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Panofsky ratio, threshold pion photoproduction, and axial-vector form factor in the $A = 3$ system

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The Panofsky ratio and photopion production cross section at threshold on ^3He are investigated in a soft-pion approach to the $^3He \to {}^3H$ weak axial-vector form factor. Results are compared with data on β decay and electron magnetic scattering, as well as with calculations based on "exact" $A = 3$ wave functions previously obtained by solving the Faddeev equations in the coordinate representation.

[NUCLEAR REACTIONS Calculations on π^- (stopped) + $^3He \to \gamma + {}^3H$, π^- (stopped) + $^3He \to \pi^0 + {}^3H$, γ(threshold) + $^3He \to \pi^+ + {}^3H$. ($^3He \to {}^3H$) weak axial-vector form factor. Model three-nucleon wave functions.]

I. INTRODUCTION

Radiative absorption of stopped pions, pion photoproduction near threshold, and charge exchange reactions induced by stopped charged pions have in recent years generated activity in the field of nuclear physics[1,2]; in certain instances, these processes have yielded some information on nuclear structure.[2] They could become quantitative probes of nuclei throughout the Periodic Table to the extent that the following conditions are reasonably fulfilled: (i) The basic amplitude for the corresponding process on individual nucleon (i.e., $\pi^- + p \to n + \gamma$, $\gamma + p \to \pi^+ + n$, $\pi^- + p \to \pi^0 + n$) is rather well known; (ii) one knows how to incorporate this amplitude in the many-body theory of the nucleus; and (iii) accurate pion wave functions are available. The agreement between theory and experiment is rather satisfactory for the three above mentioned reactions in the present framework of the pion-nucleon interaction at low energy. This somewhat satisfactory situation is due to the model independence of the bulk of the amplitude[3,4]; the model-dependent corrections have been investigated by various methods leading to results essentially consistent with one another.[2,5] The pion wave functions are obtained by solving the Klein-Gordon equation with appropriate optical potentials, the parameters of which are taken from fits to experimental data, i.e., from the energy shifts and the width of the atomic orbits for bound states and elastic scattering data for unbound states.[1,5] The various data reflect the small pion-nucleon S-wave scattering length $a_{\pi\text{-}N} \cong 0.1$ fm and the relative weakness of the π-nucleus optical potential. Again, such simplifying features come from the fact that one is close to pion threshold; while the aforementioned considerations indicate that conditions (i) and (iii) are reasonably satisfied; the answer to (ii) is more difficult. For reasons of simplicity and convenience, the nucleon-only impulse approximation (NOIA) is usually applied.[6] It is clear that the validity of NOIA or, better, the interplay between the above three conditions, can best be investigated with $A = 2$ and 3 nuclei for which the nuclear dynamics is less complicated than for heavier nuclei. The present investigation will focus on the three following reactions: (1) $^3He(\pi^-,\gamma)^3H$, (2) $^3He(\pi^-,\pi^0)^3H$, and (3) $^3He(\gamma,\pi^+)^3H$ near threshold and their relation to the $^3He \to {}^3H$ weak axial-vector form factor. New experimental data have recently come up, adding some momentum to this domain of photopion physics. Besides a new measurement of the Panofsky ratio,[7] recent investigations of the $^3He(\gamma,\pi^+)^3H$ cross section at threshold[8] and of the ^3He pionic atoms[9] give rise to a new way of getting at the $^3He \to {}^3H$ weak axial-vector form factor. Before going beyond the NOIA and embarking into detailed mesonic exchange currents (MEC) calculations, it is of interest to know to what extent the analysis of these new experimental data contributes to a reliable and overall consistent extraction of the $^3He \to {}^3H$ axial-vector form factor. Already at this level, use of adequate $A = 3$ nuclear wave functions allows preliminary investigation into the interplay between the nucleon-nucleon forces, the axial-vector form factor, and reactions (1), (2), and (3). Such an analysis is the subject of the present work.

Thus, in Sec. II, effective operators and corresponding lifetimes and cross sections are written down within the conventions given by Gibbs, Gibson, and Stephenson,[10] and including the $^3He \to {}^3H$ form factors under scrutiny. Wave functions proposed by Laverne and Gignoux for $A=3$ bound systems[11] are introduced in Sec. III, in order to relate $^3He \to {}^3H$ axial-vector form factors to specific nuclear wave functions. Then in Sec. IV, the presently available experimental data will be investigated in terms of the formalism set up in Secs. II and III. A conclusion will follow.

