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(α,^{3}He) breakup reaction on nuclei and the neutron momentum distribution in the α particle

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The (α,^{3}He) breakup reaction on ^{62}Hi and ^{209}Bi targets at incident α-particle energies of 172.5 and 140 MeV respectively has been studied by a plane-wave spectator model. The results have been compared with the Serber model calculations as well as with those of the distorted-wave Born approximation. It is found that plane wave methods are not appropriate to describe this reaction correctly. Apart from the distortions in the α and ^{3}He channels, the proper consideration of the neutron target interaction is also very important. The validity of some of the assumptions of the Serber model has also been investigated. It is found that these assumptions are not valid for the reactions under investigation and consequently some of the conclusions drawn previously from the Serber model calculations for this reaction are not justified.

[NUCLEAR REACTIONS ^{209}Bi(α,^{3}He), ^{62}Ni(α,^{3}He) breakup reactions, calculated ($d^2\sigma/d\Omega dE$); E_α=140–172.5 MeV plane wave spectator model analysis; comparison with DWBA.]

I. INTRODUCTION

The breakup of loosely bound projectiles such as deuteron, ^{6}Li, and ^{9}Be in the field of nuclei is well known.[1-3] Quite recently also the evidence for the breakup of the strongly bound α particle has been given by Wu, Chang, and Holmgren[4] and by Budzanowski *et al.*[5] Wu *et al.*[4] have used a theoretical model analogous to one proposed by Serber[6] for deuteron breakup, to analyze their experimental data. In this model the ^{3}He particle is treated as a spectator in the breakup process, and the breakup cross section is assumed to be proportional to the absolute square of the momentum space wave function, $\varphi_{^3\mathrm{He}-n}(Q)$, of the ^{3}He-neutron system. This model is, in a way, equivalent to a simple plane-wave spectator model (PWSM) (see, e.g., Ref. 7) for the breakup reactions in which wave functions for the relative motion of α and ^{3}He particles in their respective channels are taken to be plane waves and only the neutron is allowed to interact with the target. In the Serber model, however, the neutron-target interaction is also ignored. With such simplified assumptions Wu *et al.*[4] have concluded that the information about the wave function $\varphi_{^3He-n}(Q)$ can be obtained directly from the breakup cross section.

The purpose of this paper is to show that the Serber as well as the plane wave spectator models are grossly deficient[8] to describe the α-particle breakup in the energy region E_α ~150 MeV. It will be shown that even in the plane wave limit, it is not quite obvious how one can get information about the wave function $\varphi_{^3\mathrm{He}-n}(Q)$ directly from the breakup cross section.[9] By comparing the results of calculations performed with the Serber model and with the PWSM, the importance of the neutron target interaction in describing the α-particle breakup is studied.

II. METHOD OF ANALYSIS

The cross section for the breakup of the α particle describing the elastic breakup process of the type $\alpha+A\rightarrow{}^3\mathrm{He}+n+A$ (ground state) as well as the inelastic breakup process in which the target nucleus in the final channel is not in the ground state can be written in the framework of the distorted-wave Born approximation (DWBA) as[1,5]

$$\frac{d^2\sigma}{d\Omega_{^3\mathrm{He}}dE_{^3\mathrm{He}}}=\frac{2\pi}{\hbar v}\rho(\mathrm{phase})\left|\sum_{l_nm_n}\left(|T_{l_nm_n}|^2+\frac{\sigma^{\mathrm{reac}}_{l_n}}{\sigma^{\mathrm{el}}_{l_n}}|T_{l_nm_n}-T^0_{l_nm_n}|^2\right)\right|. \tag{1}$$

The relative velocity in the initial channel is denoted by v and $\sigma^{\mathrm{el}}_{l_n}$ and $\sigma^{\mathrm{reac}}_{l_n}$ are the total neutron-target elastic and reaction cross section in the l_n^{th} partial wave. ρ (phase) denotes the phase-space factor and is given by

$$\rho(\mathrm{phase})=\frac{\mu_{^3\mathrm{He}}\mu_n}{8\pi(\pi\hbar^2)^3}\,q_{^3\mathrm{He}}q_n\,, \tag{2}$$

where $q_{^3\mathrm{He}}$ and q_n are the wave numbers of the outgoing ^{3}He and neutron, respectively. It is noted that in Eq. (1) the sum over l_n values has become incoherent, because we have done the integration over the angles of the unobserved neutron. The matrix element $T_{l_n m_n}$ describes the elastic breakup,[9]

