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Pion production in relativistic heavy-ion reactions

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High-energy heavy-ion reaction data on pion production near $\theta_{lab} = 0°$ and 180° are analyzed. The result strongly suggests that the forward (backward) pions are due to materialization of the kinetic energy of the "effective projectiles" ("effective targets") in peripheral nucleus-nucleus collisions. A statistical interpretation for the observed energy distributions is discussed.

NUCLEAR REACTIONS Proton and HI projectiles on various targets (H to Pb) at $E \simeq 1.05$–7.52 GeV/nucl.; analyzed single-pion inclusive spectra at forward and backward directions ($\theta_{lab} \approx 0°$ and 180°); discussed cumulative effect and statistical interpretation in HI reactions.

Recent experiments[1-3] on pion production in relativistic heavy-ion reaction in the fragmentation regions have received much attention. One of the main reasons for this interest is the fascinating fact that some of these pions observed in the forward directions (at $\theta_{lab} = 0°$ and 2.5°) carry kinetic energies higher than the kinetic energy per nucleon of the projectile.[1,2] Furthermore, similar effects have also been seen in the reactions $p + A \to \pi^- + X$ and $d + A \to \pi^- + X$ for various target nuclei A where the pions are detected at $\theta_{lab} = 180°$.[3] These observations gave rise in particular to the question[1-3]: Are the pions "cumulatively produced?" That is, are they produced in processes in which several nucleons inside the projectile (target) nucleus participate in a cooperative fashion?

In the present paper, an attempt is made to understand the production mechanism of these pions by analyzing the above-mentioned data.[1-3] This analysis is based on the simple physical picture for multiparticle production on nuclei discussed in the previous papers.[4-6]

The main features of this picture[4-6] are

(a) The time needed for the formation of multibody final states in hadron-hadron collisions at high energies is so *long* that in a high-energy hadron-nucleus multiparticle production process, the nucleons in the path of the incident hadron inside the target nucleus can be viewed as acting *collectively*, and in first-order approximation be considered as a *single* object—an "effective target."

(b) The hadron-effective target process can be described by the *same* physical picture as that used to describe the collision between two hadrons. In particular, such a process is *either* an *F-type* (*fragmentation*)[7] *or* a *V-type* (*violent collision*)[8] process. That is to say, hadron-effective target collisions can also be classified according to the amount of energy and momentum transfer in each event into two different types [small in F-type (gentle) but large in V-type (violent) collisions]. In the picture of such a hadron-"hadron" collision, the projectile and the target are spatially extended objects. Either (in the former case) they go through each other, become excited, and fragment separately; or (in the latter case) they hit each other so hard that they arrest each other and form a conglomerate which expands and then decays when a critical volume is reached. Furthermore, in terms of the usual semiclassical picture, it means that also in collisions between these objects the average impact parameter in fragmentation processes is larger than that in the corresponding violent collisions.

(c) The mass of the effective target is proportional to the "thickness" ν_{ET} of the target nucleus. (To be more precise, $\bar{\nu}_{ET}$ is the average number of nucleons in the target nucleus along the path of the incident hadron. "The average mass number of the effective target" may be a more suitable name.) That is,

$$\bar{M}_{ET} = \bar{\nu}_{ET} M \tag{1}$$

where $\bar{M}_{ET}$ and M are the average mass of the effective target and the mass of the nucleon, respectively.

(d) *A high-energy nucleus-nucleus collision can be considered as the simultaneous collision between all possible pairs of effective targets and "effective projectiles."* (The latter are defined analogous to the former. The average mass of an effective projectile $\bar{M}_{\mathrm{EP}}$ is equal to $\bar{\nu}_{\mathrm{EP}}M$, where $\bar{\nu}_{\mathrm{EP}}$ is the counterpart of $\bar{\nu}_{\mathrm{ET}}$. We recall that the average binding energy per nucleon in nuclei in collision processes at these energies is negligible. Hence the dimension of such effective targets and effective projectiles in the directions perpendicular to the incident axis is that of a nucleon.) The collision between an effective target and an effective projectile can also be described by the *same* picture as that used to describe the collision between two hadrons which means, in particular, it is either an F-type or a V-type process. [For the implication of this statement, cf. (b).]

(e) A high-energy nucleus-nucleus collision is in general a mixture of both types of effective projectile-effective target processes. A *nucleus-nucleus* reaction is said to be *a violent collision process* if all or almost all the possible pairs of effective projectile-effective target processes are violent. The notion, "nucleus-nucleus fragmentation processes" will be used in a similar way.

(f) Since the average impact parameter in an F-type effective projectile-effective target process is larger than that in a corresponding V-type process [see (b) and (c)] it is clear that *a nucleus-nucleus fragmentation will most likely take place in a peripheral collision of the two nuclei.*

Experimental evidences for the present picture in the case of hadron-nucleus processes and in the case of violent nucleus-nucleus collisions have already been given in Refs. 4, 5, and 6. The production of pions[1-3] and to some extent also of nuclide[9,10] in nucleus-nucleus fragmentation processes will be discussed in this paper.

We consider a nucleus-nucleus collision where A_1 and A_2 are the mass numbers of the two colliding nuclei, and ϵ is the relative incident kinetic energy per nucleon. In the laboratory system, before the collision, the nucleus A_2 is at rest, every nucleon in the nucleus A_1 has the kinetic energy $\epsilon_{1\,\mathrm{lab}}=\epsilon$, and all the effective projectiles in A_1 move with the velocity

$$\beta_{1\,\mathrm{lab}}=[\epsilon(\epsilon+2M)]^{1/2}/(\epsilon+M)\ , \tag{2}$$

where M is the nucleon mass.

