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Trinucleon photoeffect: Isospin 1/2

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I modify the Levinger-Fitzgibbon calculation of electric dipole transitions to final trinucleon states of isospin 1/2 by assuming that the photon energy is equal to the sum of the threshold for *two-body* breakup and the total kinetic energy of the final state. This modification greatly improves the following: (i) the threshold behavior of the cross section; (ii) the agreement between the integrated cross section found from $\sigma(E_\gamma)$ and that found from sum rules; (iii) the agreement between the total calculated cross section (summed over two isospin states) and Gorbunov's experiments (summed over two-body and three-body breakup).

[NUCLEAR REACTIONS ^{3}He; calculated photoeffect; hyperspherical harmonics; sum rules.]

Fabre de la Ripelle and Levinger (FL)[1] used expansions in hyperspherical harmonics (h.h.) to calculate the trinucleon photoeffect for final isospin $\frac{3}{2}$. Recently Levinger and Fitzgibbon (LF)[2] (Ref. 2) applied the FL method[1] to calculate for final states of isospin $\frac{1}{2}$.

I note first a typographical error in LF, Table IV, concerning the cross section for photon absorption by ^{3}He with a final isospin $\frac{1}{2}$. For a photon energy E_γ of 11.63 MeV, and a Wigner-Bartlett exchange mixture, the cross section $\sigma(\frac{1}{2})$ should read 0.758 mb.

FL expand both initial and final wave functions in h.h. The rapid convergence of the h.h. expansion of the ground state wave function[3] justifies a severe truncation of the expansion for the final wave function. FL use a single term, with grand orbital 1; while Fang, Levinger, and Fabre de la Ripelle[4] use two terms, with grand orbitals 1 and 3, respectively.

Final states of isospin $\frac{3}{2}$ give only three-body breakup, so at large hyperradius the final state wave function is a plane wave in six dimensions. But nucleon-deuteron breakup is allowed for final isospin $\frac{1}{2}$, and the h.h. expansion for an N-d system does not converge. Hence, LF, Revai,[5] and Baz[6] face extra difficulties in dealing with "mixed boundary conditions."

Ballot and Fabre de la Ripelle[7] solve the mixed boundary condition problem by approximating the final isospin $\frac{1}{2}$ wave function as the product of a nucleon wave function and a deuteron wave function. They neglect the "three-body part" of the wave function, and also neglect the nucleon-deuteron interaction.

Gibson and Lehman[8] solve the photoeffect problem without using expansions in h.h., so the mixed boundary condition problem does not arise. They approximate the nucleon-nucleon interaction as a separable potential of Yamaguchi form.

LF bypass the difficult mathematics for final isospin $\frac{1}{2}$ states by considering a mathematical model in which the h.h. expansion of the potential energy of the system is truncated at two terms. In this model there is no two-body breakup. The LF calculation of $\sigma(\frac{1}{2})$ is just as simple as the FL calculation of $\sigma(\frac{3}{2})$. In each case, we approximate the final state wave function $\Psi_f(\xi)$ as a single partial wave with grand orbital 1, where the radial function $u_1(\xi)$ is a solution of an ordinary differential equation

$$-d^2u_1/d\xi^2+(35/4\xi^2)u_1+(M/\hbar^2)U_1^{(1)}(\xi)u_1=k^2u_1. \tag{1}$$

The effective potential $U_1^{(1)}(\xi)$ depends on the two-body force, and has different expressions for final isospin $\frac{1}{2}$ and $\frac{3}{2}$. [See LF, Eqs. (1.5) and (3.2).] The solution of Eq. (1) is normalized to the partial wave for a free system. We then use $u_1(\xi)$, together with Ballot's[3] ground state $u_0(\xi)$ and $u_2(\xi)$, for grand orbitals 0 and 2, respectively, to find the dipole matrix element between initial and final states. After integrations over five angles, we express the matrix element in terms of a one-dimensional integral ξ_{if} [(see LF, Eqs. (1.8) and (3.1)]. The cross section for a specified final isospin T_f is

$$\sigma(T_f)=(\pi^2/18)\alpha(M/\hbar^2)(E_\gamma k^4)(\xi_{if})^2\ . \tag{2}$$

LF discuss two difficulties with their mathematical model, both due to the model's omission of a final nucleon-deuteron state. (i) The calculated threshold for the photoeffect is at 7.7 MeV for the three-body breakup of ^{3}He. But the experimental threshold is at 5.5 MeV for proton-deuteron breakup. (ii) It is impossible in this model to calculate the (large) branching ratio of final isospin $\frac{1}{2}$ states for two-body breakup. LF also have two additional problems. (iii) The integrated cross section $\sigma_0(\frac{1}{2})$ is 23% above the sum-rule results, while $\sigma_0(\frac{3}{2})$ agrees with the sum rules to within 6% (LF, Table III). (iv) The calculated total cross section $\sigma(\frac{1}{2})+\sigma(\frac{3}{2})$ is not in good agreement with Gorbunov's measurement[9] of the total cross section, summed for two-body and three-body breakup.