$$T_{l_n m_n} = \int d^3 r_{^3\mathrm{He}-n} d^3 R_\alpha \chi_{^3\mathrm{He}}^{(-)*}(\vec{R}_{^3\mathrm{He}}) f_{l_n}(R_n) Y_{l_n m_n}(\hat{R}_n) \times V_{^3\mathrm{He}-n}(\vec{r}_{^3\mathrm{He}-n}) \varphi_{^3\mathrm{He}-n}(\vec{r}_{^3\mathrm{He}-n}) \chi_\alpha^{(+)}(\vec{R}_\alpha), \tag{3}$$

where the system of coordinates used is the same as given in Ref. 10. The distorted waves of the incoming α particle and the observed ^{3}He particle in the appropriate optical potentials are denoted by $\chi_i^{(\pm)}(\vec{R}_i)$. The function f_{l_n} denotes the radial wave function of the neutron in the appropriate optical model potential. $T^0_{l_n m_n}$ in Eq. (1) denotes a corresponding matrix element with f_{l_n} replaced by the spherical Bessel function j_{l_n} and describes the breakup process in the absence of the neutron-target interaction. It is well known that the six-dimensional integral Eq. (3) can be approximated by a three-dimensional integral by applying the local energy approximation (LEA).[11,12] Following these references we can, therefore, approximate Eq. (3) by

$$T_{l_n m_n}(\mathrm{LEA}) = D_0 \int d^3 r \chi_{^3\mathrm{He}}^{(-)*}(\vec{r}) f_{l_n}(r) Y_{l_n m_n}(\hat{r}) \times \chi_\alpha^{(+)}(\vec{r}) \Lambda(r), \tag{4}$$

where D_0 is the zero range constant and $\Lambda(r)$ the finite range correction factor.[12]

In order to see the connection of the DWBA cross section [Eq. (1)] with the PWSM and the Serber model, we use plane waves for α and ^{3}He particles in the matrix element instead of optical model wave functions. (As will be seen later, this is rather dangerous, the α particle is strongly absorbed by the nuclear field which is disregarded completely in the plane wave approximation.) It is instructive to express the optical model wave function for the neutron-target system $(\chi_{\vec{q}_n}^{(-)})$ in terms of the off-shell t-matrix element $t(\vec{q}, \vec{q}_n)$ $(E_n = q_n^2/2m_n)$:

$$\chi_{\vec{q}_n}^{(-)}(\vec{r}) = e^{i\vec{q}_n\cdot\vec{r}} + \frac{1}{(2\pi)^3} \int d^3 q \frac{e^{i\vec{q}\cdot\vec{r}}}{q^2 - q_n^2 + i\epsilon} t(\vec{q}, \vec{q}_n). \tag{5}$$

Neglecting the inelastic breakup mode, the full finite range DWBA matrix element is written as

$$T = \frac{t(\vec{q}_\alpha - \vec{q}_{^3\mathrm{He}}, \vec{q}_n)^*}{(\vec{q}_\alpha - \vec{q}_{^3\mathrm{He}})^2 - q_n^2} F(\tfrac{3}{4}\vec{q}_\alpha - \vec{q}_{^3\mathrm{He}}), \tag{6}$$

where

$$F(\vec{Q}) = \int e^{-i\vec{Q}\cdot\vec{r}} V_{^3\mathrm{He}-n}(r) \varphi_{^3\mathrm{He}-n}(r) d^3 r, \tag{7}$$

with $V_{^3\mathrm{He}-n}$ describing the interaction between the neutron and ^{3}He. $\vec{q}_\alpha$ is the momentum of the α particle. In Eq. (6) the integration over the angles of the unobserved neutron has not been performed. Using the Schrödinger equation for the ^{3}He-n system we can write Eq. (6) as

$$T = -\frac{\hbar^2}{2m_n}(2\pi)^{3/2} t(\vec{q}_\alpha - \vec{q}_{^3\mathrm{He}}, \vec{q}_n)^* \times \varphi_{^3\mathrm{He}-n}(\tfrac{3}{4}\vec{q}_\alpha - \vec{q}_{^3\mathrm{He}}). \tag{8}$$

The breakup cross section is obtained by multiplying the absolute square of Eq. (8) with the phase-space factor. The integration over the neutron angles can be done analytically. This leads to an incoherent sum over l_n values. Thus the DWBA cross section given by Eq. (1) contains the PWSM as a plane wave limit.