According to the above-mentioned picture, fragmentation of the projectile nucleus occurs when a peripheral collision between this and the target nucleus takes place in such a way that all or almost all of the processes between the possible effective projectile-effective target pairs are of the F type.

We now focus our attention on the effective projectile in such an F-type process. Before the collision it moves in the laboratory system with the average kinetic energy

$$\langle E_{1\,\mathrm{lab}}^{\mathrm{kin}}\rangle=\bar{\nu}_1\epsilon\ , \tag{3}$$

where $\bar{\nu}_1$ is the "average mass number" of the effective projectile. [The average mass is $\bar{\nu}_1 M$. See Eq. (1) and the discussion in (d).] In order to study the origin of the energetic pions observed in the forward directions it is useful to ask: What happens to the amount of kinetic energy $E_{1\,\mathrm{lab}}^{\mathrm{kin}}$ carried by the effective projectile after the collision? To answer this question, we recall: Firstly, in F-type (that is, gentle) processes, the energy transfer (from the effective projectile to the effective target) is very small compared to $E_{1\,\mathrm{lab}}^{\mathrm{kin}}$. Secondly, because of baryon-number conservation, it is *not* possible to convert the masses of the nucleons in the effective projectile into kinetic energy. Thirdly, since the average binding energy of nucleons in nuclei is very small compared to $E_{1\,\mathrm{lab}}^{\mathrm{kin}}$, the effect of binding energies is negligible. These arguments, taken together with the laws of energy and momentum conservation, lead us to the following answer: *After the collision, the above-mentioned amount of kinetic energy of the effective projectile can be used in retaining its high velocity and/or in particle emission in accordance with the conservation laws (baryon-number, energy, longitudinal momentum, etc.)* The limiting cases are (i) The excited effective projectile retains its velocity (which is approximately equal to that before the collision) until it fragments (isotropically in its rest frame into nucleons, α particles etc.).[11] (ii) A maximum amount of its kinetic energy is used up in emitting energetic low-mass particles in the forward direction ($\theta_{1\mathrm{ab}}=0$) while the velocity of the excited effective projectile is reduced to a minimum.[12] As we shall see in the comparison with experiments, there are strong evidences for the existence of both limiting cases. The frequency of occurrence of (i) is, however, much higher than that of (ii).

We note that in respect of pion emission, the above-mentioned mechanism is similar to the bremsstrahlung model of Heisenberg[13] and the two-fire-ball model of Tagaki[14] and Cocconi[15] proposed for hadron-hadron collisions; and in respect of the collective behavior of nucleons in nuclei, this mechanism is closely related to the cumulative production model proposed by Baldin *et al.*[1] and to a number of other models[16] for hadron-nucleus and nucleus-nucleus collision discussed in the literature. As far as nucleus-nucleus collision is concerned, the fundamental difference between the models proposed by other authors[1,16] and the present model is that the latter is based on a specific physical picture,[4-6] the characteristics of which are explicitly given in (a)–(f) above.

According to this picture, *the source of the energetic pions observed in the forward angles is the effective projectile which undergoes a gentle (that is, F-type) collision.* Since *the effective projectile behaves like a single hadron with average mass $\bar{\nu}_1 M$ and average kinetic energy $\bar{\nu}_1\epsilon$* [see (d) and Eq. (3)], in studying the energy distribution of the emitted pions, it is very natural to ask: Knowing the total kinetic energy of the effective projectile, how large is the probability for an emitted pion to carry a given fraction of the total kinetic energy of the emitting system (i.e., of the effective projectile)? This means, in order to compare the energy distributions of the observed fast pions near $\theta_{1ab}=0$ produced *in processes using different projectiles and/or at different incident energies,* the more relevant variable to use is *not* e_{1ab}^{kin}, the kinetic energy of the observed pion in lab frame, *but*

$$u_{1ab} \equiv e_{1ab}^{kin}/(\bar{\nu}_1\epsilon)\,, \tag{4}$$

the ratio of e_{1ab}^{kin} to the total average kinetic energy of the emitting system.[17]

We now focus our attention on the single-particle inclusive cross-section data at $\theta_{1ab}\approx 0$.[1,2] *If the proposed picture is correct, we should not only be able to see that it is indeed possible*[1,2] *to observe near $\theta_{1ab}=0$ particles which carry kinetic energy higher than the kinetic energy per nucleon of the projectile, but also be able to observe the following characteristic of such production process:*

I. Since the pions observed at $\theta_{1ab}\approx 0$ are emitted by the effective projectile in a gentle collision with the effective target, the inclusive cross section $(d\sigma/d^3p)_{1ab}$ can be factorized as follows:

$$\left(\frac{d\sigma}{d^3p}\right)_{1ab}(A_1,A_2,\epsilon;\theta_{1ab}\approx 0, e_{1ab}^{kin}) = F_{1\,1ab}(A_1,\epsilon;\theta_{1ab}\approx 0, e_{1ab}^{kin})F_{2\,1ab}(A_2). \tag{5}$$

Here the functions F_1 and F_2 describe the effective projectile and the effective target respectively.

II. Since the F-type processes between the effective projectiles and effective targets are associated with peripheral nucleus-nucleus collisions [see the discussions in (d), (e), and (f) above] the function $F_{1\,1ab}$ *is proportional to the area a_1 of the ring on the peripheral of the nucleus A_1 [cf. Fig. 1(c)],*

$$F_{1\,1ab} \propto a_1\ . \tag{6}$$

If the projectile nucleus A_1 in its rest system is considered as a sphere of radius R_1 it is, in the laboratory system, a thin slab perpendicular to the beam direction. The area of this ring on the slab is

$$a_1 = 2\pi(R_1-\Delta R_1/2)\Delta R_1\ , \tag{7}$$

where ΔR_1 is the depth of the ring. [See Figs. 1(a)–1(c).]