II. EFFECTIVE OPERATORS, AND LIFETIMES AND CROSS SECTIONS

Before discussing detailed expressions for the quantities of interest, a qualitative description of the processes involving the pion while forming a pionic atom with the ^3He nucleus might be helpful. Due to the very small Z of ^3He, the pion is absorbed $[\pi^-\text{(stopped)} + {}^3He \to \text{all}]$ only from 1s and 2p shells. Reasonable estimates yield a probability $\omega_s \cong 0.84$ ($\omega_p \cong 0.16$) for the pion to be absorbed while on the 1s shell (1p shell); calculations by Philipps and Roig, together with estimates of the x-ray 2p-1s transition rates in ^3He, lead to very small relative probabilities for the π to give rise either to a radiative capture ($\pi^{-3}He \to \gamma^3H$) or a charge exchange ($\pi^{-3}He \to {}^3H\pi^0$) while on the 2$p$ shell. As a numerical illustration, for a pion on the 2p shell, the probability to go to the 1s shell through x-ray emission is 1 000 times the probability of giving rise to a π^0 in a charge-exchange process. Thus, the processes which are of interest in the present work are essentially those involving pions in a relative pion-nuclear s state. Such a remark is valid for the pion photoproduction ($\gamma^3He \to \pi^{+3}H$), since one stays close to the pion emission threshold.

The effective operator $O_j^{(\mp)}$ responsible for the reactions ($\pi^{-3}He \to \gamma^3H$) and ($\gamma^3He \to \pi^{+3}H$) is known to be of the form[2]

$$O_j^{(\mp)} \sim \tau_j^{(-)}[A^{(\mp)}\vec{\sigma}_j \cdot \vec{\epsilon} + B^{(\mp)}\vec{\sigma}_j \cdot \vec{\epsilon}\vec{q} \cdot \vec{k} + C^{(\mp)}\vec{\sigma}_j \cdot \vec{k}\vec{\epsilon} \cdot \vec{q} + iD^{(\mp)}\vec{\epsilon} \cdot (\vec{q} \times \vec{k}) + E^{(\mp)}(\vec{\sigma}_j \cdot \vec{q})(\vec{q} \cdot \vec{\epsilon})], \quad (1)$$

where $\vec{\epsilon}$ is the photon polarization, while $\vec{k}$ and $\vec{q}$ are the momenta of photons and pions, respectively. This expression reduces to the first term at threshold. As mentioned in the Introduction, the various ways by which the coefficient A is obtained lead to an overall numerical agreement. Thus, starting from the static limit for the γ-π-N interaction, Gibbs, Gibson, and Stephenson (GGS) use the following effective operator[10]

$$O_j^{(\mp)} = \frac{2\pi i}{(km_\pi)^{1/2}}\left[\overline{A}(1+\xi)\left(1+\frac{|\vec{k}|}{2m_N}\right)\right]\vec{\sigma}_j \cdot \vec{\epsilon}\tau_j^{(-)}$$
$$= \frac{2\pi i}{(km_\pi)^{1/2}} A^{(\mp)}\vec{\sigma}_j \cdot \vec{\epsilon}\tau_j^{(-)}, \quad (2a)$$

where $\overline{A} = \alpha(G^2/4\pi)(1/2m_N^2) = 0.034\ m_\pi^{-2}$ for $(G^2/4\pi) = 14.8$, $\xi = 0.0034$ stands for the N^* anomalous contribution and $|\vec{k}| = |\vec{p}_p - \vec{p}_n|/2m_N \simeq m_\pi$ with $\vec{p}_p$ and $\vec{p}_n$ being the proton and neutron momenta. Normalization to the measured Panofsky ratio for hydrogen $P_1 - 1.533 \pm 0.021$,[12] yields $\overline{A}^{(-)} = 0.0374\ m_\pi^{-1}$, while the same operation for the reduced photopion production cross section $a_{p\to n} = (201 \pm 7)$ μb[13] leads to $A^{(+)} = 0.0326 m_\pi^{-1}$. In order to illustrate the near equivalence of the various approaches, provided one stays very close to the threshold, it is noted that the operator used by Vergados and Woloshyn,[14] based on the Peccei Lagrangian takes the form

$$O_j^{(\mp)} = \frac{2\pi i}{m_\pi}\left[\left(1+\frac{m_\pi}{m_N}\right)(A_{VW}^{(\mp)}m_\pi)\right]\vec{\sigma}_j \cdot \vec{\epsilon}\tau_j^{(-)}$$
$$= \frac{2\pi i}{m_\pi}\overline{A}_{VW}^{(\mp)}\vec{\sigma}_j \cdot \vec{\epsilon}\tau_j^{(-)}, \quad (2b)$$

with the numerical values $\overline{A}_{VW}^{(-)} = 0.038 m_\pi^{-1}$ and $\overline{A}_{VW}^{(+)} = 0.033 m_\pi^{-1}$ agreeing, within 3%, with $A^{(-)}$ and $A^{(+)}$.

The ($\pi^{-3}He \to \pi^{03}H$) reaction is also described by an effective operator of the form:

$$O_j^{(\pi^- \to \pi^0)} \sim \tau_j^{(-)}[a + b\vec{q}^i \cdot \vec{q}_0^f + ic\vec{\sigma}_j \cdot (\vec{q}^i \times \vec{q}_0^f)], \quad (3)$$

where $\vec{q}^i$ and $\vec{q}_0^f$ are the momenta of incoming and outgoing pions, respectively. If the pion is on the 1s shell, the effective operator gets simplified. It can be written as

$$O_j^{(\pi^-\pi^0)} = \frac{2\pi i}{m_\pi}\frac{\sqrt{2}}{3}\left(1+\frac{m_\pi}{m_N}\right)|a_1 - a_3|_j, \quad (4)$$

where the quantity $|a_1 - a_3| = (0.262 \pm 0.004)m_\pi^{-1}$ is the appropriate combination of scattering lengths.[15] Calculations involving the 2p shell necessitate the more complete form of Eq. (3).

