Application of a band crossing model for resonances in high energy ^{12}C-^{12}C scattering

Yosio Kondō
A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520

Yasuhisa Abe
A. W. Wright Nuclear Structure Laboratory, Yale University, New Haven, Connecticut 06520 and Research Institute for Fundamental Physics, Kyoto University, Kyoto, Japan

Takehiro Matsuse
Department of Physics, University of Tokyo, Tokyo, Japan
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Resonance phenomena observed in single and mutual 2_1^+ inelastic scattering and in 3_1^- inelastic scattering in the ^{12}C-^{12}C system are investigated using a band crossing model. It is shown that characteristic features of the observed resonances can be well understood and well reproduced by the model. The coupling between wide resonances is also discussed. Another important prediction of the model, i.e., transfer of the dominant components of resonances from single and mutual 2_1^+ excitations to the 3_1^- excitation with increasing scattering energy appears to be realized. These facts support the validity of the band crossing model in this system. We conclude that the prominent correlated resonances observed in high energy inelastic ^{12}C-^{12}C scattering can be considered as a strong evidence for the existence of nuclear molecular phenomena in this energy range.

[NUCLEAR REACTIONS $^{12}C(^{12}C,^{12}C^*(2_1^+))^{12}C^*(2_1^+)$, $^{12}C(^{12}C,^{12}C)^{12}C^*(2_1^+)$, $^{12}C(^{12}C,^{12}C)^{12}C^*(3_1^-)$, $10 \leq E_{c.m.} \leq 40$ MeV; calculated $\sigma(E)$. resonance mechanism for heavy ion reactions.]

I. INTRODUCTION

Since the first observation of three correlated resonances in the sub-Coulomb ^{12}C-^{12}C reaction,[1] it has been recognized that resonant phenomena in heavy ion reactions provide an interesting problem involving both the interaction between composite nuclei and the nuclear structure of the composite system in highly excited states.

To investigate this problem, many studies have been reported, both experimental and theoretical.[2] Several moderately successful attempts[3,4] for understanding the resonant phenomena have been proposed, but unfortunately no reliable prediction of their occurrences have been available, i.e., we could not predict at what energies, with what spin values and with what nuclear structure resonances would appear in heavy ion reactions. From the viewpoint of the nuclear molecule, the present authors have extended the Nogami-Imanishi model[3] and have proposed a band crossing model (BCM) to describe the mechanism involved in these resonances.[5-7] This model makes predictions as to whether certain molecular configuration will be observed as prominent resonances or not. It has already been demonstrated in the first paper[7] of this series that the model is very successful in reproducing and predicting the doubletlike resonances which have been observed in both elastic and inelastic scattering[8-10] of the ^{12}C-^{16}O system.

Recently, intermediate width resonances,[11,12] i.e., narrower than potential resonances and broader than resonancelike structure in these cross sections arising from purely statistical fluctuations, have been observed in the integrated cross sections for ^{12}C-^{12}C single and mutual 2_1^+ inelastic scattering at energies well above the Coulomb barrier [see panels (a) and (b) of Fig. 3]. Similar structure can be observed in the data of Wieland *et al.*[13] One of the important features of these resonances is that they have large yield and reach 17% of the unitarity limit.[11] This suggests that there exists strong coupling between the elastic and 2_1^+ inelastic channels. A partial width analysis of Cormier *et al.*[12] also showed the importance of the inelastic channels. Previously, several studies[4,14,15] which included coupling of elastic with inelastic channels have been reported for the higher energy ^{12}C-^{12}C scattering but there exists, as yet, no clear understanding of the mechanisms underlying the observed resonances.

The purpose of this paper is to investigate these resonant phenomena in the ^{12}C-^{12}C scattering using the BCM in an attempt to clarify an occurrence mechanism of resonances in heavy ion reactions. It will be shown that characteristic features of these resonances can be well understood and well reproduced by the BCM. (A short note has already

been reported in Ref. 16.)

In Sec. II of the paper, a recapitulation of the BCM is given and the model is applied to the ^{12}C-^{12}C system; only a schematic discussion—which can be performed without solving the coupled channel equation—is given in this section. Formulation of the BCM using a coupled channel approach in which single and mutual 2_1^+ inelastic channels and the 3_1^- inelastic channel are included is introduced at the beginning of Sec. III; results and discussion are also given in Sec. III. A summary and concluding discussion comprise Sec. IV.

II. SCHEMATIC APPLICATION OF THE BCM TO THE $^{12}C + ^{12}C$ SYSTEM

In this section we review the BCM and its application to the ^{12}C-^{12}C system. Initially we shall assume the existence of a series of bound or quasibound states in the elastic channel; these can easily be obtained with any reasonable choice of the interaction potential between the two interacting nuclei as has been demonstrated in a number of microscopic studies of interactions between composite particles.[17] The energies of these states are simply approximated by the rotational expression,

$$E^{(0)}_{J=L} = \frac{\hbar^2}{2\mathcal{I}} L(L+1) + E^{(0)}_0 , \qquad (1)$$

where $\mathcal{I}$, L, and $E^{(0)}_0$ are the moment of inertia, relative angular momentum, and band head energy, respectively. Since the ^{12}C-^{12}C incident channel is composed of two identical spinless particles, only partial waves with even value of total angular momentum and postive pairity can contribute to this system.