III. From Eq. (3) and the fact that the average mass number of the effective projectile $\bar{\nu}_1$ is *proportional* to the average thickness (we recall that $\bar{\nu}_1$ is an invariant),

$$\bar{t}_1 = \tfrac{4}{3}\left[(2R_1-\Delta R_1)\Delta R_1\right]^{1/2}\ , \tag{8}$$

of the ring-shape object [see Fig. 1(d)], we see that *the average kinetic energy of the effective projectile is proportional to the square root of the area a_1 given in Eq. (7). That is,*

$$\langle E_{1\,1ab}^{kin}\rangle \propto \sqrt{a_1}\ \epsilon\ . \tag{9}$$

IV. Equations (5), (6), (9) and the discussion in connection with Eq. (4) lead us to the conclusion: For fast pions observed at a given angle θ_{1ab} near $0°$, the quantity

$$G_{1\,1ab} = [a_1(A_1)F_{2\,1ab}(A_2)]^{-1}(d\sigma/d^3p)_{1ab} \tag{10}$$

depends only on the variable

$$u_{1ab} \propto e_{1ab}^{kin}/(\sqrt{a_1}\ \epsilon)\,. \tag{11}$$

for all A_1, A_2, and ϵ.

We now compare the proposed production mechanism with experiments. First of all, we see that the empirical[18] existence of nucleus-nucleus fragmentation events (i.e., the pure projectile-, the pure target-, and the hybrid-fragmentation events of Ref. 18), violet collision events (i.e., the central collision events of Ref. 18), and mixed events (i.e., those hybrid collision events of Ref. 18 which have the characteristic features of fragmentation as well as those of central collision events) is in good agreement with the proposed picture [cf. the general features, especially (d) and (e) given at the beginning of this paper]. Second, we see that the factorization property given by Eq. (5) is in good agreement with experiment.[1,2] Third, the ϵ dependence of $d\sigma/d^3p$ given by Eqs. (10) and (11) can be tested by plotting $(d\sigma/d^3p)_{1ab}$ from the $p+C\to\pi^- +X$ data of Papp *et al.*[2] at $\theta_{1ab}=2.5°$ and $\epsilon=1.05$, 1.73, 2.10, 2.66, 3.50, 4.20, and 4.80 GeV/nucleon. The result, as shown in Fig. 2 is satisfactory (cf. also Figs. 3 and 4 for other reactions at different incident energies).

In order to compare the A_1 dependence of $(d\sigma/d^3p)_{1ab}$ given in Eqs. (10) and (11) a more specific model is necessary. Consistent with the conventional picture for nuclear size and with the proposed production mechanism (the general features which immediately follow from this mechanism have already been tested), we consider a model in which the projectile nucleus (mass number A_1) in its rest system is taken to be a sphere of

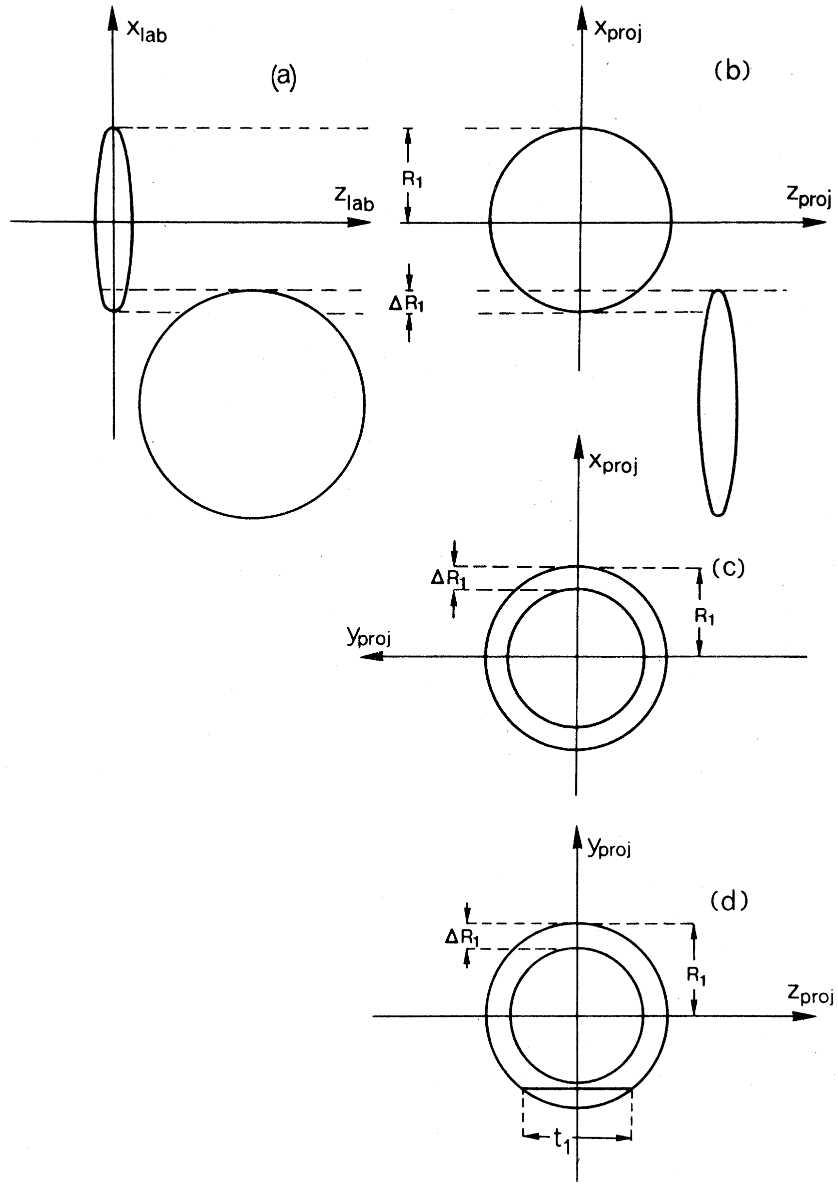

FIG. 1. A peripheral nucleus-nucleus collision is viewed from the laboratory and from the projectile system. The relations between R_1, ΔR_1, a_1, and t_1 [cf. Eqs. (6), (7), and (8)] are shown.