In this paper I propose a simple modification of the LF calculation that solves problems (i), (iii), and (iv) above. I do not attempt to solve the more difficult problem of calculating the two-body branching ratio for isospin $\frac{1}{2}$.

In the LF calculation, the photon energy E_γ is taken as $\hbar^2k^2/M + 7.7$ MeV, since in this mathematical model there is no nucleon-deuteron final state. My modification consists of saying, "Yes, there is a nucleon-deuteron final state." Then for final isospin $\frac{1}{2}$, we should have a threshold at 5.5 MeV, in agreement with experiment. In Eq. (2) I now use the Ansatz

$$E_\gamma = \hbar^2k^2/M + 5.5\ . \tag{3}$$

The value of wave number k and overlap integral ξ_{if} are taken from LF.

We cannot expect a rigorous justification of the choice, Eq. (3), since at best we are performing a good *approximate* calculation for the mixed boundary condition problem. I base my heuristic argument on Baz's "interpolation method.[6]" For final isospin $\frac{1}{2}$, the final state wave function has different asymptotic forms for small hyperradius and for large hyperradius,

$$\Psi_{\text{int}}(\vec{\xi}) = \sum_L \Psi_L(\xi)y_L(\Omega)\ , \quad \text{small } \xi\ , \tag{4}$$

$$\Psi_{\text{ext}}(\vec{\xi}) = A\sum_L \Psi_L(\xi)y_L(\Omega) + B\Psi_{Nd}, \quad \text{large } \xi\ . \tag{5}$$

(I have neglected Baz's N-d component Ψ_{Nd} at small hyperradius.) The relation (3) is compatible with the expressions (4) and (5). For calculation of the overlap integral with the ground state wave function, I use a final state wave function given by the internal (4), severely truncated at the single term $L=1$ as discussed by FL. I then use Ψ_{int} to find the outgoing current (in six-dimensional space). Of course, this current, divided by the photon flux, gives us the cross section $\sigma(\frac{1}{2})$. The current is conserved as we go from small ξ to large ξ where we use the external wave function. A fraction of the current shows up either as the first term in (5) for three-body breakup, or as the nucleon-deuteron component Ψ_{Nd}.

The choice (3) automatically gives us the experimental threshold for photon absorption [problem (i) in LF]. My solution of problems (iii) and (iv) in a particular case is shown in Table I, which consists of Table IV of LF with the modification (3) in the choice for photon energy used in (2).

The integrated cross section for a Serber mixture (spin-dependent V^x potential, zero in odd parity two-body states) is now reduced to 29.2 MeV mb, only 9% above the sum rule value of 26.7 MeV mb. For a Wigner-Bartlett mixture, the integrated cross section is reduced to 20.9 MeV mb, or only 5% above the Thomas-Reiche-Kuhn value of 19.9 MeV mb. (As noted above, LF found discrepancies of 23% for each of these sum rules.)

The value of the moment σ_{-1} is unchanged by my new expression (3) for the photon energy. The previous good agreement found by LF be-

TABLE I. $\sigma(\frac{1}{2})$ for ^{3}He, with V^x potential.

E_γ (MeV)	Born	Serber	Wigner-Bartlett
5.9	0.0009	0.0045	0.0057
6.6	0.014	0.070	0.096
7.7	0.079	0.367	0.590
9.4	0.256	0.989	1.85
12.1	0.573	1.48	2.21
16.5	0.918	1.29	0.988
24.1	1.05	0.701	0.241
38.9	0.782	0.238	0.028
74.0	0.239	0.038	0.0004
198.8	0.0020	0.00026	0.00009
Integrated cross sections	49.1 MeV mb	29.2 MeV mb	20.9 MeV mb
Sum rule		26.7 MeV mb	19.9 MeV mb