The corresponding lifetime and cross sections given, in the pion nucleus center of mass system, are

$$\tau_S^{-1}(\pi^{-3}He \to \gamma^3H) = 8|\overline{A}^{(-)}|^2 \frac{c}{(a_3^B)^3}\frac{|\vec{k}_3^f|}{m_\pi}\frac{1}{[1+(|\vec{k}_3^f|/m_3)]} C_i^{\pi s}|F_{SF}(k_3^f)^2|^2, \quad (5)$$

$$\tau_S^{-1}(\pi^{-3}\text{He}\to\pi^{03}\text{H})=\frac{8}{9}\left(1+\frac{m_\pi}{m_N}\right)^2|a_1-a_3|^2\frac{c}{(a_3^B)^3}\frac{|\vec{q}_0^f|}{m_\pi}\frac{1}{[1+m_\pi/m_3)]}C_i^{\pi s}|F_{\text{NSF}}(|\vec{q}_0^f|)^2|^2C_f^{\pi 0}, \tag{6}$$

$$\sigma(\gamma^3\text{He}\to\pi^{+3}\text{H})=4\pi|\bar{A}^{(+)}|^2\frac{|\vec{q}_+^f|}{|\vec{k}_3^i|}\frac{1}{[1+(m_\pi/m_3)][1+(|\vec{k}_3^i|/m_3)]}|F_{\text{SF}}(q_f-k_3^i)^2|^2C_f^\pi, \tag{7}$$

where a_3^B is the pionic atom 1s Bohr radius from which the pion is captured, c is the light velocity in vacuum, and m_3 is the mass of the ^{3}He nucleus. $\vec{k}_3^f$(= 136.2 MeV/c) is the momentum of the outgoing photon in reaction 1, $\vec{q}_0^f$(= 32.4 MeV/c) is the momentum of the outgoing pion in reaction 2, and $\vec{q}_+^f$ and $\vec{k}_3^i$ are the respective momenta of the outgoing pion and of the incoming photon in reaction 3.

The distortion factor $C_i^{\pi s}$ is given by the expression

$$C_i^{\pi s}=\left|\frac{\langle^3\text{He}|[\varphi_{\pi i}^{1s}(\text{optical potential})]|^3\text{He}\rangle}{\langle^3\text{He}|[\varphi_{\pi i}^{1s}(\text{Bohr})]|^3\text{He}\rangle}\right|^2, \tag{8}$$

where $\varphi_{\pi i}^{1s}$, which represents the atomic pion wave function in ^{3}He, will cancel out in the Panofsky ratio; and $C_f^\pi(C_f^{\pi 0})$ corresponds to the distortion of the outgoing charged (neutral) pion in the triton field and takes the form:

$$C_f^\pi=\left|\frac{\langle^3\text{H}|[\varphi_f^\pi(\text{optical potential})]^3\text{H}\rangle}{\langle^3\text{H}|j_0(q_+^f r)|^3\text{H}\rangle}\right|^2. \tag{9}$$

This expression will be investigated in Sec. III.

The form factors $F_{\text{SF}}((k_3^f)^2)$, $F_{\text{SF}}((k_3^i-q_+^f)^2)$(SF for spin flip) and $F_{\text{NSF}}((q_+^f)^2)$ (NSF for nonspin flip) represent the probability amplitudes that ^{3}He stays as a bound state in the corresponding reactions. With the obvious notation:

$$|^3\text{He}\rangle=|^3\text{He},J_i^\pi=\tfrac{1}{2}^+,M_i,T=\tfrac{1}{2},T_z=\tfrac{1}{2}\rangle;$$

$$|^3\text{H}\rangle=|^3\text{H},J_f^\pi=\tfrac{1}{2}^+,M_f,T=\tfrac{1}{2},T_z=-\tfrac{1}{2}\rangle,$$

one has the following equations:

$$|F_{\text{NSF}}((q_0^f)^2)|^2=2\left|\langle^3\text{H}\|\sum_{j=1}^3 j_0(|\vec{q}_0^f|r_j)\tau_j^{(-)}\|^3\text{He}\rangle\right|^2 \tag{10}$$

$$\cong\left(1-\frac{|\vec{q}_0^f|^2R_{ch}^2}{6}\right), \tag{10'}$$

where the double bars refer to the angular momentum reduction of the matrix element and R_{ch} is the $^3\text{He}\to{}^3\text{H}$ root mean square (rms) radius. The NSF expression of Eq. (8) is related to the weak polar vector current. In view of the accurate knowledge of the quantity $|a_1-a_3|$, together with its weak polar nature which implies no MEC, the NSF form factor will be considered as well known data in the forthcoming discussion. Further, in the assumption of point-like nucleons:

$$|F_{\text{SF}}(Q^2)|^2=\frac{1}{2}\sum_{Mi,Mf}\left|\langle^3\text{H}|\sum_{j=1}^3 e^{-i\vec{Q}\cdot\vec{r}_j}\vec{\sigma}_j\cdot\vec{\epsilon}\tau_j^{(-)}|^3\text{He}\rangle\right|^2, \tag{11}$$

$$=\frac{1}{6}\sum_{k=0,2}\left|\langle^3\text{H}\|\sum_{j=1}^3 j_k(|\vec{Q}|r_j)[\sqrt{4\pi}Y_k(\hat{r}_j\otimes\vec{\sigma}_j]^1\tau_j^{(-)}\|^3\text{He}\rangle\right|^2 \tag{11'}$$

$$=\frac{1}{6}\left|\langle^3\text{H}\|\sum_{j=1}^3(|\vec{Q}|r_j)\vec{\sigma}_j\tau_j^{(-)}\|^3\text{He}\rangle\right|^2+\frac{1}{6}\left|\langle^3\text{H}\|\sum_{j=1}^3 j_2(|\vec{Q}|r_j)[\sqrt{4\pi}Y_2(\hat{r}_j)\otimes\vec{\sigma}_j]\tau_j^{(-)}\|^3\text{He}\rangle\right|^2,$$

with $Q=k_3^f(q_+^f-k_3^i)$; the spin-flip expressions are obviously equivalent to the Gamow-Teller matrix element (up to a numerical factor) within the NOIA; soft-pion theorems tell us that this equivalence goes beyond the NOIA framework so that

$$F_{\text{SF}}(Q^2)=F_A^{3\text{He}\to 3\text{H}}(Q^2)\Big/g_A=\tilde{F}_A(Q^2), \tag{12}$$

g_A(= −1.24) is the nucleon Gamow-Teller constant and the $^3\text{He}\to{}^3\text{H}$ weak axial-vector form factor $F_A^{3\text{He}\to 3\text{H}}(Q^2)$ is the unifying element of reactions (1) $^3\text{He}(\pi^-,\gamma)^3\text{H}$ and (3) $^3\text{He}(\gamma,\pi^+)^3\text{H}$; such an expression also appears in weak processes involving the isodoublet $A=3$ system.[16]

III. A=3 WAVE FUNCTIONS

The ^{3}He and ^{3}H wave functions used in the present work are "exact" solutions of the Faddeev equations for the three nucleons interacting via a local interaction.[11] These wave functions permit reliable estimates of most observables, an exception being the binding energy of the $A=3$ ground states (since it comes out as the difference of large kinetic and potential energies ~50 MeV), and the charge form factor at momentum transfers much higher than far beyond the domain presently under investigation. Furthermore, the direct connection between the realistic nucleon-nucleon interaction and the resulting nuclear wave function given by the Laverne-Gignoux (LG) solution might offer more self-consistency for future calculation on mesonic exchange currents (MEC).[17] Indeed, one difficulty inherent in MEC in traditional nuclear physics is the coexistence of both a two-body operator implying a specific nucleon-nucleon interaction and a nuclear wave function built from an effective residual interaction. This latter one is not necessarily consistent with the nucleon-nucleon potential implied by the MEC.

In the reduced matrix element of Eqs. (8), (9), and (10), the three-particle system is defined either by $(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ coordinates in some frame or equivalently by $(\vec{R}, \vec{y}, \vec{x})$ related to the first system by the well-known transformation

$$\begin{pmatrix} \vec{r}_1 \\ \vec{r}_2 \\ \vec{r}_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 1 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} \vec{R} \\ \vec{y}/\sqrt{3} \\ \vec{x} \end{pmatrix}, \tag{13}$$

so that in the three-nucleon center of mass system $(\vec{R}=0)$

$$\vec{r}_1 = -\frac{\vec{y}}{\sqrt{3}}, \quad \vec{r}_2 = \tfrac{1}{2}(\vec{x}+\vec{y}/\sqrt{3}), \quad \vec{r}_3 = \tfrac{1}{2}(-\vec{x}+\vec{y}/\sqrt{3}). \tag{14}$$

In the $(\vec{x}, \vec{y})$ representation, the wave function $\Phi^{\mathcal{J}T}_{MT_z}(\vec{x}, \vec{y})$ is written as a sum over the various components characterized by orbital, spin, and total angular momenta $(\lambda \frac{1}{2} j)$ of particle 1 and $(l\sigma J)$ of particles 2 and 3, combining to yield the total angular momentum $[\mathcal{J}, M] = [\frac{1}{2}, M]$ and isospin $[T, T_z] = [\frac{1}{2}, -\frac{1}{2}]$ for ^{3}H, $= [\frac{1}{2}, \frac{1}{2}]$ for ^{3}He,