radius

$$R_1 = A_1^{1/3} r_0 \quad (r_0 = 1.2 \text{ fm}) \tag{12}$$

and

$$\Delta R_1 = \xi(A_1) r_0 , \tag{13}$$

where the function $\xi(A_1)$ has yet to be specified. From Eqs. (7), (8), (12), and (13) we obtain

$$a_1 = \pi r_0^2 s_1 ,$$

$$s_1 = (2A_1^{1/3} - \xi)\xi , \tag{7a}$$

$$\overline{t_1} = \tfrac{4}{3} r_0 \sqrt{s_1} . \tag{8a}$$

To determine $\xi(A_1)$, we note that it has to satisfy the following conditions: Firstly, it should be smaller than a certain value (≤ 1, say). This is because the processes between the effective projectile and effective target pairs are of the F type and such processes are most probable in peripheral collision between the projectile and target nuclei. Secondly, it should be a decreasing function of A_1. This means, nucleus-nucleus frag-

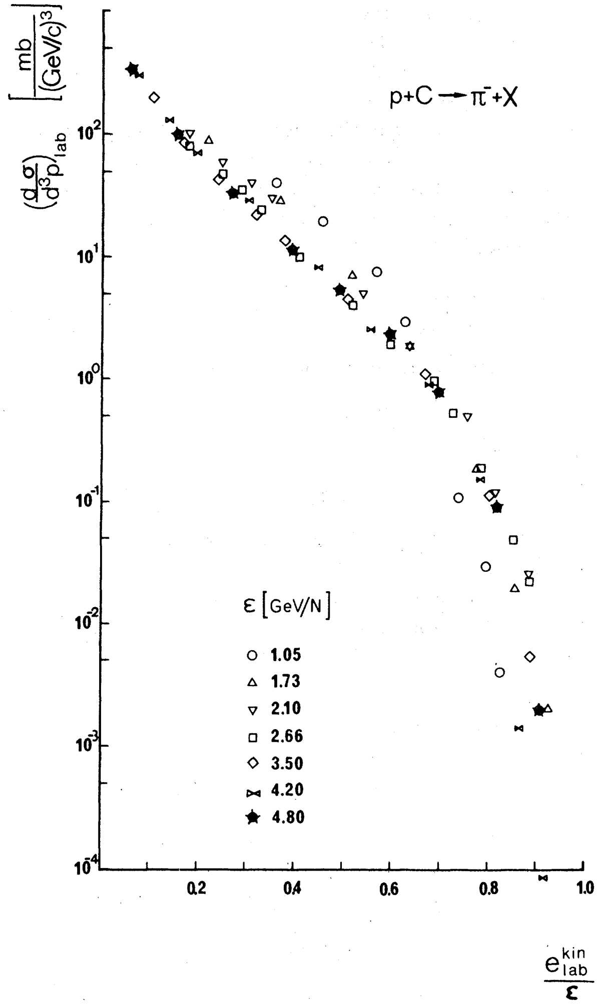

FIG. 2. $(d\sigma/d^3p)_{1ab}$ is plotted against e^{kin}_{lab} ϵ for the reaction $p+\mathrm{C}\to\pi^-+X$ at $\theta_{1ab}=2.5$ and $\epsilon=1.05$, 1.73, 2.10, 2.66, 3.50, 4.20, and 4.80 GeV/nucleon. The data are taken from Ref. 2.

mentation processes of heavier nuclei should be more peripheral. The reason is, F-type processes (in this case, between effective projectile and effective targets) are associated with large (relative to corresponding violent collision processes) impact parameters. Now [cf. Eqs. (8) and (8a)] the average thickness $\bar{t}_1$ of the effective projectile increases with the radius $R_1=A_1^{1/3}r_0$ of the projectile nucleus and hence the average impact parameter would decrease unless the depth of the ring $\Delta R_1=\xi(A_1)r_0$ decreases with increasing R_1, that is with increasing A_1. As an illustrative example, we use the simple ansatz

