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Magnetic moments of single-particle states around ^{208}Pb

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Magnetic moments of single-particle and single-hole states in the ^{208}Pb region are calculated in a large shell model space taking into account the core polarization and meson exchange effects to a high order of perturbation.

[NUCLEAR STRUCTURE ^{207}Tl, ^{209}Bi, ^{207}Pb, ^{209}Pb; magnetic dipole moments; core polarization; meson exchange currents.]

Because of the simple and well-defined electromagnetic operator involved, nuclear magnetic moments have provided one of the basic tests of the nuclear shell model. Frequently, however, a detailed comparison with the experimental data shows not only that the pure shell model gives an inadequate approximation and that the nuclear wave functions need improvements by an admixing of configurations,[1,2] but also that the electromagnetic operator, which acts on the constituent nucleons individually, may not suffice. It has been suggested[3] that an explanation, which may, we hope, apply also to a larger class of phenomena, lies in the interaction of the external field with the mesonic currents in the nuclei.

In the present paper we examine the meson exchange effects on the magnetic moments of the single-particle and the single-hole nuclei in the vicinity of ^{208}Pb. Although the magnetic properties in this region have already been investigated either on their core polarization aspects[4-7] or meson current aspects,[3,8] we are reopening the subject because new data have recently been obtained[9-15] and because we have performed systematic calculations of the mesonic effects for many states in a large model space and to a higher order of perturbation than has been done to date.

In the usual shell model the magnetic dipole operator is taken to be the one-particle operator

$$\vec{M}_{sp} = g_l \vec{l} + g_s \vec{s}\,, \tag{1}$$

where $g_l = 1\ \mu_N$, $g_s = 5.58\ \mu_N$ for a proton, and $g_l = 0$, $g_s = -3.82\ \mu_N$ for a neutron. In the series expansion of the transition amplitudes induced by the operator M_{sp}, there exist two first order terms as illustrated in Fig. 1(a). In the lead region they correspond to the excitations of the proton configuration $[(h_{11/2}^{-1} h_{9/2})1^+]$ and the neutron configuration $[(i_{13/2}^{-1} i_{11/2})1^+]$. They are the first terms of summable series. In what follows we present the results of our calculations of the core polarization effects in the random phase approximation (RPA) in the model space to be defined below.

The exchange magnetic moment operator includes a translationally invariant part, $M_{\rm rot}$, which is determined by the rotational part of the current density, and a translationally noninvariant part, $M_{\rm div}$, which is determined by the irrotational part of the current density:

$$\vec{M}_{\rm ex} = \vec{M}_{\rm rot} + \vec{M}_{\rm div}\,. \tag{2}$$

The translationally invariant terms have been classified by Chemtob and Rho[16] as follows:

$$\begin{aligned}\vec{M}_{12}^{\rm rot} = \tfrac{1}{2}\{&(\vec{\tau}_1\times\vec{\tau}_2)_z[(\vec{\sigma}_1\times\vec{\sigma}_2)g_{\rm I} + T_{12}^{(\times)}g_{\rm II}]\\ &+(\vec{\tau}_1-\vec{\tau}_2)_z[(\vec{\sigma}_1-\vec{\sigma}_2)h_{\rm I} + T_{12}^{(-)}h_{\rm II}]\\ &+(\vec{\tau}_1+\vec{\tau}_2)_z[(\vec{\sigma}_1+\vec{\sigma}_2)j_{\rm I} + T_{12}^{(+)}j_{\rm II}]\\ &+(\vec{\tau}_1\cdot\vec{\tau}_2)[(\vec{\sigma}_1+\vec{\sigma}_2)m_{\rm I} + T_{12}^{(+)}m_{\rm II}]\}\,,\end{aligned} \tag{3}$$

where T_{12}^{ϵ} $(\epsilon = \pm, \times)$ are defined as

$$T_{12}^{\epsilon} = (\vec{\sigma}_1\epsilon\vec{\sigma}_2)\cdot\hat{r}_{12}\hat{r}_{12} - \tfrac{1}{3}\sigma_1\epsilon\sigma_2\,. \tag{4}$$

The functions $g, h, \ldots$ which depend on the internucleon separation $r_{12} = |\vec{r}_1 - \vec{r}_2|$ will be defined below.

