the reaction $\pi^+ d \to pp$ reported in Ref. 8 do indeed indicate that the cross section is somewhat too small near threshold which may be a reflection of this fact.

Only a small number of the functions $J_{LL'}$ in Eq. (27) contribute appreciable amounts to the value of $\mathrm{Im}B_0$, as most of these integrals are very small. In fact the integral J_{01}, i.e., one of the functions involving an initial S-state pair wave function contributes of the order of 90% and the integral J_{12}, which involves an initial P-state wave function contributes of the order 5% of the value of $\mathrm{Im}B_0$. In Table I we list the contributions to $\mathrm{Im}B_0$ of the most important integrals $J_{LL'}$. The other integrals which are not listed in Table I can actually be neglected.

In order to illustrate the remarkable cancellation of the energy dependence of the integrals $J_{LL'}$ by the factor μ^{*4} in the expression (27) for $\mathrm{Im}B_0$ we plot in Fig. 3 the largest integral J_{01} as a function of pion kinetic energy and the relevant product $\mu^{*4} J_{01}$. Despite the singularity in J_{01} at $q = \sqrt{3}\mu$, the product of the two factors is essentially independent of energy. This result leads to the important conclusion that the resulting energy dependence in $\mathrm{Im}B_0$ is due to the explicit pion energy factors which are associated with the terms involving λ_2 in Eq. (27). The nuclear structure part of $\mathrm{Im}B_0$ is almost independent of energy.

The results above have all been obtained with Fermi gas pair wave functions without inclusion of short range correlations. We justify the neglect of short range correlations by the observation made in Refs. 4 and 12 that the threshold absorption rate is insensitive to such correlations because of the relatively long range of the two-body absorption operator. In Ref. 4 it was demonstrated that while the repulsive nucleon-nucleon interaction in the P waves does reduce the matrix elements considered somewhat, the attractive tensor correlations more or less compensate this reduction leaving the net change in $\mathrm{Im}B_0$ small. Although one might suppose that the importance of short range correlations could grow with increased pion momentum, this turns

TABLE I. Values contributed to $\mathrm{Im}B_0$ by the most important integrals $J_{LL'}$ (units μ^{-4}).

q (fm^{-1})	J_{01}	J_{12}	$\mathrm{Im}B_0$ (tot)
0.0	0.248−1	0.122−2	0.29−1
0.4	0.335−1	0.167−2	0.39−1
0.8	0.582−1	0.349−2	0.71−1
1.2	0.953−1	0.677−2	0.125
1.6	0.140	0.114−1	0.206
2.0	0.191	0.146−1	0.312

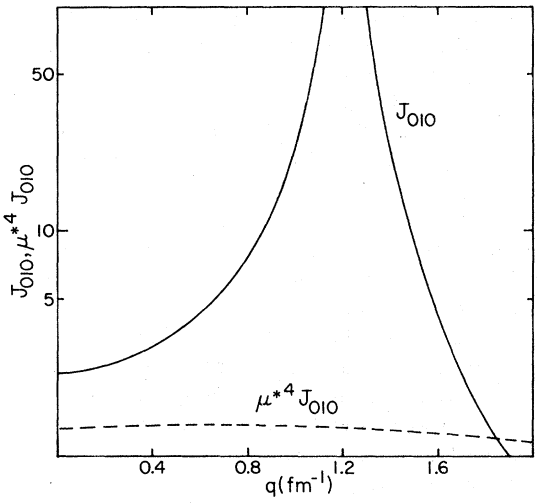

FIG. 3. The dominant integral J_{01} and the combination $\mu^{*4} J_{01}$ as functions of pion momentum.

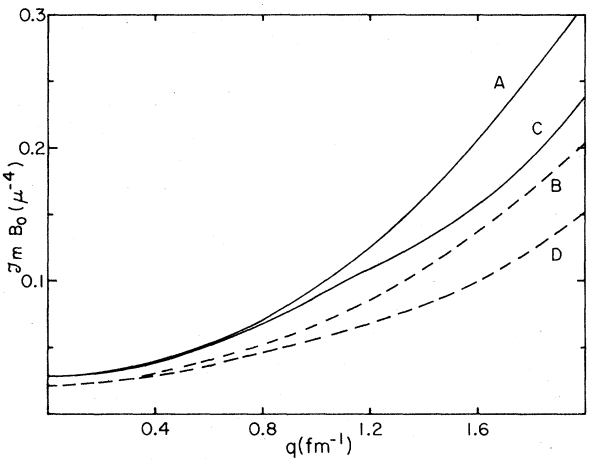

FIG. 4. The values for $\mathrm{Im}B_o$ as calculated without (*A*) and with inclusion of the vertex form factor effect (*B*). Curves *C* and *D* are the corresponding results when the energy dependence of the coupling parameters λ is taken into account.

out not to be so because of the rapid increase of the range of the absorption operator with pion energy.