$$\Phi^{\mathcal{J}T}_{MT_z}(\vec{x}, \vec{y}) = \sum_{\lambda l \sigma j t J} \frac{1}{xy} \varphi^{jJ\frac{1}{2}\frac{1}{2}}_{\lambda l \sigma}(x, y) \times [\mathcal{Y}^{j}_{\lambda\frac{1}{2}}(\hat{y}) \otimes \mathcal{Y}^{J}_{l\sigma}(\hat{x})]^{\mathcal{J}}_{M} \eta^{TT_z}_{\frac{1}{2}t}, \tag{15}$$

where antisymmetry of the wave function in the interchange of particles $2 \leftrightarrow 3$ requires that $l+\sigma+t$ be odd. The reduced matrix elements are then

$$\left\langle {}^3\mathrm{H} \left\| \sum_{j=1}^{3} j_k(|\vec{Q}| r_j)[\sqrt{4\pi}\, Y_k(\hat{r}_j) \otimes \vec{\sigma}_j]^1 \tau_j^{(-)} \right\| {}^3\mathrm{He} \right\rangle = 3 \langle {}^3\mathrm{He} \,|\, j_k(|\vec{Q}| r_1)[\sqrt{4\pi}\, Y_k(\hat{r}_1) \otimes \vec{\sigma}]^1 \tau_1^{(-)} \,\|\, {}^3\mathrm{He} \rangle \tag{16}$$

$$= 18\sqrt{2} \sum_{\substack{\lambda j \lambda' j' \\ l\sigma J t}} (-1)^{j'+J+\frac{1}{2}+t} \frac{[(2j+1)(2j'+1)]^{1/2}}{(2t+1)} \begin{Bmatrix} j' & j & 1 \\ \frac{1}{2} & \frac{1}{2} & J \end{Bmatrix} \begin{Bmatrix} \lambda & \frac{1}{2} & j \\ k & 1 & 1 \\ \lambda' & \frac{1}{2} & j' \end{Bmatrix} I_k(\lambda j; \lambda' j'; l\sigma J t) \langle \lambda \| \sqrt{4\pi}\, Y_k(\hat{r}_1) \| \lambda \rangle, \tag{17}$$

$$I_k(\lambda j; \lambda' j; l\sigma J t) = \iint dx\,dy [\varphi^{jJ\frac{1}{2}\frac{1}{2}}_{\lambda l \sigma t}(x, y)]^* j_k\left(\frac{|\vec{Q}|\, y}{\sqrt{3}}\right) \varphi^{jJ\frac{1}{2}\frac{1}{2}}_{\lambda l \sigma t}(x, y). \tag{18}$$

These latter integrals are computed from the radial parts $\varphi^{jJ\frac{1}{2}\frac{1}{2}}_{\lambda l \sigma t}(x, y)$ of the solution of the three-nucleon problem.[11] Three realistic nucleon-nucleon forces are considered for illustration, to with, the Reid soft core (RSC),[18] the Sprung de Tourreil super-soft core type c (SSCc),[19] and the Malfliet-Tjon type 1-3 (MT13)[20] interactions. Table I displays the corresponding S, S', and D components of the corresponding wave functions, the charge radii, and the allowed Gamow-Teller matrix elements which are written as:

$$\tfrac{1}{3}|M_A|^2 = |\tilde{F}_A(0)|^2 = (P_S - \tfrac{1}{3}P_{S'} + \tfrac{1}{3}P_D)^2. \tag{19}$$

Keeping in mind that, within the NOIA scheme, a decreasing $^3\mathrm{H} \to {}^3\mathrm{He}$ β-decay matrix element goes along with an increasing D component, the results are not surprising, since the RSC, the main feature of which is a strongly attractive tensor contribution in the even-triplet subspace,[18] yields the smallest $\tilde{F}_A(0)$; the SSC(c) involves a much weaker tensor interaction,[19] while the MT(13) has no tensor part.[20] However, microscopic calculations show that in the reduced matrix element

$$\left\langle {}^3\mathrm{H} \left\| \sum_j \vec{\sigma}_j \tau_j^{(-)} + \sum_{jk} (\mathrm{MEC})_{jk} \right\| {}^3\mathrm{He} \right\rangle,$$

the bulk of MEC comes with the D component[17] and has the same sign as the total NOIA contribution.

TABLE I. Quantities related to the axial form factor $P_{S'}$ and P_D represent the percentages of S' and D states in the $A=3$ system, with $(P_S+P_{S'}+P_D=1)$, R_{ch} is the root mean square charge radius, other symbols are defined in the text. The last line [Rho+SSS(c)] refers to numbers obtained from the Rho prescription for a β-decay matrix element combined with the variation of $\mathfrak{F}_M(Q^2)$ based on SSC (c). Numbers in parentheses correspond to values coming from the use of $F_M(Q^2)$, i.e., point-like nucleons, related to $\mathfrak{F}_M(Q^2)$ by Eq. (21).