In zeroth order approximation, the energies of the corresponding resonances in an inelastic channel c are given by Eq. (2).

$$E^{(J_c)}_J = \frac{\hbar^2}{2\mathcal{I}} L_c(L_c+1) + E^{(0)}_0 + \epsilon_c , \qquad (2)$$

where I_c is the channel spin and ϵ_c is the excitation of the state reached via the inelastic scattering in the target and/or projectile nuclei. Total spins $J = |L_c - I_c|, \ldots, L_c + I_c$ are then possible as a consequence of angular momentum coupling. States having the same L_c in this simple model are all degenerate in a resonance energy and they form quasirotational bands, respectively. One of these bands in which the total spin J is the sum $I_c + L_c$ and which has therefore been designated as the "aligned" rotational band plays a particularly important role, because it crosses the elastic molecular-rotational band. The energy and spin corresponding to the crossing region can be obtained easily be equating Eqs. (1) and (2). Together with the elastic molecular rotational band in the ^{12}C-^{12}C system, we show the aligned rotational bands in Fig. 1 for inelastic channels which correspond to the lowest three excited states of ^{12}C (2_1^+; $\epsilon_c = 4.44$ MeV, 0_2^+; $\epsilon_c = 7.66$ MeV, and 3_1^-; $\epsilon_c = 9.64$ MeV). Here, the band head energy $E^{(0)}_0$ has been adjusted to yield a 12^+ potential resonance in the elastic channel at $E_{\text{c.m.}} = 20$ MeV, since resonances observed in many reaction channels suggest an active 12^+ partial wave at this energy.[18,19]

As is clearly shown in this figure, the 2_1^+ and 3_1^- inelastic aligned bands cross the elastic molecular one in the regions of $J = \sim 10$ and ~ 14, respectively. On the other hand, the 0_2^+ inelastic band (which of course is unique because the intrinsic spin of this channel is zero) does not cross the elastic molecular band at all. An aligned rotational band in the mutual 2_1^+ excitation channel ($\epsilon_c = 8.88$ MeV) is also included in Fig. 1. This mutual excitation aligned band in which the channel spin I_c is $4\hbar$ also crosses the elastic one at about $J = 10$ in spite of its high excitation energy.

Crossing of these aligned rotational bands with the elastic one implies strong coupling between elastic potential resonances and quasibound states in the inelastic channels. In consequence, we expect that the quasibound states and the potential

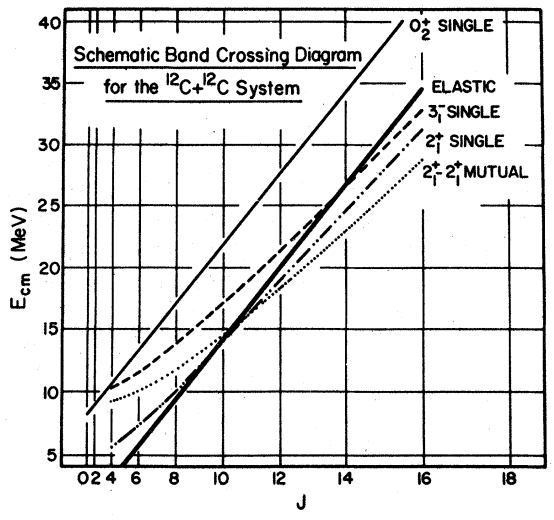

FIG. 1. Schematic band crossing diagram for the $^{12}C-^{12}C$ system. Elastic rotational band and aligned rotational bands for low lying three inelastic (2_1^+, 0_2^+, 3_1^-) and mutual 2_1^+ inelastic channels are shown. It is clearly seen that aligned rotational bands in the single and mutual 2_1^+ inelastic channels cross the elastic one at about $J^\pi = 10^+$. The 3_1^- aligned band crosses the elastic one at about $J^\pi = 14^+$. On the other hand, the band for the excited 0_2^+ inelastic channel does not cross the elastic one at all.

resonances will share partial widths of the elastic and the inelastic channels as a reflection of the strong mixing of the corresponding wave functions and that they will be observed as peaks at the band crossing region. Depending of course upon the width of the resonance to be coupled these may—or may *not*—be a discernible peak in the cross section. Where narrow resonances are coupling we would expect to observe them as a cluster of resonances in the relevant channels in the region of the band crossing energy. One of the examples is the ^{12}C-^{16}O system in which doubletlike resonances were observed. In the case of coupling between wide resonances, we as yet have no physical examples and in general it might be expected that the broad composite structure might be very difficult to identify unambiguously. We discuss this further in Sec. III of this paper. It must be emphasized that, experimentally, both the single and mutual 2_1^+ inelastic cross sections show prominent resonances at the energy region where the corresponding aligned rotational bands approach and cross the elastic one.

An important prediction[7] of the BCM is the systematic change in the dominant inelastic excitation component of the resonances as the energy hence angular momentum increases. This prediction is illustrated in Fig. 1. In the ^{12}C-^{12}C system, the dominant component of the resonances is expected to move systematically from single and mutual 2_1^+ excitations to the 3_1^- single excitation at angular momenta in the $10 \sim 14\hbar$ range. In this range, coexistence of the components of the three channels can be expected. This prediction will be examined further in Sec. III by comparing our model calculations, which includes single, mutual 2_1^+ and 3_1^- inelastic channels, with the experimental data.