If the absolute square of the off-shell t-matrix element multiplied by the phase-space factor is assumed to be constant (as is done in the Serber model), the breakup cross section reflects directly the momentum distribution of the neutron in the α particle. In the calculation of Wu *et al.*,[4] how-

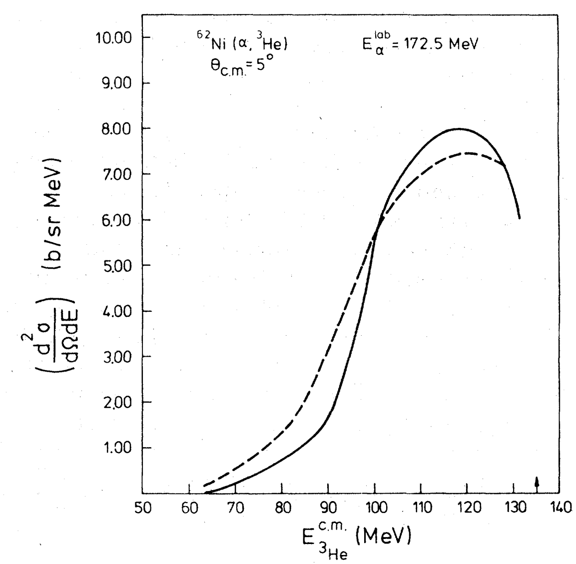

FIG. 1. Plane wave spectator model calculations for the double differential cross section for the ^{62}Ni(α, ^{3}He) reaction. Full lines indicate the results of the calculation with finite range effects taken care by means of LEA and the dotted line indicates the same with zero range approximation. The arrow indicates the three-body threshold.

ever, the absolute square of the off-shell t matrix multiplied by the neutron momentum is assumed to be constant. In the following section we shall discuss the validity of this assumption along with the general discussion regarding the inappropriateness of the plane wave methods to describe the α-particle breakup.

III. RESULTS AND DISCUSSION

We have performed the plane wave calculation for the (α, ^{3}He) breakup reaction on a ^{62}Ni target at the incident α-particle energy of 172.5 MeV. In this calculation we take plane waves for the α and ^{3}He particles instead of optical model wave functions in Eq. (4). However, the radial neutron wave function is obtained by optical potentials whose parameters have been taken from Wilmore and Hodgson.[13] In Fig. 1, we show the results of such a calculation where finite range effects have been taken into account by means of LEA. Also shown are the results obtained by the zero range approximation. By zero range approximation we mean

$$F(Q) = (\alpha^2 + Q^2)\varphi_{^3\mathrm{He}-n} \tag{9}$$

put to a constant value $F(Q=0)$, where $\alpha^2 = 2m^*\epsilon/\hbar^2$ with ϵ as the separation energy of ^{3}He in the α particle. In these calculations we have included the contributions of the inelastic breakup mode also which is ignored in the derivations of Eq. (8) as well as in Ref. 7. It has been shown[1,5] that the inelastic breakup is an important breakup mode. The experimental as well as the DWBA values of the cross section at the peak energy (~120 MeV for both) are 12.7 mb/sr MeV and 11.9 mb/sr MeV respectively. These values have been taken from Ref. 5. We note that although the shape of the cross section versus the ^{3}He energy curve for the PWSM calculation is rather similar to that of the DWBA, the PWSM calculations overestimate the absolute value of the experimental cross-section at the peak by orders of magnitude whereas the DWBA value is quite close to the experiment. It may further be noted that at the low ^{3}He energies the shapes of both DWBA and PWSM curves may differ from that of the experiment, because at these low energies other mechanisms such as multistep processes may further invalidate the simple spectator picture and also the single step mechanism assumed in the DWBA. This shows the gross deficiency of the plane-wave method to describe the α-particle breakup process in this energy range. Certainly, in plane wave calculations one misses a very important point, namely, the absorption of the α particle in the target nucleus. Thus the breakup matrix elements are far too high. This makes the plane wave method for the calculation of the α-particle breakup cross section questionable.