	$P_{S'}$ (%)	P_D (%)	R_{ch} (fm)	$\lvert\tilde{F}_A(0)\rvert^2$	$\left\lvert\frac{\mathfrak{F}_M(\tilde{m}_\pi^2)}{F_M(0)}\right\rvert^2$	$P_3(1s)$	P_3
Experiment			1.87 ±0.05 [a]	0.98 ±0.02 [b] 0.97 ±0.03 [b]	0.65 ±0.01 [a] (0.60 ±0.01)	2.35 ±0.18 2.77 ±0.13 2.91 ±0.18	2.28 ±0.18 [c] 2.68 ±0.13 [d] 2.82 ±0.07 [e]
RSC	1.60	9.3	1.86	0.855	0.575	3.37	3.27
SSC(c)	1.35	7.9	1.84	0.874	0.596	3.19	3.09
MT13	2.02	0	1.76	0.947	0.575	3.05	2.96
[Rho+SSC(c)]	1.35	7.9	1.84	0.964	0.596	2.89	2.80

[a]See Ref. 23.
[b]See Ref. 24.
[c]See Ref. 27.
[d]See Ref. 25.
[e]See Ref. 7.

Thus, the overall D-state contribution (NOIA + MEC) will tend to bring the matrix element value back to that without a D-state admixture. Thus, Rho[21] relates the presence of D states to tensor parts of the nucleon-nucleon force and N^* excitations in intermediate nuclear states, and on the basis of a simple model obtains a complete cancellation of the D contribution. This approach and its results are also included in Table I. Finally, the $A=3$ isovector magnetic form factor $F_M(Q^2) = [F_M^{\text{spin flip}}(Q^2)+F_M^{\text{orbital}}(Q^2)]$ has been calculated; the orbital contribution has been found quite negligible compared to the spin-flip contribution so that the relation

$$F_A(Q^2)=F_A(0)\,\frac{F_M(Q^2)}{F_M(0)}\,, \qquad (20)$$

is quite accurate for each of the three wave functions considered.

Before reviewing the experimental data, it is recalled that Eqs. (11), and (11′) imply point-like nucleons, so that the form factors $F_M(Q^2)$ calculated in this framework are related to the nucleus form factor $\mathfrak{F}_M(Q^2)$ by the relation:

$$\mathfrak{F}_M(Q^2)=F_M(Q^2)f_M(Q^2)\,, \qquad (21)$$

where $f_M(Q^2)$ is the corresponding nucleon form factor $[f_M(0)=1]$. The following expression,

$$f_M(Q^2)=(1+a^2Q^2)^{-2}\cong 1-2a^2Q^2\,, \qquad (22)$$

with $a^2=0.04\ \text{fm}^{-2}$, will be adopted for $\mathfrak{F}_M(Q^2)$ following Refs. (22) and (23).

IV. DETERMINATION OF THE AXIAL-VECTOR FORM FACTOR FROM EXPERIMENTAL DATA

The validity of Eq. (20) suggests considering $\tilde{F}_A(Q^2)$ as the product $[\tilde{F}_A(0)][F_M(Q^2)/F_M(0)]$, with the first factor directly connected to the experimental Triton β-decay rate,[24] and the second factor being obtained from magnetic electron scattering on ^{3}He and ^{3}H.[23] Results for $Q^2=0.473\ \text{fm}^2 = \tilde{m}_\pi^2(\simeq m_\pi^2)$ are given together with error bars in table for $\mathfrak{F}_M(\tilde{m}_\pi^2)/F_M(0)$ and also for $\tilde{F}_A(0)$. Values for $\tilde{F}_A(\tilde{m}_\pi^2)$ resulting from the three nucleon-nucleon interactions are displayed in Table I, together with the [Rho + SSS(c)] transition matrix elements. It is noted that the three sets of wave functions, and more particularly the SSS(c) wave function, yield a $\mathfrak{F}_M(\tilde{m}_\pi^2)/F_M(0)$ very close to the experimental result. The [Rho + SSC(c)] solution yields $\lvert\tilde{F}_A(\tilde{m}_\pi^2)\rvert^2=0.574$. It is recalled that this latter method amounts to a definite (NOIA + MEC) calculation.

Although the first reaction $\pi^-{}^3\text{He}\to\gamma{}^3\text{He}$ has been investigated experimentally by the Berkeley group,[25] a direct comparison with theory is not possible, the reason being that only the γ-ray energies are measured. Hence, one cannot single out the above γ rays from those coming from the reactions: $(\pi^-{}^3\text{He}\to pnn\gamma, dn\gamma, {}^3\text{H}(\pi^0\to 2\gamma))$; however, Eqs. (1) and (2) give rise to a relation between the experimentally measured Panofsky ratio and $\tilde{\mathfrak{F}}_A(Q^2)$ provided one takes into account the correction due to the absorption $\tau_{(2p)}{}^{-1}(\pi^-{}^3\text{He}\to\gamma{}^3\text{He})$ and $\tau_{(2p)}{}^{-1}(\pi^-{}^3\text{He}\to\pi^{0}{}^3\text{H})$ from $2p$ atomic states. Roig

TABLE II. Values of $|\tilde{F}_A(\tilde{m}_\pi{}^2)|^2$ as obtained through various approaches. $\tilde{m}_\pi{}^2$ stands for 0.473 fm^{-2}.