III. FORMULATION, RESULTS, AND DISCUSSION

As discussed in Sec. II, the single and mutual 2_1^+ inelastic channels and the 3_1^- inelastic channel are expected to play important roles in ^{12}C-^{12}C scattering at this energy region. We have carried out a detailed numerical calculation including these three inelastic channels. We shall only briefly summarize the formalism before presenting our new calculational results and discussion of them.

A. Formalism

The model space of the present calculation includes the elastic channel, both single and mutual 2_1^+ inelastic channels and the 3_1^- inelastic channel. In obtaining the intrinsic wave function $\chi_{I_i M_i}(\theta_i)$ of the ith carbon nucleus, we assume a rotational model for the 2_1^+ excitation and a phonon model for the 3_1^- excitation; thus we shall use

$$\chi_{I_i M_i}(\theta_i) = \begin{cases} \left(\frac{2I_i+1}{8\pi^2}\right)^{1/2} D^{I_i}_{M_i 0}(\theta_i), & \text{for } I_i = 2, \\ b^\dagger_{I_i M_i}|0\rangle, & \text{for } I_i = 3, \end{cases} \tag{3}$$

where D^I_{M0} is a usual rotation matrix element and $b^\dagger$ is a usual phonon operator. Our coupling potential is obtained in the same fashion as in Ref. 20. The effect of the intrinsic motion of the carbon nuclei is introduced via the range parameter R of the attractive potential as follows,

$$R = R_0\left(2 + \beta Y_{20}(\Omega_1) + \sum_\mu \frac{\gamma}{\sqrt{7}}\left[b_{3\mu} + (-1)^\mu b^\dagger_{3-\mu}\right] Y_{3\mu}(\hat{r}) + \beta Y_{20}(\Omega_2) + \sum_\mu \frac{\gamma}{\sqrt{7}}\left[b_{3\mu} + (-1)^\mu b^\dagger_{3-\mu}\right] Y_{3\mu}(-\hat{r})\right), \tag{4}$$

where R_0 is the mean radius of the carbon nucleus, and β and γ are the coupling parameters to the 2_1^+ and 3_1^- excitations, respectively. The angle Ω_i refers to the body fixed system in the ith carbon nucleus. As the coupling potential, we take the first order expansion in γ and up to the second order expansion in β. The second order term in β explicitly indicates the direct coupling between the elastic channel and the mutual 2_1^+ inelastic channel. As a result we obtain the following coupled channel equation:

$$\sum_{c'}\left[(\mathfrak{L}_c - E_c)\delta_{cc'} + V_{cc'}(r)\right] u_{c'}(r) = 0, \tag{5}$$

where

$$\mathfrak{L}_c = -\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dr^2} - \frac{L_c(L_c+1)}{r^2}\right) + U^c_{\text{opt}}(r). \tag{6}$$

E_c is the energy of relative motion in a channel c, $E_c = E - \epsilon_c$, $V_{cc'}(r)$ is the coupling potential (and is given in Appendix A), $u_c(r)$ is the radial wave function in channel c, and $U^c_{\text{opt}}(r)$ is the optical potential in channel c. We have adopted a Woods-Saxon shape for *both* the repulsive core and the attractive part of the optical model potential; this results in the following parametrization:

$$U^c_{\text{opt}}(r) = V_{\text{core}} F(R_{\text{core}}, a_{\text{core}}, r) - V_c F(R, a, r) - iW F(J_c, \Delta J, J) F(R, a, r) + V_{\text{Coul}}(r), \tag{7}$$

where

$$F(R, a, r) = \left[1 + \exp\left(\frac{r-R}{a}\right)\right]^{-1}, \tag{8}$$

and the Coulomb potential $V_{\text{Coul}}(r)$ is

$$V_{\text{Coul}}(r) = \begin{cases} \frac{Z_1 Z_2 e^2}{r} & \text{for } r \geq R, \\ \frac{Z_1 Z_2 e^2}{2R}\left[3 - (r/R)^2\right] & \text{for } r < R. \end{cases} \tag{9}$$

TABLE I. Potential parameters. Adopted parameters of the interaction potential are given. The part indicated "core" represents a short range repulsive core reflecting effects of the Pauli principle. Depths of the attractive potential are given by V_1, V_2, V_3, and V_4 for elastic, single 2_1^+, mutual 2_1^+ and single 3_1^- inelastic channels, respectively. The range and the diffuseness of the attractive Woods-Saxon potential are given by R and a, respectively. According to the Eq. (7) and (10), parameters W, $\bar{R}$, $\bar{Q}$, and ΔJ define the imaginary part. β and γ stand for coupling parameters to the 2_1^+ and 3_1^- excitations, respectively.

Core			Attractive					
V_{core}	R_{core}	a_{core}	V_1	V_2	V_3	V_4	R	a
100 MeV	2.4 fm	0.3 fm	17.0 MeV	16.0 MeV	15.0 MeV	17.0 MeV	5.72 fm	0.35 fm

Imaginary			Coupling		
W	$\bar{R}$	$\bar{Q}$	ΔJ	β	γ
7 MeV	5.72 fm	−12.0 MeV	1.0	−0.3	0.15

The first term in Eq. (7) represents a short range repulsive core reflecting effects of the Pauli principle,[17] the second term is the usual attractive part of the optical potential, and the third term is the explicitly angular momentum dependent imaginary potential term.