Another modification to the present results rather similar in consequences to that caused by short range correlations would be the effect of hadronic form factors at the vertices of the rescattered meson. The effect of such form factors is relatively straightforward to estimate. Assuming that both vertices of the rescattered pion in Fig. 1 would require a form factor of the monopole type

$$\frac{\Lambda^2 - \mu^{*2}}{\Lambda^2 + k'^2}, \tag{32}$$

where Λ is a mass scale and k' the momentum of the exchanged meson, the function $Y_1(\mu^* r)$ in the integrals (19) should be replaced by[13]

$$Y_1(\mu^* r) \to Y_1(\mu^* r) - \frac{\Lambda^2}{\mu^{*2}} Y_1(\Lambda r) - \frac{\Lambda^2 - \mu^{*2}}{2\mu^{*2}} e^{-\Lambda r}. \tag{33}$$

While the proper value for the mass scale Λ is not very well known, recent results indicate that it should be well above 1 GeV/c^2.[14] In order to have an estimate of the effect of this form factor modification we evaluate the matrix elements using (33) and $\Lambda = 1.4$ GeV/c^2. This value for Λ is between the natural mass scale $2m$ and the threshold value $m_\omega + \mu$ for the contribution from the $\pi\omega$ vertex triangle diagram. In Fig. 4 we compare the results as obtained with and without the form factor correction for $\mathrm{Im}B_0$ as functions of pion momentum. At threshold inclusion of the form factor leads to a 25% reduction of the value for $\mathrm{Im}B_0$. At higher energies the reduction is somewhat larger. This large effect of the form factor is a consequence of the use of uncorrelated Fermi gas pair wave functions. If the depletion of the wave functions at short range were taken into account the form factor modification would be considerably smaller. We therefore regard the present estimates of the form factor modification as maximal. In any case the form factor modification does not change the result that $\mathrm{Im}B_0$ is a rapidly increasing function of energy.

We finally turn to the question of the energy dependence of the coupling parameters λ in the rescattering interaction (5). The constant values $\lambda_1 = 0.003$ and $\lambda_2 = 0.050$ which were determined by the *S*-wave pion-nucleon scattering lengths do lead to an adequate fit for the isospin $\frac{3}{2}$ pion-nucleon phase shift S_{31} up to 250 MeV pion laboratory kinetic energy but lead to a large overrestimate of the isospin $\frac{1}{2}$ phase shift S_{11} above 50 MeV. If one allows the λ's to be energy dependent parameters one may of course obtain a good fit to both phase shifts even above 50 MeV. A parametrization for the λ's which leaves the fit to the S_{31} phase shift unchanged but improves that for the S_{11} phase shift would be

$$\lambda_1 = 0.003 + 0.0334\left(\frac{T}{\mu}\right) - 0.0058\left(\frac{T}{\mu}\right)^2,$$
$$\lambda_2 = 0.050 - \frac{\mu}{\omega}\left[0.0334\left(\frac{T}{\mu}\right) - 0.0058\left(\frac{T}{\mu}\right)^2\right]. \tag{34}$$

Here T is the pion laboratory kinetic energy in the pion-nucleon system, and ω the total pion en-

ergy in the center-of-mass system.

In the pion-nucleus system the pion kinetic energy is the same in both the laboratory and center-of-mass systems. With this modification we have used the parametrization (34) to calculate $\mathrm{Im}B_0$ as a function of the momentum of the absorbed pion with and without the form factor modification mentioned above. The results are also shown in Fig. 4 (curves C and D). At high energies the result for $\mathrm{Im}B_0$ is considerably smaller than the value obtained with constant parameters λ. The reason for the slower increase with energy for $\mathrm{Im}B_0$ in this case can be traced to the fact that with this parametrization (34) the parameter λ_1 which is very small compared to λ_2 at threshold rapidly grows with energy and becomes comparable to λ_2 which is correspondingly reduced. As in the expression for $\mathrm{Im}B_0$ the terms involving λ_1 are not multiplied by explicit energy factors ω as are the terms involving λ_2, any increase of λ_1 and decrease of λ_2 will reduce the energy dependence of $\mathrm{Im}B_0$. This situation is so much clearer once account is taken of the very weak energy dependence of the nuclear matrix elements. The pion form factor modification is of a similar magnitude as in the case of constant coupling parameters.

It is clear, however, that the introduction of energy dependence into the coupling parameters destroys the invariance of the Hamiltonian (5), thus introducing the problem of the transformation of the energy variable between the pion-nucleon and pion-nucleus center-of-mass systems. In particular that transformation when applied to P-wave rescattering mechanisms not considered here could lead to additional contributions to $\mathrm{Im}B_0$.

We finally note that our result that the energy dependence of the optical potential parameter $\mathrm{Im}B_0$ is explicitly related to the pion-nucleon rescattering vertex and not to the structure of the nuclear matrix elements gives a nice justification for the method in which the energy dependence was estimated on the basis of the S-wave pion-nucleon phase shifts in the phenomenological model for the pion-nucleus optical potentials in Ref. 5.

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