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$E0$ matrix elements in ^{40,42,44}Ca

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Simple wave functions for ^{40}Ca from Gerace and Green and for ^{42,44}Ca derived from (d,p) data give a remarkably good fit to recently measured $E0$ matrix elements in ^{40,42,44}Ca.

[NUCLEAR STRUCTURE ^{40,42,44}Ca. Calculated $E0$ matrix elements in the three-component model.]

A recent high resolution (e,e') study of several Ca isotopes by Gräf *et al.*[1] reported accurate $E0$ matrix elements between the ground and first excited 0^+ states of ^{40,42,44,48}Ca. Their results, which are summarized in Table I for ^{40,42,44}Ca, are in agreement with earlier measurements,[2-5] but are much more accurate. Prior to the work of Gräf *et al.* the $E0$ matrix element had been calculated[6-8] for ^{40}Ca and ^{42}Ca. Only the calculations of Gerace and Green[6] are in reasonable accord with the data. In an attempt to describe the trend of $E0$ matrix elements in all even Ca isotopes, Gräf *et al.* performed a shell model calculation in a $(1d_{3/2})^{-m}(1f_{7/2})^{n+m}$ model space and found it to be incapable of reproducing the $E0$ magnitudes or the observed A dependence. This failure prompted them to introduce a simple model in which the first two 0^+ states of $^{40+n}$Ca are orthogonal linear combinations of an $(fp)^n$ configuration and a collective state. The ratio of collective to $(fp)^n$ amplitudes in the various ground states is v/δ_n, where v is the interaction matrix element between the two basis states (assumed independent of n) and δ_n is obtained for each isotope separately by fitting[1] to the excitation energy Δ_n of the excited 0^+ state $\delta_n = \frac{1}{2}[(\Delta_n^2 - 4v^2)^{1/2} + \Delta_n]$. The model appears to fit the data, but close inspection reveals an inconsistency, simultaneously requiring $v/\delta_n \ll 1$ and $v/\delta_n \approx 1$. Furthermore, the model of Ref. 1 gives $\approx 50\%$ core excitation in the ground states of ^{42,44}Ca, much larger than is thought to be the case, and only 8% in ^{40}Ca, much smaller than expected. Another deficiency is that their model cannot account for the third 0^+ level, which lies about 1.5 MeV above the second in most of the even Ca nuclei. We report here on an even simpler model[9] that does not have these deficiencies and does fit the $E0$ data.

In ^{40,42,44}Ca, a large body of data on γ decay and transfer experiments (for summary, see Ref. 10) has demonstrated the need for at least two separate core-excited components in the ground state. These, together with $(fp)^n$, then give three low-lying 0^+ states. In ^{42}Ca, e.g., for the 0_i^+ states in terms of the dominant components we have

$$0_i^+ = a_i(fp)^2_{01} + b_i(fp)^4_{00}(sd)^{-2}_{01} + c_i(fp)^6_{01}(sd)^{04}_{00} ,$$

where the double subscripts denote JT. We show below that the $E0$ matrix element connecting the first two 0^+ states at ^{42}Ca depends only on the a's which are known[9] from ratios of spectroscopic factors in the ^{41}Ca(d,p) reaction.[11]

Since each core-excited component in ^{42}Ca involves exactly two protons, we have (using orthogonality: $b_0b_1 + c_0c_1 = -a_0a_1$)

$$\langle 0_1^+ | E0 | 0_0^+ \rangle = \langle 0_1^+ | \sum r_p^{\,2} | 0_0^+ \rangle ,$$

i.e.,

$$E0(^{42}\mathrm{Ca}) = +\, 2a_0a_1(r_{fp}^{\,2} - r_{sd}^{\,2}) .$$

Analysis of data[11] from ^{41}Ca(d,p) gives[9,10] $a_1^{\,2} = 0.18$, $a_0^{\,2} = 0.76$, i.e., 76%$(fp)^2$ in the ^{42}Ca(g.s.). Thus for ^{42}Ca, $\langle 0_1^+ | E0 | 0_0^+ \rangle = 0.74(r_{fp}^{\,2} - r_{sd}^{\,2})$.