Following Chatwin *et al.*,[21] the critical angular momentum J_c is defined as follows:

$$J_c=\bar{R}\left(\frac{2\mu}{\hbar^2}(E+\bar{Q})\right)^{1/2}. \tag{10}$$

Then the angular momentum dependence of the imaginary potential is determined by parameters ΔJ, $\bar{R}$, and $\bar{Q}$. To avoid obtaining much too large a width for the predicted resonances, it is essential that we use only a weak absorption potential for the resonant partial wave. On the other hand, the magnitude of the experimental fusion cross section data[22] seem to require a strong absorption potential. This is the reason why we adopt an explicitly angular momentum dependent imaginary potential. With such a potential, we can keep the predicted resonance widths reasonable while still reproducing the magnitude of the fusion cross section; it is important to recognize that the fusion cross section probably comes from complete fusion of partial waves with angular momentum lower than the grazing one of greatest importance to the surface based molecular resonance phenomena.[23]

Our coupled channel equation (5) is solved via a variational method which has been described in the first paper[7] of this series. Adopted parameters of the interaction potential are given in Table I. Before solving the coupled channel equation, in Fig. 2 we show a band crossing diagram in which the resonance energies have been calculated using the above potential. It must be emphasized that in obtaining the location of these resonances we have used only the real part of the potential and specifically have not yet included either coupling to inelastic excitations or effects of the absorption

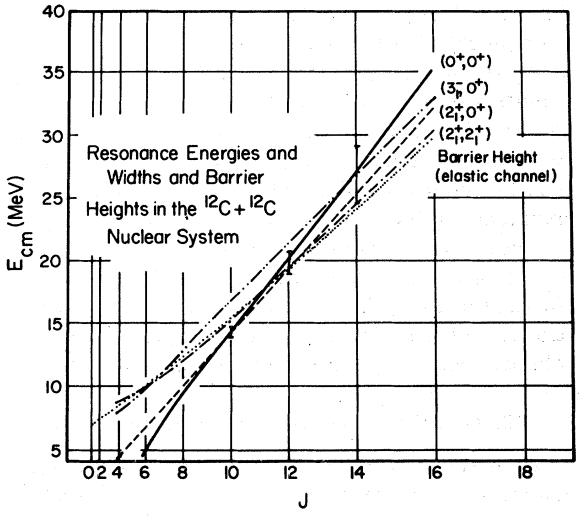

FIG. 2. Energies and widths of the calculated resonances and the height of the effective barrier in the $^{12}C-^{12}C$ system. For elastic, aligned single 2_1^+, aligned mutual 2_1^+, and aligned single 3_1^- channels, calculated resonance energies are shown. In obtaining the location of these resonances we have used only the real part of the potential and specifically have not yet included either coupling to inelastic excitations or effects of the absorption represented by the imaginary potential. Features of the schematic band crossing diagram, Fig. 1, can be reproduced. The calculated widths of resonances and the heights of the effective barrier for the elastic channel are also included. It is clear that in the band crossing region in the $^{12}C+^{12}C$ system the energies of resonances in the elastic channel become higher than the effective barrier and the widths of the resonances become wide.

represented by the imaginary potential. We use the same repulsive core for all channels, i.e., $V_{\text{core}} = 100$ MeV, $R_{\text{core}} = 2.4$ fm, and $a_{\text{core}} = 0.3$ fm. To reduce the predicted resonance widths to conform more nearly with the experimental observations, we have found it necessary to adopt a shorter range R than would be usual in a standard optical potential,[24] i.e., $R = 5.72$ fm and $a = 0.35$ fm. The depth of this attractive potential ($V_1 = 17$ MeV) was adjusted to reproduce the desired 12^+ elastic resonance at about $E_{\text{c.m.}} = 20$ MeV. This same reasoning established the choice of $E_0^{(0)}$ as discussed in Sec. II. It is obvious that the potential predicts a rotational band in the elastic channel. This was first noted by Arima *et al.*[25] and independently by Fink *et al.*[4] shortly after publication of the potential determined by Reilly *et al.*[24] as necessary to reproduce the early Yale elastic scattering data for the $^{12}C + ^{12}C$ system. It is interesting to note that the potential also requires a 2^+ resonance with one radial node at about $E_{\text{c.m.}} = 6.3$ MeV. This is consistent with a fragmentation analysis[26] of the sub-Coulomb ^{12}C-^{12}C resonances. The analysis suggested an existence of 2^+ resonance in the elastic channel. For the inelastic channels, somewhat shallower or the same attractive potential was adopted in order to reproduce observed resonance energies ($V_2 = 16$ MeV, $V_3 = 15$ MeV, and $V_4 = 17$ MeV). The coupling parameters β and γ were adjusted to reproduce the yields of the relevant inelastic scattering. The values of the parameters used, $\beta = -0.30$ and $\gamma = 0.15$, are physically reasonable.