The translationally noninvariant terms may be written according to Hyuga and Arima,[8]

$$\begin{aligned}\vec{M}_{12}^{\rm div} = &(\vec{\tau}_1\times\vec{\tau}_2)_z(\vec{x}_{12}\times\vec{y}_{12})\\ &\times(F_{\rm I} + \vec{\sigma}_1\cdot\vec{\sigma}_2 F_{\rm II} + S_{12}F_{\rm III})\,,\end{aligned} \tag{5}$$

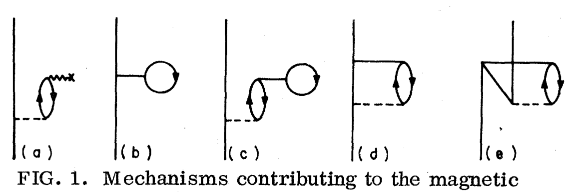

FIG. 1. Mechanisms contributing to the magnetic dipole moments in single-particle nuclei (dotted line: nucleon-nucleon interaction, wiggly line: electromagnetic interaction, horizontal line: exchange current, vertical line: nucleon): core polarization (a); meson exchange (b)-(e). Time-reversed diagrams are implied.

where $S_{12}=3(\vec{\sigma}_1\cdot\hat{r}_{12})(\vec{\sigma}_2\cdot\hat{r}_{12})-\vec{\sigma}_1\cdot\vec{\sigma}_2$, $\vec{x}_{12}=m_\pi\vec{r}_{12}$, $\vec{y}_{12}=\frac{1}{2}m_\pi(\vec{r}_1+\vec{r}_2)$, with m_π being the pion mass. The radial functions F, as well as $g, h, \ldots,$ are defined as follows.

(a) One-pion exchange. Pion current plus pair excitation plus nucleonic current:

$$\begin{aligned}
g_{\rm I}&=\left[\tfrac{2}{3}f^2\frac{m_N}{m_\pi}(2x-1)+\tfrac{1}{9}\eta_1\right]\phi_0(x)\,,\\
g_{\rm II}&=\left[-2f^2\frac{m_N}{m_\pi}(x+1)-\tfrac{1}{6}\eta_1\right]\phi_2(x)\,,\\
h_{\rm I}&=j_{\rm I}=-\tfrac{1}{9}\eta_2\phi_0(x)\,,\\
h_{\rm II}&=j_{\rm II}=-\tfrac{1}{3}\eta_2\phi_2(x)\,,\\
F_{\rm II}&=\tfrac{1}{3}f^2\frac{m_N}{m_\pi}\phi_0(x)\,,\\
F_{\rm III}&=\tfrac{1}{3}f^2\frac{m_N}{m_\pi}\phi_2(x)\,,
\end{aligned}\tag{6}$$

where $f^2=0.08$, $\eta_i=2(\mu_p-\mu_n)m_\pi^3h_i(0)$ with $h_1(0)=0.074/m_\pi^3$, $h_2(0)=0.0658/m_\pi^3$, and μ_p, μ_n are the usual anomalous magnetic moments of the proton and the neutron respectively. The radial functions $\phi(x)$ are

$$\begin{aligned}
\phi_0(x)&=\frac{e^{-x}}{x}\,,\\
\phi_2(x)&=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\phi_0(x)\,.
\end{aligned}\tag{7}$$

(b) $\rho\pi\gamma$ and $\omega\pi\gamma$ dissociation currents:

$$\begin{aligned}
m_{\rm I}&=2R\left(\frac{m_\pi}{m_\rho}\right)^3\phi_0^\rho(x)\,,\\
m_{\rm II}&=6R\left(\frac{m_\pi}{m_\rho}\right)^3\phi_2^\rho(x)\,,\\
h_{\rm I}&=j_{\rm I}=W\left(\frac{m_\pi}{m_\omega}\right)^3\phi_0^\omega(x)\,,\\
h_{\rm II}&=j_{\rm II}=3W\left(\frac{m_\pi}{m_\omega}\right)^3\phi_2^\omega(x)\,,
\end{aligned}\tag{8}$$

where m_ρ, m_ω stand for the mass of the ρ meson and the ω meson, and $R=-0.69$, $W=-4.07$, and

$$\phi_i^v=\frac{m_v^2}{m_v^2-m_\pi^2}\left[\phi_i(m_\pi r)-\left(\frac{m_v}{m_\pi}\right)^2\phi_i(m_v r)\right].\tag{9}$$