Now we want to investigate the following points: Can one get information about the momentum space wave function $\varphi_{^3\mathrm{He}-n}(Q)$ directly from the breakup cross section and is the assumption that $P(E_n) = q_n |t(\vec{q}_\alpha - \vec{q}_{^3\mathrm{He}}, \vec{q}_n)|^2 =$ constant valid? How important is the neutron-target interaction in describing the breakup cross section? We proceed as follows: We have calculated the cross section for the breakup of the α particle on ^{209}Bi target at the incident α particle energy of 140 MeV employing the Serber model (following the procedure of Wu *et al.*,[4] but we have not considered a shift of 6° due to Coulomb repulsion as has been done by these authors), and also the PWSM. The results are shown in Fig. 2, where the Serber model results are normalized to those of PWSM at the peak. We see from this figure that the Serber model overestimates the experimental value of the full width at half maximum (FWHM) (=30 MeV) by at least 10 MeV, whereas PWSM gives nearly the correct value for this. It should be noted that even if a shift of 6° due

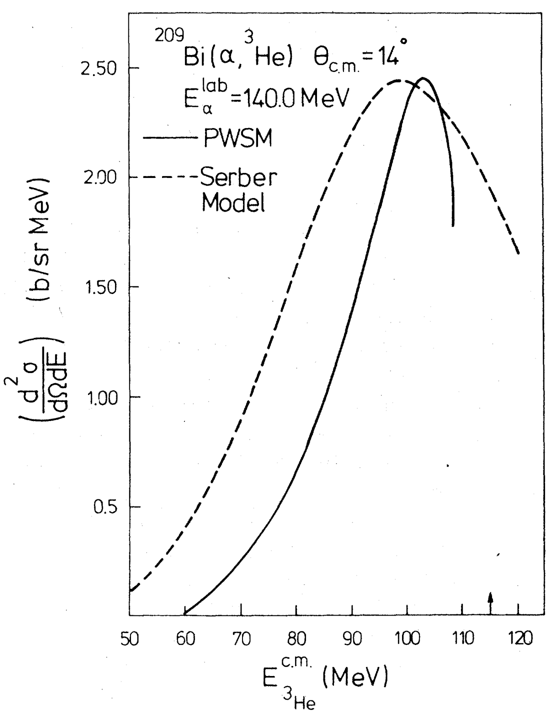

FIG. 2. Serber model and the plane wave spectator model (with finite range effects) calculations for ^{209}Bi ^{209}Bi(α, ^{3}He) reaction. The full line indicates the results for the plane wave spectator model whereas the dotted line shows the same for the Serber model which has been normalized to the peak of the PWSM. The arrow indicates the three-body threshold.

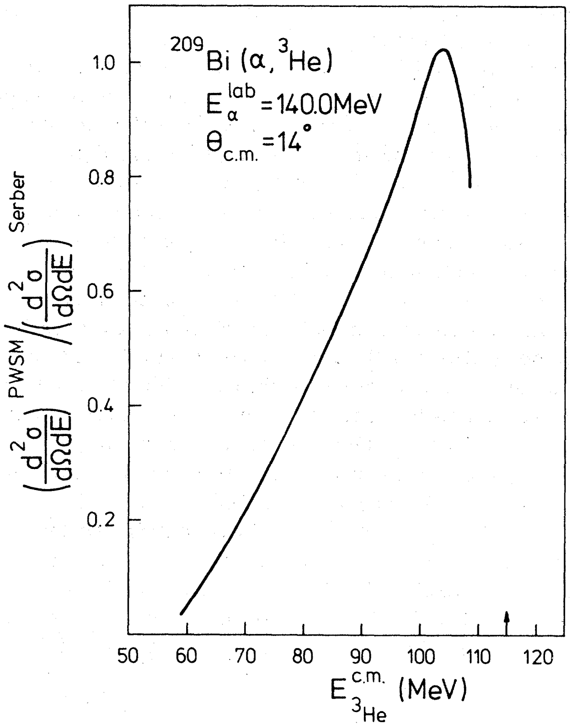

FIG. 3. The variation of the ratio of the plane wave spectator model cross section to the Serber model cross section (normalized) as a function of the ^{3}He energy for the ^{209}Bi(α, ^{3}He) reaction. The arrow indicates the three-body threshold.

to Coulomb repulsion is included in the Serber model calculations, it still overestimates the experimental FWHM by about 10 MeV. Thus not only the distortions in the α and ^{3}He channels are important but also very necessary is the consideration of the neutron-target interaction for the correct description of the α-particle breakup.

A further point against the use of the Serber model for the α-particle breakup is that the extreme tails of the cross section versus ^{3}He energy are not correct, particularly the part for which $E_{^3He} > E_{\text{threshold}}$, since in this portion of the curve breakup is no longer existing. ($E_{\text{threshold}}$ denotes the threshold for the three-body breakup shown by the arrow in Fig. 2, beyond which the breakup mode ceases to exist.) For further details the original article[6] of Serber may be looked at.