In Fig. 2, the calculated widths of resonances and the heights of the effective barrier for the elastic channel are included. Here the barrier height is somewhat arbitrarily defined as the maximum value attained in the outer potential maximum in the potential of Eq. (7). The slope of the elastic rotational band locus is steeper than that of the barrier height, because the dominant part of the resonant wave function is located at radii inside the outer potential barrier. In going to higher beam energies, hence angular momenta, the predicted resonance energies move above the barrier and the predicted widths of the resonances increase substantially to reflect greater barrier penetrability. In the energy region where prominent intermediate width resonances were observed, the predicted elastic potential resonances are rather wide. As illustrated in the upper parts of each panel in Fig. 3, the widths of resonances in the aligned rotational bands also become wide in this energy region. Consequently, in the lowest band crossing region in the $^{12}C + ^{12}C$ system the situation is one of coupling between broad resonances—an example of the second situation noted in Sec. II and one where we would not anticipate that a cluster of sharp resonance fragments would be involved.

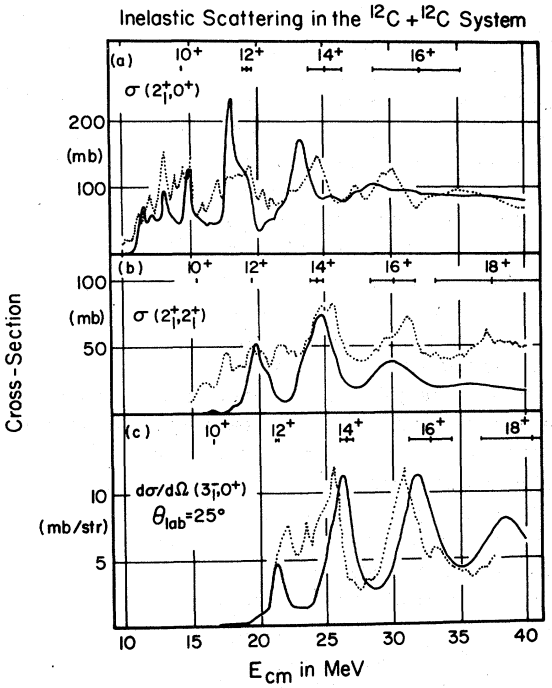

FIG. 3. Inelastic cross sections in the $^{12}C - ^{12}C$ system. Single 2_1^+, mutual 2_1^+, and single 3_1^- inelastic cross sections are shown in parts (a), (b), and (c), respectively. The solid lines show the results of our model calculation. Adopted potential parameters are those of Table I. The dotted lines show experimental data (Refs. 11, 12, 27). For single and mutual 2_1^+ inelastic scattering, angle integrated cross sections are given. For single 3_1^- inelastic scattering, the differential cross section at an angle $\theta_{\text{lab}} = 25°$ is shown. In the upper part of each panel, the predicted energies and widths of zeroth order resonances in each aligned rotational channel, which are calculated without coupling and absorption effect are shown with their spins and parities.

B. Inelastic cross sections

Our calculated inelastic cross sections are shown in Fig. 3 and are compared with the available experimental data. For single and mutual 2_1^+ inelastic scattering, angle integrated cross sections are given in panels 3(a) and 3(b), respectively. For 3_1^- inelastic scattering, an excitation function at $\theta_{\text{lab}} = 25°$ is shown in panel 3(c). The data are taken from papers of Cormier *et al.*[11,12] for single and mutual 2_1^+ inelastic scattering and from one of Reilly *et al.*[27] for the 3_1^- inelastic scattering.

It is clear from this figure that the model calculation can produce resonance structure for all three inelastic channels, as is expected from the

discussion in Sec. II. Each peak of the cross section is brought about by one partial wave, i.e., the peaks at about 20, 25, and 30 MeV are brought about by partial waves 12^+, 14^+, and 16^+, respectively. It must be emphasized that the calculated cross sections can reasonably reproduce resonance energies, widths, and yields of the observed resonances for all three inelastic scattering channels. The fact that the predicted mutual 2^+ inelastic excitation envelope is smaller than the experimental data probably reflects the fact that our second order expansion in the quadrupole deformation—which explicitly couples this mutual excitation channel to the entrance channel—underestimates the mutual excitation cross sections. Additional mechanisms to reflect enhanced probability of such mutual excitations through rather mechanical meshing of the nuclear surfaces during grazing collisions with subsequent gear-wheel-like interactions may be necessary for any formalism which seeks a quantitative reproduction of the data. Here we have focussed instead on a relatively simple mechanism and on the more qualitative reproduction of the data as a measure of the validity of the BCM.