(c) Multipion exchange currents. Although the corresponding magnetic operators can be derived from the field theory, we have used the phenomenological Sachs moment associated with the central and tensor parts of the Hamada-Johnston potential:

$$\begin{aligned}
F_{\rm I}&=\frac{f^2}{3}\frac{m_N}{m_\pi}[4.45-8.65\phi_0(x)]\phi_0^2(x)\,,\\
F_{\rm II}&=\frac{f^2}{3}\frac{m_N}{m_\pi}[1-2.31\phi_0(x)+8.77\phi_0^2(x)]\phi_0(x)\,,\\
F_{\rm III}&=\frac{f^2}{3}\frac{m_N}{m_\pi}[1-0.698\phi_0(x)+0.288\phi_0^2(x)]\phi_2(x)\,.
\end{aligned}$$

In the zeroth order correction the valence nucleon interacts with the core particles via the two-body exchange operators [Fig. 1(b)]. This correction has already been calculated in the past; we repeated the calculation including all the occupied orbits. The next order is given by six diagrams [Figs. 1(c)–1(e)], the first terms of the sequences of forward-going bubble diagrams which can be summed to all orders of perturbation as in the usual Tamm-Dancoff approximation.

With the exception of the zeroth order diagram for which the space was enlarged, we took as intermediate states in all other diagrams all the allowed particle-hole configurations of our model space spanned by the proton orbits $3s\frac{1}{2}$, $2d\frac{1}{2}$, $\frac{3}{2}$, $1g\frac{7}{2}$, $1h\frac{11}{2}$, the neutron orbits $4s\frac{1}{2}$, $3d\frac{5}{2}$, $\frac{3}{2}$, $2g\frac{7}{2}$, $\frac{9}{2}$, $1i\frac{11}{2}$, $1j\frac{15}{2}$ and the neutron-proton orbits $3p\frac{3}{2}$, $\frac{1}{2}$, $2f\frac{7}{2}$, $\frac{5}{2}$, $1h\frac{9}{2}$, and $1i\frac{13}{2}$. Matrix elements were calculated in a spherical harmonic oscillator basis with the parameter $\hbar\omega=7.6$ MeV. To simulate the short-range correlations we imposed a lower cutoff at $r=0.4$ fm in the radial integrals of the exchange operators. We used two typical phenomenological nucleon-nucleon potentials, one with a Gaussian radial shape and an attractive singlet-odd component (the Gillet force[17]), the other with a Yukawa shape and a mildly repulsive singlet-odd component (the Perez force[18]). The single-particle energies used in our calculations were taken from experimental data tables.[19]

We focus our attention first on the states with known magnetic moments. In Table I we present the core polarization effects calculated in the RPA with the Gillet potential or the Perez potential. The corrections are all in the desired direction, although insufficient in most cases even if account is taken of the spread in the predicted results due to uncertainties in the residual interaction. Vergados[7] has calculated the core polarization effects on the magnetic moments of the $\frac{1}{2}^+$ ($3s\frac{1}{2}$), $\frac{1}{2}^-$ ($3p\frac{1}{2}$), and $\frac{5}{2}^-$ ($2f\frac{5}{2}$) states using the G-matrix elements based on the Hamada-Johnston potential. His corrections are larger than ours but still insufficient. In the values presented in Table I we have included, in addition to the influence of the 1^+ phonon involved in the RPA, the effects of the vibrational states 2^+, 3^-, 4^+, and 5^- of ^{208}Pb coupled to single-particle states. These effects were calculated by Towner *et al.*[6] using the mixing amplitudes due to Hamamoto,[20] and the g values $g(3^-)=0.58$, $g(5^-)=0.02$, $g(2^+)=g(4^+)=0.5$. They are generally small with the exception of the $i\frac{13}{2}$ proton state and the $j\frac{15}{2}$ neu-

TABLE I. Core polarization effects $\delta\mu = \mu - \mu_{s.p.}$ (in μ_N); calculated in the random phase approximation with the Perez and the Gillet potentials.