	$\|\tilde{F}_A(0)\|^2 \left\| \frac{F_M(\tilde{m}_\pi{}^2)}{F_M(0)} \right\|^2$	Wave functions	Panofsky ratio	$\frac{a_{^3\text{He}\to{}^3\text{H}}}{a_{p\to n}}$	$(\tau_s^{\text{total}})^{-1}$
$\text{SIN}^{(0)}$					0.83 ±0.28
Zaimidoroga (Ref. 26)			(0.71 ±0.06)		
Berkeley (Ref. 25)			(0.60± 0.04)		
Salgo *et al.* (Refs. 23, 24)	0.59±0.02				
Bergkvist *et al.* (Refs. 23, 24)	0.58±0.03				
[Rho + SSC (c)] (Refs. 21, 23)		0.574			
TRIUMF (Ref. 8)			0.57 ±0.02		
MT13 (Ref. 19)		0.566			
SSC (c) (Ref. 18)		0.521			
RSC (Ref. 17)		0.692			
Saclay-Louvain (Ref. 8)				0.45 ±0.03	

and Phillips,[6] and we also [using the formalism set up by Vergados[26] which involves all given constants A, B, C, D, E of Eq. (1) and a, b, c, of Eq. (3)], find a correction standing between 3 and 4%, as displayed in Table I. The numerical values for $P_3(1s)$ stand for the "$2p$ corrected" experimental Panofsky ratios obtained respectively by Zaimidoroga *et al.*,[27] Trüol *et al.*,[25] and Hasinoff *et al.*[7] Those values, together with the corresponding P_3, are displayed in Table I while the corresponding $|\tilde{F}_A(\tilde{m}_\pi{}^2)|^2$ are shown in Table II.

It is recalled that the Panofsky ratio has the form

$$P_3 \cong P_3(1s) = [\tau_s^{-1}(\pi^- {}^3\text{He} \to \pi^{0\,3}\text{H}) / \tau_s^{-1}(\pi^- {}^3\text{He} \to \gamma {}^3\text{H})]\,. \tag{23}$$

Further, it is known that $\tau_s^{-1}(\pi^- {}^3\text{He} \to \pi^{0\,3}\text{H})$ is not sensitive anyway to the amount of D component in the ^{3}He(^{3}H) ground state. Therefore, the considerations of Sec. III about the dependence of the $^3\text{He} \to {}^3\text{H}$ axial-vector form factor upon the D component are applicable to the Panofsky ratio. Thus, Table I displays the sensitivity of $P_3[P_3(1s)]$ to the amount of D state, *in the NOIA approximation*, while the incorporation of MEC tends to cancel the effect of that component.

In order to unfold $\tilde{F}_A(Q^2)$ from the reduced cross section $(|\vec{k}_3^{\,i}|/|\vec{q}_+^{\,f}|)\sigma(\gamma {}^3\text{He} \to \pi^{+\,3}\text{H})$ given in Eqs. (7), (9), and (12), an optical potential, already described in Ref. (6) and (13), has been used to estimate the distortion of the plane wave in the field of ^{3}H. Since the corresponding optical parameters for $A = 3$ are not available,[14] two sets of S-wave optical parameters taken from ^{4}He and ^{6}Li were applied to the pion amplitudes involved in the ^{3}H case. Since both sets give practically identical effects, i.e., within 0.5%, this suggests that the results will not change significantly if ^{3}H optical parameters are used. Results displayed in Table IV for one sample wave function show that distortion effects are quite significant near threshold and rapidly decrease to very small amounts for pion kinetic energies $\lesssim 3.5$ MeV. The reduced cross section is then parametrized as follows:

$$a_{^3\text{He}\to{}^3\text{H}} = \left(\frac{|\vec{k}_3^{\,i}|}{|\vec{q}_+^{\,f}|}\, \sigma(\gamma {}^3\text{He} \to \pi^{+\,3}\text{H}) (S_\gamma)^{-1} \right), \tag{24}$$

with $S_\gamma = 2\pi\gamma / e^{2\pi\gamma} - 1$ and $\gamma = Z_{^3\text{H}}\, \alpha m_\pi / |\vec{q}_+^{\,f}|$, a dimensionless quantity describing the Coulomb interaction between the pion of momentum $\vec{q}_+^{\,f}$ and

TABLE III. $\gamma\pi^+$ cross sections at threshold. The quantities entering in the table are defined in the text and in Table I RSC (S) stands for a RSC wave function projected on its S component.

	$\frac{\|\vec{k}_3^{\,i}\|}{\|\vec{q}_+^{\,f}\|}\sigma_{\text{non dist}}$	$\frac{\|\vec{k}_3^{\,i}\|}{\|\vec{q}_+^{\,f}\|}\sigma_{\text{dist}}$	$a_{^3\text{He}\to{}^3\text{H}}$	$\frac{a_{^3\text{He}\to{}^3\text{H}}}{a_{p\to n}}$
Experiment			0.116± 0.006	0.59 ± 0.03
RSC	0.119	0.059	0.133	0.66
RSC (S)	0.148	0.074	0.166	0.82
SSC (c)	0.126	0.063	0.141	0.70
[Rho + SSC(c)]	0.139	0.069	0.156	0.77

TABLE IV. Variations of the $\gamma\pi^+$ cross section very close to threshold. The SSC (c) nucleon-nucleon force is taken for illustration. Other wave functions give rise to the same features. Nucleons are supposed point-like as in Table III.