In the upper parts of each panel in Fig. 3 the predicted energies and widths of zeroth order resonances in each inelastic channel are shown. The calculated resonance peaks roughly correspond to the original potential resonances in each channel. Doublet resonances do not clearly appear in this calculation except for cases with spin lower than $12\hbar$. This is also true for the experimental data. In this the situation is strikingly different from that of the ^{12}C-^{16}O system, where clear doubletlike resonances were observed most strikingly in the 3^- channel.[8-10]

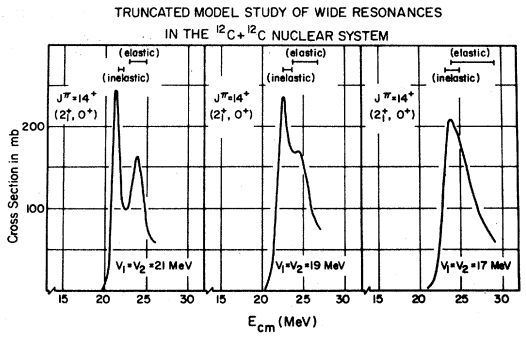

FIG. 4. Coupling between two resonances with various widths. Integrated inelastic cross sections for a partial wave $J^\pi=14^+$ are shown for the coupling between two resonances in the elastic and single 2_1^+ inelastic channels. The three panels correspond to the cases, $V_1=V_2=21$ MeV, 19 MeV, and 17 MeV, respectively. For the model space and other parameters see the text. In the upper part of each panel the energy and width of the zeroth order resonances in the elastic and single 2_1^+ inelastic channels are shown. Comparison among the three panels shows a gradual change from doublet peaks to a single wide peak.

In the previous short note[16] we have already emphasized this contrast between the $^{12}C+^{12}C$ and the $^{12}C+^{16}O$ systems and have conjectured that the absence of doublet structure in the former reflected the fact that the coupling involved was between wide resonances. To examine this question further we have carried out a simple model calculation in which the widths of two resonances to be coupled are changed systematically. In this calculation, our model space is truncated to include only elastic and single 2_1^+ inelastic channels so that only the coupling of two resonances is involved. All the potential parameters assumed are identical to those of Table I except for the depths of the attractive potentials, i.e., V_1 and V_2. To examine the coupling between resonances with various widths, we have considered three cases, i.e., $V_1=V_2=21$ MeV, 19 MeV, and 17 MeV, respectively. The calculated inelastic cross sections for $J^\pi=14^+$ are shown in Fig. 4. In the upper part of each panel the energy and width of the zeroth order resonance in the elastic and inelastic channels are shown. Comparison among the three panels of Fig. 4 shows a gradual change from doublet peaks to a single wider peak—depending of course upon the widths of resonances to be coupled. As is expected, the coupling between narrow resonances produces doublet peaks. In the limit of coupling between wide resonances, a single peak appears. This is very probably the explanation for the "incomplete" doublet structure, which is one of the features of the experimental data, and, is a new aspect of the BCM in regions where rather broad individual resonance widths are expected.

C. Change of the dominant components of resonances with scattering energy

A further prediction of the BCM is the systematic change from one inelastic excitation to another excitation strongly coupled to the elastic channel as the dominant component of the molecular resonances as the scattering energy is increased. As discussed in Sec. II, the dominant components of molecular resonances in the ^{12}C-^{12}C system can be expected to change from single and mutual 2_1^+ excitations to 3_1^- excitation at $E_{c.m.}=15$–25 MeV and angular momentum $J=10$–14. The calculated cross sections clearly show this tendency, i.e., the single and mutual 2_1^+ inelastic cross sections are dominant at lower energy and the 3_1^- inelastic one is dominant at higher energy. In the energy region between

them the three inelastic cross sections are of equal importance. It must be recognized that this tendency is consistent with the available experimental data shown in Fig. 3.

Unfortunately the 3_1^- inelastic cross sections have, as yet, only been measured at one angle. Precise measurement of angle integrated 3_1^- inelastic cross sections would be extremely valuable. Recently Erb *et al.*[28] have found that at even higher energies, $E_{c.m.} > 35$ MeV, the inelastic excitation to the 4^+ member of the ^{12}C ground state rotational band ($\epsilon_c = 14.08$ MeV) and mutual inelastic $(4_1^+, 2_1^+)$ excitation ($\epsilon_c = 18.52$ MeV) are strongly enhanced in the ^{12}C-^{12}C scattering. The BCM reproduces this observation in natural fashion in that it predicts that the aligned bands corresponding to these excitations cross the elastic molecular one in this energy region characterized by $J^\pi = 16^+$.

IV. SUMMARY AND CONCLUSIONS

We have shown that the band crossing model can reproduce characteristic features, i.e., energies, widths, and yields, of the correlated resonances with intermediate widths which were observed in the single and mutual 2_1^+ and 3_1^- inelastic scattering in the ^{12}C-^{12}C system. An angular momentum dependent imaginary potential in which the grazing partial wave feels weak absorption and lower partial waves feel strong absorption plays an important role in reproducing both the widths of the resonances and the magnitude of the fusion cross section. Incomplete doublet features of the observed resonances are probably attributed to the coupling between wide resonances. A further important prediction of the band crossing model, i.e., systematic change of the dominant inelastic components of molecular resonances from single and mutual 2_1^+ excitations to 3_1^- excitation with increasing scattering energy appears to be supported by the experimental data. From the fact that the observed energies and widths of the experimentally observed resonances are rather close to those of the original potential resonances in each channel, we conclude that the prominent correlated resonances observed in high energy inelastic ^{12}C-^{12}C scattering can be considered as a strong evidence for the existence of molecular resonances in the $^{12}C + ^{12}C$ interaction at higher energies and for the validity of the band crossing model in this system.

One of the additional predictions of the band crossing model,[7] i.e., a magnetic substate population is currently being developed in the hope of stimulating the experimental studies necessary for its testing.