	Orbits	$\mu_{s.p.}$	Perez	Gillet	Expt.
^{207}Tl	$3s\frac{1}{2}$	2.79	−0.683	−0.403	−0.96 ± 0.18 [a]
^{209}Bi	$1h\frac{9}{2}$	2.62	0.529	0.307	1.49 ± 0.01 [b]
	$1i\frac{13}{2}$	8.79	−1.340	−1.122	−0.78 ± 0.10 [c]
^{207}Pb	$3p\frac{1}{2}$	0.64	−0.068	−0.012	−0.045 ± 0.001 [b]
	$2f\frac{5}{2}$	1.37	−0.334	−0.155	−0.58 ± 0.03 [d]
	$1i\frac{13}{2}$	−1.91	0.757	0.527	0.90 ± 0.03 [e]
^{209}Pb	$2g\frac{9}{2}$	−1.91	0.503	0.240	0.58 ± 0.06 [f]

[a] Reference 12.
[b] Reference 11.
[c] References 9 and 10.
[d] Reference 13.
[e] Reference 14.
[f] Reference 15.

tron state.

The zeroth order exchange correction is rather sensitive to the truncation of space and shows no sign of stabilizing even at the limits of our relatively large model space. Consequently we used the full space (all occupied orbits) in the calculation of this term (Fig. 2). In the one-pion range the exchange processes are rather well defined, whereas in the multipion range uncertainties in the exchange mechanism and the coupling constants will make the results somewhat less reliable. We list their contributions separately in Table II. For each group the corrections in the zeroth order [Fig. 1(b)] and in the higher orders [infinite series based on Figs. 1(c)–1(e)] are shown. The contributions in the one-pion range, which result from the opposing effects of the pionic and pair current on the one hand and the nucleonic current on the other, generally act in the right direction in moving the calculated moments toward the experimental values, with the exception of the $\frac{13}{2}^+$ state in ^{207}Pb and the $\frac{9}{2}^+$ state in ^{209}Pb. They are further enhanced in all cases (except in the $\frac{1}{2}^-$ state of ^{207}Pb for which the mesonic corrections are small) by the combined effects of the ρ, ω dissociations and the multipion exchange. These effects are substantial and, although their importance may have been overestimated in the phenomenological approach we used, the same conclusion was reached by Konopka *et al.*,[21] who calculated the multipion range effects in magnetic moments of nuclei in the oxygen region by using heavy boson exchange operators. Higher order contributions, neglected in most previous calculations, turned out to be generally small except for states lying near the Fermi surface for which they are comparable to the lowest order terms.

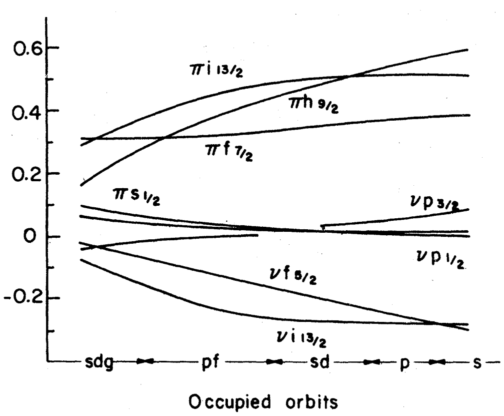

FIG. 2. Effects of space truncation. The one-pion exchange contributions in the zeroth order [Fig. 1(b)] are shown as functions of the number of occupied orbits with the results corresponding to the full space being given at the extreme right.

Both core polarization and meson effects on magnetic moments of many single-particle and single states in $A = 207$, 209 nuclei are presented in Table III. We show the sum of the core polarization effects and the one-pion exchange effects calculated with the Perez potential in column 3, to which we add the $\rho\omega$ and multipion effects in column 4; the same effects with the Gillet potential are shown respectively in columns 5 and 6. The last column lists the available experimental data. With the exception of the $\frac{13}{2}^+$ and $\frac{9}{2}^+$ states already mentioned, and the $\frac{13}{2}^+$ state of ^{209}Bi known to have large vibrational components, the mesonic effects are seen to play an important role in explaining the magnetic moments of simple nuclei in the lead region. Within the limitations of the nuclear model and the uncertainties of the two-nucleon interaction, our present calculation of the magnetic moments and a similar calculation

TABLE II. Meson exchange contributions to magnetic moments (in μ_N).