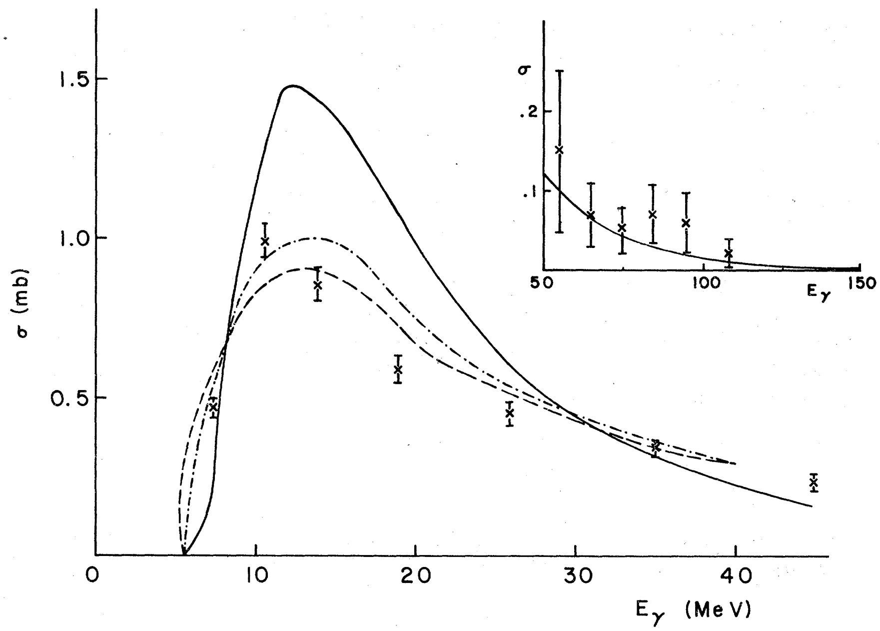

FIG. 1. The curves show calculated $\sigma(\frac{1}{2})$ for final isospin $\frac{1}{2}$. The dashed curve is from Gibson and Lehman, the dotted curve from Ballot and Fabre de la Ripelle and the solid curve from Table I. The points with errors show Gorbunov's experiment: $\sigma(2)$ for two-body breakup.

tween different calculated values of σ_{-1} and the experimental value persists in my present calculation.

Figures 1 and 2 compare $\sigma(\frac{1}{2})$ with experiment. In the former figure I compare the results of Table I, for a Serber mixture, with Gorbunov's experimental results[9] for $\sigma(2)$, the cross section for two-body breakup. We would expect the calculated $\sigma(\frac{1}{2})$ to lie somewhat above the experimental points, since some fraction of isospin $\frac{1}{2}$ states will decay by three-body breakup, for photon energies above 7.7 MeV. This expectation is generally satisfied.

I also show as a dashed curve in Fig. 1 the cross section $\sigma(\frac{1}{2})$ found by Gibson and Lehman[8] calculated using a separable approximation to the two-body t-matrix. Finally, I show as a dotted curve the Ballot Fabre calculations[7] of $\sigma(\frac{1}{2})$ in their nucleon-deuteron model of the final state. The BF and GL (Gibson and Lehman) curves agree well with each other, and also agree well with Gorbunov's experiments on two-body breakup.

We can avoid the problem of our inability to

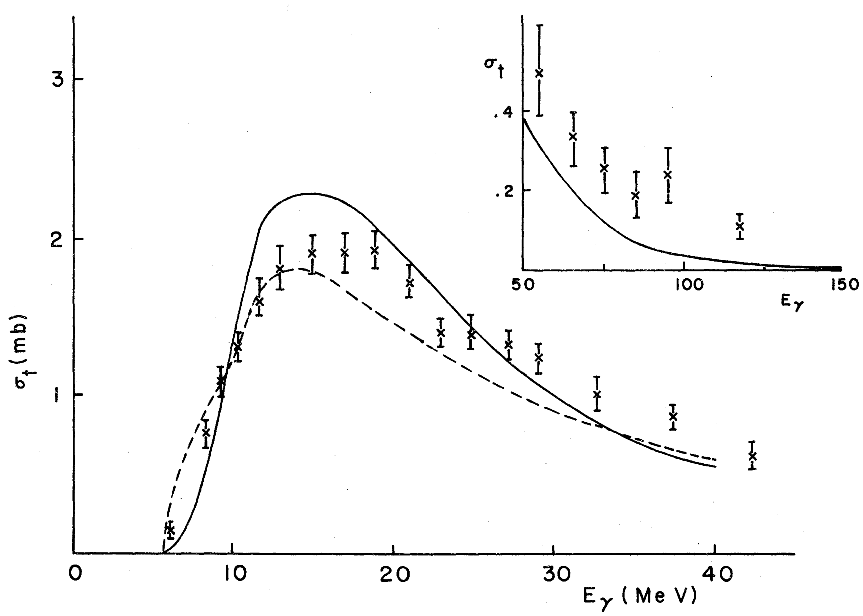

FIG. 2. The curves show calculated total cross sections, summed over final isospins. The dashed curve is from Gibson and Lehman; the solid curve is from this paper. The points with errors show Gorbunov's experiment for the cross section summed for two-body and three-body breakup.

calculate branching ratios for decay of the isospin $\frac{1}{2}$ state, by comparing calculated and experimental *total* cross sections. The calculated total cross section consists of Table I for $\sigma(\frac{1}{2})$ added to the FL results for $\sigma(\frac{3}{2})$, both for a V^x potential with Serber exchange. The latter consists of the sum of Gorbunov's $\sigma(2)$ for two-body breakup and $\sigma(3)$ for three-body breakup.[9] Figure 2 shows rather good agreement between the calculated (solid) curve and the experimental points. The GL calculation,[8] shown dotted, has a similar agreement with experiment. The solid curve gives an integrated total cross section of 58 MeV mb, while Gorbunov finds an integrated cross section of (70 ± 5) MeV mb.

In summary, I modify the LF calculation of the cross section for transitions to isospin $\frac{1}{2}$ by choosing the photon energy as $\hbar^2 k^2/M + 5.5$ MeV where 5.5 MeV is the threshold for p-d breakup. I find good agreement both with sum rule calculations and with Gorbunov's experiments.

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