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APPENDIX A

The coupling potential $V_{cc'}(r)$ is given as follows:

$$V_{cc'}(r) = \beta V^{(1)}_{\text{coupl}}(r) K^{(1)}_{cc'} + \beta^2 V^{(2)}_{\text{coupl}}(r)(K^{(2)}_{cc'} + K^{(3)}_{cc'}) + \gamma V^{(3)}_{\text{coupl}}(r) K^{(4)}_{cc'} . \quad \text{(A1)}$$

Radial parts, $V^{(1)}_{\text{coupl}}(r)$, $V^{(2)}_{\text{coupl}}(r)$, and $V^{(3)}_{\text{coupl}}(r)$ are given by Eqs. (A2), (A3), and (A4), respectively.

$$V^{(1)}_{\text{coupl}}(r) = \frac{RV_2}{2} \frac{d}{dr} F(R, a, r) , \quad \text{(A2)}$$

$$V^{(2)}_{\text{coupl}}(r) = -\frac{R^2 V_2}{8} \frac{d^2}{dr^2} F(R, a, r) , \quad \text{(A3)}$$

$$V^{(3)}_{\text{coupl}}(r) = -\frac{RV_4}{2} \frac{d}{dr} F(R, a, r) . \quad \text{(A4)}$$

We use a notation for channels; $c = ((I_1^c, I_2^c) I_c, L_c)$. Then angular momentum dependent terms, $K^{(1)}_{cc'}$, $K^{(2)}_{cc'}$, $K^{(3)}_{cc'}$, and $K^{(4)}_{cc'}$ are given by Eqs. (A5), (A7), (A8), and (A9), respectively.

$$K^{(1)}_{cc'} = \frac{1}{\sqrt{4\pi}} K^{(0)}_{cc'}(2) , \quad \text{(A5)}$$

where

$$K^{(0)}_{cc'}(N)=\left(\frac{1}{(1+\delta_{I_1^cI_2^c})(1+\delta_{I_1^{c'}I_2^{c'}})}\right)^{1/2}\hat{I}_c\hat{I}_{c'}\hat{L}_c\hat{L}_{c'}(L_cL_{c'}00|N0)W(I_cL_cI_{c'}L_{c'};JN)$$

$$\times[\hat{I}_1^{c'}(NI_1^{c'}00|I_1^c0)W(I_2^{c'}I_1^{c'}I_cN;I_{c'}I_1^c)\delta_{I_2^cI_2^{c'}}+\hat{I}_1^{c'}(NI_1^{c'}00|I_2^c0)W(I_2^{c'}I_1^{c'}I_cN;I_{c'}I_2^c)\delta_{I_1^cI_2^{c'}}$$

$$+\hat{I}_2^{c'}(NI_2^{c'}00|I_1^c0)W(I_1^{c'}I_2^{c'}I_cN;I_{c'}I_1^c)\delta_{I_2^cI_1^{c'}}+\hat{I}_2^{c'}(NI_2^{c'}00|I_2^c0)W(I_1^{c'}I_2^{c'}I_cN;I_{c'}I_2^c)\delta_{I_1^cI_1^{c'}}]\,. \tag{A6}$$

Symbols, $\hat{I}$, $(abcd|ef)$ and $W(abcd;ef)$ stand for a factor $(2I+1)^{1/2}$, Clebsch-Gordan coefficients, and Racah coefficients, respectively.

$$K^{(2)}_{cc'}=\frac{1}{4\pi}\sum_{N=0}^{4:\text{even}}\frac{5}{\hat{N}}(2200|N0)^2K^{(0)}_{cc'}(N)\,, \tag{A7}$$

$$K^{(3)}_{cc'}=\sum_{N=0}^{4:\text{even}}\frac{1}{2\pi}\left(\frac{1}{(1+\delta_{I_1^cI_2^c})(1+\delta_{I_1^{c'}I_2^{c'}})}\right)^{1/2}\hat{I}_1^c\hat{I}_1^{c'}\hat{I}_2^c\hat{I}_2^{c'}\hat{I}_c\hat{I}_{c'}\hat{L}_c\hat{L}_{c'}(2200|N0)(L_cL_{c'}00|N0)W(I_cL_cI_{c'}L_{c'};JN)$$

$$\times\left[\begin{Bmatrix}2 & I_1^{c'} & I_1^c\\ 2 & I_2^{c'} & I_2^c\\ N & I_{c'} & I_c\end{Bmatrix}(I_1^cI_1^{c'}00|20)(I_2^cI_2^{c'}00|20)+\begin{Bmatrix}2 & I_1^{c'} & I_2^c\\ 2 & I_2^{c'} & I_1^c\\ N & I_{c'} & I_c\end{Bmatrix}(I_1^cI_2^{c'}00|20)(I_2^cI_1^{c'}00|20)\right]\,, \tag{A8}$$

where $\begin{Bmatrix}abc\\ def\\ ghi\end{Bmatrix}$ stands for 9-J symbols.

$$K^{(4)}_{cc'}=\left(\frac{1}{2\pi}\right)^{1/2}\hat{L}_c\hat{L}_{c'}(L_cL_{c'}00|30)W(I_cL_cI_{c'}L_{c'}|J3)(1-\delta_{I_cI_{c'}})\,. \tag{A9}$$

[1]E. Almqvist, D. A. Bromley, and J. A. Kuehner, Phys. Rev. Lett. 4, 515 (1960).