$$\xi(A)=A^{-1/6} \tag{14}$$

to obtain an explicit expression for

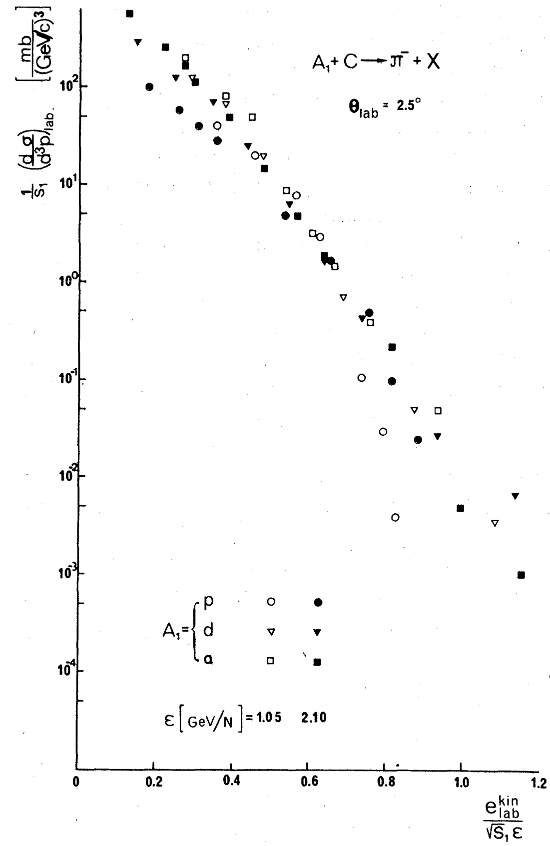

FIG. 3. $s_1^{-1}(d\sigma/d^3p)_{1ab}$ is plotted against $e^{kin}_{lab}/(\sqrt{s_1}\epsilon)$ for the reactions $p+\mathrm{C}\to\pi^-+X$, $d+\mathrm{C}\to\pi^-+X$, and $\alpha+\mathrm{C}\to\pi^-+X$ at $\theta_{1ab}=2.5°$ and $\epsilon=1.05$ and 2.10 GeV/nucleon. The data are taken from Ref. 2.

$$s_1=A_1^{-1/3}(2A_1^{1/2}-1) \tag{15}$$

and thus, from Eqs. (7a) and (8a), we obtain the corresponding expressions for a_1 and $\bar{t}_1$.

Using the results obtained above, we plot in Fig. 3,

$$\frac{1}{s_1}\left(\frac{d\sigma}{d^3p}\right)_{1ab}=\pi r_0^2 F_{2\,1ab}G_{1\,1ab}\ , \tag{10a}$$

against the variable

$$e^{kin}_{1ab}/(\sqrt{s_1}\,\epsilon)\propto u_{1ab}\ , \tag{11a}$$

for the data[2] of the reactions $p+\mathrm{C}\to\pi^-+X$, $d+\mathrm{C}\to\pi^-+X$, and $\alpha+\mathrm{C}\to\pi^-+X$ at $\theta_{1ab}=2.5°$ and $\epsilon=1.05$ and 2.1 GeV per nucleon. It is very interesting to see that the data points for different reactions and at different incident energies indeed fall approximately on one curve. [We note that since the same target nucleus C is used in all three reactions, $F_{2\,1ab}(A_2)$ is a common factor.]

We now turn out attention to the pion production data at $\theta_{1ab}=180°$,[3] and consider again a nucleus-

nucleus collision where A_1 is the mass number of the projectile nucleus which moves with the kinetic energy $\epsilon_{1ab}=\epsilon$ per nucleon, and A_2 is that of the target nucleus which is originally at rest in the laboratory system.

If the proposed physical picture for the production of fast pions in the forward direction is correct, trivial kinematics dictates that pions observed at $\theta_{lab}=180°$ in such a collision are due to the materialization of the kinetic energy of the effective target in the projectile system. This is because, viewed from the reference system in which the projectile nucleus A_1 is at rest, all the effective targets in A_2 move with the velocity $\beta_{2\,\mathrm{proj}}=[\epsilon(\epsilon+2M)]^{1/2}/(\epsilon+M)$ towards the projectile, the average kinetic energy $\langle E^{\mathrm{kin}}_{2\,\mathrm{proj}}\rangle$ is $\bar{\nu}_2\epsilon$, where $\bar{\nu}_2$ is the average mass number of the effective target [cf. Eq. (3)]. Very fast pions emitted by the effective target at $\theta_{\mathrm{proj}}=0$ (forward direction viewed from the projectile at rest) will be observed at $\theta_{lab}=180°$. In fact, all the statements made in connection with the forward pions should also be valid for the backward pions provided that the role of projectile and target is interchanged (which means formally, laboratory system is replaced by projectile system, effective projectile is replaced by effective target, A_1 is replaced by A_2, etc). In particular, the dependence of single-pion distribution on the incident energy (per nucleon, ϵ) and on the mass number of the target (A_2) should be such that

$$\frac{1}{s_2}\left(\frac{d\sigma}{d^3p}\right)_{\mathrm{proj}}=\pi r_0^{\,2}F_{1\,\mathrm{proj}}\,G_{2\,\mathrm{proj}} \tag{16}$$

is a universal function of the scaling variable

$$e^{\mathrm{kin}}_{\mathrm{proj}}/(\sqrt{s_2}\,\epsilon)\propto u_{\mathrm{proj}} \tag{17}$$

alone, where

$$s_2=A_2^{-1/3}(2A_2^{\,1/2}-1) \tag{18}$$

and $F_{1\,\mathrm{proj}}(A_1)$ should have the same form as $F_{2\,\mathrm{lab}}(A_2)$ for $A_1=A_2$.[19]

In Figs. 4 and 5 we plot the data of Baldin *et al.*[3] for the reactions $p+A_2\rightarrow\pi^-+X$ for $A_2=d$, C, Al,