For ^{44}Ca, the three 0^+ states are expected to be of the form

$$0_i^+ = a_i(fp)^4_{02} + b_i(fp)^6_{01}(sd)^{-2}_{01} + c_i\,(fp)^6_{01}\,(sd)^{04}_{00} .$$

Again each of the core-excited components involves the excitation of two protons, so that $E0(^{44}\mathrm{Ca}) = 2a_0a_1(r_{fp}^{\,2} - r_{sd}^{\,2})$. Data[12] from ^{43}Ca$(d,p)$ give $a_0^{\,2} = 0.83$, $a_1^{\,2} = 0.17$, and hence $E0(^{44}\mathrm{Ca}) = 0.75(r_{fp}^{\,2} - r_{sd}^{\,2})$.

Thus, the ratio of $E0$'s for ^{42}Ca and ^{44}Ca de-

TABLE I. Monopole transition matrix elements connecting ground and first-excited 0^+ states in ^{40,42,44}Ca.

Nucleus	$E0$ (fm^2) Measured[a]	Calculated[b]
^{40}Ca	2.53 ± 0.41	2.11
^{42}Ca	5.24 ± 0.39	5.38
^{44}Ca	5.45 ± 0.41	5.45

[a] Reference 1.
[b] Using $r_{fp}^{\,2} - r_{sd}^{\,2} = 7.27$ fm^2.

TABLE II. Percentages of core excitation and magnitudes of charge radii R_n for ground states of ^{40,42,44}Ca.

	$1-a_0^2$		R_n (fm)			R_n-R_0 (fm)	
Nucleus	Ref. 1	Present	Ref. 1	Present	Exp	Ref. 1	Present
^{40}Ca	0.08	0.18	3.43	3.430	3.43		
^{42}Ca	0.50	0.24	3.46	3.445	3.46	0.032	0.015
^{44}Ca	0.39	0.16	3.45	3.442	3.46	0.023	0.012

pends only on the (d,p) data and is predicted to be about unity. To fit the absolute magnitude for ^{44}Ca requires $r_{fp}^2 - r_{sd}^2 = 7.27$ fm^2. For wave functions in a standard Woods-Saxon well ($r_0 = 1.26$ fm, $a = 0.60$ fm, $\lambda = 25$), this value of Δr^2 corresponds to a $1f_{7/2} - 1d_{3/2}$ binding-energy difference of about 5 MeV.

The situation in ^{40}Ca is only slightly more complicated. Here, to be consistent we use $0_i^+ = a_i(fp)^0(sd)^{-0} + b_i(fp)^2(sd)^{-2} + c_i(fp)^4(sd)^{-4}$, but now the effective number of proton excitations is one for the 2p-2h part and two for the 4p-4h component, so that

$$E0(^{40}\mathrm{Ca}) = (b_0 b_1 + 2c_0 c_1)(r_{fp}^2 - r_{sd}^2)\,,$$

and orthogonality alone is not sufficient to reduce it to a single term. Instead we use the wave functions of Gerace and Green[6] in this basis for ^{40}Ca. The result is $E0(^{40}\mathrm{Ca}) = 0.29(r_{fp}^2 - r_{sd}^2)$. Using the value of $r_{fp}^2 - r_{sd}^2$ needed to fit ^{44}Ca gives $E0(^{40}\mathrm{Ca}) = 2.11$ fm^2, close to the experimental value (see Table I). Unlike Ref. 1, we find no need to include 6p-6h and 8p-8h components in order to get the experimental value for ^{40}Ca.

We can also use this model to compute ground state charge radii

$$R^2(^{42}\mathrm{Ca}) = 2a_0^2(r_{fp}^2 - r_{sd}^2)/20 + R_{\mathrm{core}}^2\,,$$

$$R^2(^{44}\mathrm{Ca}) = 2a_0^2(r_{fp}^2 - r_{sd}^2)/20 + R_{\mathrm{core}}^2\,,$$

$$R^2(^{40}\mathrm{Ca}) = (b_0^2 + 2c_0^2)(r_{fp}^2 - r_{sd}^2)/20 + R_{\mathrm{core}}^2\,.$$

We have used the measured ground state charge radius of ^{40}Ca($R_0 = 3.43$ fm) and the above expression to evaluate R_{core}(3.42 fm) and then used this value of R_{core} to compute the other radii. R_n results are given in Table II, where they are compared with similar results from Ref. 1. (We have corrected a small arithmetical error for the ^{44}Ca entry in Ref. 1.)

It can be seen that not all of the isotope shift can be accounted for in this model. This is expected, since other effects, such as coupling to the breathing mode, are known to make small contributions also.

In summary, the wave functions of Gerace and Green for ^{40}Ca and the ones derived from (d,p) data for ^{42,44}Ca, which give reasonable fits[9,10] to a variety of data, also quite adequately reproduce the measured $E0$'s in these nuclei.

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