[2]D. A. Bromley, in *Nuclear Molecular Phenomena*, edited by N. Cindro (North-Holland, Amsterdam, 1978), p. 3, and references therein.

[3]B. Imanishi, Nucl. Phys. A125, 33 (1969).

[4]H.-J. Fink, W. Scheid, and W. Greiner, Nucl. Phys. A188, 259 (1972).

[5]Y. Kondō, T. Matsuse, and Y. Abe, in Proceedings of the INS-IPCR Symposium on Cluster Structure of Nuclei and Transfer Reactions Induced by Heavy-Ions, Tokyo, edited by H. Kamitsubo, I. Kohno, and T. Marumori (The Institute of Physical and Chemical Research, Wako-shi, Saitama, Japan, 1975), p. 280; Y. Abe, in *Proceedings of the Second International Conference on Clustering Phenomena in Nuclei*, edited by D. Goldman (University of Maryland, College Park, MD, 1975), p. 500.

[6]T. Matsuse, Y. Kondō, and Y. Abe, Prog. Theor. Phys. 59, 1009 (1978).

[7]T. Matsuse, Y. Abe, and Y. Kondō, Prog. Theor. Phys. 59, 1904 (1978).

[8]R. E. Malmin, J. W. Harris, and P. Paul, Phys. Rev. C 18, 163 (1978).

[9]D. Shapira, R. M. DeVries, M. R. Clover, R. N. Boyd, and R. N. Cherry, Jr., Phys. Rev. Lett. 40, 371 (1978).

[10]K. Katori, F. Furuno, and T. Ooi, Phys. Rev. Lett. 40, 1489 (1978).

[11]T. M. Cormier, J. Applegate, G. M. Berkowitz, P. Braun-Munzinger, P. M. Cormier, J. W. Harris, C. M. Jachcinski, L. L. Lee, Jr., J. Barrette, and H. E. Wegner, Phys. Rev. Lett. 38, 940 (1977).

[12]T. M. Cormier, C. M. Jachcinski, G. M. Berkowitz, P. Braun-Munzinger, P. M. Cormier, M. Gai, J. W. Harris, J. Barrette, and H. E. Wegner, Phys. Rev. Lett. 40, 924 (1978).

[13]R. Wieland, A. Gobbi, L. Chua, M. W. Sachs, D. Shapira, R. Stokstad, and D. A. Bromley, Phys. Rev. C 8, 37 (1973); see also K. A. Erb, in *Proceedings of the Symposium on Heavy-Ion Elastic Scattering, University of Rochester, 1977,* edited by R. M. DeVries (University of Rochester, Rochester, New York, 1977), p. 123.

[14]J. Y. Park, W. Greiner, and W. Scheid, Phys. Rev. C 16, 2276 (1977).

[15]O. Tanimura, Nucl. Phys. A309, 233 (1978).

[16]Y. Abe, Y. Kondō, and T. Matsuse, Prog. Theor. Phys. 59, 1393 (1978).

[17]I. Shimodaya, R. Tamagaki, and H. Tanaka, Prog. Theor. Phys. 27, 793 (1962); T. Ando, K. Ikeda, and A. Tohsaki-Suzuki, Prog. Theor. Phys. 59, 2168 (1978), and references therein.

[18]E. R. Cosman, T. M. Cormier, K. Van Bibber, A. Sperduto, G. Young, J. Erskine, L. R. Greenwood, and O. Hansen, Phys. Rev. Lett. 35, 265 (1975).

[19]N. R. Fletcher, J. D. Fox, G. J. Kekelis, G. R. Morgan, and G. A. Norton, Phys. Rev. C 13, 1173 (1976).

[20]T. Tamura, Rev. Mod. Phys. 37, 679 (1965).

[21]R. A. Chatwin, J. S. Eck, D. Robson, and A. Richter, Phys. Rev. C 1, 795 (1970).

[22]P. Sperr, T. H. Braid, Y. Eisen, D. G. Kovar, F. W. Prosser, Jr., J. P. Schiffer, S. L. Tabor, and S. Vigdor, Phys. Rev. Lett. 37, 321 (1976).

[23]Y. Abe, in *Proceedings of the Third International Conference on Clustering Aspects of Nuclear Structure*

and Nuclear Reactions, edited by W. T. H. Van Oers, J. P. Svenne, J. S. C. McKee, and W. R. Falk (University of Manitoba, Manitoba, Canada, 1978), p. 132; Y. Abe, T. Matsuse, and Y. Kondō, Phys. Rev. C 19, 1365 (1979).

[24]W. Reilly, R. Wieland, A. Gobbi, M. W. Sachs, and D. A. Bromley, Nuovo Cimento 13A, 897 (1973).

[25]A. Arima, G. Scharff-Goldhaber, and K. W. McVoy, Phys. Lett. 40B, 7 (1972).

[26]Y. Kondō, T. Matsuse, and Y. Abe, Prog. Theor. Phys. 59, 465 (1978).

[27]W. Reilly, R. Wieland, A. Gobbi, M. W. Sachs, J. Maher, R. H. Siemssen, D. Mingay, and D. A. Bromley, Nuovo Cimento 13A, 913 (1973).

[28]K. A. Erb *et al.*, private communication.