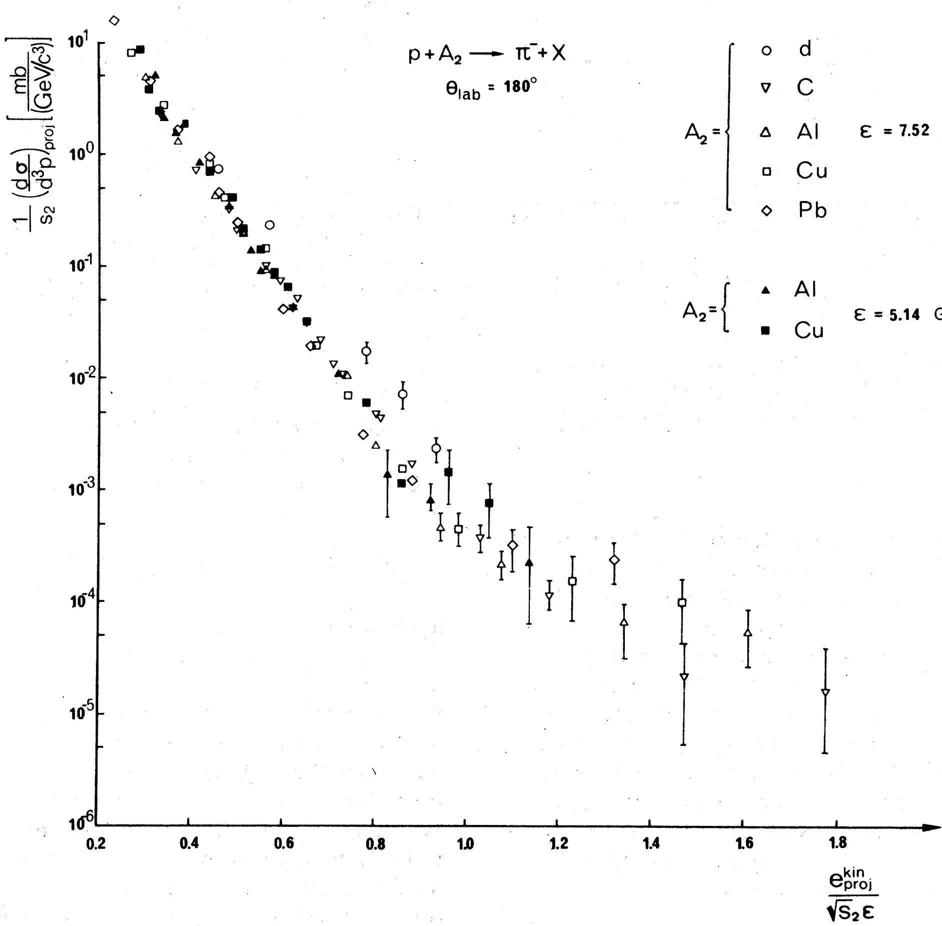

FIG. 4. $s_2^{-1}(d\sigma/d^3p)_{\mathrm{proj}}$ is plotted against $e^{\mathrm{kin}}_{\mathrm{proj}}/(\sqrt{s_2}\,\epsilon)$ for the reactions $p+A_2\rightarrow\pi^-+X$ where A_2 denotes d, C, Al, Cu, and Pb, at incident proton momentum $P_p=8.4$ and 6.0 GeV/c, angle of observation $\theta_{lab}=180°$. The data are taken from Ref. 3.

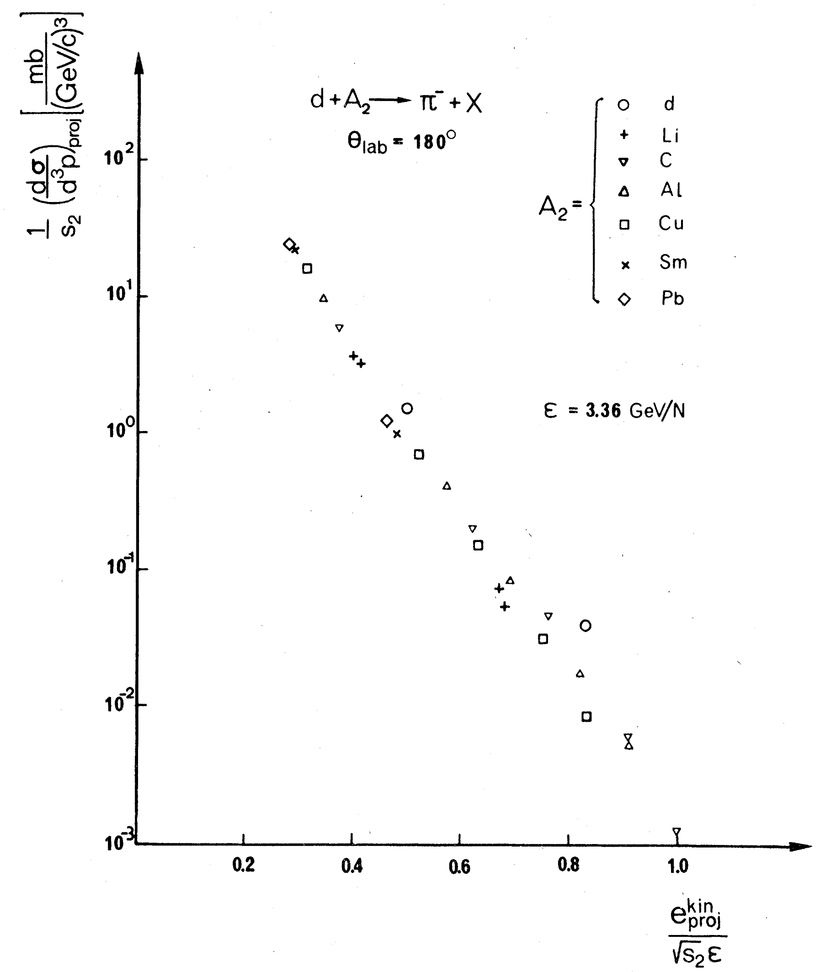

FIG. 5. $s_2^{-1}(d\sigma/d^3p)_{\rm proj}$ is plotted against $e_{\rm proj}^{\rm kin}/(\sqrt{s_2}\,\epsilon)$ for the reactions $d+A_2 \to \pi^- + X$ for $A_2 = d$, ^{6}Li, ^{7}Li, C, Al, Cu, ^{144}Sm, and Pb at deuteron incident momentum $P_d = 8.4$ GeV/c and $\theta_{\rm lab} = 180°$. The data are taken from Ref. 3.