E_π (MeV)	E_γ (MeV)	$\frac{\lvert\vec{k}_s^f\rvert}{\lvert\vec{q}_f\rvert}\cdot\sigma_{\text{non dist}}$	$\frac{\lvert\vec{k}_s^f\rvert}{\lvert\vec{q}_f\rvert}\cdot\sigma_{\text{dist}}$	$a_{^3\text{He}\to^3\text{H}}$
0.07	13.63	0.126	0.063	0.136
1.82	13.81	0.125	0.116	0.134
3.57	13.98	0.124	0.121	0.133
5.32	141.6	0.124	0.123	0.132
7.07	143.8	0.123	0.122	0.132

the ^{3}H (assumed to be point-like). The coefficients thus extracted are compared with our experiment in Table III. Further, since the experimental measurement involves the ratio $\sigma(\gamma^3\text{He}\to\pi^{+3}\text{H})/\sigma(\gamma p\to\pi^+ n)$, the quantity $(a_{^3\text{He}\to^3\text{H}})/(a_{p\to n})$ is included. The axial-vector form factor corresponding to the experimental result is given in Table II, and is found to be about 35% lower than results based on other experimental data.

Incidentally, it has been noticed that the momentum-dependent terms of Eq. (1) do not significantly contribute to the (γ,π^+) cross section near threshold. Even at energies $E_\pi=20$ MeV (in the center of mass frame) for the outgoing pion, the momentum-dependent terms represent only 18% of the total cross section. This contribution arises primarily from the p-wave pions.

Recently reported measurements on the total absorption width $(\tau_s^{\text{total}})^{-1}$ of 1s levels in ^{3}He pionic atoms[9] open a new possibility of getting at the ^{3}He $\to$ ^{3}H axial-vector form factor, provided it is combined with other experimental data on the branching ratio $R((\pi^{-3}\text{He}\to{}^3\text{H}\gamma)/(\pi^{-3}\text{He}\to All))$, and provided that the probability is that the pion in the pionic ^{3}He atom will be absorbed in the s or p states. Thus, one has altogether

$$(\tau_s)^{-1}=(\tau_s^{\text{total}})^{-1}\frac{R((\pi^{-3}\text{He}\to{}^3\text{H}\gamma)/(\pi^{-3}\text{He}\to All))}{W_s}$$

$$=(5.2\pm1.8)\,\text{sec}^{-1}\,,$$

which corresponds, in view of Eq. (5), to $|\tilde{\mathfrak{F}}_A(m_\pi^2)|^2=0.83\pm0.28$, as reported in the last column of Table II.

V. CONCLUSION

In the framework of assumptions based on soft-pion theorems, the ^{3}He$\to$^{3}H axial-vector form factor values extracted, on the one hand, from β-decay measurements[24] and magnetic electron scattering,[23] and, on the other hand, from the Panofsky ratio as measured at Berkeley,[25] display a good overall agreement, as illustrated in Table II. In particular, the Triumf measurement of the Panofsky ratio, although still preliminary, seems to confirm the Berkeley data as opposed to the pionneer experiment of Zaimidoraga *et al.*[27] Despite the large error in the total width of the 1s levels in the ^{3}He pionic atom, these latter data lead to values of the ^{3}He$\to$^{3}H axial-vector form factors consistent with values extracted from other experiments, the only exception being the value given by the photopion production at threshold, which is at variance of other results by 30–40%.

The use of Laverne-Gignoux wave functions leads to the same conclusions as those of Phillips and Roig, at the level of NOIA adopted in this work. Thus, an increase of the D-state component gives rise to a decrease of the resulting axial-vector form factor; this explains why the values calculated on a NOIA basis are systematically below the values suggested by experiment. Furthermore, since these calculated values are still 20% higher than the number extracted from the photopion cross section, it shows that the discrepancy cannot be due to a breakdown of the NOIA. It is recalled that the NOIA based calculation on $^3\text{H}(\gamma,\pi^+)pn$ and $^6\text{Li}(\gamma,\pi^+)^6\text{He}$ yield results in agreement with experimental data, within 10%.[28,29]

Finally, it appears that the role of the D state in the reaction $\pi^-(^3\text{He},{}^3\text{H})\gamma$ rate tends to cancel out because of the interplay of NOIA and MEC contributions. In other words, the sensitivity of the Panofsky ratio to the D state admixture in the ^{3}He wave function is a feature of the NOIA and gets destroyed by the presence of MEC: so, hope that a very precise measurement of the Panofsky ratio would yield information of the D component of the ^{3}He wave function should be given up.

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