	Orbits	One-pion exchange Zeroth	One-pion exchange Series	$\rho\omega$ + multipion Zeroth	$\rho\omega$ + multipion Series	Total
^{207}Tl	$3s\frac{1}{2}$	−0.013	−0.025	−0.077	0.014	−0.101
^{209}Bi	$1h\frac{9}{2}$	0.578	−0.015	0.560	−0.010	1.113
	$1i\frac{13}{2}$	0.513	0.002	0.572	0.013	1.100
^{207}Pb	$3p\frac{1}{2}$	0.001	−0.006	−0.026	0.001	−0.032
	$2f\frac{5}{2}$	−0.300	−0.018	−0.233	0.002	−0.549
	$1i\frac{13}{2}$	−0.281	0.012	−0.365	−0.008	−0.642
^{209}Pb	$2g\frac{9}{2}$	−0.143	0.003	−0.219	−0.001	−0.360

TABLE III. Predicted magnetic moments (in μ_N) including core polarization and meson exchange effects.

	Orbits	Perez + pion	Perez $+\rho\omega+n\pi$	Gillet + pion	Gillet $+\rho\omega+n\pi$	Expt.
^{207}Tl	$3s\frac{1}{2}$	2.07	2.01	2.35	2.29	1.83 ± 0.18[a]
	$2d\frac{3}{2}$	0.58	0.79	0.45	0.65	
	$1h\frac{11}{2}$	7.23	7.77	7.45	7.98	
	$2d\frac{5}{2}$	4.29	4.50	4.54	4.75	
	$1g\frac{7}{2}$	2.76	3.28	2.72	3.23	
^{209}Bi	$1h\frac{9}{2}$	3.80	4.26	3.60	4.04	4.117 ± 0.011[b]
	$2f\frac{7}{2}$	5.55	5.86	5.78	6.09	
	$1i\frac{13}{2}$	8.03	8.55	8.25	8.77	8.01 ± 0.10[c]
	$2f\frac{5}{2}$	1.60	1.89	1.43	1.73	
	$3p\frac{3}{2}$	3.23	3.25	3.26	3.50	
	$3p\frac{1}{2}$	0.149	0.164	0.08	0.09	
^{207}Pb	$3p\frac{1}{2}$	0.563	0.538	0.619	0.594	0.593 ± 0.001[b]
	$2f\frac{5}{2}$	0.714	0.483	0.89	0.66	0.786 ± 0.03[d]
	$3p\frac{3}{2}$	−1.29	−1.30	−1.48	−1.49	
	$1i\frac{13}{2}$	−1.42	−1.80	−1.65	−2.03	−1.01 ± 0.03[e]
	$2f\frac{7}{2}$	−1.55	−1.74	−1.75	−1.94	
	$1h\frac{9}{2}$	0.75	0.35	0.99	0.59	
^{209}Pb	$2g\frac{9}{2}$	−1.55	−1.77	−1.81	−2.03	−1.33 ± 0.03[f]
	$1i\frac{11}{2}$	0.61	0.22	0.83	0.22	
	$1j\frac{15}{2}$	−1.02	−1.35	−1.20	−1.53	
	$3d\frac{5}{2}$	−1.42	1.52	−1.68	−1.78	
	$4s\frac{1}{2}$	−1.25	−1.20	−1.52	−1.47	
	$2g\frac{7}{2}$	0.82	0.57	1.02	0.77	
	$3d\frac{3}{2}$	0.80	0.68	0.92	0.81	

[a]Reference 12.
[b]Reference 11.
[c]References 9 and 10.
[d]Reference 13.
[e]Reference 14.
[f]Reference 15.

of dipole magnetic transitions[22] have succeeded in explaining, at least in a qualitative way, a large body of data on the magnetic properties of simple nuclei in this region.

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