Cu, and Pb at incident proton momentum $P_p = 8.4$ GeV/c, the same reactions for $A_2 =$ Al and Cu at $P_p = 6.0$ GeV/c as well as $d + A_2 \to \pi^- + X$ for $A_2 = d$, ^{6}Li, ^{7}Li, C, Al, Cu, ^{144}Sm, and Pb at deuteron incident momentum $P_d = 8.4$ GeV/c. In all these cases the pions are observed at $\theta_{\rm lab} = 180°$. Here we see that all the data points of the reactions $p + A_2 \to \pi^- + X$ for different A_2 and different ϵ fall on one curve, and all the data points of the reactions $d + A_2 \to \pi^- + X$ for different A_2 (only data at $P_d = 8.4$ GeV/c are available) fall on one curve. Furthermore, we also see that the curves in Figs. 4 and 5 have approximately the same shape (in first approximation, straight lines with the same slope). This means, the scaling functions $s_2^{-1}(d\sigma/d^3p)_{\rm proj}$ for the processes $p + A_2 \to \pi^- + X$ and $d + A_2 \to \pi^- + X$ in terms of the variable given in Eq. (17) differ from each other only by a factor which depends on the mass number of the projectile nucleus A_1. This shows that the factorization property of $(d\sigma/d^3p)_{\rm proj}$ [the counterpart of Eq. (5), cf. also Eq. (16)] is also valid for the reactions mentioned above.

Several remarks should be made in connection with the observed energy distribution of the forward and the backward pions[1-3] discussed in this paper.

1. The results obtained in Figs. 2–5 show that the proposed production mechanism, in particular the specific "scaling behavior" given in Eqs. (10a) and (16) is in agreement with the forward (near $\theta_{\rm lab} = 0°$) and the backward (near $\theta_{\rm lab} = 180°$) pion-production data[1-3] for a rather large energy range ($\epsilon = 1.05$ to 7.52 GeV/nucleon) and for light as well as for heavy nuclei (p, d, ..., Pb). These results can, in particular, be considered as a strong indication that such pions are cumulatively produced.[1]

2. The steep exponential decrease of the foward

pion distribution $(d\sigma/d^3p)_{\rm lab}$ with increasing kinetic energy ($e^{\rm kin}_{\rm lab}$), taken together with the fact that the multiplicity in fragmentation processes are relatively low, lead us to the following conclusion: There is not much probability that an effective projectile will lose most of its kinetic energy through the creation and subsequent emission of low-mass particles and thus reduce its velocity to a minimum before it decays [cf. the extreme case (ii) discussed in connection with Eq. (3)]. This and the close relationship between fragmentation processes and peripheralness [cf. (d), (e), and (f) discussed at the beginning of this paper] explain, in the present framework, why most of the nuclear isotopes observed[9,10] near the forward direction have nearly the beam velocity.[20]

3. The results obtained from this analysis suggest a statistical interpretation. Eqs. (16), (17), and (18) and Figs. 4 and 5 show that the function $G_{2\,\rm proj}$ can approximately be written as

$$G_{2\,\rm proj} \approx {\rm const}[(F_{1\,\rm proj}(A_1)]^{-1}\exp[-e^{\rm kin}_{\rm proj}/(kT_2)]\ , \quad \text{for}\ \ \theta_{\rm proj}\approx 0°\ , \tag{19}$$

where k is the Boltzmann constant, T_2 is the temperature of the pion gas, at the time of emission. The relation between T_2 and the average kinetic energy of the effective target (viewed from the projectile system) is

$$kT_2 \propto \sqrt{s_2}\,\epsilon\ . \tag{20}$$

Similarly, we obtain from Eqs. (10a), (11a), and (15) and Figs. 2 and 3,

$$G_{1\,\rm lab} \approx {\rm const}[F_{2\,\rm lab}(A_2)]^{-1}\exp[-e^{\rm kin}_{\rm lab}/(kT_1)]\ , \quad \text{for}\ \ \theta_{\rm lab}\approx 0° \tag{21}$$

$$kT_1 \propto \sqrt{s_1}\,\epsilon\ . \tag{22}$$

This means, viewed from the laboratory system, that the fast pions observed in the forward direction may in a first-order approximation be considered as being emitted from a system of free pion gas, the maximum total energy of which is equal to the maximum total kinetic energy of the effective projectile. The average total kinetic energy of this system can be approximated by $\bar{\nu}_1\epsilon$ where ϵ is the kinetic energy per nucleon and $\bar{\nu}_1$ is the average mass number of the effective projectile. (Note that the total momentum of the system of gas in lab is not zero, and hence no isotropic distribution is expected.)

4. In connection with the A_1 dependence in Eqs. (10a) and (11a) and the A_2 dependence in Eqs. (16) and (17), it should be emphasized that *not* the ansatz given in Eq. (14) *but rather* the relationship between the normalization factor s_1 (s_2) in Eq. (10a) [Eq. (16)] and the average mass number of the effective projectile $\sqrt{s_1}$ (effective target, $\sqrt{s_2}$) in the variable $u_{\rm lab}$ ($u_{\rm proj}$) is a characteristic feature of the proposed picture. This point may be of some importance when we compare this picture with future experiments.