In Fig. 3 the variation of the ratio of the PWSM cross section to the Serber model cross section with ^{3}He energy is shown. It can be seen that in the peak region (where the neutron energies vary from $E_n = 0$ MeV to $E_n = 30$ MeV), this ratio is off by nearly 50 to 20% from the value expected if $P(E_n)$ were constant. Thus the validity of the assumption that $P(E_n)$ is constant seems doubtful.

Even in the plane wave limit, therefore, it is not quite obvious how one can get information about $\varphi_{^3He}(Q)$ directly from the breakup cross section. Actually the real situation is even more complicated. The strong absorption of α and ^{3}He waves in their respective channels mixes in other momentum components with $\vec{q}_\alpha$ and $\vec{q}_{^3He}$ giving rise to some "local momenta" which may be very much different from $\vec{q}_\alpha$ and $\vec{q}_{^3He}$ (see for example Ref. 14). Hence to make the conclusion that some information about the momentum distribution of the neutron in the α particle could be directly obtained from the breakup cross section is, at all, not possible.

IV. CONCLUSIONS

The applicability of the plane wave methods to analyzing the breakup cross sections of α particles in the field of nuclei has been examined here. The plane wave methods for calculating the cross section for these reactions are unrealistic because of the fact that α particles are strongly absorbed inside the target nucleus. It seems quite impossible to extract *directly* the information about the momentum space wave function for the ^{3}He-n system from the breakup cross section. Apart from the distortions of the α and ^{3}He waves in their respective channels, the proper consideration of the neutron target interaction is also necessary to describe correctly the breakup of the α particles.

The classic Serber model, introduced some thirty years ago, has proved to be very useful for an understanding of the high energy deuteron breakup reactions. Certainly the essential points can be carried over to the description of the breakup of other composite particles. The location of the peak and the dependence of the breakup cross section on the target mass number is well described by the Serber model. However, as has been remarked by Satchler[8] in the case of stripping to bound states that, while the plane wave Butler theory gives insight into the physical origin of the observed characteristics of this reaction, this theory is grossly deficient to account for the detailed nature of the observed cross-section (specially its absolute magnitude). In a similar fashion we want to argue here that while the plane wave theories (e.g., Serber or PWSM) describe some gross features of the breakup of the composite particles, the detailed nature of the observed ed cross section could be fully described by the distorted wave methods.

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[1]J. Pampus, J. Ernst, T. Mayer-Kucuk, J. Rama Rao, G. Baur, F. Rösel and D. Trautmann, in *Proceedings of the International Conference on Nuclear Structure, Tokyo, 1977,* edited by T. Marumori (Physical Society of Japan, Tokyo, 1978); and Nucl. Phys. A 311, 141 (1978).

[2]H. Gemmeke, B. Deluigi, L. Lassen, and D. Scholz, Z. Phys. A286, 73 (1978).

[3]J. Unternährer, J. Lang, and R. Müller, Phys. Rev. Lett. 40, 1077 (1978).

[4]J. R. Wu, C. C. Chang, and H. D. Holmgren, Phys. Rev. Lett. 40, 1013 (1978).

[5]A. Budzanowski, G. Baur, C. Alderliesten, J. Bojowald, C. Mayer-Böricke, W. Oelert, P. Turek, F. Rösel, and D. Trautmann, Phys. Rev. Lett. 41, 635 (1978).

[6]R. Serber, Phys. Rev. 72, 1008 (1947).

[7]G. Baur, Z. Phys. A277, 147 (1976).

[8]G. R. Satchler, Rev. Mod. Phys. 50, 1 (1978).

[9]G. Baur and D. Trautmann, Phys. Rep. 25C, 293 (1976).

[10]G. Baur, F. Rösel, and D. Trautmann, Nucl. Phys. A265, 101 (1976).

[11]P. J. A. Buttle and L. J. B. Goldfarb, Proc. Phys. Soc. London 83, 701 (1964).

[12]J. R. Shepard, W. R. Zimmermann, and J. J. Kraushaar, Nucl. Phys. A275, 189 (1977).

[13]D. Wilmore and P. E. Hodgson, Nucl. Phys. 55, 673 (1964).

[14]D. F. Jackson, *Nuclear Reactions* (Methuen, London, 1970).