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[8]Meng Ta-chung, Phys. Rev. D 9, 3062 (1974); Kwan-Wu Lai and Meng Ta-chung, Phys. Rev. Lett. 37, 241 (1976); Meng Ta-chung and E. Moeller, Phys. Rev. D 14, 1449 (1976).

[9]H. H. Heckman *et al.*, Phys. Rev. Lett. 28, 926 (1972).

[10]D. E. Greiner *et al.*, Phys. Rev. Lett. 35, 152 (1975); B. Cork, in *High Energy Physics and Nuclear Structure—1975* (Ref. 2), p. 598.

[11]The observations made in Refs. 9 and 10 (see also the papers cited therein) are in agreement with the existence of this limiting case.

[12]The observations made in Refs. 1 and 2 show that this limiting case also exists. In order to illustrate the kinematics of such a particle-emission process and to estimate the limits of the kinematical variables of such emitted particles, we consider the following simplified example: An effective projectile a with mass M_a, lab energy E_a and lab momentum $(P_{a\parallel}, \vec{0})$ interact with an effective target b of mass M_b which is initially at rest in lab. At the final stage we have the effective projectile with mass M_a, lab energy E'_a and lab momentum $(P'_{a\parallel}, \vec{P}'_{a\perp})$ the effective target with mass M_b and energy and momentum E'_b, $(P'_{b\parallel}, \vec{P}'_{b\perp})$ and a meson of mass m which has energy and momentum e and $(p_\parallel, \vec{p}_\perp)$, respectively. It follows from energy and momentum conservation $P_{a\parallel} = P'_{a\parallel} + P'_{b\parallel} + p_\parallel$, $\vec{P}'_{a\perp} + \vec{P}'_{b\perp} + \vec{p}_\perp = \vec{0}$, and $E_a + M_b = E'_a + E'_b + e$. Since this is an F-type

process and because the meson is observed in the forward direction and is very energetic, we have $|\vec{P}'_{a\perp}| = |\vec{P}'_{b\perp}| \ll p_{\parallel}$, $m \ll p_{\parallel} \approx P_{a\parallel}$, and thus, $E'_a \approx M_a$, $E'_b \approx M_b$, $e_{\rm kin} \approx E_a - M_a$. This means, the kinetic energy of the emitted meson in the forward direction has almost all the available kinetic energy of the effective projectile before the collision. It is clear that when more than one mesons are produced, they have to share the total available kinetic energy. Furthermore, in case one or more baryons are produced in this manner, the baryon number of the rest part of the effective projectile has to be such that the total baryon number is conserved. (This case is of some importance when we discuss fast protons observed at $\theta_{\rm lab} \approx 0$).

[13]W. Heisenberg, in Vorträge über Kosmische Strahlung (Springer, Berlin, 1953), p. 148.

[14]S. Tagaki, Prog. Theor. Phys. 7, 123 (1952).

[15]G. Cocconi, Phys. Rev. 111, 1699 (1958).

[16]See for example: K. Gottfried in *High Energy Physics and Nuclear Structure, Proceedings of the Fifth International Conference, Uppsala, Sweden, 1973*, edited by G. Tibell (North-Holland, Amsterdam/American Elsevier, New York, 1974), p. 79; A. Z. Pataschinskii Zh. Eksp. Teor. Fiz. Pis'ma Red. 19, 546 (1974) [Sov. Phys.—JETP 19, 338 (1974)]; F. Tagaki Lett. Nuovo Cimento 14, 559 (1975); A. Dar, in *Proceedings of the Topical Meeting on Multiparticle Production on Nuclei at Very High Energies, Trieste, 1976* see (Ref. 4), p. 591; S. Fredriksson, Nucl. Phys. B111, 167 (1976); Y. Afek, G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D 15, 2622 (1977); and the references given in these papers.

[17]As an illustrative example, we discuss the following simple question: What is the probability for an effective projectile with total kinetic energy 4 GeV to emit at $\theta_{\rm lab} \approx 0$ a pion which carries $\frac{3}{4}$ of the total available kinetic energy of this system? Since, according to this picture, the effective projectile behaves like a single object, such a pion can be produced if the effective projectile is a single nucleon carrying 4 GeV kinetic energy but it can also be produced by an effective projectile, which consists of two nucleons with 2 GeV kinetic energy per nucleon. Hence, if the proposed physical picture is correct, we should see:
(A) A pion with 3 GeV kinetic energy can *indeed* be produced by an effective projectile which consists of two nucleon with 2 GeV kinetic energy per nucleon.
(B) The *relative probability* (note that geometrical considerations should be taken into account in order to calculate the cross sections) of emitting such a pion is the *same* in the two cases mentioned above. It is interesting to see that both (A) and (B) are confirmed by the experimental data.

[18]H. H. Heckman *et al.*, An Atlas of Heavy Ion Fragmentation Topology, Berkeley report (unpublished).

[19]We note that the function $F_{2\,{\rm lab}}(A_2)$ depends explicitly on the geometry of the target nucleus A_2 (which is not simply a Lorentz-contracted thin slab in this reference system). This dependence may also be quite different for heavy as that for light nuclei.

[20]Theoretical interpretations have already been given by several authors. See, for example, H. Feshbach and K. Huang, Phys. Lett. 47B, 300 (1973); A. S. Goldhaber, *ibid.* 53B, 306 (1974); H. Feshbach and M. Zabek, Ann. Phys. 107, 110 (1977); I. A. Schmidt and R. Blankenbecler, Phys. Rev. D 15, 3321 (1977) and the references given therein. [In the last paper cited above, forward pion-production data (Ref. 1 and 2) are also discussed.]